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Isobaric analog impurity from total and differential neutron scattering cross sections of silicon

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Resonance parameters for four resonances in Si have been obtained by analyzing measured differential neutron scattering cross sections combined with the total neutron cross section in the neutron energy range 1.05–1.40 MeV and applying R – matrix single channel multilevel theory. The resonance at 1254 keV has been identified as the s – wave analog resonance in ^{29}Si. The identification also takes into account experimental radiative neutron capture data and shell model calculations for the radiative width. Estimates of the isospin mixing matrix elements are given.

[NUCLEAR REACTIONS ^{28}Si(n), (n,n), E_n = 1.05–1.40 MeV, measured $\sigma_{nt}(E)$, $\sigma(E)$, $\sigma(E,\theta)$. R matrix analyses, ^{29}Si deduced resonances, J, π, Γ. Isobaric analog isospin impurities.]

Recently Weigmann, Macklin, and Harvey[1] for the first time in a neutron induced reaction, have observed isobaric analog states in the total and capture cross sections of neutrons on ^{24}Mg. Such an observation is quite important to obtain isospin mixing matrix elements which measure the deviation of isospin symmetry of nuclear forces. Moreover, Ikossi *et al.*[2] have observed an intriguing periodic behavior with a period of eight masses in the proton reduced widths of analog states excited in proton isospin-forbidden elastic scattering from the self-conjugate even-even nuclei. As a further step it is also interesting to investigate the neutron reduced widths of analog states in neutron isospin-forbidden elastic scattering on the same targets. Therefore, the ^{28}Si $+n$ system has been studied here to identify an isobaric analog state corresponding to the first excited state in the parent nucleus ^{29}Al.

Previously Detraz and Richter[3] have observed the $T=\frac{3}{2}$ states in ^{25}Mg and ^{29}Si by (^{3}He, α) reactions. Weigmann *et al.* agree with Ref. 3 in energy assignment in ^{25}Mg for two analog states corresponding to the ground state and the first excited state in the parent nucleus, while the third one with $J^\pi=\frac{1}{2}^+$ is lower by 46 keV in Ref. 3. The energy assignment of Ref. 3 is also 59 keV lower than the energy difference between the second excited state and the ground state in ^{25}Na. This latter difference was attributed by Detraz and Richter to the Thomas-Ehrmann shift[4] (TE shift) and is not borne out by measurements of Weigmann *et al.* For the case of ^{29}Si the relative energies of all the other analogs with respect to the ground state analog are depressed by 57–80 keV when compared with the parent ^{29}Al. The TE shift can be large only for $l=0$ resonances and the energy discrepancy in the work of Detraz and Richter for ^{29}Si cannot be accounted for by two states out of the four observed analog states. According to the measurements of Weigmann *et al.* the TE shift for the $J=\frac{1}{2}^+$ level in ^{25}Mg is about 20 keV. So it can be concluded that some of the assignments in Ref. 3 may be off by ~50 keV which is beyond the quoted experimental error of 7 keV. The lowest $T=\frac{3}{2}$ state in ^{29}Si is below the neutron separation energy and therefore cannot be observed. The analog corresponding to the first excited state in ^{29}Al should appear as a resonance at E_n (lab) = 1262 keV without any TE shift, at 1242 keV if the TE shift is the same as for ^{25}Mg and at 1197 keV according to the data of Ref. 3. Consequently, we considered a neutron energy range between 1.05 and 1.40 MeV. High resolution total neutron cross sections for these energies have already been reported by Cierjacks *et al.*[5] and Schwartz, Schrack, and Heaton[6]. Both these sets are in good agreement with each other and we have used the data of Ref. 5 in this work. Further Schouky and Cierjacks[7] have measured differential cross sections at 10 angles in 1 keV steps using the Karlsruhe neutron time-of-flight facility. In addition to these measurements we have also taken into account the neutron capture cross section measurements on Si reported by Boldeman *et al.*[8] BNL-325, Vol. I (1973)[9] lists the resonance parameters obtained by Mughabghab based on the measurements of Refs. 5 and 6. The BNL-325 lists three $\frac{1}{2}^+$ resonances for ^{28}Si $+n$ at 1163.9 ± 0.9, 1186.0, and 1252.0 ± 2.0 keV. An inspection of the data of Refs. 5 and 6 did not indicate any resonance at 1186 keV. Also, the total neutron cross section and differential cross section between 1.05 and 1.40 MeV were calcul-

