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Short-range correlations and pseudopotentials. V

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Deformed Hartree-Fock-Bogoliubov calculations have been carried out with an effective interaction derived from the different Hamada-Johnson, Yale, Tabakin, and Reid potentials and from the potential A calculated by us. Short-range pseudopotentials are introduced to these potentials to achieve healing in the correlated wave functions and to produce zero-phase shift in the pair state. The deformed Hartree-Fock-Bogoliubov method has been applied to the $2p$-$1f$ shell even-even nuclei. Intrinsic quadrupole moments $\langle Q_0{}^2 \rangle$, quadrupole moments of the first excited states Q_{2_+}, and the reduced transition probabilities $B(E2)$ $(0^+ \rightarrow 2^+)$ have been calculated. The angular momentum projected spectra for these nuclei have also been obtained. The present calculations of quadrupole moments, transition probabilities, and angular momentum spectra for the different Ti, Cr, Fe, and Ni isotopes are in good agreement with the experimental results.

[NUCLEAR STRUCTURE ^{44,46,48,50}Ti, ^{48,50,52,54}Cr, ^{52,54,56,58}Fe, ^{58,60,62,64}Ni; calculated intrinsic quadrupole moments, quadrupole moments of the first excited states, reduced transition probabilities, projected angular momentum spectra and energy levels; pseudopotentials. Hartree-Fock-Bogoliubov method.]

I. INTRODUCTION

The main purpose of the study of nuclear structure is to reach and to obtain satisfactory values of the static properties of nuclei. These properties depend basically upon the nature of the nuclear forces between nucleons inside the nucleus. These nuclear forces are described by nuclear nucleon-nucleon interaction as a long-range attractive potential surrounded by a very repulsive short-range potential. Different interesting forms for the nucleon-nucleon potentials have been suggested such as the Hamada-Johnston,[1] Yale,[2] Livermore,[3] Tabakin,[4] and Reid[5] potentials and the potential A calculated by us[6] as a different meson exchange process. In these potentials, the nuclear nucleon-nucleon interaction has been separated into a short-range and a long-range part. To carry out this separation process, short-range pseudopotentials are introduced and calculated for each case.[7-10] These short-range pseudopotentials are required to achieve healing in the correlated wave functions and to produce zero-phase shift in the pair state. With these short-range pseudopotentials the calculated static properties such as the binding energy per particle for the spherical closed-shell nuclei ^{16}O and ^{40}Ca are in good agreement with the experimental measurements.

In the present paper, our main interest will concern the even-even nuclei in the $2p$-$1f$ shell. The study of these nuclei is very interesting since the $2p$-$1f$ shell nuclei have a tendency towards $1f_{7/2}$ subshell closure if N or (and) $Z = 28$. The study of the known and identified low-lying states of the $1f_{7/2}$ nuclei is an important test of the states which are selectively excited in α transfer experiments,[11,12] and the energy of which is often so high that the spins are not identified. In the present work, the deformed Hartree-Fock-Bogoliubov method[13-15] is applied for the different even-even Ti, Cr, Fe, and Ni isotopes. The intrinsic quadrupole moments $\langle Q_0{}^2 \rangle$ for these nuclei have been calculated. Also, to study the variation of nuclear deformation in a given shell,[16] it is interesting to know the values of the quadrupole moments of the first excited states Q_{2^+} and the reduced transition probabilities $B(E2)$ $(0^+ \rightarrow 2^+)$. $\langle Q_0{}^2 \rangle$, Q_{2^+}, $B(E2)$ $(0^+ \rightarrow 2^+)$, and the angular momentum projected spectra have been calculated carrying out the Hartree-Fock-Bogoliubov calculations for different even-even Ti, Cr, Fe, and Ni isotopes employing an effective interaction derived from the Hamada-Johnston, Yale, Tabakin, and Reid potentials and from the potential A calculated by us due to the meson exchange process.

In Sec. II we introduce the used Hartree-Fock-Bogoliubov equations. Section III is left for calculations and discussion.

