Systematics of two-neutron-transfer cross sections near closed shells: A sum-rule analysis of (p,t) strengths on the lead isotopes*

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We report the study of the (p,t) reaction on ^{208}Pb, ^{206}Pb, and ^{204}Pb at $E = 35$ MeV and on the application of the two-neutron-transfer sum rule of Bayman and Clement to these data. Analysis of the sum rule shows that the observed dramatic increase in $J = 0$ (p,t) cross sections as one goes from targets of ^{208}Pb to ^{206}Pb to ^{204}Pb and the nearly-as-dramatic decrease in $J \neq 0$ cross sections reflect general features of the shell-model character of these nuclei. Application to the present series of nuclei shows that this two-neutron-transfer sum rule can be applied with nearly the same accuracy as the corresponding single-nucleon-transfer sum rules. The relationship between the shell-model and pairing-vibration (p,t) strengths is discussed.

[NUCLEAR REACTIONS ^{208}Pb(p,t), ^{207}Pb(p,t), ^{206}Pb(p,t) and ^{204}Pb(p,t), E_p = 35 MeV; measured $\sigma(E_t, \theta)$; deduced energies, spins and parities and strengths; sum rule analysis; relationship between shell-model and pairing-vibration model.]

I. INTRODUCTION

It has long been commonly believed that because the two-neutron-transfer reaction cross section to a particular final state reflected correlations between nucleons in that state, the sum-rule methods which contributed so greatly to our understanding of single-nucleon-transfer reactions could not be usefully applied to (p,t) and (t,p) reactions. This is not the case. Bayman and Clement[1] have derived a sum rule for (p,t) and (t,p) reactions on even-even targets which is analogous to the single-particle-transfer sum rule in that it relates, for a particular target nucleus, the total (p,t) and (t,p) strengths for a given angular momentum transferred to the orbit occupancies of the target nucleus. While these authors, and others,[2] have emphasized that this is only an approximate sum rule because it involves the assumption that all the angular distributions are (to within a multiplying factor) the same for a given angular momentum transferred, this assumption is well known to be accurately fulfilled in almost all measured cases. Perhaps it should be emphasized that the same assumption is made implicitly in the single-nucleon-transfer case.

To our knowledge this paper reports on the first application of *this* sum rule to the (p,t) and (t,p) reactions connecting a series of nuclei. As such, we present a test of the accuracy of the Bayman-Clement relationship. The comparison between experimental and theoretical sum rule strengths indicates that in the best cases this sum rule can be applied with nearly the same degree of accuracy as the corresponding single-nucleon-transfer one.

Perhaps the most interesting result of the present study is the clarification of the fundamental model assumptions which must be made to explain the systematic trends in (p,t) or (t,p) strengths as one studies these reactions on, and near, closed-shell nuclei. It is widely known that the $J=0$ (p,t) strength dramatically increases as one changes from a target which corresponds to a closed shell to targets with two or four less neutrons. This dramatic increase for $J=0$ is often cited as evidence for Boson models, such as the pairing-vibration model.[3] However, the sum rule shows that this increase is a general shell-model result; any model which recognizes that a given nucleus can be considered a closed core with adjacent nuclei considered as corresponding to a few valence particles or holes orbiting the closed core must predict this feature in (p,t) strengths. In addition, it should be noted that the almost as striking decrease in $J \neq 0$ (p,t) cross sections as one goes away from a closed shell should also be a general shell feature. From this point of view, the systematic trends in (p,t) cross sections observed near closed shells can be considered as yet another demonstration of the validity of the shell model. That the shell model predicts these systematics is, of course, not surprising. What may surprise some is that the sum rule provides a simple method of understanding the (p,t) systematics in the shell model. There is no need to go to a more qualitative model to find a simple explanation.

The (p,t) reaction of the lead isotopes has been investigated previously by Reynolds, Maxwell, and Hintz[4] (with rather poor energy resolution ~ 220

keV) but, where comparisons can be made, their results are in substantial agreement with ours.

II. EXPERIMENTAL PROCEDURE

The angular distributions for triton groups from the (p,t) reactions on the even isotopes of lead were measured using the 35-MeV proton beam from the Michigan State University (MSU) cyclotron. The outgoing tritons were detected using either a position-sensitive proportional counter[5] or nuclear emulsion plates in the focal plane of the MSU split-pole magnetic spectrometer. The experimental procedures used in the present experiment are the same as those used in the study of the (p,d) reactions on these targets; hence, only a brief description is given here. More details are given in Refs. 6 and 7.

Sample spectra recorded using the proportional counter and nuclear emulsions are shown in Figs. 1 and 2. Each of these detection systems has characteristic relative advantages; both were used and the final results were obtained by combining the results. The proportional counter system has the advantage of being an on-line device which allows monitoring of the experiment as it proceeds. In addition, it provides adequate resolution (30 keV, full width at half maximum) and allows one to obtain good statistical accuracy. For these reasons, the bulk of the angular distribution data was taken with the proportional counters. However, for each reaction studied, spectra were recorded at laboratory angles of 6°, 18°, and 30° using nuclear plates. These plate data were taken to allow us to make full use of the accurately known energy calibration of the spectrograph. For the present experiment, excitation energies were determined to an uncertainty of 1 keV per MeV of excitation. In addition, these plate data have better resolution, typically 15 keV. See Fig. 2.

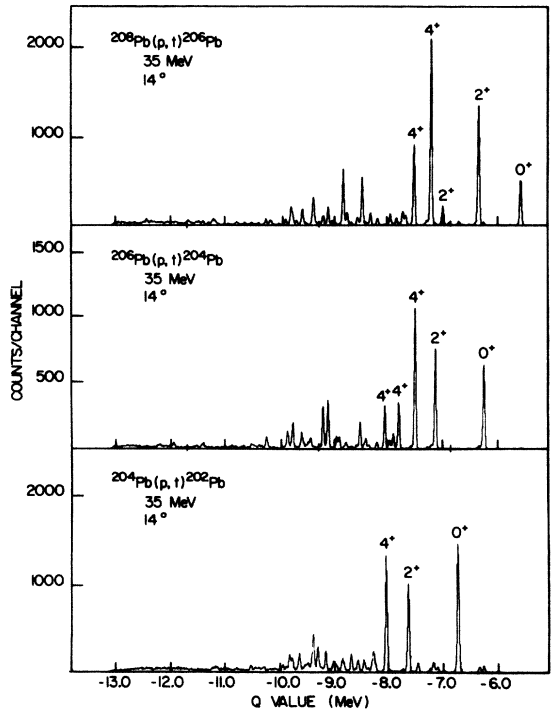

FIG. 1. Spectra of the ^{208}Pb$(p,t)^{206}$Pb, ^{206}Pb$(p,t)^{204}$Pb, and ^{204}Pb$(p,t)^{202}$Pb reactions recorded using a current division position sensitive proportional counter in the focal plane of a split-pole spectrograph. Some of the peaks are labeled with the J^{π} of the final states. These spectra are plotted with a common Q-value scale.

