PHYSICAL REVIEW C VOLUME 17, NUMBER 3 MARCH 1978

Recoil-distance measurement of lifetimes of rotational states in ^{164}Dy

R. O. Sayer
Computer Sciences Division, Oak Ridge National Laboratory, Union Carbide Corporation-Nuclear Division, Oak Ridge, Tennessee 37830*

E. Eichler,[†] N. C. Singhal,[‡] R. Sturm,[§] N. R. Johnson, and M. W. Guidry[¶]
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830
(Received 25 October 1977)

Lifetimes of the 4^+ to 12^+ levels in the ground state rotational band of ^{164}Dy have been measured by means of the Doppler-shift recoil-distance technique. Our best values for the mean lives of the 4^+–12^+ states are 286 ± 15, 39.0 ± 1.9, 10.4 ± 0.7, 3.7 ± 0.8, and 1.6 ± 0.8 ps, respectively. The corresponding $B(E2)$ values exhibit a spin dependence consistent with rigid-rotor predictions. Within the respective experimental uncertainties, previous results obtained by multiple Coulomb excitation and Doppler-broadened line shape methods are in agreement with the present recoil-distance values.

[NUCLEAR REACTIONS ^{164}Dy(^{40}Ar,^{40}Ar′), E=152.6 MeV; measured τ, E_γ; deduced $B(E2)$.]

I. INTRODUCTION

For more than two decades the properties of low-spin rotational levels in deformed nuclei have been investigated extensively. Since the recent discovery[1] of backbending, the abrupt increase in nuclear moment of inertia at about spin 12, intense experimental activity has led to identification of many new high-spin levels.[2] Nuclear structure alterations that affect level energies may also cause the intraband $B(E2)$ values to deviate from the rigid-rotor predictions. Certainly it is important to test the theoretical models for deformed nuclei by measurement of $B(E2)$ values for low- and high-spin states. Traditional techniques—principally Coulomb excitation, Doppler-broadened line shape analysis (DBLS), and recoil distance (RD)—have been utilized in $B(E2)$ measurements, primarily for low-spin levels. However, more energetic beams of heavy ions are rapidly becoming available to enhance population probabilities for high spins and to increase recoil velocities, thus permitting $B(E2)$ measurements for these short-lived states.

It is also important to assess the accuracy and reliability of different techniques for $B(E2)$ measurements. For example, the DBLS method is limited by the accuracy to which heavy-ion stopping powers are known, typically 10% or worse. Because the RD method yields a rather direct measure of the lifetime of a state, it is generally felt to suffer from fewer ambiguities than multiple Coulomb excitation (MCEX), which often requires more intricate analysis to disentangle the effects of other states from the state of interest and which lacks a complete quantal treatment for high-spin states. However, it should be noted that the precision of RD $B(E2)$ values is often limited by uncertainties in dealignment and feeding as well as the usual statistical uncertainties.

Several of us participated in a previous MCEX measurement[3] of $B(E2)$ values of states in ^{160,162,164}Dy in which the ground bands were Coulomb excited with ^{20}Ne and ^{35}Cl ions. These results and the results of Oehlberg *et al.*[4] indicate rotational $B(E2;\,2\rightarrow4)$ values but $B(E2;\,4\rightarrow6)$ values that are $15\pm5\%$ below rotational. Although the analysis included estimates of quantal effects, Coulomb-nuclear interference, and several other perturbing effects, there is still a legitimate concern regarding the accuracy of MCEX measurements. Therefore we have used RD to measure lifetimes of levels up to spin 12 in the ground-state rotational band of ^{164}Dy. We expected that the RD results could provide an evaluation of MCEX for high-spin $B(E2)$ measurements as well as possible insight into the nature of states of high angular momentum.

II. EXPERIMENTAL PROCEDURE

A detailed description of the recoil-distance apparatus, herein referred to as the plunger, used in these measurements has been given by Johnson *et al.*[5] The plunger consists of two precision-machined coaxial cylinders: one affixed to the beam line and terminated in a conical projection over which the thin target is stretched, and a second having a precision-lapped copper end plate (the stopper) coated with about 40 μm of lead. The second cylinder slides on the first to vary the relative target-stopper separation which can be measured with a Boeckler micrometer to a precision of about 1 μm.

A 1.0 mg/cm^2 foil enriched to 98.4% in ^{164}Dy was bombarded with 152.6 MeV ^{40}Ar ions from the Oak Ridge isochronous cyclotron (ORIC). An 18% efficient Ge detector located at 0° relative to the beam axis and 5 cm from the target was used to detect the ensuing γ rays in coincidence with ^{40}Ar ions scattered into an annular Si counter that subtended the angular range 159–175° in the laboratory. The corresponding recoil angular range was 1.7–8.1°, and the average recoil velocity was $v = 0.03226c$ in the case where the recoiling nucleus was left in the 6^+ state.