II. HARTREE FOCK-BOGOLIUBOV EQUATIONS

The intrinsic deformed Hartree-Fock-Bogoliubov (HFB) state for axial symmetry can be written as[15]

$$|\phi\rangle = \prod_{im} (u_i^m + v_i^m b_{im}^\dagger b_{\bar{i}m}^\dagger)|0\rangle, \quad (1)$$

where i is the band for the same m and the operators $b_{im}^\dagger$ are expanded as

$$b_{im}^\dagger = \sum_j C_{ji}^m a_{jm}^\dagger; \quad b_{\bar{i}m}^\dagger = \sum_j (-)^{j-m} C_{ji}^m a_{j,-m}^\dagger. \quad (2)$$

The wave function $|\phi\rangle$ can be written as[17,18]

$$|\phi\rangle = N \exp\left(\frac{1}{2}\sum_{\alpha\beta} f_{\alpha\beta} a_\alpha^\dagger a_\beta^\dagger\right)|0\rangle, \quad (3)$$

where N is the normalization constant, α stands for the quantum numbers (j_α, m_α), and

$$f_{\alpha\beta} = \sum_i C_{j_\alpha i}^{m_\alpha} C_{j_\beta i}^{m_\beta} \frac{v_i^{m_\alpha}}{u_i^{m_\alpha}} \delta_{m_\alpha, -m_\beta}. \quad (4)$$

The expectation value of the Hamiltonian is

$$H = \sum_\alpha \epsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|v|\gamma\delta\rangle a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta. \quad (5)$$

Then for a state with angular momentum J, the energy is given as

$$E_J = \frac{\langle\phi|HP_{MM'}^J|\phi\rangle}{\langle\phi|P_{MM'}^J|\phi\rangle} = \frac{\int_0^\pi h(\theta) d_{00}^J(\theta)\sin\theta d\theta}{\int_0^\pi n(\theta) d_{00}^J(\theta)\sin\theta d\theta}. \quad (6)$$

$P_{MM'}^J$ is the projection operator and, is given by

$$P_{MM'}^J = \frac{2J+1}{8\pi^2}\int D_{MM'}^{J*}(\Omega) R(\Omega) d\Omega. \quad (7)$$

$D_{MM'}^J(\Omega)$ and $R(\Omega)$ are the rotation matrix and the rotation operator, respectively. The overlap integrals $h(\theta)$ and $n(\theta)$ are given as[19]

$$h(\theta) = n(\theta)\left[\sum_\alpha \epsilon_\alpha \left(\frac{K}{1+K}\right)_{\alpha\alpha} + \frac{1}{4}\sum_{\alpha\beta\gamma\delta}\langle\alpha\beta|v|\gamma\delta\rangle\left\{2\left(\frac{K}{1+K}\right)_{\gamma\alpha}\left(\frac{K}{1+K}\right)_{\delta\beta} + \sum_{\nu\rho}\left(\frac{1}{1+K}\right)_{\nu\rho} F_{\rho\delta}\left(\frac{1}{1+K}\right)_{\nu\alpha} f_{\nu\beta}^*\right\}\right], \quad (8)$$

$$n(\theta) = \{\det[1+K(\theta)]\}^{1/2}, \quad (9)$$

where

$$F_{\alpha\beta}(\theta) = \sum_{m'_\alpha m'_\beta} d_{m_\alpha m'_\alpha}^{j_\alpha}(\theta) d_{m_\beta m'_\beta}^{j_\beta}(\theta) f_{j_\alpha m'_\alpha, j_\beta m'_\beta} \quad (10)$$

and the matrix

$$K = Ff^\dagger. \quad (11)$$

TABLE I. The intrinsic quadrupole moments of $\langle Q_0^2\rangle$ for Ti, Cr, Fe, and Ni isotopes calculated for different nucleon-nucleon effective interactions. The values of the intrinsic quadrupole moments are given here in units of ν^2.