Because, as will be noted below, we are primarily interested in the ratio of cross sections of the (p,t) reaction on the lighter isotopes of lead to the cross section on ^{208}Pb, special care was taken to measure these ratios more accurately than would have been the case for the absolute values. Absolute cross sections were determined by measuring both the (p,t) and elastic scattering cross sections using the identical detection and beam monitoring systems. The elastic scattering cross sections were assumed to be reproducible by an optical model using the proton parameters of Becchetti and Greenlees.[8] The absolute (p,t) cross sections were determined from the measured ratio of elastic to transfer cross sections. This procedure is believed to result in absolute (p,t) cross sections accurate to ± 20%. In addition, the *relative* (p,t) cross sections were determined by measuring the (p,t) yields from all the stable isotopes of lead under *identical* experimental conditions. This was done by sequentially recording the triton yields from targets of ^{208}Pb, ^{207}Pb, ^{206}Pb, and ^{204}Pb without changing any of the experimental conditions except the target. In addition to these targets, a spectrum of a natural lead target of known isotopic abundances was also recorded. By comparing the yields from these five targets, the relative cross sections were determined. These relative cross sections are believed to be accurate to better than ± 10%. The four spectra shown in Fig. 2 comprise one of the sets of data recorded as described above.

This procedure is not only useful for determining relative cross sections but also provides a very sensitive means of determining which peaks in a spectrum are due to "impurities." By "impurity" we mean either some element other than lead present in the target or one of the lead isotopes other than that for which the isotopically enriched target was made. All the targets were highly enriched (^{208}Pb:99%, ^{207}Pb:99%, ^{206}Pb:97%, ^{204}Pb:90%) but there still can be present, especially in the ^{204}Pb

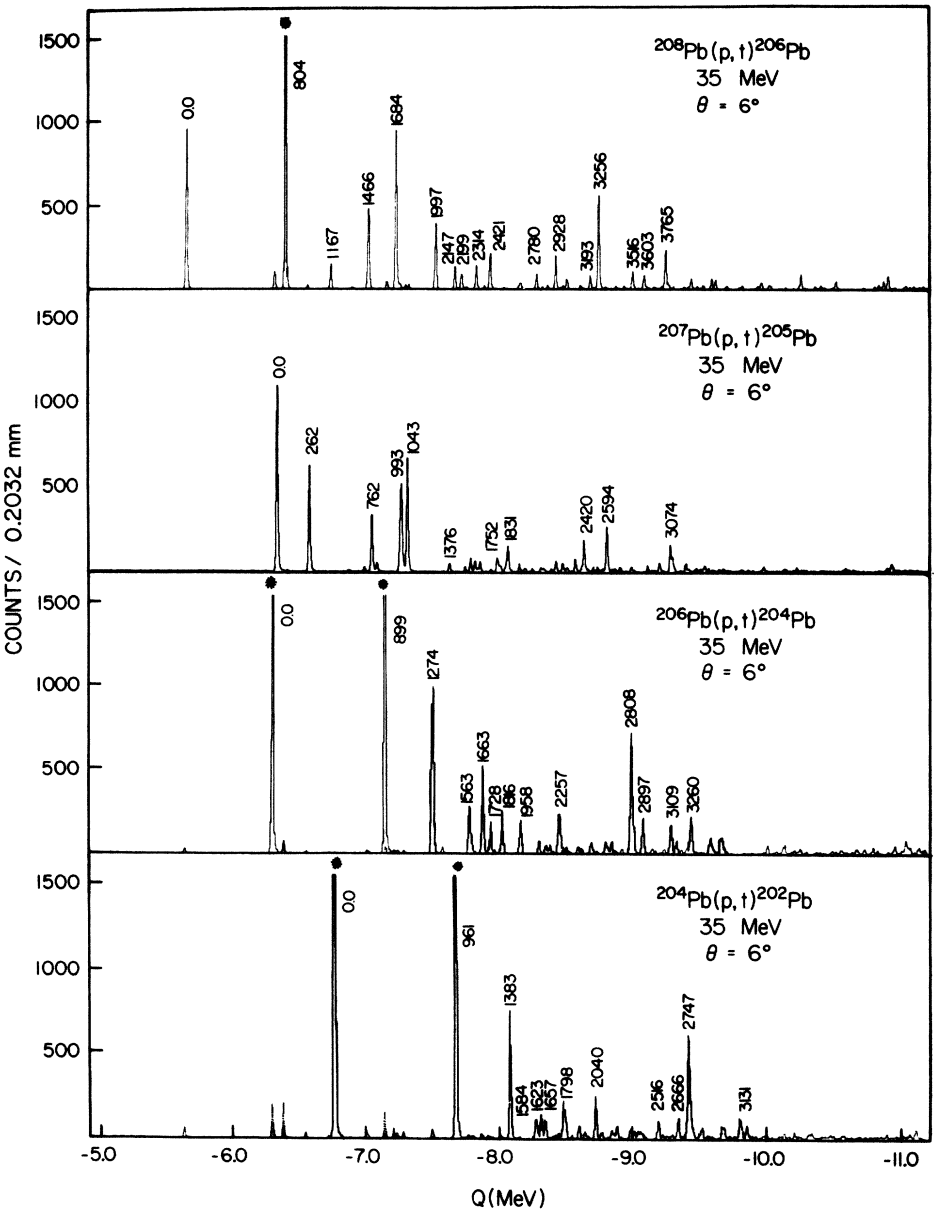

FIG. 2. Spectra of the $^{208}Pb(p,t)^{206}Pb$, $^{207}Pb(p,t)^{205}Pb$, $^{206}Pb(p,t)^{204}Pb$, and $^{204}Pb(p,t)^{202}Pb$ reactions recorded using nuclear emulsions in the focal plane of a split-pole spectrograph. Some of the peaks are labeled with the excitation energy (keV) of the final states. Some of the strong peaks go off scale at the top; this is indicated by an asterisk. These spectra are plotted with a common Q-value scale.

target, up to a few percent of other stable isotopes. By recording spectra from targets of all the stable isotopes of lead, however, both elemental and isotopic impurity peaks are clearly identified. Since each target was made using the same method and materials (all were evaporated onto similar carbon foil backings), any elemental impurity should be present in all (or at least more than one) of the spectra and, hence, can be identified by simple comparison of the different spectra. Likewise, an impurity peak resulting from a few percent of another lead isotope is easily identified because (as in Fig. 2) the precise position and intensity of each such peak is known from comparison of the spectra of each of the four stable isotopes.

III. EXCITATION ENERGY, SPIN-PARITY ASSIGNMENTS, AND COMPARISON WITH PREVIOUSLY KNOWN LEVELS

It is well known that for (p,t) reactions the angular distributions are sensitive to the angular momentum (J) transferred during the reaction. In the present case, since we are studying (p,t) reactions on the even lead isotopes having spin zero, the angular momentum transferred is necessarily equal to the spin of the final state. In addition, since only natural parity transitions are observed with appreciable cross section, the measured angular distributions permit assignment of both spin (J) and parity of $\pi=(-1)^J$ to the states populated.

Shown in Fig. 3 are representative angular distributions for $J=0,2,4,5,6,7,9$. The solid curves shown are the results of distorted-wave Born-approximation (DWBA) calculation which will be discussed below. It is clear from this figure that the angular momentum transferred can be determined uniquely by comparing the DWBA predication with the measured angular distribution. Furthermore, since all transitions with the same angular momentum transfer show essentially the same angular distribution, spin and parity assignments can be made independently of any calculation simply by comparing measured angular distributions corresponding to states of unknown spin and parity with those of states with previously measured spin and parity.

