Complex-particle emission in the pre-equilibrium exciton model

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The complex-particle ($A \leq 4$) emission in the particle induced reaction was analyzed in the pre-equilibrium reaction mechanism. A simple closed form pre-equilibrium exciton model is employed. The complex particle is assumed to form with a certain formation probability from the excited particles (protons and neutrons) which correlate to each other with the right combination of proton and neutron numbers and the momentum in order to form a cluster. The complex-particle formation probability extracted from the proton induced reactions on several nuclei from ^{12}C to ^{209}Bi shows strong A dependence.

[NUCLEAR REACTIONS ^{12}C, ^{16}O, ^{27}Al, ^{54}Fe, ^{89}Y, ^{120}Sn, ^{197}Au, $^{209}Bi(p,x)$, $E_p = 62$ MeV; $^{54}Fe(p,x)$; $E_p = 29$ and 39 MeV, $^{58}Ni(p,x)$, $E_p = 90$ MeV; pre-equilibrium exciton model.]

I. INTRODUCTION

Experimental data such as d, t, τ, and α particles are important nuclear reaction channels.[1-3] Furthermore, the yield of high energy complex particles is larger than that predicted on the basis of statistical compound nucleus theory. Following the success of the pre-equilibrium exciton model in interpreting nucleon emission,[4] a few attempts have been made to use this model to account for the high energy component of the complex-particle energy spectra.[5-7] In the analyses of α-particle emission, two different approaches were used: (i) An α particle is preformed in the target nucleus and is treated as one exciton (one single particle) in the emission and/or in the nuclear equilibration process.[7] (ii) An α particle is formed from among the excited nucleons with a certain formation probability, and the "α particle" as a whole is treated as four excitons, i.e., two protons and two neutrons.[5] Recently, a quasifree scattering approach for the α particle in either the entrance or the exit channels has been developed.[8] In this model, a master equation is used to predict the spectra of (α, α') or (N, α) under the assumption that the mechanism is a quasifree intranuclear α-nucleon scattering process.

Based on the first approach, Milazzo-Colli and Marcazzan-Braga[7] introduced a preformation factor f into the expression for the α-particle emission rate in the exciton model. Chevarier *et al.*[9] used the hybrid model, taking into account the α preformation, in the analysis of (p, α), (d, α), $(^3He, \alpha)$, and (α, α') reactions. These calculations were successful in reproducing the spectral shape at the high energy end. However, this approach is not adequate because competition between nucleons and complex particles is omitted in the processes of nuclear equilibration and particle emission. Furthermore, questions concerning the particle-hole state density with a mixture of nucleons and α particles, and the nuclear transition rates resulting from the two-body residual interactions between a pair of nucleons or a nucleon and an α particle are ambiguous. Because of these ambiguities, no attempt was made to treat the emission of complex particles other than α particles.

The second approach for the emission of complex particles assumes that the nucleon particle-hole state densities can be used to generate probabilities for the existence of clusters in the composite nucleus. Such a formulation was first proposed by Blann and Lanzafame.[10] Calculations based on this assumption predicted neither the spectral shape nor the magnitude for the α-particle spectrum. The work of Blann and Lanzafame was modified by Cline.[6] In this latter calculation, an empirical factor was arbitrarily introduced into the emission rate expression in the master equation to enhance the complex-particle emission. However, the physical significance of this empirical constant is not obvious. Although this calculation improved the fit to the data, it was unable to predict the general spectral shape. Later, Ribansky, Oblozinsky, and Betak[5] reformulated the above approach by taking into account all the distinguishable configurations of the excited nucleons from which a complex particle can then be formed with a certain formation probability. These calculations provide good fits to the data. Folllowing the work of Ribansky *et al.*,[5] we will propose an alternative method to extract the cluster formation probabilities.

Up to now, most of the pre-equilibrium calculations have been for the emission of α particles and very little effort was directed toward the emission of d, t, and τ. The experimental results from the proton induced reactions show that the deuteron yield is approximately one-tenth that of the proton yield and reasonably large yields were also seen for t and τ.[1,2] For this reason, it is important to see whether the emission of complex particles other than α particles can be understood in the pre-equilibrium exciton model.

In Sec. II a brief review of the exciton model is given, and in Sec. III a detailed description of the method is provided. Section IV compares the experimental results with the theoretical calculations.

II. PRE-EQUILIBRIUM EXCITON MODEL

A detailed description of the pre-equilibrium exciton model formulated with a closed-form expression is given in Refs. 3 and 11. In the following, only the formulas needed are outlined briefly.

The pre-equilibrium exciton model assumes that a composite nucleus is formed in an initial particle-hole state following the projectile-target-nucleus interaction. The composite nucleus then proceeds from this initial particle-hole state through a series of more complex particle-hole states via energy-conserving two-body residual interactions until a statistical equilibrium is reached. During this equilibration process, particles may be emitted from each intermediate state.

