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Charge asymmetry in ^{3}He-^{3}H and the neutron-proton mass difference

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The dynamical effect of the neutron-proton mass difference on charge asymmetry in the trinucleon system is investigated using perturbation theory and the Faddeev equations. The mixed symmetry components of the wave function play an unexpectedly critical role and reduce the effect by roughly a factor of 2 from simple estimates. A reduction rather than an enhancement occurs because the neutron-proton force is stronger than the neutron-neutron or proton-proton forces.

[NUCLEAR STRUCTURE ^{3}He-^{3}H charge asymmetry, n-p mass difference.]

I. INTRODUCTION

In his review of charge-symmetry-breaking (CSB) effects in nuclei, Henley[1] divides physical CSB processes into two categories: direct and indirect. The direct processes include direct one-photon exchange, which contains the dominant static Coulomb interaction, and the dynamical effect of the neutron-proton mass difference. Thus the direct processes do not explicitly involve the strong nucleon-nucleon force and are easier to evaluate, while the indirect processes are more complicated and model-dependent and in most cases poorly understood.

A similar division arises naturally when considering single-virtual-photon processes inside a complex nucleus.[2] In addition to direct one-photon exchange between nucleons (the Breit interaction)[3] and the n-p mass difference, retardation effects (finite photon propagation time) and exchange-current contributions also arise, which are model-dependent but are better understood than the majority of the exotic indirect CSB processes. The retardation and exchange processes, therefore, occupy an intermediate position between direct and indirect contributions. Recently, Brandenburg, Coon, and Sauer[4] adopted a similar division in their exhaustive discussion of charge asymmetry in the trinucleon system (see Tables 1–3 of that work).

Because of the complexity and difficulty of calculation of many CSB processes, it is clearly in our best interest to calculate the simpler direct processes using the most reliable methods available. The dominant Coulomb part of the CSB energy is often calculated using the hyperspherical formula,[5,6] which appears to be considerably more accurate than using wave functions calculated from realistic two-body potentials; these wave functions are too diffuse[7] and thus underestimate the Coulomb energy. Although an error of uncertain size is made in writing the hyperspherical formula, the subsequent evaluation of the Coulomb energy is model-independent; one uses directly the elastic electron scattering data. The result[4] is 638 KeV compared with the experimental ^{3}He-^{3}H mass difference of 764 keV. The size of the remaining direct processes is much smaller.

Several estimates have been made of the contribution of the dynamical effect of the n-p mass difference ΔE_{np} to the ^{3}He-^{3}H mass difference. All but one have implicitly or explicitly assumed that the kinetic energies of each of the three nucleons are equal. Folk[8] estimated $\Delta E_{np} = 28$ keV, Ohmura[9] estimated $\Delta E_{np} \geqslant 20$ keV, Fabre de la Ripelle[6] quotes 20 keV, while Okamoto and Pask[10] guess 20–30 keV. Using perturbation theory with Fadeev wave functions, Brandenburg, Coon, and Sauer[4] estimated $\Delta E_{np} = 12$ keV, which is surprisingly small. They surmised that this resulted from an *unequal* distribution of kinetic energies among the nucleons; we will show in Sec. II that ΔE_{np} can be quite sensitive to the kinetic energy distribution among the nucleons and that the *decrease* of ΔE_{np} from the result of assuming a symmetric distribution, ΔE^s_{np}, is due to $|V_{np}| > |V_{nn}|$. An *increase* would result if the inequality were reversed. A discussion of our numerical methods is presented in Sec. III and our numerical results confirm the supposition concerning the unequal kinetic energy distribution. We find $\Delta E_{np} \cong 9$ keV for our model.