Given in Tables I–III are the excitation energies, spin and parity assignments, and relative cross sections for the states excited in the (p,t) reaction at 30 MeV. Also given is information[6,9-11] from the literature about states which have been previously studied. The literature results are given when there is some possibility that a state quoted in the literature is the same as that excited in the present study. Also given in Table IV is a list of excitation energies of states observed in the ^{207}Pb$(p,t)^{205}$Pb reaction together with the literature[7,12,13] results for ^{205}Pb. Since no angular distributions were analyzed for this reaction, no spin assignments can be made; the table is included insomuch as it will be of interest to others studying the level structure of ^{205}Pb.

Because we are primarily interested in relative (p,t) cross sections for states with the same J^π, we quote the cross sections in Tables I–III in terms of the relative cross section to the state of lowest excitation in ^{206}Pb excited by the ^{208}Pb$(p,t)^{206}$Pb reaction with the same J^π transfer. Namely, the cross sections to all 0^+ states are quoted relative to the ^{208}Pb$(p,t)^{206}$Pb cross section to the ground state of ^{206}Pb, and all 2^+ states relative to the ^{208}Pb$(p,t)^{206}$Pb cross section to the 2^+

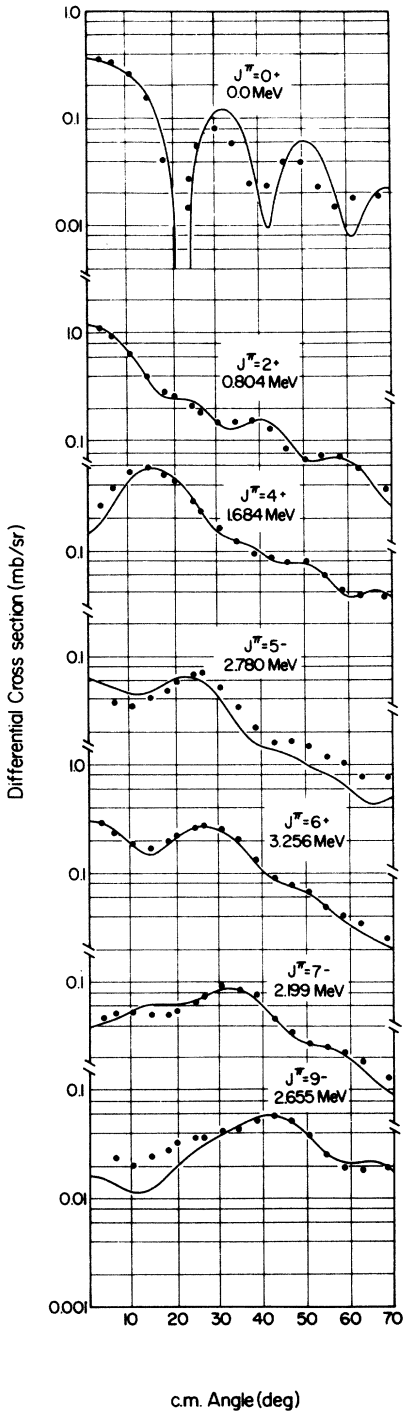

FIG. 3. Angular distributions for the ^{208}Pb$(p,t)^{206}$Pb reaction to the lowest states with $J^\pi=0^+,2^+,4^+,5^-,6^+,7^-,9^-$ in ^{206}Pb. The solid lines are DWBA calculations using the optical-model parameters for protons from Becchetti and Greenlees and for tritons from Flynn *et al.*

TABLE I. $^{208}Pb(p,t)^{206}Pb$ reaction data: excitation energies, spin and parity assignments, relative cross sections, and comparison with literature.

Present results			Literature[a]	
Excitation energy	J^π	Relative cross section[b]	Excitation energy	J^π
0.0	0^+	1.000*	0.0	0^+
0.804	2^+	1.000*	0.803	2^+
1.167	0^+	0.099	1.165	0^+
1.339			1.341	3^+
1.466	2^+	0.151	1.459	2^+
1.684	4^+	1.000*	1.684	4^+
1.783	2^+	0.014	1.784	2^+
1.997	4^+	0.42	1.998	4^+
2.147	2^+	0.081	2.149	(2^+)
2.199	7^-	1.000*	2.200	7^-
2.314	0^+	0.174	2.314	(0^+)
2.379			2.384	6^-
2.421	2^+	0.098	2.428	2^+
2.644			2.647	3^-
2.655	9^-	1.000*	2.659	(9^-)
2.780	5^-	1.000*	2.782	5^-
2.827			2.826	$(4)^-$
2.865	(7^-)	0.012	2.864	7^-
2.928	4^+	0.25	2.930	4^+
2.979				
3.014	5^-	0.74	3.016	5^-
3.119			3.117	$(2,3,4)^-$
3.193			3.191	
3.256	6^+	1.000*		
3.390	(7^-)	(0.15)	3.383	(7^-)
3.452				
3.516				
3.603	2^+	0.080		
3.765	(7^+)	(0.181)		
3.958	4^+	0.10		
4.113	4^+	0.14		
4.140				
4.225	(4^+)	(0.04)		
4.484				
5.317				
5.348				
5.383				

[a]References 6 and 9.

[b]States labeled with an asterisk are those used as units of strength.

state at 0.803 MeV, etc. Note that since the angular distributions used to define the unit cross section for a given J^π transfer are given explicitly in Fig. 3, and since, as mentioned above, to within a multiplying constant all cross sections to states with a given J^π are the same, the angular distribution for any state can be reconstructed by multiplying the relative cross section given in Table I, II, or III by the angular distribution given in Fig. 3.

There was only one case where, even with the higher resolution of the plate data (see Fig. 2), a relatively strong transition was observed which did not seem to have an angular distribution in agreement with any single J value, and that corresponds to an excitation of 2.257 MeV in ^{204}Pb. Close inspection of the plate data showed that the peak corresponding to the transition to this state was consistently wider than were adjacent peaks, indicating that it was at least a doublet. Inspection of the measured angular distribution for this state (see Fig. 4) indicates a relatively high J transfer. Assuming this angular distribution results for two J values, one finds an excellent fit, assuming a mixture of $J=5$ and 7. See Fig. 4. It is perhaps surprising that this mixture results in an excellent fit but that a $J=6$ does not. But, as can be seen in Fig. 3, a $J=6$ angular distribution has a rather strong peak at zero degrees which is not present in the data. The mixture of $1.0\sigma(J=5)+0.7\sigma(J=7)$ fits the forward angles, as well as the rest of the distribution, very well. It should also be noted that because of the inherently very large cross sections for low J values, there cannot be a significant amount of low J strength hidden in this doublet.

IV. TWO-NEUTRON-TRANSFER SUM RULE

The necessary sum rule is the one derived by Bayman and Clement for (p,t) and (t,p) reactions on even-even nuclei.[1] This is an extension of the special case of (p,t) or (t,p) on a closed-shell nucleus which was derived earlier by Smith *et al.* and applied to the $^{208}Pb(p,t)^{206}Pb$ reaction.[2] The one assumption needed to derive the sum rule is that the shape of the angular distribution for a given angular momentum transfer (J) is independent of the microscopic wave functions of the nuclear states involved, but depends only on the J transferred in the reaction. Both experimental and calculational results indicate this to be a good approximation.