The general expression for the pre-compound decay probability per unit time of a particle β with channel energy ϵ from a certain p-particle h-hole state [(p,h) state] is given by[11]

$$W_\beta(p,h,E,\epsilon)d\epsilon = \frac{\Gamma_\beta(p,h,E,\epsilon)}{\hbar}\,d\epsilon = \left[\frac{\omega(p-p_\beta,h,U)}{\omega(p,h,E)}R_\beta(p)\gamma_\beta\,\omega(p_\beta,0,E-U)d\epsilon\right]\lambda_\beta^c(\epsilon)$$
$$= \frac{2S_\beta+1}{\pi^2\hbar^3}\,\mu_\beta\sigma_\beta(\epsilon)\epsilon\,d\epsilon\,\frac{\omega(p-p_\beta,h,U)}{\omega(p,h,E)}\,.\,\frac{\omega(p_\beta,0,E-U)}{g_\beta}R_\beta(p)\gamma_\beta\,, \quad (1)$$

where the quantity in the square bracket gives the particle populations in each energy interval in terms of the particle-hole state densities ω, and $\lambda_\beta^c(\epsilon)$ is the emission rate into the continuum for a particle β at energy ϵ. S_β, μ_β, σ_β, and g_β are the spin, the reduced mass, the inverse reaction cross section, and single-particle state density for the emitted particle β. U and E are the excitation energies of the residual and the composite nuclei, and p_β is the nucleon number of the emitted particle. The factor $R_\beta(p)$, which is a pure combinatorial probability, gives the probability that p_β nucleons chosen at random from among the p excited particles has the right combination of protons and neutrons to form the outgoing particle β, and γ_β is the formation probability for the particle β in the composite nucleus to have the right momentum to undergo emission an an entity.

The total pre-equilibrium decay probability of a particle β with channel energy ϵ is given by:

$$I_\beta^{\text{PEQ}}(E,\epsilon) = \sum_{\substack{p=p_0\\ \Delta p=+1}}^{\bar{p}}\left[\frac{\Gamma_\beta(p,h,E,\epsilon)}{\Gamma(p,h,E)}\right]\left[\prod_{\substack{p'=p_0\\ \Delta p'=+1}}^{p-1}\frac{\Gamma_+(p',h',E)}{\Gamma(p',h',E)}\right]\left[1+\frac{\Gamma_+(p,h,E)}{\Gamma(p,h,E)}\,\frac{\Gamma_-(p+1,h+1,E)}{\Gamma(p+1,h+1,E)}\cdots\right], \quad (2)$$

where

$$\Gamma(p,h,E) = \Gamma_+(p,h,E)+\Gamma_-(p,h,E)+\Gamma_c(p,h,E)\,, \quad (3)$$

$$\Gamma_c(p,h,E) = \sum_\nu \Gamma_\nu(p,h,E) = \sum_\nu \int_0^{E-B_\nu}\Gamma_\nu(p,h,E,\epsilon)d\epsilon\,. \quad (4)$$

The Γ's are the transition widths and are related to the transition rates for creating a particle-hole pair ($\Delta p=\Delta h=+1$), λ_+, and for annihilating a particle-hole pair ($\Delta p=\Delta h=-1$), λ_-, by[12]

$$\lambda_+(p,h,E) = \frac{\Gamma_+(p,h,E)}{\hbar} = \frac{\pi}{\hbar}|M|^2\frac{g}{(p+h+1)}(gE-C_{p+1,h+1})^2\,, \quad (5)$$

$$\lambda_-(p,h,E) = \frac{\Gamma_-(p,h,E)}{\hbar} = \frac{\pi}{\hbar}|M|^2 gph(p+h-2)\,, \quad (6)$$

where $C_{p,h}=\frac{1}{2}(p^2+h^2)$ and $|M|^2$ is the square of the average two-body transition matrix element given by an empirical expression $|M|^2 = KE^{-1}A^{-3}$.[13]

Finally, the fraction of pre-equilibrium emission (one particle out only) is given by

$$F_{\mathrm{PEQ}}(E) = \sum_{\nu} \int_0^{E-B_\nu} I_\nu^{\mathrm{PEQ}}(E,\epsilon)d\epsilon \ . \tag{7}$$

III. EXTRACTION OF THE FORMATION PROBABILITY

The nuclear equilibration process is assumed to proceed from the simplest particle-hole state, (p_0, h_0) state, to more complicated states via residual two-body interactions which are limited to nucleon-nucleon interactions. Complex particles, such as d, t, τ, and α particles, are formed from these excited nucleons with certain formation probability, if the state has the right combination of neutrons and protons with the proper momentum. These formation probabilities γ_β appear in Eq. (1). In the following, we propose a method to obtain the values of γ_β's unambiguously from the experimental data.

As one can see from Eq. (2), the formation probabilities do not enter linearly into this equation. Therefore, one cannot obtain the γ_β's by a direct comparison of the experimental data with the calculation. The method proposed is based on the fact that the high energy end of an energy spectrum results mainly from particle emission during the earlier stages of the nuclear equilibration. In other words, only a few simple particle-hole states contribute to the high energy portion of the energy spectrum (see Fig. 1). For simplicity, we assume that γ_β is energy independent, but varies with the type of particle emitted, and the target nucleus.

A. Empirical estimation of $f_\beta^{\mathrm{emp}}(p,h,E)$

From Eq. (1), the differential cross section for the pre-equilibrium emission of a particle β from a (p,h) state with kinetic energy ϵ is given by

$$\begin{aligned} d\sigma_\beta^{\mathrm{emp}}(p,h,E,\epsilon)/d\epsilon &= N_\beta \frac{2S_\beta+1}{\pi^2\hbar^3} \mu_\beta \sigma_\beta(\epsilon)\epsilon \frac{\omega(p-p_\beta,h,U)}{\omega(p,h,E)} \\ &\quad \times \frac{\omega(p_\beta,0,E-U)}{g_\beta} R_\beta(p) \\ &= N_\beta \phi_\beta(p,h,E,\epsilon)/\hbar \ . \end{aligned} \tag{8}$$

The constant N_β is a function of all the γ's and the transition widths. If (p,h) state is the simplest particle-hole state from which a certain type of particle can be emitted, N_β can be determined by normalizing $d\sigma_\beta^{\mathrm{emp}}(p,h,E,\epsilon)/d\epsilon$ to the high energy end of the experimental angle-integrated energy spectrum.