II. PERTURBATION THEORY

We begin by examining the kinetic energy operator for a system of three particles in the three-nucleon center-of-mass system. Two of the particles (denoted "even") have the same mass, m_e, while the third particle (denoted "odd") has a different mass, m_0. The system is depicted in Fig. 1, showing different forces between two even (like) particles (e.g., two neutrons) and between even

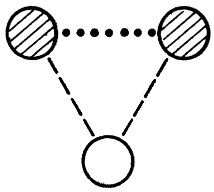

FIG. 1. Trinucleon system depicting even (shaded) and odd (unshaded) particles. The odd and even forces between nucleons are n-p (dashed) and n-n or p-p (dotted).

and odd (unlike) particles (i.e., a neutron and a proton). The kinetic energy is given by

$$T=\frac{\vec{\pi}_1^{\,2}+\vec{\pi}_2^{\,2}}{2m_e}+\frac{\vec{\pi}_3^{\,2}}{2m_0}\equiv 2T_e+T_0, \tag{1}$$

where $\vec{\pi}_i$ is the momentum of the ith particle relative to the center of mass. These momenta satisfy the mass-independent constraint, $\Sigma_i\vec{\pi}_i=0$. Note the factor of 2 in the second definition. The Hamiltonian H is constructed by adding a potential V to T. Variations of the total energy of the system, E, with respect to small variations of the masses can be obtained using $E=\langle H\rangle$ and the following identities

$$\frac{\partial E}{\partial m_e}=\left\langle\frac{\partial H}{\partial m_e}\right\rangle=\left\langle\frac{\partial T}{\partial m_e}\right\rangle=\frac{-\langle 2T_e\rangle}{m_e}, \tag{2a}$$

$$\frac{\partial E}{\partial m_0}=\left\langle\frac{\partial H}{\partial m_0}\right\rangle=\left\langle\frac{\partial T}{\partial m_0}\right\rangle=\frac{-\langle T_0\rangle}{m_0}, \tag{2b}$$

where we have *assumed* for this problem that the potential is mass-independent (it is not). The crucial step is the first and is known as the Hellmann-Feynman theorem.[11-13] We further define the average and difference of the neutron and proton masses m_n and m_p as

$$m=\tfrac{1}{2}(m_n+m_p), \tag{3a}$$

$$\Delta m=m_n-m_p. \tag{3b}$$

The change in energy of ^{3}He in going from three equal (average) masses to two proton masses and one neutron mass is given by

$$\Delta E_{np}(^3\text{He})\cong-\frac{\partial E}{\partial m_e}\frac{\Delta m}{2}+\frac{\partial E}{\partial m_0}\frac{\Delta m}{2}\cong\langle 2T_e-T_0\rangle\frac{\Delta m}{2m}, \tag{4a}$$

where Eq. (2) has been used, and similarly

$$\Delta E_{np}(^3\text{H})\cong\frac{\partial E}{\partial m_e}\frac{\Delta m}{2}-\frac{\partial E}{\partial m_0}\frac{\Delta m}{2}\cong-\langle 2T_e-T_0\rangle\frac{\Delta m}{2m}. \tag{4b}$$

Taking differences, the ^{3}He-^{3}H mass difference effect is given by

$$\Delta E_{np}=\langle 2T_e-T_0\rangle\frac{\Delta m}{m}. \tag{4c}$$

Equations (4) are the primary relationships in this work.

It is worthwhile examining a special case. If we assume that the trinucleon system is described *solely* by the dominant symmetric S state (the $[\bar{4}]$ spin-isospin state) we find $\langle T_e\rangle=\langle T_0\rangle=\frac{1}{3}\langle T\rangle$ and thus

$$\Delta E_{np}^s=\langle T\rangle\frac{\Delta m}{3m}. \tag{5}$$

This is also the result of assuming that ^{3}He is composed of identical particles each of mass $\frac{1}{3}(2m_p+m_n)$ and similarly for ^{3}H, $\frac{1}{3}(2m_n+m_p)$. If $\langle T\rangle\cong 50$ MeV, Eq. (5) yields $\Delta E_{np}^s\cong 23$ keV, in agreement with the first four estimates discussed in the Introduction.