From the assumption of a common angular distribution for transitions of the same angular momentum transferred, it follows that a strength $g^\pm(J)$ for a transition can be defined as

$$g^\pm(J)=\frac{d\sigma^\pm}{d\Omega}(\Theta)_{\text{expt}}/\sigma^J_{\text{DW}}(\Theta)\,, \tag{4.1}$$

where $d\sigma/d\Omega$ is the experimental differential cross section to the state involved, the "+" and "−" signs refer to (t,p) and (p,t) reactions, respectively, and $\sigma^J_{\text{DW}}(\Theta)$ is the assumed common angular dependence of all transitions with angular-momentum transfer J. In principle, $\sigma^J_{\text{DW}}(\Theta)$ should be calculable from reaction theory, e.g., DWBA. For our purposes it is sufficient to consider *relative strengths* and, consequently, it is not necessary to calculate absolute two-neutron-transfer cross

TABLE II. ^{206}Pb$(p,t)^{204}$Pb reaction data: excitation energies, spin and parity assignments, relative cross sections, and comparison with literature.

Present results			Literature[a]	
Excitation energy	J^π	Relative cross section	Excitation energy	J^π
0.0	0^+	1.74	0.0	0^+
0.899	2^+	0.63	0.899	2^+
1.274	4^+	0.65	1.274	4^+
1.351	2^+	0.015	1.353	
1.563	4^+	0.22	1.563	(4)
1.582		(0.10)	1.584	(0^+)
1.663	2^+	0.12	1.663	(4^+)
1.728	0^+	0.17		
1.816	4^+	0.05	1.817	(4^+)
1.958	2^+	0.05	1.932	(2^+)
2.103	2^+	0.015		
2.156				
2.186	9^-	0.88	2.186	9^-
2.257	$J=5+J=7$ doublet	1.0 ($J=5$) 0.7 ($J=7$)	2.258	(5^-)
2.399	(7^-)	0.11		
2.430			2.434	($4,5,6^-$)
2.505	5^-	0.48	2.507	
2.620				
2.660			2.642	(2^+)
2.808	6^+	0.77		
2.829			2.831	
2.897	4^+	0.20		
3.147				
3.226				
3.949				

[a]Reference 10.

sections. Below, where this sum rule is applied to reactions on ^{208}Pb, ^{206}Pb, and ^{204}Pb, the sum of the transition strengths on the closed-core nucleus ^{208}Pb will be used to obtain a normalization for the strengths which can then be used to check the *absolute* predictions of the sum rule for reactions on ^{206}Pb and ^{204}Pb.

The definition given above does not take into account explicitly any Q-value dependence of the (p,t) cross section. In the present case, because we are studying the reaction at a relatively high energy and because the average Q value for a given J transfer does not change very much as one goes from ^{208}Pb to ^{206}Pb to ^{204}Pb targets (see Fig. 1), the effects of Q dependence on (p,t) cross sections are small. In cases where the Q values for a given J transfer vary greatly or where the triton energy is near the Coulomb barrier, the Q dependence of $g^\pm(J)$ can be removed approximately by estimating the dependence of cross section on Q value using DWBA and including this Q dependence in $\sigma^J_{\rm DW}(\Theta)$. The effects of Q values on the sum rule are discussed further in an appendix to Ref. 2. In the present case no corrections for the Q dependence were made. All DWBA cross sections were calculated assuming a Q value corresponding to the mean Q value for that particular J transfer.

If the *total strength* for (p,t) or (t,p) on a given target is defined

$$G^\pm(J)=\sum_{\text{all transitions}} g^\pm(J)\,, \tag{4.2}$$

where the sum is over transitions to all states reached with angular momentum J, the sum rule can be written[1]:

$$G^+(J)-G^-(J)=\sum_{j_1\le j_2} D^2_{j_1j_2}(J)\left[1-\frac{\langle n(j_1)\rangle}{2j_1+1}-\frac{\langle n(j_2)\rangle}{2j_2+1}\right], \tag{4.3}$$

where $\langle n(j)\rangle$ is the number of neutrons in the ground state of the target in orbit j, and $D^2_{j_1j_2}(J)$ is a factor which is proportional to the relative two-neutron-transfer cross section for a pure *configuration* of two neutrons in orbit j_1 and j_2, i.e., $D^2_{j_1j_2}(J)$ is essentially the DWBA cross section for a pure $(j_1\otimes j_2)_J$ transfer but with the angular dependence factored out. See Ref. 1. To obtain the $D^2_{j_1j_2}(J)$ the DWBA code DWUCK was used.[14]

TABLE III. $^{204}Pb(p,t)^{202}Pb$ reaction data: excitation energies, spin and parity assignments, relative cross sections, and comparison with literature.

Present results Excitation energy	J^π	Relative cross section	Literature[a] Excitation energy	J^π
0.0	0^+	2.31	0.0	0^+
0.961	2^+	0.56	0.961	2^+
1.383	4^+	0.48	1.383	4^+
1.584	(2^+)	0.026		
1.623	4^+	0.090	1.623	4^+
1.657	2^+	0.020		
1.798				
1.815			1.817	$(3,4)^+$
1.915	4^+	0.067	1.915	$4^+(3^-)$
1.963				
2.040	5^-	1.28	2.040	5^-
2.172	9^-	0.93	2.170	9^-
(2.185)			2.185	9^-
2.207				
(2.307)				
2.364				
2.389				
2.516	4^+	0.086		
2.666	4^+	0.115		
2.747	6^+	0.55		
2.995				
3.131				
3.180	4^+	0.070		

[a]Reference 11.

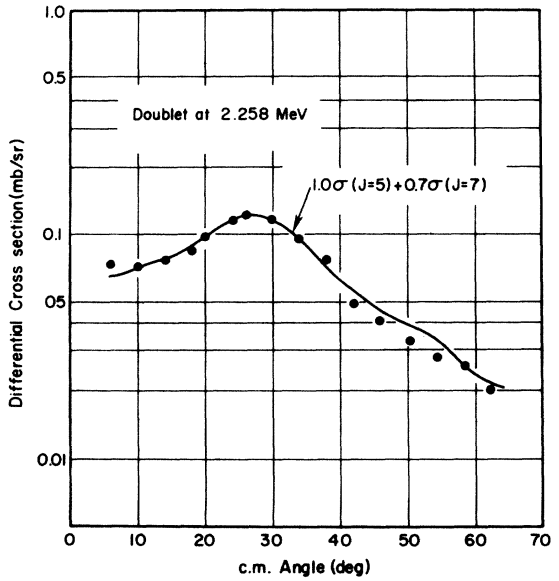

FIG. 4. Measured angular distribution to the doublet at 2.258 MeV observed in the $^{206}Pb(p,t)^{204}Pb$ reaction. The solid line is 1.0 times the differential cross section given in Fig. 3 for a $J=5$ transfer plus 0.7 times the differential cross section for $J=7$ also given in Fig. 3. See text.

TABLE IV. States excited in the $^{207}Pb(p,t)^{205}Pb$ reaction.