The total cross section for the emission of particle β from the (p,h) state is obtained by integrating Eq. (8) over ϵ; that is

$$\begin{aligned} \sigma_\beta^{\mathrm{emp}}(p,h,E) &= N_\beta \int_0^{E-B_\beta} \frac{\phi(p,h,E,\epsilon)}{\hbar} d\epsilon \\ &= N_\beta \Phi_\beta(p,h,E)/\hbar \ . \end{aligned} \tag{9}$$

From Eq. (9), the empirical estimation of the fraction of total reaction cross section resulting from the (p,h) state is defined as

$$f_\beta^{\mathrm{emp}}(p,h,E) = \sigma_\beta^{\mathrm{emp}}(p,h,E)/\sigma_R(E) \ , \tag{10}$$

where $\sigma_R(E)$ is the total reaction cross section.

B. Theoretical calculation of $f_\beta^{\mathrm{theo}}(p,h,E)$

The pre-equilibrium exciton model predicts the total cross section for the emission of a particle β from the (p,h) state, according to Eq. (2), as

$$\begin{aligned} \sigma_\beta^{\mathrm{theo}}(p,h,E) &= \sigma_R(E)\left[\frac{\Gamma_\beta(p,h,E)}{\Gamma(p,h,E)}\right] \\ &\quad \times \left[\prod_{\substack{p'=p_0 \\ \Delta p'=+1}}^{p-1} \frac{\Gamma_+(p',h',E)}{\Gamma(p',h',E)}\right] \\ &\quad \times \left[1 + \frac{\Gamma_+(p,h,E)}{\Gamma(p,h,E)} \right. \\ &\quad\quad \left. \times \frac{\Gamma_-(p+1,h+1,E)}{\Gamma(p+1,h+1,E)} + \cdots \right] \ . \end{aligned} \tag{11}$$

From Eq. (11) the theoretical prediction of the fraction of total reaction cross section from the (p,h) state is given by

$$f_\beta^{\mathrm{theo}}(p,h,E) = \sigma_\beta^{\mathrm{theo}}(p,h,E)/\sigma_R(E) \ . \tag{12}$$

C. Estimation of formation probabilities

We limit our discussion to the emission of particles up to α particles. As mentioned previously, the high energy end of a particle energy spectrum is mainly due to the emission from the lowest possible particle-hole state. For this simple state, $\Gamma_- \ll \Gamma_+$ and the last bracket in Eq. (11) can be approximated as unity. Under this approximation, one obtains from Eqs. (10)–(12):

$$f_\beta^{\mathrm{emp}}(p,h,E)=f_\beta^{\mathrm{theo}}(p,h,E)=\frac{\Gamma_\beta(p,h,E)}{\Gamma(p,h,E)}\prod_{\substack{p'=p_0\\ \Delta p'=+1}}^{p-1}\frac{\Gamma_+(p',h',E)}{\Gamma(p',h',E)}$$

or (13)

$$f_\beta^{\mathrm{emp}}(p,h,E)=\left[\frac{\gamma_\beta\Phi_\beta(p,h,E)}{\sum_\nu\gamma_\nu\Phi_\nu(p,h,E)+\Gamma_+(p,h,E)+\Gamma_-(p,h,E)}\right]\prod_{\substack{p'=p_0\\ \Delta p'=+1}}^{p-1}\frac{\Gamma_+(p',h',E)}{\Gamma(p',h',E)}\,.$$

The summation is taken over all particles that can be emitted from the (p,h) state. One should note that γ_p and γ_n are equal to unity and that the product in Eq. (13) is equal to unity for $p=p_0$ and $h=h_0$. γ_β can be found from Eq. (13) once the value of $f_\beta^{\mathrm{emp}}(p,h,E)$ is determined empirically.

D. Determination of γ_β for the proton induced reactions

Starting from the initial particle-hole state, (p_0,h_0) state, only particle β (particle with p_β nucleons) with $p_\beta\le p_0$ and $p_\beta<p_0+h_0$ can be emitted. In the case of nucleon induced reaction, the initial particle-hole state is generally taken to be 2p-1h. From this initial particle-hole state, only p, n, and d can be emitted. Tritons and τ can only be emitted from the next 3p-2h states and α particles from the following 4p-3h states.

From Eq. (13) one can solve for the γ_β's:

$$\gamma_d=\frac{f_d^{\mathrm{emp}}(2,1,E)\left[\sum_{\nu=n,p}\Phi_\nu(2,1,E)+\Gamma_+(2,1,E)+\Gamma_-(2,1,E)\right]}{\Phi_d(2,1,E)[1-f_d^{\mathrm{emp}}(2,1,E)]}\,, \tag{14}$$

$$\gamma_{t\text{ or }\tau}=\frac{f_{t\text{ or }\tau}^{\mathrm{emp}}(3,2,E)\left[\sum_{\nu=n,p}\Phi_\nu(3,2,E)+\gamma_d\Phi_d(3,2,E)+\Gamma_+(3,2,E)+\Gamma_-(3,2,E)\right]}{\Phi_{t\text{ or }\tau}(3,2,E)\,[P(3,2,E)-\sum_{\nu=t,\tau}f_\nu^{\mathrm{emp}}(3,2,E)]}\,, \tag{15}$$

$$\gamma_\alpha=\frac{f_\alpha^{\mathrm{emp}}(4,3,E)\left[\sum_{\nu=n,p}\Phi_\nu(4,3,E)+\sum_{\eta=d,t,\tau}\gamma_\eta\Phi_\eta(4,3,E)+\Gamma_+(4,3,E)+\Gamma_-(4,3,E)\right]}{\Phi_\alpha(4,3,E)[P(4,3,E)-f_\alpha^{\mathrm{emp}}(4,3,E)]}\,, \tag{16}$$

where

$$P(p,h,E)=\prod_{\substack{p'=p_0\\ \Delta p'=+1}}^{p-1}\frac{\Gamma_+(p',h',E)}{\sum_\nu\Gamma_\nu(p',h',E)+\Gamma_+(p',h',E)+\Gamma_-(p',h',E)}\,. \tag{17}$$

f_β^{emp} are known from Eq. (10), P's are calculated from Eqs. (1)–(6), and $\Phi_\beta(p,h,E)$ can be obtained from Eqs. (8) and (9).

