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Nucleon form factors, Lorentz invariance, and nuclear photoabsorption*

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The role of the intrinsic nucleon size in a phenomenological treatment of photon absorption by a nucleus is discussed. It is suggested that the complete transition matrix element should be written as a product of the purely nuclear matrix element and the intrinsic nucleon form factor. The latter is written as a function, not of q^2 (the four-momentum transfer), but of $q^2+\omega^2$, where ω is the excitation energy of the nucleus. We note that for real photons where $q^2=0$ this is essentially equivalent to $\vec{q}^{\,2}$, i.e., the three-momentum transfer.

[NUCLEAR REACTIONS Nucleon form factor, phenomenology, relativistic invariance, photonuclear cross sections.]

One of the frequently asked questions at photonuclear discussions is how to incorporate the nucleon finite size in photonuclear calculations. Probably the earliest example of phenomenology in the description of nuclear electromagnetic interactions was the introduction of the proton form factor $f_p(\vec{q}^{\,2})$ as a function of the three-vector momentum transfer $\vec{q}$ in electron-scattering calculations.[1] In such calculations the complete charge form factor of a nucleus has the form $f_p(\vec{q}^{\,2})F_0(\vec{q}^{\,2})$, where F_0 is the matrix element of the Fourier transform of the purely nuclear charge operator. This expression has the form dictated by Galilean invariance and is clearly at variance with Lorentz invariance, since one expects any form factor to be a function of $q^2\equiv\vec{q}^{\,2}-q_0^{\,2}$, the square of the four-momentum transfer, and depend on q_0, the energy of recoil and excitation transferred to the nucleus by the electron. Since the energy transfer q_0 has the dimensional character, 1/mass or $(1/m)$, the factor $q_0^{\,2}/\vec{q}^{\,2}$ is of the form $(1/m)^2$ or a $(v/c)^2$ relativistic correction.[2] Should one naively replace $\vec{q}^{\,2}$ by q^2 in the above form factors? If photoabsorption matrix elements have the same basic structure as the charge matrix elements, they would not depend on f_p at all, since $q^2=0$ for photoabsorption and $f_p(0)\equiv1$. This is the question we address ourselves to.

Recently, the approximate relativistic corrections of order $(v/c)^2$ arising from nuclear motion were investigated [3,4] and to this order they were found to produce an effective nuclear charge form factor (in addition to other terms) of the form $F_0(\bar{q}^{\,2})+\Delta F(\vec{q}^{\,2})$. The argument $\bar{q}^{\,2}$ is given to order $(v/c)^2$ by

$$\bar{q}^{\,2}=\vec{q}^{\,2}-q_0^{\,2}+\omega^2-\vec{q}^{\,4}/4m_t^{\,2}, \qquad (1a)$$

$$q_0\cong\omega+\omega_R, \qquad (1b)$$

$$\omega_R=(\vec{P}_f^{\,2}-\vec{P}_i^{\,2})/2m_t, \qquad (1c)$$

where ω is the intrinsic energy difference (mass difference) of nuclear final and initial states, ω_R is the recoil energy, and $\vec{q}=\vec{P}_f-\vec{P}_i$ denote the momentum transfer and the nuclear final and initial momenta, respectively, while m_t is the sum of the masses of the constituents. The wave functions used to calculate matrix elements are *rest-frame* wave functions and ΔF is a correction of order $(1/\text{mass})^2$. We see that the first two terms in Eq. (1a) are the invariant momentum transfer q^2, while the third term guarantees that $\bar{q}^{\,2}$ vanish as the three-momentum transfer vanishes and this preserves the nonrelativistic long-wavelength limit[4,13]; the remaining term is a Lorentz contraction factor. To order $(v/c)^2$, $\bar{q}^{\,2}$ is an invariant and this illustrates an important point: Although it is necessary for matrix elements to be functions of invariants, both q^2 and ω^2 are invariants and they can *a priori* occur in any combination.