Present results excitation energy	Literature[a] Excitation energy	J^π
0.0	0.0	$\frac{5}{2}^-$
	0.002	$\frac{1}{2}^-$
0.262	0.263	$\frac{3}{2}^-$
0.580	0.576	$\frac{3}{2}^-$
0.703	0.703	$\frac{7}{2}^-$
0.762	0.761	$\frac{5}{2}^-$
0.803	0.803	$(\frac{1}{2},\frac{3}{2})$
0.993 (doublet)	0.987	$\frac{9}{2}^-$
	0.997	$(\frac{1}{2},\frac{3}{2})^-$
	1.014	$\frac{13}{2}^+$
1.043	1.044	$\frac{7}{2}^-$
1.376	1.375	$(\frac{1}{2},\frac{3}{2})^-$
1.498	1.499	$\frac{9}{2}^-$
1.541		
1.575	1.575	$(\frac{7}{2},\frac{9}{2})^-$
(1.595)	1.593	$\frac{9}{2}^+$
1.616	1.614	$\frac{7}{2}^-$
1.700	1.705	$(\frac{11}{2},\frac{9}{2})^-$
1.752 (doublet)	1.758	$\frac{9}{2}^+$
	1.764	$\frac{7}{2}^-$
	1.776	$\frac{7}{2}^-$
1.831 (doublet)		
1.921		
(1.964)	1.966	$\frac{9}{2},\frac{7}{2}$
(2.019)		
(2.091)	2.089	$(\frac{3}{2}^-)$
2.204	2.204	$\frac{11}{2}^+$
2.255	2.252	$\frac{7}{2}^+$
2.291		
2.353	2.353	$(\frac{1}{2},\frac{3}{2})^-$
2.420		
2.594		
2.697	2.692	$\frac{7}{2}^-$
2.779		
2.905	2.903	$\frac{9}{2}^-$
2.993		
3.074		
3.091		
3.192		
3.774		
4.715		

[a]References 7, 12, and 13.

This sum rule is useful when discussing the two-neutron-transfer strengths on a nucleus or series of nuclei for which all the states involved can be described by the action of neutrons in a set of shell-model orbits. This is generally not a serious restriction, since the set of active single particle orbits can be expanded until this assumption is satisfied. The number of orbits necessary can be determined (and usually has been) by studying the single-neutron pickup and stripping reactions on the same target nuclei. Since the sum rule, Eq. (4.3), uses the orbit occupancies $\langle n(j)\rangle$ which can be obtained from the single-neutron-transfer data, it is generally necessary to have the single-nucleon-transfer data before the two-neutron-transfer sum rules can be applied. Below we will apply this sum rule to the (p,t) and (t,p) reactions on ^{208}Pb, ^{206}Pb, and ^{204}Pb. These nuclei have been extensively studied, both experimentally and theoretically. In particular, all the necessary (p,d) or (d,p) (or both) data are available.

A. Relationship between shell-model and pairing-vibration (p,t) strengths

To put this sum rule into context we first consider the case of reactions on a closed-shell nucleus. Since, by assumption, we have a closed-core nucleus, it is convenient to separate orbits which are empty for the closed-shell nucleus (they will be denoted with numeric subscripts: $j_1, j_2, \ldots$) and orbits which are full for the closed-shell nucleus (they will be denoted with alphabetic subscripts: $j_a, j_b, \ldots$). We first examine the case of (t,p) reactions on the closed-shell nucleus. Since all the active orbits have $\langle n(j_i)\rangle = 0$; $i = 1, 2, 3, \ldots$, $G^-(J) = 0$ because there are no neutrons to be removed. Equations (4.3) becomes

$$G_A^+(J) = \sum_{j_1 j_2} D^2_{j_1 j_2}(J)\,, \tag{4.4}$$

where we will use the label A to denote the mass of the closed-shell nucleus.

Another simple case is the (p,t) on a closed-shell nucleus. Here for all the active orbits $\langle n(j_i)\rangle = 2j_i + 1$ for $i = a, b, c, \ldots$, and, consequently, $G^+(J) = 0$ because there are no holes into which neutrons can be added. Equation (4.3) becomes

$$G_A^-(J) = \sum_{j_a j_b} D^2_{j_a j_b}(J)\,. \tag{4.5}$$

Note that the summations in Eqs. (4.4) and (4.5) are over different single particle orbits. As an example, consider the case of the ^{208}Pb closed-shell nucleus. The sum for the (t,p) reactions is over the empty orbit in the next major shell for neutrons, i.e., $2g_{9/2}, i_{11/2}, \ldots$, while for the (p,t) reaction the sum is over the last filled neutron major shell, i.e., $3p_{1/2}, 2f_{5/2}, \ldots$ orbits. This sum rule for (p,t) and (t,p) on closed-shell nuclei was derived by Smith *et al.*[2] before the more general result was reported. Smith *et al.* applied the sum rule to the ^{208}Pb$(p,t)^{206}$Pb reaction and found that it worked well.

We now consider the case of (t,p) on a nucleus which is a closed shell plus two neutrons. We shall first consider only $J = 0$ transitions. The only active orbits are then the valence orbits $j_1, j_2, \ldots$; the target has two neutrons distributed over these orbits. The final nucleus resulting from the (t,p) reaction has four neutrons distributed over these orbits, whereas the final nucleus resulting from (p,t) has zero neutrons, i.e., the (p,t) reaction can go only to the ground state $(J = 0)$ because this is the only state with zero valence neutrons. But by detailed balance, this $J = 0$ transition is related to the (t,p) cross section on the closed-shell nucleus. That is, we have $g^-_{A+2}(\text{g.s.}) = G^-_{A+2}(0) = g^+_A(\text{g.s.})$ where g.s. indicates ground state transitions. Since for $J = 0$ two-neutron-transfer reactions most of the cross section is to the ground state, let us write

$$\begin{aligned} G_A^+(J=0) &= g^+_A(\text{g.s.}) + \epsilon \\ &= G^-_{A+2}(0) + \epsilon\,, \end{aligned} \tag{4.6}$$

where we expect $\epsilon \ll g^+(\text{g.s.})$. The sum rule applied to nucleus $A+2$ is

$$G^+_{A+2}(J) - G^-_{A+2}(J) = \sum_{j_1 j_2} D^2_{j_1 j_2}(J)\left[1 - \frac{\langle n(j_1)\rangle}{2j_1+1} - \frac{\langle n(j_2)\rangle}{2j_2+1}\right], \tag{4.7}$$

where at present we are interested in the $J = 0$ case. Using Eqs. (4.4), (4.6), and (4.7), the (p,t) strength can be eliminated to give

$$\begin{aligned} G^+_{A+2}(J=0) = 2G^+_A(J=0) - \epsilon \\ - \sum_{j_1 j_2} D^2_{j_1 j_2}(J=0)\left[\frac{\langle n(j_1)\rangle}{2j_1+1} + \frac{\langle n(j_2)\rangle}{2j_2+1}\right]. \end{aligned} \tag{4.8}$$

In this form, we can immediately identify the term corresponding to the simple pairing-vibration model.[3] The first term on the right-hand side is the largest term, and the two additional terms can be considered "correction terms." This first term in Eq. (4.8) would correspond to the assertion that the total $J = 0$ (t,p) strength on nucleus $A+2$ (the one-boson nucleus in the language of the pairing-vibration model) is approximately 2 times the total (t,p) strength on the vacuum. If the sum rule were applied to (t,p) on the nucleus $A+4$ (a two-boson nucleus), it is easy to see that again detailed

balance would require that the leading term in the equation for the total $J=0$ (t,p) strength would be 3 times the (t,p) strength on the vacuum, and there would be a different correction term. In general to first order, the total (t,p) strength is simply proportional to the number of bosons in the final state—the prediction of the simple pairing-vibration model.[3,15] In addition, the sum rule gives correction terms which provide additional information concerning the deviations from such a simple boson model. As we shall show, these correction terms rapidly become large, in agreement with experiment, but in qualitative disagreement with the boson model.