Timing signals from the γ-ray and particle counters were fed to a time-to-amplitude converter (TAC). A valid TAC output permitted linear signals from the γ-ray and particle counters and the TAC to be digitized by a fast analog to digital converter, buffered to disk, and subsequently copied onto magnetic tape. In the post-experiment playback analysis, suitable windows were set on the backscattered ^{40}Ar energy and the time correlation to obtain random-corrected γ-ray spectra.

Data were collected for nine target-stopper distances over the range 25–670 μm appropriate for the $6 \to 4$ and faster transitions and for three longer distances up to 6352 μm suitable for the $4 \to 2$ transition.

The relative efficiency of the γ-ray counter was determined by calibration with both a ^{226}Ra source and a mixed source of several radionuclides from the National Bureau of Standards.

III. DATA ANALYSIS AND RESULTS

In Fig. 1 we present γ-ray spectra obtained at the five smallest target-stopper separations. The marked growth of the shifted fractions with increasing separation D and the wide variation in shifted fractions for different transitions are evident. In zeroth order the intensities of the shifted (S) and unshifted (U) peaks for a given transition depend exponentially on the mean life τ of the upper level so that the ratio $R = U/(U+S)$ may be written

$$R = e^{-D/v\tau}, \qquad (1)$$

where v, the average recoil velocity component on the axis, may be computed from the expression

$$\Delta E/E_0 = v/c. \qquad (2)$$

Here E_0 is the energy of γ rays emitted by stopped nuclei and ΔE is the observed Doppler shift. Hence, in principle, measurement of R as a function of D yields a direct evaluation of the nuclear lifetime.

Determination of the peak areas and the corresponding uncertainties plays a crucial role in the data reduction. Both a hand analysis and computer fits were used to extract peak areas so as to account for the small amount of low-energy tailing and for the "unshifted" component line shape due to γ rays emitted by nuclei slowing down in the stopper material. Each "unshifted" component was analyzed to give essentially identical fractional intensities in the high-energy tails for all distances.

In practice the extraction of accurate (±5%) lifetimes from recoil-distance data requires extension of the simple zero-order theory described above to include several corrections and perturbing influences. The remainder of this section will be devoted to brief discussions of the magnitudes of effects considered in this work. More complete

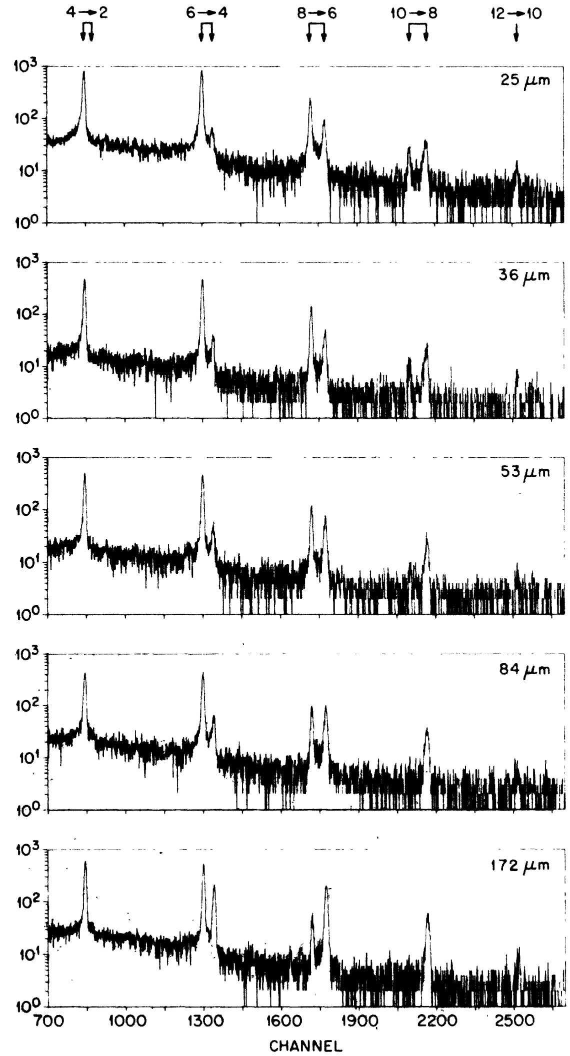

FIG. 1. Spectra of γ rays in coincidence with ^{40}Ar ions backscattered from a ^{164}Dy target for five target-stopper distances.

descriptions of many of these effects have been given previously.[5-7]

A. Velocity and distance distribution effects

In general the zero-order treatment must be modified to account for the distribution of recoil velocities due to finite target thickness and spread in recoil angle and for the distribution in distance due to slight nonparallelism of target and stopper. These effects were estimated to affect the extracted lifetimes by less than 1% by examination of the observed γ-ray line shape and the behavior of the data acquired at close distances. Of more importance is the effect of the finite size of the γ-ray detector on the velocity distribution.