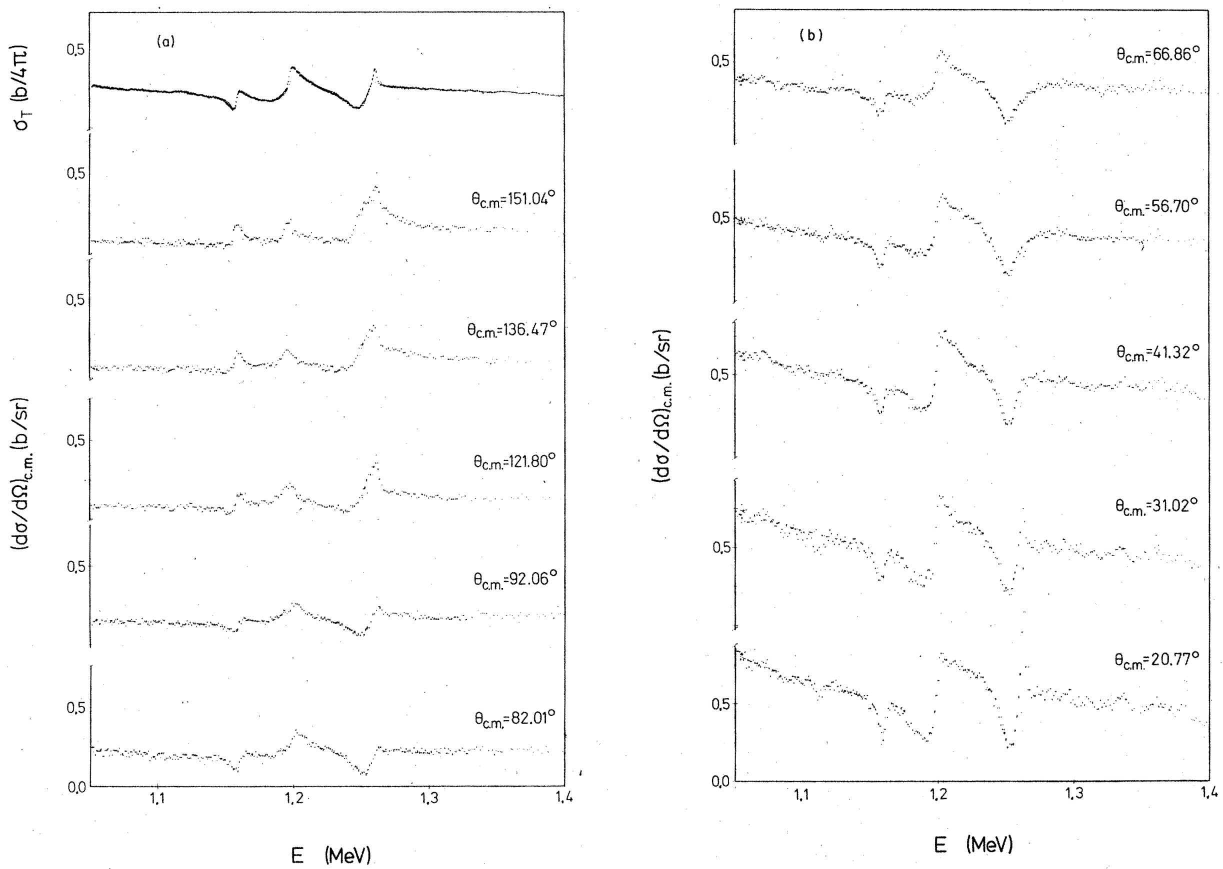

FIG. 1. (a), (b) The total and the differential elastic neutron scattering cross section for natural silicon. The total cross sections were measured in energy steps of ~0.6 keV with a total uncertainty of ±2%. The differential cross section has errors ranging between 5–10% and the energy steps are ~1 keV. All these data were measured with the Karlsruhe neutron time-of-flight facility using a flight path of about 57 m. The solid curves were calculated with the Karlsruhe single channel multilevel R-matrix code (Ref. 10) using a 2 keV smearing width. The resonance parameters for the resonances in the figure are given in Table I. Other resonance parameters beyond the analyzed range were taken from BNL-325 (1973).