Nucleus	$\langle Q_0^2\rangle$ (Present calculations) H-J	Yale	Tabakin 1	Tabakin 2	Reid	A	ν^2 (fm^2)
^{44}Ti	13.2	13.3	13.1	13.2	13.3	13.1	4.006
^{46}Ti	19.1	19.1	19.3	19.2	19.2	19.3	4.006
^{48}Ti	14.3	14.2	14.3	14.1	14.2	14.1	4.006
^{50}Ti	2.5	2.5	2.4	2.4	2.5	2.4	4.006
^{50}Cr	27.4	27.3	27.2	27.2	27.3	27.4	4.108
^{52}Cr	25.2	25.3	25.2	25.1	25.3	25.1	4.108
^{54}Cr	5.3	5.2	5.3	5.1	5.1	5.2	4.108
^{56}Cr	26.4	26.3	26.5	26.6	26.6	26.4	4.108
^{52}Fe	19.2	19.3	19.1	19.4	19.1	19.4	4.232
^{54}Fe	3.4	3.4	3.4	3.6	3.5	3.6	4.232
^{56}Fe	25.7	25.6	25.6	25.5	25.4	25.5	4.232
^{58}Fe	26.6	26.7	26.6	26.8	26.7	26.5	4.232
^{58}Ni	4.8	4.9	4.8	4.7	4.9	4.8	4.314
^{60}Ni	9.6	9.7	9.5	9.3	9.6	9.6	4.314
^{62}Ni	9.3	9.2	9.4	9.1	9.3	9.2	4.314
^{64}Ni	15.8	15.9	15.7	15.8	15.7	15.9	4.314

The projected state with good angular momentum J from Eq. (7) can be written as

$$|\psi^J_{MM'}\rangle = P^J_{MM'}|\phi\rangle = \frac{2J+1}{8\pi^2}\int D^{J*}_{MM'}(\Omega)R(\Omega)d\Omega. \tag{12}$$

We then obtain

$$\langle\psi^J_{MM'}|Q_0{}^2|\psi^{J'}_{MM'}\rangle = \frac{2\sqrt{2}}{(2J'+1)\sqrt{7}}a_J{}^2a_{J'}{}^2\int_0^\pi\sum_\mu\begin{bmatrix} J & 2 & J' \\ -\mu & \mu & 0\end{bmatrix}d^J_{\mu 0}(\theta)n(\theta)\nu^2 e\Big[\sum_{\alpha\beta\tau_3}e_{\tau_3}\langle\alpha|Q_\mu{}^2|\beta\rangle\rho^{\tau_3}_{\alpha\beta}(\theta)\Big]\sin\theta d\theta. \tag{13}$$

The quadrupole moment for the first excited states is given by

$$Q_{2^+} = \langle\psi_{22}{}^2|Q_0{}^2|\psi_{22}{}^2\rangle \tag{14}$$

and the reduced transition probabilities are given as

$$B(E2)(0^+\to 2^+) = \frac{1}{16\pi}|\langle\psi_0{}^2||Q^2||\psi_0{}^2\rangle|^2, \tag{15}$$

with

$$Q_\mu{}^2 = \left(\frac{16\pi}{5}\right)^{1/2}\frac{r^2}{\nu^2}Y_\mu{}^2(\Omega) \tag{16}$$

and the normalizations or the intensities of the various angular momentum states are given by

$$a_J{}^2 = \tfrac{1}{2}(2J+1)\int_0^\pi n(\theta)d^J_{00}(\theta)\sin\theta d\theta \tag{17}$$

and where ν is the oscillator length parameter.

III. CALCULATIONS AND DISCUSSION

The Hartree-Fock-Bogoliubov method has been applied here for different even-even nuclei. We have considered the different even-even isotopes of the nuclei Ti, Cr, Fe, and Ni. Firstly, the wave function $|\phi\rangle$ is obtained by solving the HFB equation by iteration.[15] In our present calculations, only the pairing correlations between identical particles have been included. The calculations are applied to the isotopes ^{44,46,48,50}Ti, ^{48,50,52,54}Cr, ^{52,54,56,58}Fe, and ^{58,60,62,64}Ni. These calculations are carried out using the deformed Hartree-Fock-Bogoliubov calculations employing the effective interaction. The calculations are performed when the effective interaction is derived from different nuclear nucleon-nucleon potentials. The potentials considered here are the Hamada-Johnston,[1] Yale,[2] Tabakin 1[4] (with Yamaguchi[20] parameters), Tabakin 2[4] (with Breit *et al.*[21] parameters), Reid[5]

TABLE II. The $B(E2)$ $(0^+\to 2^+)$ values for Ti, Cr, Fe, and Ni isotopes calculated for different nucleon-nucleon effective interactions. The isoscalar effective charge is taken as $e = 0.6$. The values of $B(E2)$ $(0^+\to 2^+)$ are given here in units of $10^{-50}\ e^2\,\mathrm{cm}^4$.