The relativistic Doppler shift for a particular γ ray, $\Delta E'$, is given by the formula

$$1+\frac{\Delta E'}{E_0}=\frac{(1-\beta^2)^{1/2}}{1-\beta\cos\psi}, \quad (3)$$

where $\beta=v/c$ and ψ is the angle between the vectors of the γ ray and the recoiling nucleus. The observed average shift in γ-ray energy ΔE may be computed by an average of Eq. (3) over the detector solid angle Ω, each projection being weighted by the probability $P(\theta)$ that the full γ-ray energy is deposited. The angle θ is the polar angle of the γ ray with respect to the beam axis. Hence,

$$1+\frac{\Delta E}{E_0}=(1-\beta^2)^{1/2} \times \frac{\int d\phi_v \int d\Omega\, W(\theta)P(\theta)(1-\beta\cos\psi)^{-1}}{\int d\phi_v \int d\Omega\, W(\theta)P(\theta)}, \quad (4)$$

where $W(\theta)$ is the γ-ray angular distribution and φ_v is the recoil azimuthal angle. Integration over azimuthal angles gives

$$1+\frac{\Delta E}{E_0}=(1-\beta^2)^{1/2}\,\frac{\int_0^{\theta_c} W(\theta)P(\theta)\sin\theta\, d\theta\{1-2\beta\cos\theta_v\cos\theta+\beta^2(\cos^2\theta-\sin^2\theta_v)\}^{-1/2}}{\int_0^{\theta_c} W(\theta)P(\theta)\sin\theta\, d\theta}, \quad (5)$$

where θ_v is the recoil angle and θ_c is the maximum γ counter half-angle. In the limit $W(\theta)=1$, $P(\theta)=1$, expression (5) may be reduced to that given by Quebert, *et al.*[8] Their expression may be further reduced, in the small β limit, to

$$\frac{\Delta E}{E_0}=\tfrac{1}{2}\,\beta\cos\theta_v\,(1+\cos\theta_c). \quad (6)$$

Equation (5) was evaluated numerically to determine the average recoil velocity for each transition of interest. The $W(\theta)$ were obtained from a Winther-de Boer calculation,[9] and the $P(\theta)$ were computed using the full-energy absorption coefficients of Krane.[10] For the transitions of interest the resultant average velocities from Eq. (5) were 2.6–3.5% smaller than those given by Eq. (6) and 1.2–2.0% smaller than those computed with the formula of Quebert *et al.*[8]

B. Solid angle and efficiency effects

Geometrical differences in solid angle of shifted and unshifted γ rays are caused by emission of the former from points in the region between target and stopper. This effect required a correction to the shifted areas of less than 2% except in the case of the $4\rightarrow 2$ transition, where it was 16.5% at $D=6352$ μm.

A correction of magnitude 6.1% was applied to the area of the shifted peak to account for the alteration in solid angle due to the emission of the shifted γ rays from a moving coordinate system. The ratio of solid angles given by the relativistic transformation is

$$\Omega_u/\Omega_s=\left(1-\frac{v}{c}\cos\theta_c\right)\bigg/\left(1+\frac{v}{c}\right). \quad (7)$$

A further correction was applied to take into account the small variation in counter efficiency between shifted and unshifted γ-ray energies. Since a graded shield was used, this correction was -3.2% for the $4\rightarrow 2$ shifted area and ranged from $+1.1$ to $+2.2\%$ for other transitions.

C. Feeding

In this work the lower spin levels were populated significantly by feeding from higher levels in the ground band. Indeed, it was necessary to consider the cascade feeding through two or more higher levels to the level of interest. Feeding was incorporated into the analysis by solving the appropriate Batemann equations in an iterative manner discussed in the last subsection. The direct population probabilities were obtained by summing the shifted and unshifted components for each transition and applying the usual corrections for counter efficiency, internal conversion, and absorption. In the case of the $6\rightarrow 4$ transition, inclusion of cascade feeding produced the largest alteration (13%) in the extracted lifetime.

What about feeding of ground band levels from other states? The only non-ground band states observed, the $2^+, 4^+, 6^+$ members of the γ-vibrational band, were excited so weakly that the $6' \rightarrow 4$ transition was less than 3% as strong as the $6 \rightarrow 4$ transition. Winther-de Boer calculations indicated that the feeding from unobserved γ-band states, e.g., $8' \rightarrow 6$, was negligible in comparison with the corresponding "direct" feeding through the ground band.

D. Attenuation of angular distributions

It is well known that considerable loss of the nuclear spin alignment produced by a nuclear reaction can occur when the residual atom recoils into vacuum. Fluctuations in the large hyperfine magnetic fields at the nucleus due to the atomic transitions in the highly stripped atom are believed to produce a time-dependent perturbation of the nuclear alignment. This type of perturbation leads to a greater loss of alignment for the unshifted peak because the recoil nuclei are, on the average, in flight longer than the recoils corresponding to the shifted peak.

