PHYSICAL REVIEW C VOLUME 17, NUMBER 4 APRIL 1978

Volume integral of particle-particle collision probability in nuclear matter

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(Received 14 March 1977; revised manuscript received 10 August 1977)

Average volume integrals per nucleon of particle-particle collision probability in nuclear matter are evaluated using the preequilibrium exciton model. The results obtained are in quite reasonable accord with the volume integrals of optical model absorptive potentials.

[NUCLEAR REACTIONS Exciton model evaluation of volume integrals of collision probabilities; comparison with optical model results.]

I. INTRODUCTION

The collision probability per unit time, $W_{1p}(\epsilon, \vec{r})$, of a nucleon inside a nucleus is related to the imaginary part of the nuclear complex potential, $W(\epsilon, \vec{r})$, by the equation

$$W(\epsilon, \vec{r}) = \tfrac{1}{2} \hbar W_{1p}(\epsilon, \vec{r}) . \tag{1}$$

Here $W_{1p}(\epsilon, \vec{r}) = v(\rho_n \overline{\sigma}_{Nn} + \rho_p \overline{\sigma}_{Np})$; ϵ and v are, respectively, the nucleon energy and velocity within the nucleus, ρ_n and ρ_p the neutron and proton densities, and $\overline{\sigma}_{Nn}$ and $\overline{\sigma}_{Np}$ the average nucleon-neutron and nucleon-proton cross sections. In 1955 Lane and Wandel,[1] using values for $W(\epsilon, \vec{r})$ obtained in early optical model (OM) analyses for neutrons of energies $0 \lesssim E \lesssim 22$ MeV, reported satisfactory fulfillment of (1) when $W_{1p}(\epsilon, \vec{r})$ was calculated in the framework of the Fermi gas model with free nucleon-nucleon cross sections assumed isotropic. The calculation, which takes into account in an approximate way the Pauli principle, is reproduced, e.g., in the treatise of Kikuchi and Kawai.[2] However, the conclusion of Lane and Wandel was not supported by successive studies. Greenlees, Pyle, and Tang[3] found in 1968 that for protons of energies around 30–40 MeV interacting with ^{58}Ni, the figures obtained for $W_{1p}(\epsilon, \vec{r})$ with the above procedure gave $W(\epsilon, \vec{r})$ values appreciably greater (by a factor of ~2 on the average for the various $\vec{r}$'s) than those derived from phenomenological OM analyses.

We think that the situation can now be reconsidered for the following reasons:

(a) In a recent work Agrawal and Sood,[4] following an early suggestion of Feshbach,[5] calculated the average volume integral per nucleon of the absorbing (imaginary) part of the proton OM potential J_W^p/A. They showed that, in spite of severe ambiguities often encountered in the choice of the parameters defining the phenomenological imaginary potential, J_W^p/A remained remarkably constant for proton energies from 10 to 60 MeV, and in practice did not depend on the mass of the target nucleus for $A > 40$. Subsequently, their findings have been extended and on the whole confirmed by Hodgson[6] using a wider and updated selection of material. As remarked by this author, the near constancy of J_W^p/A with A suggests that one attempts to reproduce it theoretically, rather than the values of $W(\epsilon, \vec{r})$ for the various $\vec{r}$.

(b) After 1968, pre-equilibrium reaction models have been developed in which the nucleon collision probability per unit time plays an essential role. Therefore it is now possible, albeit somewhat indirectly, to extract from analyses of suitable experimental data *phenomenological* values of $W_{1p}(\epsilon, \vec{r})$.

It is then of interest to establish whether a relation similar to (1), connecting the volume integrals of $W(\epsilon, \vec{r})$ and $W_{1p}(\epsilon, \vec{r})$, can be satisfied by using phenomenologically derived quantities to evaluate both integrals. Moreover, if to treat pre-equilibrium reaction phenomena the exciton model (EM) (exposed, e.g., in Ref. 7) is used, the comparison of volume integrals is quite naturally indicated because there the analysis of the experiments yields values of $W_{1p}(\epsilon, \vec{r})$ averaged over nuclear volumes.[8] The purpose of this communication is indeed to report an attempt in this direction.