	$B(E2)$ $(0^+\to 2^+)$ (Present calculations)							ν^2
Nucleus	H-J	Yale	Tabakin 1	Tabakin 2	Reid	A	$B(E2)$ (exp)	(fm^2)
^{44}Ti	6.5	6.6	6.8	6.7	6.5	6.6	···	4.006
^{46}Ti	6.2	6.3	6.5	6.6	6.4	6.2	8.0±1.7	4.006
^{48}Ti	4.8	4.9	4.6	4.5	4.7	4.8	7.0±1.4	4.006
^{50}Ti	0.8	0.7	0.9	0.8	0.7	0.6	3.2±0.8	4.006
^{48}Cr	14.6	14.8	14.5	14.7	14.6	14.7	···	4.108
^{50}Cr	13.9	13.8	13.9	13.8	13.7	13.7	12.0±0.8	4.108
^{52}Cr	6.2	6.4	6.3	6.2	6.1	6.2	6.7±0.7	4.108
^{54}Cr	9.1	9.4	9.2	9.3	9.1	9.3	10.0±0.7	4.108
^{52}Fe	10.3	10.4	10.3	10.3	10.1	10.2	···	4.232
^{54}Fe	1.9	1.8	1.7	1.9	1.6	1.6	5.1±0.5	4.232
^{56}Fe	6.8	6.7	6.9	6.7	6.8	6.8	9.0±1.0	4.232
^{58}Fe	9.3	9.6	9.4	9.3	9.6	9.5	13.0±3.0	4.232
^{58}Ni	5.8	5.7	5.9	5.8	5.8	5.7	7.3±0.2	4.314
^{60}Ni	8.4	8.3	8.4	8.4	8.3	8.2	9.3±0.3	4.314
^{62}Ni	7.8	7.7	7.8	7.7	7.9	7.9	8.8±0.3	4.314
^{64}Ni	7.8	7.7	7.8	7.7	7.9	7.9	8.4±0.5	4.314

TABLE III. The $B(E2)$ $(0^+ \rightarrow 2^+)$ values for Ti, Cr, Fe, and Ni isotopes calculated for different nucleon-nucleon effective interactions. The isoscalar effective charge is taken as $e = 0.7$. The values of $B(E2)$ $(0^+ \rightarrow 2^+)$ are given here in units of $10^{-50}\ e^2\ \mathrm{cm}^4$.

	$B(E2)$ $(0^+ \rightarrow 2^+)$ (Present calculations)							ν^2
Nucleus	H-J	Yale	Tabakin 1	Tabakin 2	Reid	A	$B(E2)$ (exp)	(fm^2)
^{44}Ti	7.5	7.3	7.4	7.5	7.4	7.3	···	4.006
^{46}Ti	6.9	6.7	6.8	6.9	6.8	6.9	8.0 ± 1.7	4.006
^{48}Ti	5.3	5.2	5.4	5.3	5.1	5.2	7.0 ± 1.4	4.006
^{50}Ti	0.5	0.7	0.5	0.6	0.6	0.7	3.2 ± 0.8	4.006
^{48}Cr	16.3	16.5	16.3	16.4	16.5	16.4	···	4.108
^{50}Cr	15.3	15.1	15.2	15.3	15.2	15.1	12.0 ± 0.8	4.108
^{52}Cr	7.1	7.3	7.1	7.2	7.2	7.3	6.7 ± 0.7	4.108
^{54}Cr	9.6	9.5	9.6	9.7	9.6	9.5	10.0 ± 0.7	4.108
^{52}Fe	11.4	11.3	11.4	11.2	11.4	11.2	···	4.232
^{54}Fe	2.1	2.0	2.1	2.2	2.1	2.2	5.1 ± 0.5	4.232
^{56}Fe	7.2	7.1	7.3	7.1	7.2	7.3	9.0 ± 1.0	4.232
^{58}Fe	9.8	9.9	9.8	9.7	9.8	9.7	13.0 ± 3.0	4.232
^{58}Ni	6.4	6.6	6.4	6.6	6.6	6.5	7.3 ± 0.2	4.314
^{60}Ni	9.1	9.2	9.3	9.1	9.2	9.3	9.3 ± 0.3	4.314
^{62}Ni	8.8	8.9	8.8	8.7	8.9	8.6	8.8 ± 0.3	4.314
^{64}Ni	8.5	8.4	8.5	8.3	8.4	8.3	8.4 ± 0.5	4.314