We emphasize here that a complete treatment of the vacuum deorientation must include the time-differential behavior of the angular distribution. Although no time-differential measurements for high-spin states have been reported, Ward, *et al.*[11] have measured time-integral attenuation coefficients G_k for the 2^+ and 4^+ states in ^{150}Sm and the 6^+ and 8^+ states in ^{156}Gd. On the basis of previous time-differential measurements for 2^+ states[12] they assumed an Abragam-Pound[13] type exponential decay

$$A_k(t) = A_k(0)e^{-t/\tau_k} \qquad (8)$$

for the perturbed angular distribution coefficients $A_k(t)$. With this assumption Ward *et al*. extracted the relaxation times, τ_k from their time-integral data. Within experimental errors they find equal τ_k values for the 6^+ and 8^+ states of ^{156}Gd, which does not backbend below spin 16.

Since ^{164}Dy does not backbend below spin 14, we have applied deorientation corrections of the form given by Eq. (8) to our data with equal relaxation times for all levels. The magnitudes of the corrections are displayed in Fig. 2 for the levels of interest as a function of τ_2 with $\tau_4 = \frac{3}{10}\tau_2$. Although the correction values are quite sensitive to τ_2, the maximum value is less than 7%. We chose $\tau_2 = 25$ ps, a value consistent with the experimental results for ^{156}Gd, and $\tau_4 = \frac{3}{10}\tau_2$ as predicted by the Abragam-Pound theory.

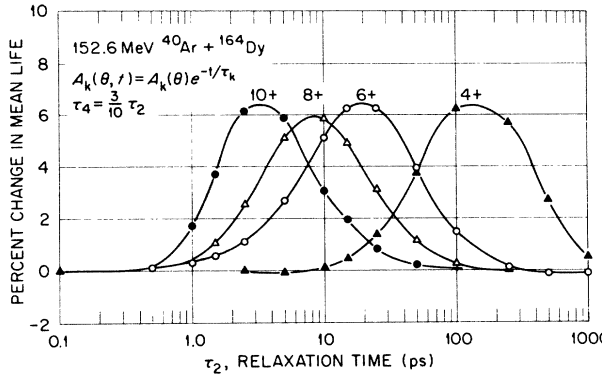

FIG. 2. Percent change in mean life as a function of relaxation time τ_2 for the 4^+, 6^+, 8^+, and 10^+ states in ^{164}Dy. Equal relaxation times were assumed for all states (4^+ through 12^+), and lifetimes were extracted in the manner described in Sec. III F of the text.

E. Line shape corrections

A potential problem in peak area analysis is the fact that nuclei slowing down in the stopper material emit γ rays that contribute to a "tail" that extends from the unshifted (U) peak to the shifted (S) peak. This tail must be included in the U area. Fortunately, this contribution was small enough for the lifetimes of interest to permit the following procedure. Following careful background subtraction, the "total" ($U+S$) areas were determined to relatively high precision with conventional techniques. An initial U area was extracted using a region of the spectrum that included the U peak but excluded the S peak and the tail of the U peak. This region was fixed for all separations D. A complete analysis as described in the following section was performed to obtain lifetimes that were used to predict the U line shapes with the computer program DOPCO.[14] The predicted line shapes were used to compute the fractional intensity in the original U region and hence the correction factors for the initial U areas. With corrected U areas a second complete analysis yielded new lifetimes based on the appropriate U line shapes.

With the above procedure the line shape corrections have the effect of changing only the zero intercept and thus the feeding and alignment corrections. Introduction of line shape corrections changed the extracted lifetimes by less than 1% except in the case of the $10 \rightarrow 8$ transition where a 3% change occurred.

F. Application of corrections and extraction of lifetimes

Our recoil-distance data for ^{164}Dy were analyzed with the aid of a computer code FILIP2,[15] which incorporated the corrections discussed in Secs. III A–III D. The data were also analyzed with the computer code ORACLE,[7] which takes into account multistep feeding from all levels above the level of interest. In FILIP2 only two-step feeding is considered; for example, the 8^+ and 6^+ feeding to the 4^+ state. However, the two analyses were in agreement for all lifetimes to better than 2%.

Starting lifetimes were obtained from linear least-squares fits for each transition in which velocity distribution, solid angle, and counter efficiency effects were included. Feeding and angular attenuation effects were built into an iterative procedure that yielded final lifetime values. Typically, only 3–5 iterations were required for convergence. The $R=0$ intercept was a free parameter for each transition, and the so-called "true zero" value was obtained by adjusting the nominal distance scale to give zero weighted average intercept for the 4^+, 6^+, 8^+, and 10^+ states. Our best fits with all corrections included are displayed in Fig. 3 as plots of the unshifted fractions versus target-stopper separation. Three large distance points for the $4\to 2$ transition were omitted from Fig. 3. A single point for the $12\to 10$ transition, when extrapolated backward to "true zero," gave a mean life near the rotational value. The large uncertainty in 12^+ lifetime introduced less than 3% uncertainty in the 10^+ lifetime because the feeding contributed only 14% to the population of the 10^+ state.