As illustrated in Fig. 1 the odd particle is acted upon by a stronger force than the even particles feel, since the n-p force is stronger than the n-n or p-p force ($\overline{V}_{np}\cong\frac{3}{4}V_{np}^t+\frac{1}{4}V_{np}^s$). This means that the wave function of the odd particle is more confined (compact) in space than those of the even particles and consequently we expect its kinetic energy to be greater.[14,15] This difference is also reflected in the appearance of mixed symmetry components of the wave function (e.g., S' and D states). If $\langle T_0\rangle>\langle T_e\rangle$, we will find $\Delta E_{np}<\Delta E_{np}^s$. Were the inequality of forces reversed, we should expect that $\Delta E_{np}>\Delta E_{np}^s$. If $V_{np}=V_{nn}$ (or V_{pp}) there is no mixed symmetry component of the wave function and $\Delta E_{np}=\Delta E_{np}^s$.

In Sec. III we will calculate T_E and T_0 by varying m_e and m_0; that is, the first identity in Eqs. (4) will be used directly as our computational algorithm.

III. RESULTS

Because we are seeking large qualitative effects, we make the simplifying assumption that the N-N interactions can be represented by separable potentials. We choose the original Yamaguchi-Yamaguchi[16] form for the triplet:

$$V^t(k,k')=-(\lambda_t/m)g_t(k)g_t(k'), \tag{6a}$$

where

$$g_t(k)=g_c(k)+[S_{ij}(\hat{k})/\sqrt{8}]g_T(k),$$

$$g_c(k)=(k^2+\beta_c^2)^{-1},$$

$$g_T(k)=-\xi_T k^2(k^2+\beta_T^2)^{-2},$$

$$S_{ij}(\hat{k})=3\vec{\sigma}_i\cdot\hat{k}\vec{\sigma}_j\cdot\hat{k}-\vec{\sigma}_i\cdot\vec{\sigma}_j.$$

Here the subscripts c and T refer to the central and tensor components, respectively, and ξ_T is the ratio of tensor to central strengths. For the singlet potentials (n-n = p-p and n-p) we have utilized the simple Yamaguchi[17] form:

$$V^s(k,k') = \frac{\lambda_s}{m} g_s(k) g_s(k'), \tag{7a}$$

where

$$g_s(k) = (k^2 + \beta_s^2)^{-1}. \tag{7b}$$

The singlet phases are adequately described up to 100 MeV. However, such a simple rank-1 potential cannot reproduce the known phase shift zero near 300 MeV; we have tested the sensitivity of our conclusions to the use of rank-1 singlet potentials by comparing such quantities as kinetic energies with those from a calculation utilizing a rank-2 singlet potential[18]:

$$V^s(k,k') = -(\lambda_a/m) g_a(k) g_a(k') + (\lambda_r/m) g_r(k) g_r(k'), \tag{8a}$$

where

$$g_a(k) = (k^2 + \beta_a^2)^{-1}, \quad g_r(k) = k^2 (k^2 + \beta_r^2)^{-2} \tag{8b}$$

are the form factors for the attractive and repulsive components of the potential. For the preferred potential case discussed below with a 7% deuteron D state, the kinetic energy with a rank-2 singlet interaction differs from that for the corresponding rank-1 singlet interaction by only 1.4 MeV (the former is smaller than the latter), while the 4% D-state case difference is 2.4 MeV. This difference is rather slight and we expect that a rank-2 singlet will not greatly affect our results. We will use a rank-1 singlet potential.

For ^{3}H the set of coupled linear integral equations for the spectator functions comprising the three-body bound state wave function can be expressed schematically as

$$\begin{aligned}
u^s_{nn} &= \tau^s_{nn} \int \left[\tfrac{1}{2} I^{ss}_{nn,np} u^s_{np} + \tfrac{3}{2}\left(I^{sc}_{nn,np} u^c_{np} + I^{sT}_{nn,np} u^T_{np} \right) \right], \\
u^s_{np} &= \tau^s_{np} \int \left[I^{ss}_{np,nn} u^s_{nn} - \tfrac{1}{2} I^{ss}_{np,np} u^s_{np} \right. \\
&\qquad \left. + \tfrac{3}{2}\left(I^{sc}_{np,np} u^c_{np} + I^{sT}_{np,np} u^T_{np} \right) \right], \\
u^c_{np} &= \tau^t_{np} \int \left[I^{cs}_{np,nn} u^s_{nn} + \tfrac{1}{2} I^{as}_{np,np} u^s_{np} \right. \\
&\qquad \left. + \tfrac{1}{2}\left(I^{cc}_{np,np} u^c_{np} + I^{cT}_{np,np} u^T_{np} \right) \right], \\
u^T_{np} &= \tau^t_{np} \int \left[I^{Ts}_{np,nn} u^s_{nn} + \tfrac{1}{2} I^{Ts}_{np,np} u^s_{np} \right. \\
&\qquad \left. + \tfrac{1}{2}\left(I^{Tc}_{np,np} u^c_{np} + I^{TT}_{np,np} u^T_{np} \right) \right],
\end{aligned} \tag{9}$$