Equations (14)–(16) are then used successively to determine the formation probabilities for d, t, τ, and α. The values thus obtained are used for the calculations of the total cross section as well as differential energy cross section for all the complex particles considered.

E. General expression for γ_β

Equations (14)–(17) can be generalized to include reactions induced by other projectiles, such as α particles. The general expression for γ_β is

$$\gamma_\beta=\frac{f_\beta^{\mathrm{emp}}(p,h,E)\left[\sum_\nu\gamma_\nu\Phi_\nu(p,h,E)+\Gamma_+(p,h,E)+\Gamma_-(p,h,E)\right]}{\Phi_\beta(p,h,E)\,[P(p,h,E)-\sum_\mu f_\mu^{\mathrm{emp}}(p,h,E)]}\,. \tag{18}$$

The summation ν in the numerator is taken over all the particles which can be emitted from the states preceding the (p,h) state, i.e., from the $(p-1,h-1)$ state. The summation μ in the denominator is taken over all the particles for which the (p,h) state is their simplest possible emitting state. $P(p,h,E)$ is the fraction of total reaction cross section [or the probability of populating the (p,h) state] and is given by

$$P(p,h,E)=\prod_{\substack{p'=p_0\\ \Delta p'=+1}}^{p-1}\frac{\Gamma_+(p',h',E)}{\sum_\nu\Gamma_\nu(p',h',E)+\Gamma_+(p',h',E)+\Gamma_-(p',h',E)}$$

$$=P(p-1,h-1,E)\frac{\Gamma_+(p-1,h-1,E)}{\sum_\nu\gamma_\nu\Phi_\nu(p-1,h-1,E)+\Gamma_+(p-1,h-1,E)+\Gamma_-(p-1,h-1,E)}\,. \tag{19}$$

The summation ν is taken over particles emitted from the $(p-1,h-1)$ state. Of course $P(p_0,h_0,E)=1$.

IV. RESULTS AND DISCUSSION

The calculations based on the method described in Sec. III were performed for the proton induced reactions on the target nuclei ^{12}C, ^{16}O, ^{27}Al, ^{54}Fe, ^{89}Y, ^{120}Sn, ^{197}Au, and ^{209}Bi at bombarding energy $E_p=62$ MeV. The experimental data were taken from Bertrand and Peele.[1] The computer program PREQC2[14] was used to compute each individual energy spectrum. $f_\beta^{\text{emp}}(p,h,E)$ was first obtained according to Eq. (10), and then the γ_β's were obtained from Eqs. (14)–(16).

In this calculation, the total reaction cross section $\sigma_R(E)$ is taken from Ref. 1. The pairing energy is taken from Gilbert and Cameron,[15] the binding energy is from the tabulations of Ref. 16, and those not listed in the tabulations were calculated from a semiempirical mass formula of Wing and Fong.[17] The inverse reaction cross sections were calculated using an empirical formula of Ref. 3 for evaporation calculation and a global set of optical model parameters[18,19] for pre-equilibrium calculation. The level density parameter a is taken to be $A/8$ which is equivalent to $g=3A/4\pi^2$ MeV^{-1}. The average two-body transition matrix element $|M|^2$ was estimated according to an empirical formula $|M|^2=KE^{-1}A^{-3}$ with $K_N=200$ MeV^3 for nucleon induced reactions.[13] In all cases, a 2p-1h initial state is assumed for proton induced reactions.

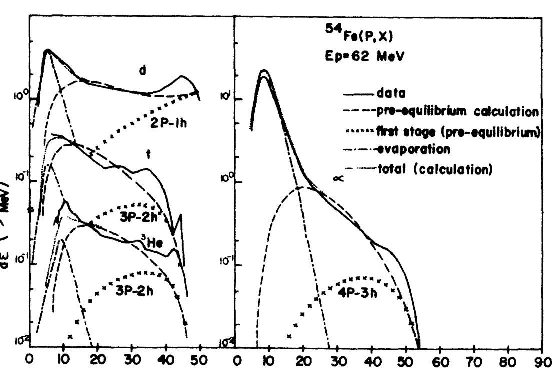

FIG. 1. Comparison of the pre-equilibrium and evaporation calculations with the experimental complex-particle energy spectra for the reaction $^{54}Fe(p,x)$ at $E_p=62$ MeV. The initial particle-hole number is 2p–1h.