II. VOLUME INTEGRAL OF THE IMAGINARY PART OF THE NUCLEON OPTICAL MODEL POTENTIAL

Agrawal and Sood, and Hodgson, evaluated the average volume integral per nucleon of the absorbing part of the proton OM potential only. This quantity, however, cannot be compared directly with the volume integral of the nucleon-nucleon collision probability, as extracted from EM analyses of reaction data, because the latter quantity includes collisions of neutrons, in addition to those of protons, with the other nucleons of the nucleus.

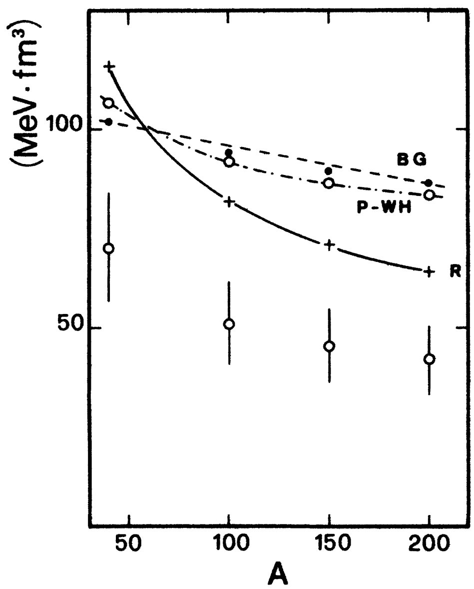

FIG. 1. Values of J_W/A deduced from the optical potentials of Becchetti and Greenlees (BG), Perey and Wilmore-Hodgson (PWH), Rosen *et al.* (R); and values of $\frac{1}{2}\hbar J_+/A$ (circlets with error bars) resulting from the EM approach.

The results of Agrawal and Sood and of Hodgson, have then been supplemented by evaluating also the average volume integral per nucleon of the absorbing part of the neutron OM potential J_W^n/A. To this end we considered some very frequently used potentials (those of Wilmore and Hodgson,[9] of Rosen *et al.*,[10] and of Becchetti and Greenlees[11]) and found for J_W^n/A properties quite similar to those of J_W^p/A: viz., that J_W^n/A depends very little on energy for $10 \lesssim E \lesssim 50$ MeV and only weakly on target mass. The resulting numerical values of J_W^n/A were somewhat lower than those of J_W^p/A: namely $J_W^n/A \cong 0.7(J_W^p/A)$; and hence we inferred a mean of the average volume integrals $J_W/A = \frac{1}{2}(J_W^n/A + J_W^p/A)$. This mean is plotted vs A in Fig. 1 where the three curves shown represent the results obtained with the OM potentials of Becchetti and Greenlees (BG), Rosen *et al.* (R), and Perey[12] and Wilmore-Hodgson (PWH); the proton and the neutron energies used were not always the same, but they were well inside the energy interval, quoted in Sec. I (a), where the volume integrals depend little on energy.

III. VOLUME INTEGRAL OF THE NUCLEON-NUCLEON COLLISION PROBABILITY PER UNIT TIME

By analogy with what has been done in the case of the imaginary part of the phenomenological nuclear potential, one can now define the integral

$$\int W_{1p}(\epsilon, \vec{r})d\vec{r} = J_+ . \qquad (2)$$

The calculated values of $W_{1p}(\epsilon, \vec{r})$, as shown, e.g., in Figs. 12 and 13 of the paper by Greenlees *et al.*, vary quite weakly inside the nuclear volume and exhibit appreciable changes only in a very peripheral part of the nucleus; thus their averages over the nuclear volume appear to be significant quantities [the near constancy of $W_{1p}(\epsilon, \vec{r})$ with $\vec{r}$ is due to the fact that in its expression the variations of ρ_n and ρ_p are to a large extent compensated by the opposite ones of $\bar{\sigma}_{Nn}$ and $\bar{\sigma}_{Np}$].

Now, the quantities that one obtains from the EM analysis of the experimental data are the values of $W_{1p}(\epsilon, \vec{r})$ averaged over the nuclear volume: henceforth we shall denote these values as $W_{1p}(u)$, where $u = E + B$, E being the nucleon energy outside the nucleus and B its binding energy. By this time, there is a considerable amount of experimental data concerning many different reactions (produced by incident nucleons of energy variable from ~10 to 100 MeV[7,13,14] and by π^- at rest[15]) that can be used to obtain such $W_{1p}(u)$ values. All the data analyzed with the EM concur in indicating that the values of $W_{1p}(u)$ are insensitive to mass changes of target nuclei, and have an energy dependence conformable to that predicted by the Fermi gas model for a Fermi energy $E_F = 20$ MeV. This prediction is derived, as usual, by employing isotropic free nucleon-nucleon cross sections [however, the absolute values of $W_{1p}(u)$ thus calculated turn out to be systematically higher by a factor of about 4 than the phenomenological ones: cf., e.g., Ref. 7].