Although the argument described above was developed for the nuclear charge form factor, we expect a similar form for the transverse form factor as well, and this has an immediate bearing on photon absorption. For electron scattering, $|\vec{q}|$ and q_0 are independent variables, while photon absorption is restricted to $q^2=0$. For the latter case $\bar{q}^{\,2}\cong\omega^2\cong\vec{q}^{\,2}$ to order $(v/c)^2$; since $|\vec{q}|=q_0\sim(1/m)$, we can neglect the last term in Eq. (1a). Thus for photoabsorption, the standard nonrelativistic approach gives the same result as a more sophisticated treatment including $(v/c)^2$ corrections, if the argument above is valid.

The preceding argument based on analogy with prior calculations of *charge* form factors does not completely answer the question we posed earlier since we still have to justify the above expectation concerning the *current* matrix elements. In addition, the previous argument involved only nuclear physics and not the nucleon form factor. It does point the way, however, to a possible answer. In order to proceed further, we make use of the fact that we are dealing with relativistic corrections and that the charge and current are components of a conserved four-vector. According to Close and Osborn[5] this implies the following relationship[6] between the boost operator $\vec{K}$, which generates infinitesimal Lorentz transformation, the Hamiltonian H, and the charge and current densities $\rho(\vec{x})$ and $\vec{J}(\vec{x})$:

$$[\vec{K}, \rho(\vec{x})] = i\vec{J}(\vec{x}) + \vec{x}[H, \rho(\vec{x})], \tag{2a}$$

$$[K^\alpha, J^\beta(\vec{x})] = i\delta_{\alpha\beta}\,\rho(\vec{x}) + x^\alpha[H, J^\beta(\vec{x})], \tag{2b}$$

$$\vec{\nabla}\cdot\vec{J}(\vec{x}) = -i[H, \rho(\vec{x})]. \tag{2c}$$

Regardless of the dynamics we expect these equations to hold. We are, however, neglecting the nucleon dynamics which leads to a nucleon charge distribution, and are restricting H to be the nuclear Hamiltonian in the usual spirit of phenomenology.

Rather than continue in coordinate space, we Fourier transform Eq. (2) and will try to connect the result with our previous discussion. We define $\rho(\vec{q})$ and $\vec{J}(\vec{q})$ to be the appropriate transforms, and note that $\rho(\vec{q})$ is essentially the form factor F_0 discussed earlier. This leads to

$$[\vec{K}, \rho(\vec{q})] = i\vec{J}(\vec{q}) - i\vec{\nabla}_q[H, \rho(\vec{q})], \tag{3a}$$

$$[K^\alpha, J^\beta(\vec{q})] = i\delta_{\alpha\beta}\,\rho(\vec{q}) - i\nabla_q^\alpha\,[H, J^\beta(\vec{q})], \tag{3b}$$

$$\vec{q}\cdot\vec{J}(\vec{q}) = [H, \rho(\vec{q})]. \tag{3c}$$

These equations apply to *both* the "bare" charges and currents ρ_0 and $\vec{J}_0$ and the ones which contain the proton charge distribution. If we multiply each equation by a common nucleon form factor $f_N(\vec{q}^{\,2})$ and define $\bar{\rho} \equiv f_N\rho_0$, $\bar{\vec{J}} = f_N\vec{J}_0$, we see that Eq. (3c) is satisfied directly for $\bar{\rho}$ and $\bar{J}$ while Eqs. (3a) and (3b) can be easily manipulated into the appropriate form, but with the *extra* terms $+i(\vec{\nabla}_q f_N)[H, \rho_0(\vec{q})]$ in Eq. (3a) and $+i(\nabla_q^\alpha f_N)[H, J_0^\beta(\vec{q})]$ in Eq. (3b). In order to remove these terms, it is necessary to modify both $\bar{\rho}$ and $\bar{J}$. We define