Figure 1 shows the comparison of the calculation with the experimental data for the reaction $^{54}Fe(p,x)$ at $E_p=62$ MeV.[1] The crossed curve is the contribution only from the lowest particle-hole state that can emit the particle of interest. It is obvious that this spectrum alone accounts for most of the high energy portion of the energy spectrum. The dotted curve is the calculated total energy spectrum. The evaporation component generally results from two sources. The first is the so-called "pure evaporation" with a fraction $[1-F_{\text{PEQ}}(E)]$ of the total reaction cross section. The second is the evaporation following the pre-equilibrium emission. The multiple precompound emission, which presumably has a spectral shape similar to that of evaporation, was replaced by the evaporation cascade. A computer code EVAPOR,[20] which includes the emission of six different kinds of particles, namely, n, p, d, t, τ, and α, and the fission competition, was used for the evaporation calculation. The agreement is quite good. Figures 2(a) and 2(b) show the comparisons of calculated results with the experimental data for the (p,x) reactions on ^{12}C, ^{16}O, ^{27}Al, ^{89}Y, ^{120}Sn, ^{197}Au, and ^{209}Bi at $E_p=62$ MeV. The dashed curves represent the calculated pre-equilibrium spectra.

As can be seen from Figs. 1 and 2, the pre-equilibrium model predicts most of the reaction cross section above the evaporation peak. The region between sharp peaks in the high energy end and the evaporation peak constitutes a large fraction of total yield of each spectrum. A pure evaporation model calculation alone is not able to predict sufficient yield above the evaporation peak. The pre-equilibrium model calculation shows that nearly all of the complex particles are emitted during the pre-equilibrium stage. The importance of pre-equilibrium decay for the complex particles is even more apparent for heavier nuclei because the increase in Coulomb barrier inhibits the evaporation of low energy charged particles.

The values of complex-particle formation probabilities γ_β for d, t, τ, and α particles were extracted. These values were subjected to uncertainty because a number of approximations were made in the calculation. In Table I, instead of γ_β, we listed $\gamma'_\beta=\gamma_\beta\, g/g_\beta$ where g_β is the single-particle

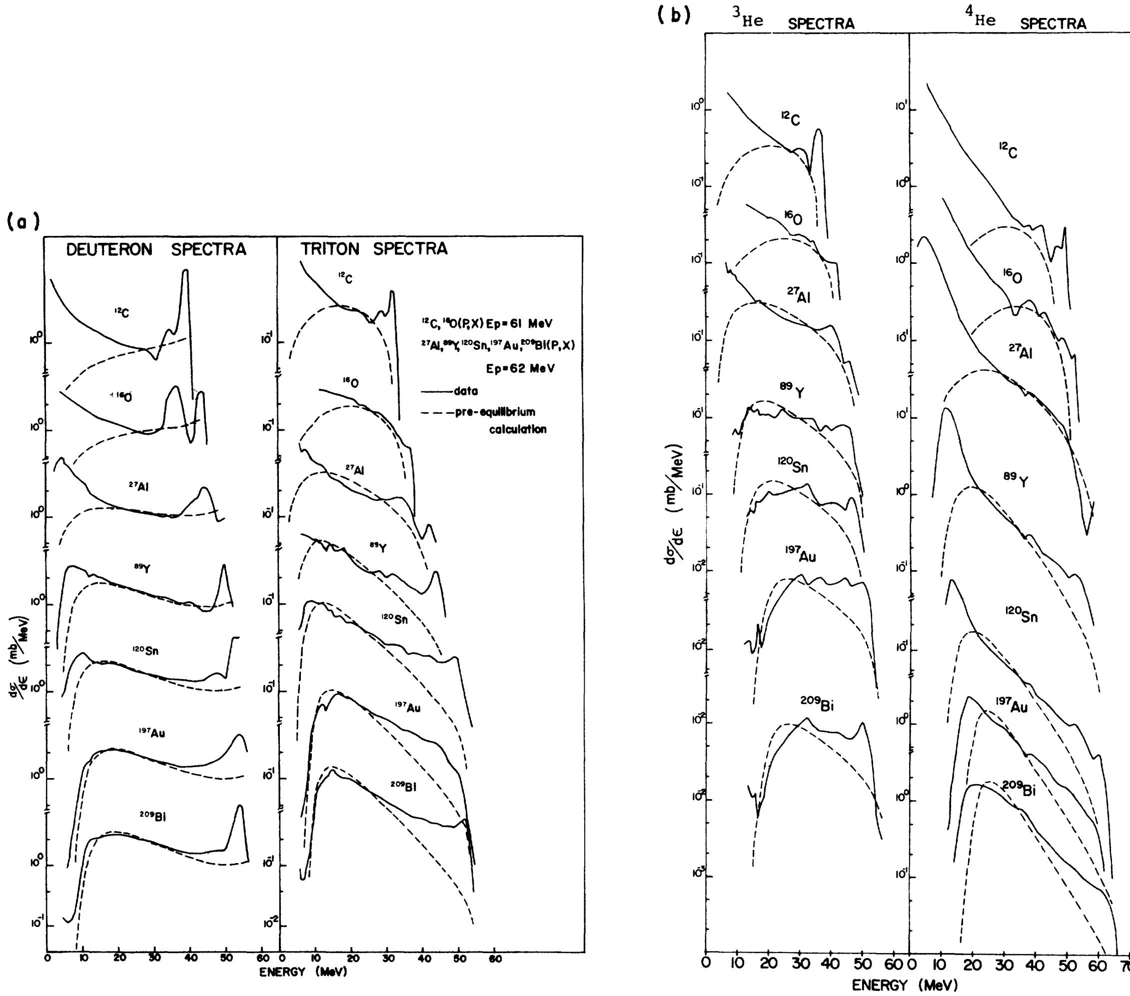

FIG. 2. (a) Comparison of the pre-equilibrium calculations with the experimental deuteron and triton energy spectra on several target nuclei. (b) Same as (a) for ^{3}He and α.