Table I presents typical percentage changes in lifetimes found by turning various corrections on and off in the computer analysis. Only two changes are larger than 5%, and these two effects

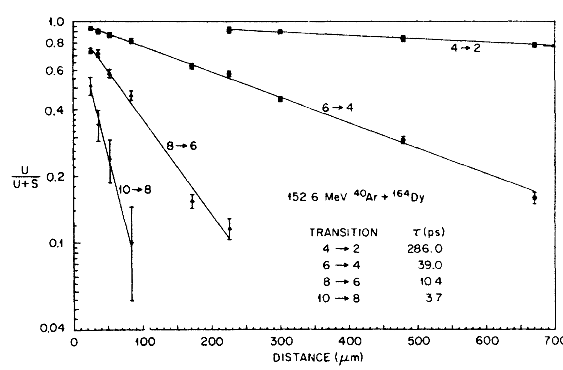

FIG. 3. Plot of the unshifted fraction $U/(U+S)$ as a function of target-stopper distance for the $4\to 2$, $6\to 4$, $8\to 6$, and $10\to 8$ transitions in ^{164}Dy. Excitation was produced by a 152.6 MeV ^{40}Ar beam, and the average recoil velocity corresponded to $v/c=0.032\,26$ for excitation of the 6^+ state. Three large distance points for the $4\to 2$ transition are not plotted. All corrections discussed in the text have been applied to the data. The lines represent least squares fits to the data.

TABLE I. Typical corrections to lifetime values for ^{164}Dy.

Effect \ Transition	$4\to 2$	$6\to 4$	$8\to 6$	$10\to 8$
Velocity smearing	−4.1%	−3.6%	−3.4%	−3.3%
Geometric solid angle	−4.8	−0.6	−0.2	−0.1
Relativistic solid angle	3.1	3.6	2.4	2.1
Ge efficiency	1.8	−0.6	−0.8	−0.7
Feeding	−4.7	−11.8	−4.9	−2.9
$W(\theta)$ attentuation ($\tau_2=25$ ps, $\tau_4=7.5$ ps)	1.4	6.3	3.1	0.8
Sum	−7.3	−6.7	−3.8	−4.1
All effects included in final analysis	−3.3	−4.0	−0.6	−2.8

partially cancel each other in the $6\to 4$ transition.

Our best values for the mean lives and corresponding $B(E2)$ ratios are given in Table II. Quoted uncertainties include contributions from uncertainties in peak areas and in corrections discussed previously. An uncertainty of ±1.3% was assigned to the $B(E2;0\to 2)$ value used to determine $B(E2)$ ratios. The transition energies and internal conversion coefficients used to compute $B(E2)$ values are also listed in Table II.

IV. DISCUSSION

Mean lives for the 4^+ through 12^+ members of the ground band in ^{164}Dy were measured by the recoil-distance technique, and $B(E2)$ values were determined to accuracies of about 6% for the $4\to 2$, $6\to 4$, and $8\to 6$ transitions. Comparisons of experimental and rigid-rotor $B(E2)$ values are presented in Table II. Although no large excursions from rotational behavior were observed, the $B(E2)$ ratios in column six are systematically less than unity by about 9%. This systematic reduction, comparison of present and previous results, and comparisons with rotational predictions will be discussed in separate subsections.

A. Systematic influences on extracted $B(E2)$ ratios

Effects that could influence all $B(E2)$ ratios by a common factor include the adopted $B(E2;0\to 2)$ value, average recoil velocity, relative distance scale, peak area analysis, and spin-dependent relaxation times.

It seems unlikely that the adopted $B(E2;0\to 2)$ value is suspect since it is a weighted average of four high precision (±2%) internally consistent measurements. Furthermore, a conservative ±4% uncertainty in the total internal conversion coefficient α_T corresponds to a ±1% uncertainty in $B(E2;2\to 4)$ and to much smaller values for higher-spin transitions.

TABLE II. Mean lives and $B(E2)$ values for ^{164}Dy.