ated by a single channel multilevel R matrix analysis using the Kirouac-Nebe code[10] starting with the parameters given in BNL-325. The theoretical cross sections were averaged over an interval of 2.5 keV. Eleven resonances below and seven resonances above the energy range considered were also included. It was essential to put the contribution due to distant levels as $R^\infty = -0.25$ for both $l=1$, $J=\frac{1}{2}$ and $J=\frac{3}{2}$ waves to obtain the theoretical curves close to the data. For other partial waves these were set to zero though the channel radius was chosen as 4.2 fm as so to yield the correct value of the total cross section for thermal neutrons, thereby describing the s-wave behavior properly through the choice of the channel radius. It was found that the theoretical curves produce pronounced minima by including a $J=\frac{1}{2}^+$ resonance at 1186 keV. Such minima were not seen in the data at this energy. Therefore the 1186 keV resonance was found to be redundant in our analysis. The resonance at 1227 keV was also not observed in neutron scattering. This supports the tentative $\frac{7}{2}^-$ assignment of Medsker, Jackson, and Yntema[11] and indicates that a higher partial wave might be involved. The final fit was chosen based on visual comparison. It is shown in Figs. 1(a) and 1(b) and seems quite satisfactory. Table I compares the resonance parameters, used in the above-mentioned energy range, obtained in the present analysis with those given in BNL-325. Only two $\frac{1}{2}^+$ resonances at 1160 and 1254 keV are present in the data. Table II lists all the $\frac{1}{2}^+$ resonances observed below 1.9 MeV, their widths and reduced widths Γ_n^0 calculated using the expression

$$\Gamma_n^0 = \left| \frac{1\ \text{eV}}{E_n\,(\text{c.m.})} \right|^{1/2} \Gamma_n\,(\text{c.m.}) . \qquad (1)$$

TABLE I. Resonance parameters for $^{28}Si+n$ for E_n = 1.05–1.40 MeV. The energy positions are accurate to ±1 keV while Γ is accurate to 10%. Being single channel analysis, the 1201 keV level is assumed to be $\frac{1}{2}^-$ because $\frac{3}{2}^-$ assignments lead to very high values of the cross sections compared with the experimental data. Distant level parameters are quoted in the text.

Present calculation E_0 (keV)	Γ (keV)	J^π	BNL-325 E_0 (keV)	Γ (keV)	J^π
1161	2.5	$\frac{1}{2}^+$	1163 ± 0.9	3.3 ± 0.4	$\frac{1}{2}^+$
			1186.0	1.8	$\frac{1}{2}^+$
1201	15	$\frac{1}{2}^-$	1203.7 ± 0.6	12 ± 3	$\frac{3}{2}^-$
			1227 ± 5	<5.7	
1254	9	$\frac{1}{2}^+$	1252.0 ± 2.0	7.0 ± 1.0	$\frac{1}{2}^+$
1263	1	$\frac{5}{2}^-$	1264.0 ± 0.7	6 ± 4	$\frac{1}{2}^-$

This table will be used to estimate isospin impurities as discussed later.