state density for particle β, $g_\beta=g$, and $\gamma_\beta=1$ for nucleons. The reason for listing $\gamma_\beta g/g_\beta$ is because this ratio appears in Eq. (1). Furthermore, two different approaches were assumed for g_β: (i) it was assumed that $g_\beta=g/4$ for α particles; (ii) the number of cluster states per MeV is calculated as for nucleons, that is $g_\beta(\epsilon)=\int_{\epsilon-1/2}^{\epsilon+1/2}\rho_\beta(\epsilon)d\epsilon$,[8,21] where the density of states is $\rho_\beta(\epsilon)=V/4\pi^2\hbar^3(2S_\beta+1)\times(2\mu_\beta)^{3/2}\epsilon^{1/2}$. In the second approach, $g_\beta(\epsilon)$ is energy dependent. Since γ_β itself may also be energy dependent, we have chosen, without a detailed knowledge of the energy dependence of γ_β, to list γ'_β instead of γ_β.

Figure 3 shows the plot of $\log(\gamma_\beta g/g_\beta)$ vs $\log A$ (A is the composite nucleus mass number). The values of γ'_β decrease as A increases. For comparison, a family of curves with different A dependence are also shown in Fig. 3(a). In order to obtain more reliable information on the dependence of γ'_β on N or Z, the experimental data resulting from a set of isotopes or isotones as target nuclei are needed

The pairing effect may play a significant role in the pre-equilibrium calculations.[22] The calculations taking into accout odd-even effects were repeated by subtracting a pairing energy from the excitation energy. The values for γ'_β thus obtained are listed in Table II, and are shown in Fig. 3(b). These values of γ'_β are slightly different from those obtained without including the pairing effects, and are varied according to the odd-even character of the residual nucleus. A stronger dependence of γ'_β on A was observed and the γ_β's fall approximately on a straight line, as can be seen in Fig. 3(b). In facc, two different slopes are observed for t, τ, and α particles, one for light

TABLE I. Values of $\gamma_\beta g/g_\beta$ for complex particles without pairing correction.

Reaction type	Excitation energy (MeV)	$\gamma_\beta g/g_\beta$ d	t	^{3}He	α
$^{12}C(p,x)$	58.22	4.72×10^{-2} $\pm3.4\times10^{-3}$	1.42×10^{-1} $\pm1.1\times10^{-2}$	8.08×10^{-2} $\pm6.1\times10^{-3}$	8.81×10^{-1} $\pm8.9\times10^{-2}$
$^{16}O(p,x)$	57.98	4.54×10^{-2} $\pm3.5\times10^{-3}$	6.45×10^{-2} $\pm4.8\times10^{-3}$	3.02×10^{-2} $\pm2.2\times10^{-3}$	5.43×10^{-1} $\pm4.8\times10^{-2}$
$^{27}Al(p,x)$	71.34	3.60×10^{-2} $\pm2.4\times10^{-3}$	1.59×10^{-2} $\pm1.0\times10^{-3}$	8.24×10^{-3} $\pm4.6\times10^{-4}$	1.23×10^{-2} $\pm0.8\times10^{-3}$
$^{54}Fe(p,x)$	65.91	2.72×10^{-2} $\pm1.8\times10^{-3}$	6.11×10^{-3} $\pm3.4\times10^{-4}$	5.28×10^{-3} $\pm3.3\times10^{-4}$	1.14×10^{-2} $\pm0.7\times10^{-3}$
$^{89}Y(p,x)$	69.68	1.98×10^{-2} $\pm1.1\times10^{-3}$	4.05×10^{-3} $\pm2.3\times10^{-4}$	1.84×10^{-3} $\pm1.0\times10^{-4}$	2.68×10^{-3} $\pm1.6\times10^{-4}$
$^{120}Sn(p,x)$	67.25	1.78×10^{-2} $\pm1.0\times10^{-3}$	3.09×10^{-3} $\pm1.8\times10^{-4}$	2.00×10^{-3} $\pm1.1\times10^{-4}$	2.25×10^{-3} $\pm1.3\times10^{-4}$
$^{197}Au(p,x)$	68.76	1.58×10^{-2} $\pm0.9\times10^{-3}$	2.49×10^{-3} $\pm1.4\times10^{-4}$	8.00×10^{-4} $\pm5.4\times10^{-5}$	8.44×10^{-4} $\pm4.9\times10^{-5}$
$^{209}Bi(p,x)$	66.68	1.51×10^{-2} $\pm0.9\times10^{-3}$	2.68×10^{-3} $\pm1.5\times10^{-4}$	6.77×10^{-4} $\pm3.9\times10^{-5}$	6.93×10^{-4} $\pm4.0\times10^{-5}$
$^{58}Ni(p,x)$	91.87	2.73×10^{-2} $\pm1.0\times10^{-3}$	8.47×10^{-3} $\pm3.0\times10^{-4}$	5.58×10^{-3} $\pm1.8\times10^{-4}$	9.87×10^{-3} $\pm3.3\times10^{-4}$

TABLE II. Values of $\gamma_\beta g/g_\beta$ for complex particles with pairing correction.