$J \to I$	Mean life (ps)	(E_γ) (keV)	α_T	$B(E2; J\to I)$ ($e^2\, 10^{-48}$ cm^4) [a,b]	$\frac{B(E2; J\to I)}{B(E2)_{\rm rot}}$
$2\to 0$				1.116 ± 0.014 [c]	1.0
$4\to 2$	286.0 ± 15	168.84	0.426	1.459 ± 0.075	0.92 ± 0.05
$6\to 4$	39.0 ± 1.9	259.09	0.103	1.625 ± 0.080	0.93 ± 0.05
$8\to 6$	10.4 ± 0.7	342.35	0.044	1.594 ± 0.107	0.87 ± 0.06
$10\to 8$	3.7 ± 0.8	417.6	0.025	$1.72^{+0.49}_{-0.31}$	$0.91^{+0.26}_{-0.16}$
$12\to 10$	1.6 ± 0.8	484.0	0.016	$1.9^{+1.9}_{-0.6}$	$1.0^{+1.0}_{-0.3}$

[a] Relaxation times $\tau_2 = 25$ ps and $\tau_4 = \frac{3}{10}\tau_2$ were used to compute alignment attenuations. See Table I for the approximate magnitudes of the attenuation effect.

[b] Quoted uncertainties include a ±2% contribution from the alignment attenuation.

[c] Weighted average of four measurements. See Refs. 25–28.

The observed energy shift, $\Delta E_\gamma/E_0 = 0.03143 \pm 0.00026$, and the procedure described in Sec. III A yielded an average recoil velocity $v/c = 0.032\,20 \pm 0.000\,30$ for the 8^+ state. Another calculation with a simple formula[16] based on the beam energy, target thickness, and stopping powers gave $v/c = 0.0318 \pm 0.0006$. The consistency of these values indicates the absence of sizable error in velocity determination. Jones *et al.*[6] discussed the effect of the velocity distribution on the extracted lifetime. For a rectangular distribution of full width $2\Delta v \ll v$, we find that the lifetimes differ by a factor $1+\frac{1}{3}\Delta^2$ from the values expected for a sharp distribution. For our experimental value, $\Delta = 0.13$, the lifetimes would be altered by approximately 0.6%. As mentioned previously, possible nonparallelism of target and stopper was estimated to affect the extracted lifetimes by less than 1%. A distance error of several percent appears to be rather improbable.

Two or more independent area determinations were made for each peak of interest, and numerous cross checks showed statistical uncertainties to be dominant. Often the "nonstatistical" factors do not affect the lifetimes to first order. An example is the line shape correction to the unshifted peak area which changes the true zero and thus the feeding coefficients to produce rather small corrections to the extracted lifetimes. Indeed, large area changes would be required to affect lifetimes by several percent. Arbitrary decreases of 4% in all unshifted areas with no changes in shifted areas reduce the extracted lifetimes by 1–2%, and we estimate systematic reading errors to be no larger than 4% for the $8\to 6$ and lower-spin transitions.

Unfortunately, a knowledge of the correct relaxation times for deorientation must await the outcome of time-differential measurements. However, we note that the extracted $B(E2)$ values are largest when perfect alignment is assumed. In that case the $B(E2)$ ratios in Table II for the 4^+ through 10^+ states would increase by 1%, 6%, 3%, and 1%, respectively. This would be highly artificial because time-integral measurements[11] show that some dealignment occurs. Only the extracted 6^+ lifetime could be increased by more than 3%, and this would require a relaxation time $\tau_2 > 60$ ps.

It is plausible that various combinations of the effects discussed above could account for perhaps 3–5% of the reduction below unity of the $B(E2)$ ratios in Table II, leaving a reduction of about 1.5 standard deviations.

B. Comparison of RD, MCEX, and DBLS results

Three techniques have been utilized to measure $B(E2)$ values in ^{164}Dy, and the recent MCEX[3,4] and DBLS[17] results are compared with the present RD results in Fig. 4. "Quantal" corrections used

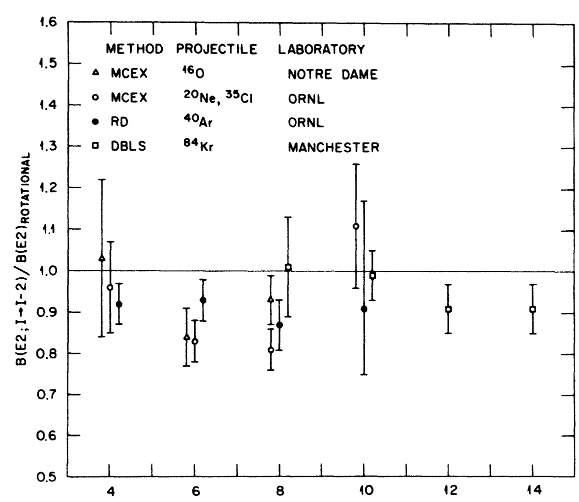

FIG. 4. Plot of ratio of experimental to rotational $B(E2)$ values as a function of spin of upper level for transitions in ^{164}Dy. The data are taken from Refs. 3, 4, 17, and the present work.

in the MCEX analysis[3] were improved by computing the theoretical Coulomb excitation probabilities according to the classical-limit S-matrix method (CLSM)[18] which treats properly the dynamical orbit distortions. The CLSM analysis changed the extracted $B(E2)$ values of Ref. 3 by −3%, +2%, +5%, and +5% for the 4^+–10^+ states, respectively. In addition, use of a more accurate $B(E2;0\rightarrow 2)$ value than those used previously[3,4] altered the extracted $B(E2)$ values of Ref. 3 by −3% and the values of Ref. 4 by −2%.