$$\rho = f_N\rho_0 + \Delta\rho, \tag{4a}$$

$$\vec{J} = f_N\vec{J}_0 + \Delta\vec{J}, \tag{4b}$$

where $\Delta\rho$ and $\Delta\vec{J}$ are the charge and current density modifications. Furthermore, because H in the extra terms above can be taken to be the nonrelativistic Hamiltonian [of order $(1/m)$] and the nonrelativistic parts of ρ_0 and $\vec{J}_0$ are of order (1) and $(1/m)$, respectively, we need only consider Eq. (3a) to order $(1/m)$ and Eq. (3b) to order $(1/m^2)$. Since the leading (nonrelativistic) term[6] in $\vec{K}$ is $\vec{K}_0 = m_t\vec{R}$, where $\vec{R}$ is the usual nonrelativistic center of mass coordinate, we see that $\Delta\rho$ will be of order $(1/m^2)$ and $\Delta\vec{J}$ of order $(1/m^3)$ [i.e., $(v/c)^2$ corrections].

Clearly $\vec{K}_0$ commutes with everything but factors of $\vec{P}$, the *total* momentum operator of the nucleus, and for this reason any part of $\Delta\rho$ or $\Delta\vec{J}$ independent of $\vec{P}$ will not contribute and our "solution" for $\Delta\rho$ and $\Delta\vec{J}$ will necessarily be ambiguous. In order to cancel $\Delta\rho$ with the extra term we found before, we separate the nuclear nonrelativistic Hamiltonian into two pieces, $H_R + h_0$, where h_0 is the internal Hamiltonian and H_R is the Hamiltonian for overall motion. A satisfactory form for $\Delta\rho$ can then be seen to be

$$\Delta\rho(\vec{q}) = -f_N'(\vec{q}^{\,2})[H_R + 2h_0, [H_R, \rho_0(\vec{q})]], \tag{5a}$$

where $f_N'(\vec{q}^{\,2}) \equiv (d/d\vec{q}^{\,2})f_N(\vec{q}^{\,2})$ and for $\Delta\vec{J}$ a similar form can be obtained

$$\Delta\vec{J}(\vec{q}) = -f_N'(\vec{q}^{\,2})[H_R + 2h_0, [H_R, \vec{J}_0(\vec{q})]]. \tag{5b}$$

It is easy to verify for example that $[m_t\vec{R}, \Delta\rho(\vec{q})] = -2if_N'(\vec{q}^{\,2})\vec{q}[H, \rho_0(\vec{q})]$ and also that any $\vec{P}$-independent term may be added to $\Delta\rho$ without affecting the commutation relation. Furthermore, by taking matrix elements of $\Delta\rho(\vec{q})$, we find that $\langle\Delta\rho\rangle = -(\omega_R^{\,2} + 2\omega\omega_R)f_N'\langle\rho_0\rangle$ which is what one would obtain by modifying the nucleon form factor $f_N(\vec{q}^{\,2})$ to the form

$$f_N(\vec{q}^{\,2}) \to f_N(\vec{q}^{\,2} - \omega_R^{\,2} - 2\omega\omega_R) = f_N(q^2 + \omega^2) \tag{6}$$

and expanding the $(1/m)^2$ terms to first order. Except for the $\vec{q}^{\,4}/4m_t^{\,2}$ terms, this is precisely the form we displayed in Eq. (1). The $\vec{q}^{\,4}$-dependent term would not contribute to the commutator of $\Delta\rho$ and $\vec{K}_0$ and thus its presence here cannot be argued on general grounds. Indeed, we have not connected f_N with the experimentally measured nucleon form factor, and it could be any arbitrary function of $\vec{q}^{\,2}$. The commutator result for $\Delta\vec{J}$ can be shown to cancel the extra current term we found earlier; it also generates the $i\delta_{\alpha\beta}\Delta\rho$ term required in Eq. (3b) if we use the definition Eq. (4a). Note that this result applies to the exchange parts of the current, as well, which are also expected to be proportional to an electromagnetic form factor.[7,8] The current continuity equation is also satisfied for the terms $\Delta\vec{J}$ and $\Delta\rho$ if this equation holds for $\vec{J}_0$ and