The phenomenological values of $W_{1p}(u)$ are displayed vs u in Fig. 2. It is seen that the function there represented grows first with u, but then flattens out at $W_{1p}^{\max}(u) \cong 0.77 \times 10^{22}$ s^{-1} for $u \gtrsim 30$ MeV. By assuming as a constant this

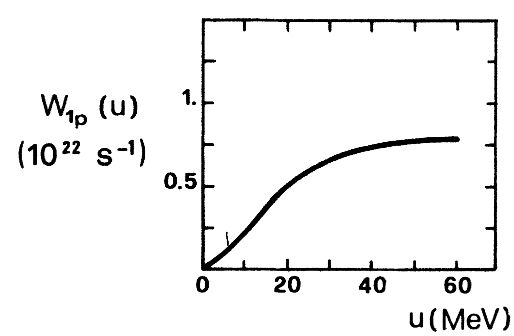

FIG. 2. Collision probability per unit time of a particle in a state above the Fermi energy ($E_F = 20$ MeV) with a particle of the Fermi sea.

$W_{1p}(u)$ value in (2), and limiting ourselves to consider the energy region for $E \gtrsim 20$ MeV ($E = u - B, B \sim 10$ MeV), we get

$$J_+ = \tfrac{4}{3}\pi R^3 W_{1p}^{\max}(u)\,, \tag{3}$$

where R is the radius of the constant density distribution introduced in the EM approach. Our choice for this parameter, in keeping with the suggestion put forward in an earlier communication from this laboratory,[16] is $R = (1.16A^{1/3} + 2.4)$ fm.

The values of $\frac{1}{2}\hbar J_+/A$ calculated with such R's for $A = 40, 100, 150, 200$, are reported in Fig. 1 together with their estimated uncertainties. But before discussing how they compare with the volume integrals per nucleon of the imaginary part of the OM potential, we would like to expose the reasons behind our choice of the R parameter.

IV. NUCLEAR RADIUS IN THE CONSTANT DENSITY APPROXIMATION

The values of the measured proton reaction cross sections on various nuclei in the energy range $30 \leq E \leq 60$ MeV[17] show that in a sharp density distribution approximation [$\rho(r) = \rho_0$ if $r \leq R$, $\rho(r) = 0$ if $r > R$] the radius must noticeably exceed the value $R \sim 1.35A^{1/3}$ fm. In fact this value of the radius is the one that approximately allows one to fit the experimental cross sections by means of the expression

$$\sigma_R = \pi(R+\lambda)^2(1 - C_b/E)\,, \tag{4}$$

where λ is the de Broglie wavelength of the proton and C_b the Coulomb barrier, that is justifiable only if one assumes the nucleus to be completely absorbing. A common feature of all the calculations of σ_R based on nucleon-nucleon collisions is however that, at the energies considered, the nuclei—especially the medium weight ones—show a non-negligible transparency (cf., e.g., the results of Refs. 18 and 19). A quantitative indication of the values to be used can be derived from the variation of $W_{1p}(\epsilon, \vec{r})$ with $\vec{r}$ in the case of a realistic density distribution $\rho(r)$, like the one given by a Fermi distribution. If, e.g., the collision probability per unit time decreases strongly with increasing $\vec{r}$, or with decreasing density, the outer regions of the nucleus are not effective in absorbing the incident particles, and the effective radius of the constant density distribution has to be correspondingly reduced; if, on the contrary, the collision probabilities do not show a significant variation with the density, but remain practically constant with decreasing density up to, say, a fraction f of the central value ρ_0, the nuclear volume, in a constant density approximation, has to contain also those regions that in the more realistic distribution correspond to a density $f\rho_0$. In this second instance, any other choice would appear arbitrary.