In their neutron capture measurement Boldeman *et al.* did not observe 1186 and 1254 keV levels while the 1160 keV level was observed with $\Gamma_\gamma = 3.8 \pm 1.1$ eV. De Voigt and Wildenthal[12] have carried out for $A = 29$ nuclei a shell model calculation with the $d_{5/2}, s_{1/2}, d_{3/2}$ basis space. They calculated the *MI* transition matrix elements for the various electromagnetic transitions from the $\frac{1}{2}^+$, $T = \frac{3}{2}$ level. Their values of the matrix elements correspond to $\Gamma_\gamma = 1.85$ eV. This work predicts a value 5 times higher than the experimental one for the absolute transition strength from the lowest $J^\pi = \frac{5}{2}^+$, $T = \frac{3}{2}$ state. If a similar factor is present for the transition from the $\frac{1}{2}^+$, $T = \frac{3}{2}$ level, the Γ_γ will become 0.37 eV. Thus electromagnetic transitions can be too weak to be observed in the neutron capture data. Combining all the facts we suggest that the 1254 keV resonance is the isobaric analog resonance having an excitation energy of 9685 keV in ^{29}Si. This assignment amounts to a TE shift of 11 keV which is comparable to the one of 20 keV observed in ^{25}Mg for the $T = \frac{3}{2}$ *s*-wave level.

TABLE II. *s*-wave resonance parameter for $^{28}Si+n$ between 0–1.9 MeV.

E_0 (keV)	Γ (keV)	Γ_n^0 (eV)	Remarks
55.6	1.5 ± 0.3	6.5	Taken from BNL-325
188.0	56 ± 7	131	Taken from BNL-325
1161	2.5	2.4	Based on present analysis
1254	9	8.2	Based on present analysis

We can estimate isospin impurities for this state in a manner similar to Weigmann *et al.* In this estimate the small isospin admixture in the ^{28}Si ground state have been ignored. From the average of Γ_n^0 of three $T = \frac{1}{2}$ *s*-wave resonances we obtain an estimate of the isospin impurity of the 1254 keV $T = \frac{3}{2}$, $J = \frac{1}{2}^+$ resonances as $\Gamma_n^0(1254\ \text{keV})/\bar{\Gamma}_n^0(T = \frac{1}{2}) = 18\%$ which is comparable to the value of 18% obtained for $T = \frac{3}{2}$ *s*-wave resonance in ^{25}Mg.

Using first order perturbation theory Weigmann *et al.* give two expressions for a zeroth order guess and a lower limit value for the average isospin mixing matrix elements $\overline{\langle T = \frac{3}{2}|V|i\rangle}$ where the average is over all the $T = \frac{1}{2}$ states of the same J^π labeled by i. The expression for the zeroth order value is

$$\Gamma_n^0(T = \tfrac{3}{2}) = |\overline{\langle T = \tfrac{3}{2}|V|i\rangle}_0|^2 \sum_i \frac{\Gamma_0^n(i)}{[E_i - E(T = \frac{3}{2})]^2}, \tag{2}$$

while the lower limit is given by

TABLE III. Estimates of isospin impurity for $^{28}Si+n$ and comparison with $^{24}Mg+n$.

Quantity	$^{28}Si+n$ for the assigned IAR at 1254 keV	$^{28}Si+n$ if 1160 keV level would be the IAR	$^{24}Mg+n$ for 1567 keV $J=\frac{1}{2}^+$, $T=\frac{3}{2}$ level from Ref. 1
$\frac{\Gamma_n^0(T=\frac{3}{2})}{\bar{\Gamma}_n^0(T=\frac{1}{2})}$	18%	5%	18%
$\overline{\langle T=\frac{3}{2}\|V\|i\rangle}_0$ in keV	144	47	150
$\overline{\langle T=\frac{3}{2}\|V\|i\rangle}_{\min}$ in keV	97	35	90

$$\Gamma_n^0(T=\tfrac{3}{2}) = |\overline{\langle T=\tfrac{3}{2}|V|i\rangle}_{\min}|^2 \left| \sum_i \frac{\Gamma_n^0(i)^{1/2}}{|E_i - E(T=\tfrac{3}{2})|} \right|^2 . \quad (3)$$

Using data given in Table II we calculated both these estimates. They are listed in Table III along with the s-wave $T=\frac{3}{2}$ resonance data of ^{25}Mg. Also the values for the other possible choice of the 1160 keV resonance are quoted. The isospin mixing matrix elements for the 1254 keV resonance are comparable to the s-wave resonance data of ^{25}Mg.

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