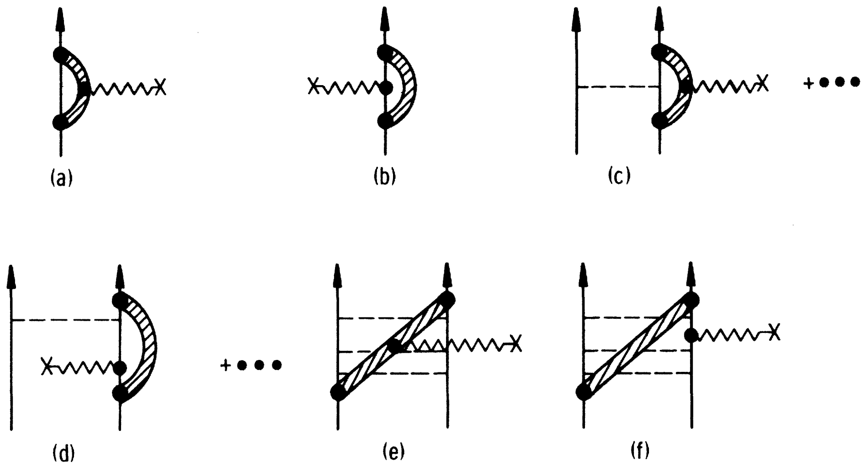

FIG. 1. Typical free nucleon form factor graphs are shown in (a) and (b), while analogous graphs for interacting nucleons are illustrated in (c) and (d). Two-nucleon exchange graphs of the same type are depicted in (e) and (f). Vertical lines represent nucleons, a wavy line with cross illustrates an external interaction while shaded and dashed lines depict mesons.

ρ_0. Thus Eq. (3) is satisfied for the charges and currents defined by Eqs. (4) and (5), provided it is satisfied in the absence of f_N.

Even though our previous arguments based on form factors and the boost commutation relations yield essentially identical results, it is somewhat unsettling to have a nucleon form factor which depends on purely nuclear quantities, ω and ω_R. It must be borne in mind, however, that the nucleons in the nucleus are continuously interacting with each other and this will necessarily have an effect on the form factor for bound nucleons.[9] It is instructive to examine Figs. 1(a) and 1(b) which depict typical nucleon (solid line) form factor graphs containing a meson (double shaded line) loop and an external electromagnetic field (wavy line and cross). These diagrams are meant to illustrate a free nucleon, and the corresponding form factor graphs for interacting nucleons are given in Figs. 1(c) and 1(d), where the dashed line indicates an unspecified meson exchange. The corrections arising from the binding in the latter graphs are retardation corrections; and they are intimately connected to the effect we have been discussing. The retardation effects are also illustrated for the corresponding exchange graphs, Figs. 1(e) and 1(f) which have been discussed recently in connection with exchange currents.[10]

We may generalize our results somewhat by relaxing the restriction of a common nucleon form factor by using Eq. (6) in each form factor. For photoabsorption, as we argued earlier, $q^2=0$ and there is no difference between the nonrelativistic result $f_N(\vec{q}^{\,2})$ and $f_N(q^2+\omega^2)$ to order $(1/m^2)$, since the nuclear recoil terms are at least of order $\vec{q}^{\,2}\omega/m_t \sim (1/m)^4$ and should be unimportant. We also note that using a nucleon form factor $f_N(\vec{q}^{\,2})$ in the nonrelativistic Compton amplitude is necessary in order to obtain the proper low-energy limit,[11] but this static form factor affects the analytic properties adversely.[12]

For the electron scattering case we cannot use the arguments we developed for photoabsorption since $q^2 \neq 0$. The nucleon form factor must be a function of q^2 and ω^2 but the precise function is not clear. The most conservative thing to do, in the absence of further information[10] is to use $f_N(q^2+\omega^2)$, although it is clear that binding will affect the nucleon form factor in the nucleus in ways we have not considered here.

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