As we already mentioned, detailed calculations by Greenlees *et al.*[3] in the case of ^{58}Ni show a weak variation of $W_{1p}(\epsilon, \vec{r})$ up to values of the density around $0.1\rho_0$ (cf., Figs. 12 and 13 of Ref. 3). The values of $W_{1p}(\epsilon, \vec{r})$ are far from being negligible also for lower ρ's. For $\rho \sim 0.02\rho_0$, $W_{1p}(\epsilon, \vec{r})$ is still ~ 0.2 times the value for $r = 0$, and its tail extends up to r of ~ 8 fm. It is important to stress that the calculation shows that the imaginary OM potential also extends up to such large distances, and that its radial dependence is the same as that which characterizes $W_{1p}(\epsilon, \vec{r})$. The result of these authors, i.e., a relatively weak dependence of $W_{1p}(\epsilon, \vec{r})$ on the distance, that makes low density regions as effective as the central region of the nucleus in the absorption of the incident particles, is confirmed by the findings of authors that used the intranuclear cascade model (ICM). Bertini summarized the results of his calculations of nuclear reaction cross sections performed with the ORNL ICM code by stating that "in regard to nuclear configuration it appears that the bulk of the effect in going from a uniform density distribution [characterized by $R = 1.3A^{1/3}$ fm] to a nonuniform distribution (diffuse nuclear edge) [with a notably greater outer radius] comes from the increased nuclear dimension. The shape of the distribution yields second order effects."[18]

We also verified this conclusion by comparing the values of the absorption cross section one gets by using the VEGAS code in the version STEP[19] and collision probabilities like the ones we suggest, with the values one obtains by means of formula (2) of Ref. 16, that has been derived in the hypothesis of a sharp density distribution (in performing these calculations with the original VEGAS code we used values of free nucleon-nucleon cross sections reduced by a factor 4). We considered the case of 39 and 62 MeV protons on ^{54}Fe and of 62 MeV protons on Y. At 39 MeV, using the VEGAS code we obtained for the Fe absorption cross section 592 mb; and with the analytical calculation 619 mb, using $W_{1p}(u) = 0.77 \times 10^{-22}$ s^{-1} and a radius equal to $(1.07A^{1/3} + 2.5)$ fm, which is the outer radius of the step density distribution utilized in VEGAS, and corresponds to densities far below $0.05\rho_0$. At 62 MeV, for the absorption cross sections of the two nuclei we obtained with VEGAS the values 492 and 768 mb, and with the analytical calculation 571 and 732 mb, respectively.[20] In both cases the agreement is sufficiently close. With a radius definitely smaller than that used in the analytical calculation (e.g., with a radius of the order of $1.3A^{1/3}$ fm), we would have obtained cross sections noticeably lower than those computed with VEGAS.

This rather detailed discussion should help to clarify our choice of the radius parameter (3). We may point out finally that the value used is in good agreement (namely, within 5% for nuclei ranging from C to Bi) with the medium outer radius suggested several years ago by Bertini,[18] and adopted by him also in a subsequent work[21] (cf. Table I of this last reference).

V. COMPARISON OF THE VOLUME INTEGRALS AND DISCUSSION

We turn now to the comparison of the quantities J_W/A (Sec. II) and $\frac{1}{2}\hbar J_+/A$ (Sec. III). One can notice that both quantities behave similarly in respect to their dependence on energy and mass number.

It will be recalled from Sec. II that the empirical value of J_W/A remains practically constant as the incident nucleon energy changes between approximately 10 and 60 MeV. The same happens for J_+/A, whose energy insensitivity beyond $u \sim 30$ MeV (i.e., beyond $E \sim 20$ MeV) is secured by the saturation of $W_{1p}(u)$. Notice that this result would not be reached if in the calculation of $W_{1p}(u)$ a value of E_F definitely greater than 20 MeV were utilized. In the case, e.g., of $E_F = 40$ MeV, $W_{1p}(u)$ would increase with u up to values of u of the order of 110 MeV;[22] and so would J_+/A.

It is seen from Fig. 1 that both volume integrals show a weak dependence on A. For J_+/A this comes out directly from the fact that $W_{1p}(u)$ does not depend on A: This property of the collision probabilities is well known.[7,8] Relations (1) and (3) can then offer a simple explanation of the weak mass dependence of J_W/A, a property not to be expected (cf., e.g., the discussion reported in Ref. 23).