Within experimental uncertainties there is reasonable agreement between the three types of measurements when each $B(E2)$ ratio is considered separately. In no case do the error bars for RD and MCEX fail to touch, and there is good overlap in the $4\rightarrow 2$ and $8\rightarrow 6$ cases. However, all RD and DBLS values taken together indicate rather less variation in $B(E2)$ with spin than might be inferred from the MCEX values alone. The magnitudes of the uncertainties do not permit a conclusive quantitative statement of the limitation of MCEX measurements for ^{164}Dy. For example, the rms difference between RD and MCEX $B(E2)$ values for the $4\rightarrow 2$, $6\rightarrow 4$, and $8\rightarrow 6$ transitions is $8\pm 6\%$. Nonetheless, it seems to us that carefully designed and analyzed MCEX experiments will continue to yield useful quantitative information on transition rates. Such information, however, may be displayed with a brush stroke, perhaps 10–15 % wide, broad enough to cover possible ambiguities arising from the multiplicity of matrix elements involved in the population of a single state. In this connection, further theoretical work toward full quantal MCEX computations[19-21] and experimental probes for Coulomb-nuclear interference for high-spin states could have the desirable effect of reducing the brush size.

C. Comparison of $B(E2)$ values with rotational predictions

The experimental $B(E2)$ ratios displayed in Fig. 4 are not dramatically different from the rigid-rotor predictions. Indeed, these data are consistent with the interpretation that the transition rates, relative to rotational, for states of spin 4–14 are independent of spin, particularly if the subset of RD and DBLS values is considered.

A quantitative, albeit narrow, assessment of rotational purity was attained with a procedure analogous to the formalism of Symons and Douglas.[22] We used the expansion

$$\langle I \,\|\, M(E2) \,\|\, J\rangle \sim 1+\tfrac{1}{2}\alpha\{I(I+1)+J(J+1)\}\ , \qquad (9)$$

where $\langle I \,\|\, M(E2)\,\|\, J\rangle$ is the reduced matrix element for the transition $I\rightarrow J$ and the generalized parameter α includes all nonrotational effects with the $I(I+1)$ dependence. In the (incorrect) view that all of the level depression is caused by centrifugal stretching, the level energy $E(I)$ is

$$E(I)\sim I(I+1)\{1-\alpha I(I+1)\}\ . \qquad (10)$$

An upper limit for α of 4×10^{-4} was obtained from a weighted least squares fit of Eq. (9) to the $B(E2)$ measurements shown in Fig. 4. A larger α value, 7×10^{-4}, was obtained from the energy levels with Eq. (10). Of course it is well known that small changes in the pairing interactions and fourth-order cranking model corrections may alter significantly the moment of inertia but not the quadrupole deformation. In this regard, recent measurements by Ward *et al.*[23] and Inamura *et al.*[24] indicate essentially rotational lifetimes up to spin 14 in ^{174,176}Yb.

Below spin 16 the $B(E2)$ values for the nucleus ^{164}Dy deviate by less than 10% from rigid-rotor predictions. It appears that reductions in experimental uncertainties by about a factor of 2 will be required to expose possible deviations in transition rates below the backbend region. Time-consuming time-differential measurements of alignment attenuation for high-spin states may be mandatory for achievement of the desired accuracy from recoil-distance measurements.

Note added in proof. Recently S. H. Sie and D. W. Gebbie [Nucl. Phys. A289, 217 (1977)] reported recoil-distance measurements for 4^+-8^+ states in the ground bands of ^{164}Dy, ^{170}Er, and ^{174}Yb. Within experimental uncertainties their mean lives in ps of 294 ± 9, 37.7 ± 1.4, and 9.8 ± 0.7 for the 4^+, 6^+, and 8^+ levels of ^{164}Dy are in good agreement with our RD lifetimes.

ACKNOWLEDGMENTS

We are grateful to Dr. M. L. Mallory and Dr. E. D. Hudson, and to the ORIC operating staff for providing the ^{40}Ar beam and to Dr. G. D. O'Kelley and Dr. D. C. Hensley for assistance in acquisition of the data. We also acknowledge, with pleasure, discussions with Dr. L. L. Riedinger. Financial support from Research Corporation (ROS), the Max Kade Foundation (RJS), and Oak Ridge Associated Universities (MWG) is gratefully acknowledged.

*Prime contractor for the Department of Energy.

†Deceased. The other authors wish to express their gratitude for the opportunity of having had Gene Eichler as a colleague.

‡Research Associate from Vanderbilt University supported in part by a grant from the National Science Foundation. Present address: Department of Physics, McMaster University, Hamilton, Ontario.