Reaction type	Excitation energy (MeV)	$\gamma_\beta g/g_\beta$ d	t	^{3}He	α
$^{12}C(p,x)$	58.22	4.70×10^{-2} $\pm3.4\times10^{-3}$	1.41×10^{-1} $\pm1.0\times10^{-2}$	8.01×10^{-2} $\pm6.1\times10^{-3}$	8.78×10^{-1} $\pm8.8\times10^{-2}$
$^{16}O(p,x)$	57.98	4.62×10^{-2} $\pm3.6\times10^{-3}$	6.78×10^{-2} $\pm5.0\times10^{-3}$	3.20×10^{-2} $\pm2.4\times10^{-3}$	4.37×10^{-1} $\pm3.5\times10^{-2}$
$^{27}Al(p,x)$	71.34	3.05×10^{-2} $\pm2.0\times10^{-3}$	1.81×10^{-2} $\pm1.2\times10^{-3}$	8.67×10^{-3} $\pm5.2\times10^{-4}$	2.38×10^{-2} $\pm1.7\times10^{-3}$
$^{54}Fe(p,x)$	65.91	2.67×10^{-2} $\pm1.7\times10^{-3}$	7.60×10^{-3} $\pm4.6\times10^{-4}$	4.03×10^{-3} $\pm2.5\times10^{-4}$	1.04×10^{-2} $\pm0.6\times10^{-3}$
$^{89}Y(p,x)$	69.68	1.79×10^{-2} $\pm1.0\times10^{-3}$	4.24×10^{-3} $\pm2.5\times10^{-4}$	1.83×10^{-3} $\pm1.0\times10^{-4}$	3.83×10^{-3} $\pm2.3\times10^{-4}$
$^{120}Sn(p,x)$	67.25	1.78×10^{-2} $\pm1.0\times10^{-3}$	3.98×10^{-3} $\pm2.3\times10^{-4}$	1.68×10^{-3} $\pm0.9\times10^{-4}$	2.28×10^{-3} $\pm1.3\times10^{-4}$
$^{197}Au(p,x)$	68.76	1.39×10^{-2} $\pm0.8\times10^{-3}$	2.36×10^{-3} $\pm1.4\times10^{-4}$	7.66×10^{-4} $\pm4.9\times10^{-5}$	9.65×10^{-4} $\pm5.6\times10^{-5}$
$^{209}Bi(p,x)$	66.68	1.38×10^{-2} $\pm0.9\times10^{-3}$	2.41×10^{-3} $\pm1.3\times10^{-4}$	6.88×10^{-4} $\pm4.0\times10^{-5}$	7.53×10^{-4} $\pm4.4\times10^{-5}$
$^{58}Ni(p,x)$	91.87	2.68×10^{-2} $\pm1.0\times10^{-3}$	9.19×10^{-3} $\pm3.4\times10^{-4}$	4.85×10^{-3} $\pm1.5\times10^{-4}$	9.32×10^{-3} $\pm3.2\times10^{-5}$

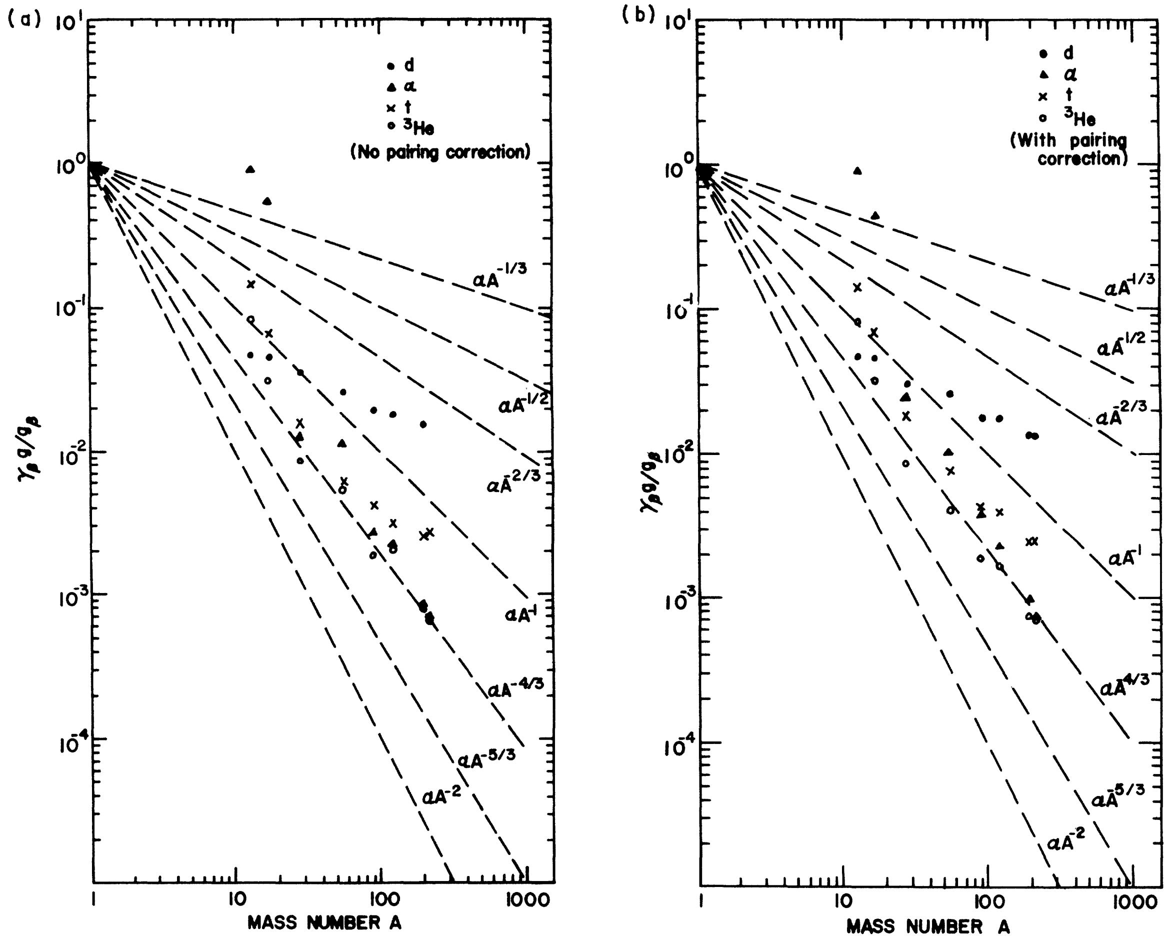

FIG. 3. (a) Plot of $\gamma_\beta\, g/g_\beta$ as a function of mass number A. A family of straight lines is plotted showing the possible A dependence. No pairing correction is included in the calculation. (b) Same as (a) except with the pairing correction included in the calculation.

nuclei and the other for medium (and heavy) mass nuclei.