where the subscripts describe the interacting pair and the superscripts denote the singlet, central-triplet, or tensor-triplet interaction of that pair. These equations are well known[19]; we repeat them here so that it will be clear which approximation has been made when they are discussed below. It should be obvious that the usual momentum variables arising in these integral equations have been replaced by more complicated quantities which account for the neutron-proton mass difference.[20]

Because the coupling of the tensor component of the triplet force to itself in the determination of the spectator function has little effect upon the binding energy, we have set $I^{TT}_{np,np} \equiv 0$ in Eq. (9) for all of the results that we quote. [The binding energy is increased by less than 0.6% by this approximation (i.e., less than 50 keV), and it should have

TABLE I. Charge asymmetry effect of the n-p mass difference for various potential cases.

	B (MeV)	T_e (MeV)	T_0 (MeV)	T (MeV)	ΔE^s_{np} (keV)	ΔE_{np} (keV)
I. $V^s = V^t$ V_T (P_D = 0%)	8.99	12.95	12.95	38.86	17.84	17.84
II. $V^s_{nn} \neq V^s_{np} \neq V^t_{np}$ V_T (P_D = 0%)	10.42	12.87	15.87	41.61	19.10	13.60
III. $V^s_{nn} \neq V^s_{np} \neq V^t_{np}$ V_T (P_D = 7%)	7.82	13.45	20.63	47.53	21.82	8.63
IV. $(V^s_{nn} = V^s_{np}) \neq V^t_{np}$ V_T (P_D = 7%)	7.63	13.32	20.45	47.09	21.61	8.53
V. $V^s_{nn} \neq V^s_{np} \neq V^t_{np}$ V_T [P_D = 7% (2nd order)]	7.24	12.52	18.65	43.69	20.06	8.80
VI. $V^s_{nn} \neq V^s_{np} \neq V^t_{np}$ V_T (P_D = 4%)	8.80	12.29	17.23	41.86	19.21	10.04

TABLE II. Separable potential parameters used in cases I–VI of Table I. In case V the same parameters were used as in case III, as discussed in the text. The units of λ are fm^{-3} and the units of β are fm^{-1}.

	I	II	III	IV	VI
λ^s_{nn}	0.2355	0.1323	0.1323	0.1323	0.1323
β^s_{nn}	1.2722	1.13	1.13	1.13	1.13
λ^s_{np}	0.2355	0.1456	0.1456	0.1323	0.1456
β^s_{np}	1.2722	1.15	1.15	1.13	1.15
λ^t_{np}	0.2355	0.3815	0.142 97	0.142 97	0.2489
β^c_{np}	1.2722	1.406	1.2412	1.2412	1.3338
ξ^T_{np}	0	0	4.4949	4.4949	1.784
β^T_{np}	···	···	1.9476	1.9476	1.5682

an insignificant effect upon the binding energy differences of interest.] We also examine the approximation in which only the tensor force contribution to $I^{cc}_{np,np}$ is retained; i.e., $u^T_{np}\equiv 0$ in Eq. (9). It will be seen that this approximation is useful in understanding the effect of the tensor nature of the triplet interaction upon the n-p mass difference effect. Finally, it should be remarked that we do consider a triplet interaction without a tensor component as well as singlet interactions which are identical for both n-n and n-p systems. We note that in the latter situation, the n-p mass difference leads to a difference in scattering lengths and effective ranges of some 0.23 and 0.0026 fm, respectively, in our rank-1 separable potential model.