If we follow the same procedure in using the sum rule to predict the total (t,p) strengths for $J\neq 0$ transfers, we get nearly the same result but with one important difference. For $J\neq 0$, no longer does detailed balance require that there be a large leading term which increases as we go away from the closed-shell target analogous to the "boson" term described above for $J=0$. To see this, we look at the example of (t,p) $J\neq 0$ strength on the closed-shell (A) and closed-shell + 2 neutrons $(A+2)$ targets. For the reaction on the closed shell, again the valence orbits are empty $[\langle n(j_i)\rangle = 0; i = 1,2,3,\ldots]$ hence, $G_A^- = 0$ and the sum rule is $G_A^+(J\neq 0) = \sum_{j_1 j_2} D^2_{j_1 j_2}(J)$. On the target corresponding to a closed shell plus 2 neutrons, the sum rule is again very simple. Here (in contrast to the $J=0$ case) we have $G^-(J\neq 0)=0$ because the only nonzero pickup strength is to the ground state which is $J=0$, since this is the only state with zero valence neutrons. So the sum rule becomes

$$G^+_{A+2}(J\neq 0) = \sum_{j_1 j_2} D^2_{j_1 j_2}(J)\left[1 - \frac{\langle n(j_1)\rangle}{2j_1+1} - \frac{\langle n(j_2)\rangle}{2j_2+1}\right] \quad (4.9)$$

$$= G^+_A(J) - \sum_{j_1 j_2} D^2_{j_1 j_2}(J)\left[\frac{\langle n(j_1)\rangle}{2j_1+1} + \frac{\langle n(j_2)\rangle}{2j_2+1}\right]. \quad (4.10)$$

Since all the $D^2_{j_1 j_2}(J)$ and $\langle n(j)\rangle$ and $\langle n(j)\rangle$ are positive numbers, the sum rule shows that the $J\neq 0$ (t,p) strength on the target corresponding to the closed-shell plus two neutrons must be smaller than the strength on the closed shell, in contrast to the large increase predicted for the $J=0$. Again, however, we can identify the leading term in the sum rule prediction with the pairing-vibration model. The pairing-vibrational model predicts strengths proportional to the number of bosons in the final state, which for $J\neq 0$ is one. That is, the pairing-vibration prediction is that $G^+_{A+2n} = G^+_A$, which again is the leading term in the sum rule prediction.

Throughout the above discussion, we have been using the Bayman-Clement sum rule to derive results for (p,t) or (t,p) total strengths in a model with a fixed set of active orbits. Hence, when these results are compared with experiment, only transitions to states which arise from nucleons moving in the set of orbits should be included. There are interesting transitions not of this nature. For example, below we will use the sum rule to analyze the (p,t) reaction on ^{208}Pb, ^{206}Pb, and ^{204}Pb. To do this we will make use of (t,p) results from the literature. However, the only transitions needed are those to the low excitation states not the transitions to the high excitation (~5 MeV) "multiple phonon" states. These latter transitions clearly result from the addition of neutrons to the next major shell. Since there is such a large gap between major shells, there is little mixing between these states and those of low excitation. Because of this large gap, it is very clear as to which transitions should be included in the sums.

As an example of how well this sum rule works and how large the various terms in Eq. (4.3) are in practice, we have applied this analysis procedure to the (p,t) reactions on the even isotopes of lead. It should be remembered that these reactions provide one of the best sets of data supporting the validity of the pairing-vibration model.[3] These nuclei are also well understood in the shell model.[16,17]

In order to make a comparison between the sum rule prediction and experiment we will adopt the same units of strength as are conventional for the pairing-vibration model. Namely, our unit of strength is defined to be 1 for the ^{208}Pb$(p,t)^{206}$Pb transition to the ground state in ^{206}Pb, and all other $J=0$ (p,t) and (t,p) transition strengths are defined as the ratio of the cross section to the particular state in question relative to this ground-state cross section. The (t,p) cross sections are related to (p,t) cross sections by detailed balance, or, equivalently, measured relative to the ^{206}Pb$(t,p)^{208}$Pb cross section to the ground state of ^{208}Pb. We define the $J\neq 0$ strengths in an analogous way, i.e., the ^{208}Pb$(p,t)^{206}$Pb transitions to the lowest states of a given J^π are defined as our units of strength. With this definition, the relative cross sections quoted in Tables I–III now become strengths.

B. Relative normalizations for different J values

As mentioned above, the factors $D^2_{j_1 j_2}(J)$ are taken from a DWBA calculation of the (p,t) cross section for picking up neutrons in a $(j_1 \otimes j_2)J$ configuration. Since there is an overall normalization in DWBA calculations of two-neutron transfer which is normally determined empirically, we deter-

mined this overall normalization by requiring that the total sum rule strength on the closed-core nucleus (^{208}Pb) for a given J transfer equal the sum rule limit. One would hope that the normalizations obtained in this way for the various J values would be nearly the same. This analysis has already been done by Smith *et al.* for the ^{208}Pb$(p,t)^{206}$Pb reaction of 40 MeV.[2] They found that the total (p,t) strength for the various J transfers agreed with the sum rule prediction to about ±30%. We repeated this analysis using our data and using the DWBA optical model parameters which we had used to analyze the (p,t) angular distributions. The optical-model parameters used in the DWBA calculation were proton parameters from Becchetti and Greenlees[8] and the triton parameters from Flynn *et al.* (the set with $r_0 = 1.16$ fm).[18] Since the proton parameters had been used successfully in analyzing the (p,d) reaction on these targets at the same bombarding energy[6] and since they were derived from elastic scattering from these targets at nearly the same energy, we did not want to change to another set, such as that used by Smith *et al.* unless there were problems in fitting the (p,t) angular distributions. Similarily, the triton parameters of Flynn also seemed, *a priori*, to be more soundly based than the alternatives. As can be seen in Fig. 3, the fits to the angular distributions are rather good with these optical-model parameters.

When we compared our experimentally determined summed ^{208}Pb$(p,t)^{206}$Pb cross sections for different J values with the sum rule limit $=\sum_{j_a j_b} D^2_{j_a j_b}(J)$ to determine the normalization factor required to relate absolute cross sections to DWBA cross sections, we obtained results similar to those of Smith *et al.* with one exception. We found that while one could choose a single factor for all $J \neq 0$ transfers and obtain agreement to ±30% between the sum rule and experiment, the $J = 0$ normalization factor was almost exactly twice as large. While this result is in disagreement with that of Smith (who used different optical model parameters), it is not very surprising. Assuming the DWBA analysis to be correct, the sum rule results immediately imply that the six orbit (one major shell) shell-model space is not sufficient to fully describe $J = 0$ states. But this is just as would be expected from the results of Vary, Ascuitto, and Ginocchio[19] These authors calculated the ^{208}Pb$(p,t)^{206}$Pb cross sections to the ground (0^+) and first 2^+ states using microscopic wave functions. These ^{206}Pb wave functions were calculated assuming nucleons were free to move either (1) only in the six shell-model orbits (one major shell) or (2), using the random phase approximation, over two major shells, i.e., some particle-hole configurations were allowed. They found that, while there was some sensitivity to the interaction used, the ground state cross section approximately doubled when the space was expanded. The cross sections to lowest 2^+ and 4^+ states also increased, but only slightly. Since most of the $J = 0$ (p,t) cross section is in the ground state, the results of Vary *et al.* indicate that the $J = 0$ sum rule should be exceeded by about a factor of 2, as we have observed.