While the main features of the OM results seem thus adequately recovered, the numerical values of $\frac{1}{2}\hbar J_+/A$ and J_W/A do not coincide. In fact, as for numerical accord, Fig. 1 shows that the magnitude of the difference between their values is of the order of, say, 60% with reference to the curve for the R_1 potential, and of the order of a factor of 2 with reference to the curves for the PWH and BG potentials. In our opinion, this kind of accord is already encouraging, but it can be argued, in addition, that the numerical values of the quantities considered cannot coincide, and that J_W/A should be somewhat greater than $\frac{1}{2}\hbar J_+/A$. In the optical model, the absorbing part of the potential represents the effect of all nonelastic processes, while the decay rates for the exciton-exciton interaction [whence the values of $W_{1p}(u)$ derive] are deduced from the analysis of only those processes accountable by the inherently statistical exciton model. It has already been shown that the total cross section σ_a calculated in the framework of the EM is smaller (by some 10%, as estimated in Ref. 16) than the reaction one σ_R, that is the experimental datum on which the most precise determinations of J_W/A can be based. The inability of the EM to reproduce the whole σ_R is traceable to the fact that it takes into account all the possible two-body interactions inside the nucleus, but cannot register other types of interactions that occur when the incident particles approach the target nuclei. Such are in particular the many-body interactions, most likely to happen near the surface region of the nuclei [just to quote a well known finding, the intervention of collective interactions in the (p,p') scattering has been pointed out, e.g., by Cohen *et al.*[24]]. Precisely because they are localized at or near the nuclear surface, these types of interactions can be thought on the other hand to contribute appreciably to the formation of the J_W/A values. An estimate of the amount whereby $\frac{1}{2}\hbar J_+/A$ would be increased if the total cross section for the processes described by the EM were σ_R instead of σ_a can be reached by means of formulas reported in Ref. 16: it turns out to be around 30%.

Therefore, an appreciable part of the discrepancy appearing in Fig. 1 can be accounted for. The remaining part can hardly be considered, in our opinion, to be a really significant difference. The experimental data whose analysis provides the ground for the numerical evaluation of J_W/A are not the same as those used to derive $\frac{1}{2}\hbar J_+/A$. In the OM preponderant importance is given to the analysis of elastic scattering, while this process cannot figure among the contributions taken into account in the statistical approach of the EM. In addition it should be remarked that the elastic scattering is rather insensitive to W,[23] the depth of the imaginary part of the potential. We maintain also that the consideration of σ_R only cannot allow an unambiguous determination of W in OM analyses. In the calculation of the absorption cross section σ_a by means of a semiclassical approach the experimental data do not fix a unique value of $W_{1p}(u)$, but can be fitted by different combinations of W_{1p} and of the radius.[16] To give an extreme example, a proper choice of the radius can allow one to fit σ_a by using a $W_{1p}(u)$ approaching infinity (in this case the nucleus would be completely absorbing). To some extent, such an ambiguity certainly affects also the OM results.

By the way, we would like to remark that in principle the data considered in several EM analyses[13-15] can remove the ambiguity mentioned above. In fact it has been shown there that the reproduction of the excitation functions of many different processes, induced by protons of energy up to about 100 MeV, is satisfactory only if, irrespective of the choice of the nuclear radius, the numerical values of the nucleon-nucleon collision probabilities are close to the ones we suggested. This is so because, once the value of the absorption cross section is given, the yields of preequilibrium particles and of particles evaporated from the compound nucleus depend critically on the nucleon-nucleon collision probabilities.

As an obvious final consideration it should be pointed out that several parameters entering both OM and EM calculations are not known with great precision, and even small readjustments of their values might bring closer the results of the two models. On the contrary, it appears that if values of $W_{1p}(u)$ calculated with the Fermi gas model and isotropic nucleon-nucleon cross sections were used in (2), $\frac{1}{2}\hbar J_{+}/A$ would come out much greater than J_W/A. This result confirms and extends the earlier conclusions of Greenlees, Pyle, and Tang.

To conclude, we feel that this comparison between the volume integrals of the EM collision probabilities per unit time and of the imaginary part of the complex OM potential, can help to clarify the similarities existing between these two widely used and successful models.

It is a pleasant duty to acknowledge several interesting discussions with Professor J. J. Hogan.

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