§Max Kade Foundation Fellow from the University of Marburg, Marburg, Germany.

¶Oak Ridge Graduate Fellow from the University of Tennessee under appointment with Oak Ridge Associated Universities. Present address: Department of Physics, University of Tennessee, Knoxville, Tennessee 37916.

[1]A. Johnson, H. Ryde, and J. Sztarkier, Phys. Lett. 34B, 605 (1971).

[2]R. O. Sayer, J. S. Smith, III, and W. T. Milner, At. Data Nucl. Data Tables 15, 85 (1975).

[3]R. O. Sayer, E. Eichler, N. R. Johnson, D. C. Hensley, and L. L. Riedinger, Phys. Rev. C 9, 1103 (1974).

[4]R. N. Oehlberg, L. L. Riedinger, A. E. Rainis, A. G. Schmidt, E. G. Funk, and J. W. Mihelich, Nucl. Phys. A219, 543 (1974).

[5]N. R. Johnson, R. J. Sturm, E. Eichler, M. W. Guidry, G. D. O'Kelley, R. O. Sayer, D. C. Hensley, N. C. Singhal, and J. H. Hamilton, Phys. Rev. C 12, 1927 (1975).

[6]K. W. Jones, A. Z. Schwarzschild, E. K. Warburton, and D. B. Fossan, Phys. Rev. 178, 1773 (1969).

[7]R. J. Sturm and M. W. Guidry, Nucl. Instrum. Methods 138, 345 (1976).

[8]J. L. Quebert, K. Nakai, R. M. Diamond, and F. S. Stephens, Nucl. Phys. A150, 68 (1970).

[9]A. Winther and J. de Boer, *Coulomb Excitation*, edited by K. Alder and A. Winther (Academic, New York, 1966), p. 303.

[10]K. S. Krane, Nucl. Instrum. Methods 98, 205 (1972).

[11]D. Ward, H. R. Andrews, R. L. Graham, J. S. Geiger, and N. Rud, Nucl. Phys. A234, 94 (1974).

[12]D. Ward, R. L. Graham, J. S. Geiger, H. R. Andrews, and S. H. Sie, Nucl. Phys. A193, 479 (1972); H. Spehl, S. G. Steadman, A. Weckherlin, H. A. Doubt, K. Hagemeyer, G. J. Kumbartzki, and K. H. Speidel, Nucl. Phys. A215, 446 (1973).

[13]A. Abragam and R. V. Pound, Phys. Rev. 92, 943 (1953).

[14]W. T. Milner (private communication).

[15]R. O. Sayer (unpublished).

[16]M. W. Guidry, P. A. Butler, P. Colombani, I. Y. Lee, D. Ward, R. M. Diamond, F. S. Stephens, E. Eichler, N. R. Johnson, and R. Sturm, Nucl. Phys. A266, 228 (1976).

[17]T. Inamura, F. Kearns, and J. C. Lisle, Nucl. Instrum. Methods 123, 529 (1975); F. Kearns, G. Varley, G. D. Dracoulis, T. Inamura, J. C. Lisle, and J. C. Willmott, Nucl. Phys. A278, 109 (1977).

[18]H. Massmann and J. O. Rasmussen, Nucl. Phys. A243, 155 (1975); R. Donangelo, M. W. Guidry, J. P. Boisson, and J. O. Rasmussen, Phys. Lett. 64B, 377 (1976); M. W. Guidry, R. Donangelo, and J. O. Rasmussen, Nucl. Phys. A (to be published).

[19]M. Ichimura, M. Igarashi, S. Landowne, C. H. Dasso, B. S. Nilsson, R. A. Broglia, and A. Winther, Phys. Lett. 67B, 129 (1977).

[20]K. Alder, F. Roesel, and R. Morf, Nucl. Phys. A284, 145 (1977).

[21]M. W. Guidry (unpublished).

[22]G. D. Symons and A. C. Douglas, Phys. Lett. 24B, 11 (1967).

[23]D. Ward, P. Colombani, I. Y. Lee, P. A. Butler, R. S. Simon, R. M. Diamond, and F. S. Stephens, Nucl. Phys. A266, 194 (1976).

[24]T. Inamura, F. Kearns, J. C. Lisle, G. Varley, and J. C. Willmott, Annual Report 1974, Schuster Laboratory, University of Manchester (unpublished), p. 75.

[25]J. D. Kurfess and R. P. Scharenberg, Phys. Rev. 161, 1185 (1967).

[26]K. A. Erb, J. E. Holden, I. Y. Lee, J. X. Saladin, and T. K. Saylor, Phys. Rev. Lett. 29, 1010 (1972).

[27]H. J. Wollersheim, W. Wilcke, T. W. Elze, and D. Pelte, Phys. Lett. 48B, 323 (1974).

[28]A. H. Shaw and J. S. Greenberg, Phys. Rev. C 10, 263 (1974).