While the indication that a basis of neutrons moving in only one major shell with no particle-hole excitations is not sufficient to describe the correlations present in the ground state of ^{206}Pb might suggest that the sum rule should not work for $J = 0$, we will proceed and compare the sum rule predictions with experiment. As will be noted below, the sum rule is rather successful in predicting summed cross sections both for $J = 0$ and for $J \neq 0$ (p,t) reactions on ^{206}Pb and ^{204}Pb relative to those measured on ^{208}Pb. The agreement seems more than coincidental. It may be that the result discussed above—that the $J = 0$ ^{208}Pb$(p,t)^{206}$Pb summed cross section is about twice the sum rule prediction assuming a single normalization factor for (p,t) cross sections independent of J—may result from a failure of DWBA to predict relative cross sections for different J values. The fact that Smith *et al.* found no such enhancement for $J = 0$ over $J \neq 0$ transitions when he did the sum rule analysis of essentially equivalent data, but using different proton and triton optical-model parameters, at least indicates considerable uncertainty in DWBA's prediction of the relative cross sections for different J values.

On the other hand, assuming that the present analysis is more accurate than that of Smith *et al.*[2] and that the $J = 0$ enhancement is real, the theoretical results of Vary *et al.*[19] show that this factor of 2 increase in the ground state cross section probably results from very small changes in the nuclear wave functions. As the basis space is expanded from one major shell, the ground-state wave function of ^{206}Pb in the larger basis calculation has only about 2% of four-particle–two-hole components in it. Since the $J = 0$ states in ^{206}Pb are still so dominantly two-neutron-hole states, it may be possible to incorporate the effects of these small admixtures of particle-hole configurations simply by renormalizing the $J = 0$ (p,t) cross sections relative to $J \neq 0$. The philosophy behind such an approach is in some ways analogous to the concept of effective charge used to describe the electromagnetic decay of the single or few neutron states around ^{208}Pb. In the case of (p,t) reactions, it has already been found that the shell model does a reasonable job of predicting

relative cross sections to different states of the same J transfer, both for $J=0$ and $J\neq 0$.[17]

C. Application of the sum rule

Because we are using the sum rule principally to analyze (p,t) data on ^{208}Pb, ^{206}Pb, and ^{204}Pb, it is convenient to write the sum rule [Eq. (4.3)], in terms of the number of neutron holes outside ^{208}Pb. But the number of particles $=2j+1-$ number of holes. With this substitution, the sum rule becomes

$$G^-(J)-G^+(J)=\sum D^2_{j_aj_b}(J)\left[1-\frac{\langle n(j_a)\rangle}{2j_a+1}-\frac{\langle n(j_b)\rangle}{2j_b+1}\right], \tag{4.11}$$

where now $\langle n(j)\rangle$ is the number of neutron *holes* in orbit j and, as before, the "+" and "−" signs refer to (t,p) and (p,t) reactions, respectively.

In order to check this relationship, the orbit occupancies must be known. In the case of ^{206}Pb, they are taken from Ref. 7 where the analysis of a number of data, including (p,d) and (d,p) on ^{206}Pb, is summarized and the orbit occupancies are deduced. In the case of ^{204}Pb, the $\langle n(j)\rangle$'s are determined from the analysis of the ^{204}Pb$(d,p)^{205}$Pb reaction. Reference 13 gives spectroscopic factors for transitions to "neutron hole" configurations in ^{205}Pb, and from these, using the usual monopole sum rule, the orbit occupancies are determined. Doing this gave a total number of neutron holes in ^{204}Pb of 5.01 instead of the expected 4.00. The DWBA absolute cross section was renormalized to make the number of holes equal to 4.00. The results of this analysis are given in Table V.

Where needed, the (t,p) strengths $[G^+(J)]$ were taken from Flynn *et al.*[20] There are actually very few (t,p) cross sections needed. None are needed to apply the sum rule to the ^{206}Pb target because the only transition which enters is the ^{206}Pb$(t,p)^{208}$Pb reaction to the ground state, but this is identical to the ^{208}Pb$(p,t)^{206}$Pb strength. For the ^{204}Pb target, again the (t,p) ground-state transition was taken from the ^{206}Pb$(p,t)^{204}$Pb ground-state cross section. The only other (t,p) cross sections to hole states in ^{206}Pb with measurable strength are to the 1.167 0^+ state ($g^+=0.17$), the 0.804 2^+ state ($g^+=0.07$), and the 1.684 MeV 4^+ state ($g^+=0.01$) where these (t,p) strengths were determined by comparing the measured (t,p) cross sections to the unit (p,t) cross sections.

The results of using the strengths data in Tables I–III and the orbit occupancies of Table V in the sum rule are given in Table VI. Note that to check the accuracy of this relationship we have used it to make *absolute* predictions of the total (p,t) strength (cross sections) for various J transfers on targets ^{206}Pb and ^{204}Pb. This prediction was made by writing Eq. (4.3) as

$$G^-(J)=G^+(J)+\sum D^2_{j_aj_b}-\sum D^2_{j_aj_b}\left[\frac{\langle n(j_a)\rangle}{2j_a+1}+\frac{\langle n(j_b)\rangle}{2j_b+1}\right] \tag{4.12}$$

and substituting in the known quantities on the right-hand side, and hence obtaining a *prediction* for $G^-(J)$. The ^{208}Pb$(p,t)^{206}$Pb data are used only to obtain the DWBA absolute normalization factors.

TABLE V. Number of neutron holes in the ground states of ^{206}Pb and ^{204}Pb:

Orbit	$\frac{\langle n(j)\rangle}{2j+1}$ ^{206}Pb [a]	^{204}Pb [b]
$P_{1/2}$	0.600	0.70
$f_{5/2}$	0.056	0.23
$P_{3/2}$	0.070	0.13
$i_{13/2}$	0.007	0.035
$f_{7/2}$	0.007	0.022
$h_{9/2}$	0.002	0.0

[a] From Ref. 7.
[b] From Ref. 13, see text.

TABLE VI. Comparison between the sum rule prediction and experiment for total (p,t) strengths on ^{206}Pb and ^{204}Pb:

$$G^-_{sr}(J)=G^+(J)+\sum D^2_{j_aj_b}(J)-\sum D^2_{j_aj_b}(J)\left[\frac{\langle n(j_a)\rangle}{2j_a+1}+\frac{\langle n(j_b)\rangle}{2j_b+1}\right].$$

J^π	G^+	$\sum D^2$	$-\sum D^2\left[\frac{\langle n\rangle}{2j+1}+\frac{\langle n\rangle}{2j+1}\right]$	G^-_{sr}	G^-_{expt}
			^{206}Pb target		
0^+	1.000	1.27	−0.47	1.80	2.01
2^+	0.0	1.42	−0.47	0.95	0.83
4^+	0.0	1.95	−0.49	1.46	1.25
5^-	0.0	1.74	−0.11	1.63	1.48
6^+	0.0	1.0	−0.06	0.94	0.77
7^-	0.0	1.28	−0.40	0.88	0.81
9^-	0.0	1.0	−0.05	0.95	0.88
			^{204}Pb target		
0^+	1.91	1.27	−0.66	2.52	2.31
2^+	0.07	1.42	−0.69	0.80	0.64
4^+	0.01	1.95	−0.79	1.17	0.91
5^-	0.0	1.74	−0.26	1.48	1.28
6^+	0.0	1.0	−0.19	0.81	0.55
7^-	0.0	1.28	−0.55	0.73	···
9^-	0.0	1.0	−0.24	0.76	0.93

Hence, the sum rule results in Table VI are *absolute predictions* with no free parameters. Notice also that in Table VI we have included not only the total sums, but also some of the individual terms. The magnitude of the individual terms will be useful when comparing the sum rule with other predictions such as those of the pairing-vibration model discussed below.