Equation (4) was used directly to calculate T_e and T_0, where the expectation value of the operators is implied. A standard nucleon mass of 939 MeV was assumed for m and binding energies $B(=-E)$ were calculated for $m_e=m$, $m\pm 1.0$ MeV, and $m_0=m, m\pm 1.0$ MeV. Appropriate energy differences directly yield right and left numerical derivatives of B with respect to m_e and m_0, which are found in all cases to be nearly equal. Taking the averages of right and left derivatives yields $\partial B/\partial m_e$ and $\partial B/\partial m_0$, from which T_e and T_0 are calculated using Eqs. (2a) and (2b). The charge asymmetry effect is calculated from Eq. (4c) and, similarly the estimate for a totally symmetric wave function is determined from Eq. (5). Results for a variety of potential combinations are presented in Table I and the corresponding parameters which were used are listed in Table II.

Case I has no tensor force and equal singlet and triplet forces. Since V_{nn} (or V_{pp}) $=V_{np}$, there are no S or D states and thus we find $T_e=T_0$, as shown in the first row. The total kinetic energy is rather low by comparison with results for realistic local potentials.[7] The solution of the ^{3}H problem with the Reid soft-core potential yields $T=49.1$ MeV, for example.[21] The latter potentials underbind the trinucleon system, however. Case II has no tensor force, but different singlet and triplet forces as well as charge dependence ($V^s_{np}\neq V^s_{nn}$). This case has an S' state but no D state. Although T_e changes little, T_0 changes dramatically and $\Delta E_{np}<\Delta E^s_{np}$, as discussed earlier. If the singlet and triplet forces are reversed we find that the inequality is reversed, as expected. Adding a tensor force in case III, which produces a deuteron with a 7% D state, increases T_e slightly but changes T_0 even more, which is reflected in a further reduction in ΔE_{np}. The binding energy is somewhat smaller than case I, although the total kinetic energy is almost 25% greater. The odd particle's kinetic energy is more than 50% larger than the kinetic energies of the even particles. Case IV examines the effect of charge dependence by equating V^s_{nn} and V^s_{np}. A slight reduction of both T and B results and ΔE_{np} is only marginally affected. The mixed symmetry effect (ratio of T_0 to T_e) is essentially unchanged. Since the tensor force plays an important role, we investigated the effect of the D state probability on charge asymmetry by deleting all coupling between the tensor and central forces in the Fadeev equations. The sole effect of the tensor force is then a second-order (tensor) force in the central channels [see the discussion following Eq. (9)] and there is *no D state*. The result of this approximation is a drop in the binding and all kinetic energies. Comparing cases III and V we see that T_0 is lowered by a somewhat greater fraction than T_e or T, which is the reason why ΔE_{np} is slightly greater in case V than in case III. The ratio T_0/T_e decreases from the latter to former cases and the mixed-symmetry effect on ΔE_{np} decreases; that is, the D states increase the mixed symmetry effect, as expected. From the numerical results in the table, it seems likely that a substantial part of the tensor effect on charge asymmetry in our model is due to an enhanced S' state. We note that the kinetic energy has no matrix elements between S and D states, while there exist S-S' matrix elements. Thus although the S' state has a very modest *probability* compared to that of the D states, the *amplitude* of the S' state is not similarly small.

For comparison, we have also listed results corresponding to a tensor force which generates a 4% deuteron D state. The binding energy is similar to case I. Comparing cases III and VI, we see that the decrease of the mixed symmetry effect (T_0/T_e) more than compensates for the decrease in all kinetic energies.

Our best estimate of ΔE_{np} is 8.6 keV, which compares with 12 keV in Ref. 4. We believe that the separable potential approach to this aspect of the trinucleon problem is not necessarily unrealistic. Our results indicate that the mixed symmetry effect is appreciable and must be taken into account. It is also likely to be potential-dependent. Because ΔE_{np} is completely calculable when T_e and T_0 are known, we recommend that these numbers be determined, if possible, when Faddeev calculations of trinucleon properties are performed and published.

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