Figure 4 shows the comparison of calculations with the experimental data for reactions ^{54}Fe-(p,x) at $E_p=29$ and 39 MeV. In these calculations, the same values for γ_β' as obtained from the $E_p=62$ MeV data were used. The comparison of calculations with the ^{58}Ni(p,x) at $E_p=90$ MeV[23] was also made and is shown in Fig. 5. The γ_β's extracted from 90 MeV proton on ^{58}Ni are listed in Tables I and II. These values are very close to those for ^{54}Fe(p,x) reactions at $E_p=62$ MeV. The agreement between calculation and experimental data suggests that γ_β's are nearly independent of incident (or excitation) energy E.

The problem of the internal consistency of Eq. (1) has been raised.[4] Using the ^{197}Au(p,α) reac-

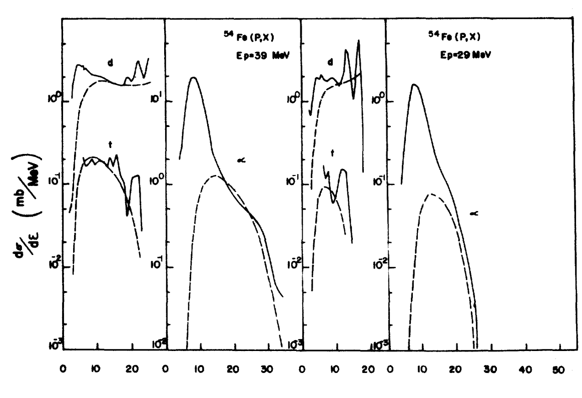

FIG. 4. Comparison of the pre-equilibrium calculations with the experimental complex-particle energy spectra for the reaction ^{54}Fe(p,x) at $E_p=29$ and 39 MeV.

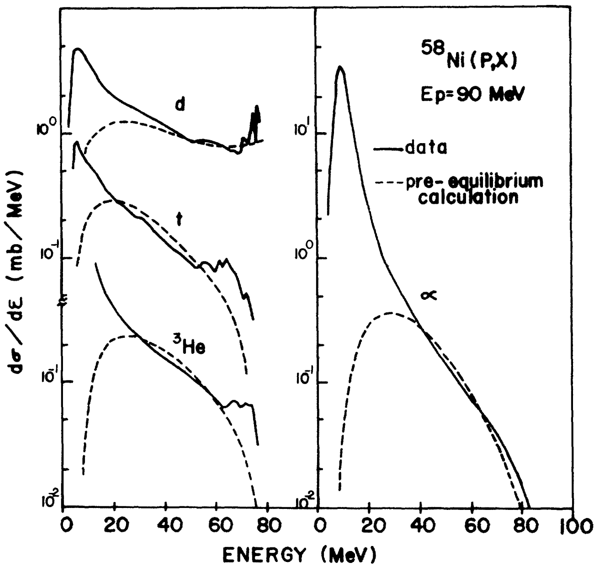

FIG. 5. Same as Fig. 4 for the reaction $^{58}Ni(p,x)$ at $E_p = 90$ MeV.

tion at $E_p = 29$ and 62 MeV as an example, it was found that the calculated ratio of the cross section to first order at $E_\alpha = 26$ MeV disagrees with the experimental ratio at the same α-particle energy by a factor of 400 : 1.

We would like to point out that the ratio of 400 : 1 was based on an error in Ref. 4, i.e., the expected ratio P_1/P_2 was found to be equal to $(U_1/U_2)^{p-\beta-1} \times (E_2/E_1)^{n-1}$ instead of the correct one which is $(U_1/U_2)^{n-\beta-1}(E_2/E_1)^{n-1}$. With the latter expression, the ratio becomes 6 : 1. Summing up the contributions from all of the particle-hole states, one finds that the experimental cross sections at different incident energies can be predicted using Eqs. (1) and (2) (see Figs. 1 and 4). Thus, the question of the lack of internal consistency is resolved.[24]

V. CONCLUSION

As mentioned earlier, the present method estimates γ_β unambiguously because the depletions of nucleon emission as well as other complex-particle emission in each stage are properly taken into account. The neglect and improper treatment of these depletions will lead to incorrect estimates of γ_β. This is because the probability of particle emission from a given particle-hole state not only depends on the particle emission rates from that state, but also on the probability of populating that particular state. This probability is determined by the internal transition widths and the particle emission widths from the previous particle-hole state which is particularly critical for complex particle emission.

One should note that the method developed here does not include any assumption on the preformation of clusters, such as α clusters, in the nucleus. Therefore, any process such as direct α knockout is not included in this calculation. Furthermore, the collective excitation and the direct pickup or stripping reactions to the low lying discrete states are not included in the present calculation either.

The method presented here gives an empirical estimate of the cluster formation probability γ_β by comparing theoretical calculation with experimental data. Although some discrepancies exist, the results are encouraging.

The application of this extended exciton model to nuclear reactions induced by complex projectiles, such as deuterons and α particles, is presently underway.

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