The root-mean-squared percentage deviation between experiment and the sum rule prediction is 17%. This accuracy is comparable with the accuracy typically obtained when single-neutron-transfer sum rules are applied to obtain the orbit occupancies. However, it should be remembered that the inaccuracy of single-neutron-transfer sum rules results largely from uncertainties in the ability of the DWBA to predict absolute single particle transfer cross sections. If we had not used the ^{208}Pb$(p,t)^{206}$Pb reaction to calibrate the DWBA, the deviation between the two-neutron-transfer sum rule and experiment would have been much greater.

However, by making this calibration, the sum rules can be applied with remarkable accuracy. Notice that a number of prominent trends in the (p,t) data as the targets become further removed from the closed-shell nucleus ^{208}Pb are accurately predicted by the sum rule. A plot of these total strengths for each J relative to the total strength in ^{208}Pb$(p,t)^{206}$Pb is given separately in Fig. 5. In particular, the dramatic *increase* in the total $J=0$ (p,t) cross section and the almost as rapid decrease in the $J\neq 0$ total cross sections as one goes from ^{208}Pb to ^{206}Pb to ^{204}Pb are well reproduced. The dramatic increase in the $J=0$ strength has been well publicized and is one of the important experimentally observed features predicted by the pairing-vibration model. The general decrease in the total $J\neq 0$ cross sections has not been so widely discussed. Again examination of the sum rule indicates that it should be a general feature since both $D^2_{j_a j_b}$ and $\langle n(j)\rangle$ are positive numbers.

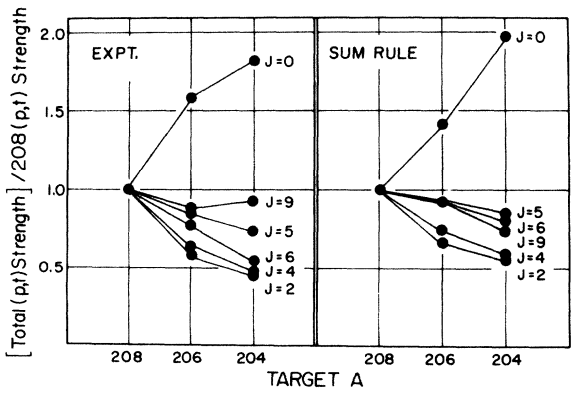

FIG. 5. The ratio of total strengths (G_A^-) for (p,t) reactions on ^{208}Pb, ^{206}Pb, and ^{204}Pb for various J transfers relative to that observed in ^{208}Pb$(p,t)^{206}$Pb. Notice the rapid increase for $J=0$ and the rapid decrease for $J\neq 0$.

That these two trends can legitimately be described as dramatic can be seen not only by looking at the strengths in Tables I–III and Fig. 5 but also by looking at the changes in the raw (p,t) spectra as one goes away from the closed shell. See Fig. 1. As can be seen in Fig. 1, the character of the spectrum has changed markedly as one goes from ^{208}Pb$(p,t)^{206}$Pb to ^{204}Pb$(p,t)^{206}$Pb. The ^{208}Pb$(p,t)^{206}$Pb is typical of (p,t) on good closed-shell nuclei: Many states are excited, and several have much larger cross sections than the $J=0$ ground-state cross section. The change as one goes to a ^{204}Pb target is striking. The total $J=0$ cross section has greatly increased while the total $J\neq 0$ cross sections have markedly decreased. The ground-state cross section is now larger than that to any other state. The net result is that the spectrum seen in ^{204}Pb$(p,t)^{202}$Pb appears much more like the typical (p,t) cross section observed in nuclei far from closed shells where the cross section is concentrated in the ground-state $J=0$ transition.

V. DISCUSSION

It has been commonly held that because (p,t) reaction cross sections are so intimately related to correlations in the nuclear wave functions of the states involved, sum rule methods, which have been so powerful in analyzing single particle transfer data, could not be usefully applied to (p,t) data. The sum rule derived by Bayman and Clement[1] relating the total (p,t) and (t,p) strengths on a given target to the orbit occupancies of that target showed that this was not the case. However, the inaccuracy of DWBA in predicting absolute cross sections made it difficult to apply their sum rule. One of the initial motivations of the present study was to make a meaningful check of the accuracy of their sum rule by using the (p,t) reaction on the closed-shell nucleus ^{208}Pb to calibrate the DWBA and, hence, to reduce greatly the uncertainty introduced by the DWBA. The present results indicate that, when such a procedure is available, the two-neutron-transfer sum rules can be applied with nearly the same accuracy as the corresponding single-nucleon-transfer sum rule.

The application of sum rules often leads to much less model dependent confrontations between theory and experiment. If one compares theoretical and experimental results for *individual* (p,t) transitions, it is difficult (at best) to know what meaning to attach to a certain discrepancy. When theo-

retical and experimental sums are compared, there is much less ambiguity. In the present case, the J dependence observed in the normalization factor relating DWBA to experimental cross sections can result from either an inaccuracy of DWBA or an inadequacy of the assumed basis. While our feeling is that this results at least in part from insufficient basis, the results of Smith *et al.* indicate that one can find a set of optical-model parameters for which there is little or no J dependence. In any case, there is a clear question. If one were simply comparing theory and experiment for individual (p,t) cross sections, an occasional factor of 2 discrepancy would probably go unnoticed. See, for example, Ref. 17.

However, it is in clarifying what is the minimum set of assumptions needed to create a model which predicts certain observables correctly that the sum rule approach can often make its greatest contribution. In an earlier report[17] on the relative (p,t) cross sections on ^{208}Pb, ^{206}Pb, and ^{204}Pb, comparison was made for individual transitions strengths between experiment and both the shell-model and the simplest pairing-vibration model. It was concluded there that there was a remarkable degree of agreement between these seemingly different models. A natural question is what is the minimum set of assumptions needed to obtain this agreement. In the discussion in Sec. IV above, it was pointed out that the sum rule prediction for the total (p,t) strengths for both $J=0$ and $J\neq 0$ transitions looked remarkably like the simplest pairing-vibration model. It was observed that the leading term in the sum rule expression for total (p,t) strength on target $A+2n$ is equal to (in the language of the pairing-vibration model) the number of bosons (n) in the final state times the strength on the closed-shell nucleus (A) with correction terms involving the orbit occupancies and (t,p) cross sections to excited states. Examination of the individual numbers in Table VI shows that the correction terms rapidly become significant. Hence, when considering summed strengths (cross sections) for given J transfers, the sum rule is much more accurate than the pairing-vibration picture.

The fact that the qualitative trends in (p,t) cross sections as one goes away from the closed shell are contained in the sum rule means that the *only necessary assumption* needed to predict the dramatic increase in total $J=0$ (p,t) cross sections and rapid decrease of $J\neq 0$ cross sections is that the nuclei involved can be considered as a closed shell plus a number of valence neutron holes outside this closed shell. Any model which makes the closed-core assumption must predict these trends.

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