potentials and also the potential A calculated by us[6] as the meson exchange process. For all the nuclei considered here, the lowest energy minima are seen to have prolate deformations. In Table I, the quadrupole moments of the intrinsic states have been introduced. The values of ν^2 in the different nuclei have been obtained from the root mean square radii employing the uniform density model. The effective charges e_{τ_3} are used such that the effective charge for protons is $e_p = 1 + e$, and for neutrons it is $e_n = e$. The calculated reduced transition probability values $B(E2)(0^+ \rightarrow 2^+)$ for the

TABLE IV. The Q_{2^+} values for Ti, Cr, Fe, and Ni isotopes calculated for different nucleon-nucleon effective interactions. The isoscalar effective charge is taken as $e = 0.6$. The values of Q_{2^+} are given here in units of $e\,\mathrm{fm}^2$.

	Q_{2^+} (Present calculations)							ν^2
Nucleus	H-J	Yale	Tabakin 1	Tabakin 2	Reid	A	Q_{2^+}(exp)	(fm^2)
^{44}Ti	−12.5	−12.7	−12.5	−12.6	−12.7	−12.6	···	4.006
^{46}Ti	−16.5	−16.3	−16.4	−16.6	−16.4	−16.3	−19 ± 10	4.006
^{48}Ti	−9.1	−9.2	−9.3	−9.1	−9.2	−9.3	−22 ± 8	4.006
^{50}Ti	−6.3	−6.2	−6.3	−6.4	−6.5	−6.6	−2 ± 9	4.006
^{48}Cr	−25.6	−25.4	−25.6	−25.4	−25.4	−25.5	···	4.108
^{50}Cr	−23.4	−23.1	−23.4	−23.1	−23.3	−23.2	···	4.108
^{52}Cr	−15.6	−15.7	−15.6	−15.8	−15.7	−15.8	···	4.108
^{54}Cr	−18.2	−18.3	−18.1	−18.2	−18.2	−18.3	···	4.108
^{52}Fe	−21.4	−21.5	−21.4	−21.5	−21.6	−21.6	···	4.232
^{54}Fe	−6.2	−6.1	−6.2	−6.3	−6.1	−6.2	···	4.232
^{56}Fe	−15.7	−15.5	−15.7	−15.6	−15.6	−15.5	···	4.232
^{58}Fe	−17.3	−17.2	−17.3	−17.2	−17.3	−17.1	···	4.232
^{58}Ni	−4.4	−4.2	−4.3	−4.2	−4.3	−4.2	−12 ± 13	4.314
^{60}Ni	−1.6	−1.6	−1.5	−1.6	−1.5	−1.5	0 ± 13	4.314
^{62}Ni	−7.3	−7.3	−7.4	−7.5	−7.5	−7.4	−8 ± 17	4.314
^{64}Ni	−4.2	−4.2	−4.1	−4.2	−4.2	−4.1	···	4.314

TABLE V. The Q_{2^+} values for Ti, Cr, Fe, and Ni isotopes calculated for different nucleon-nucleon effective interactions. The isoscalar effective charge is taken as $e = 0.7$. The values of Q_{2^+} are given here in units of $e\,\mathrm{fm}^2$.

Nucleus	Q_{2^+} (Present calculations) H-J	Yale	Tabakin 1	Tabakin 2	Reid	A	Q_{2^+}(exp)	ν^2 (fm^2)
^{44}Ti	−13.3	−13.5	−13.4	−13.3	−13.5	−13.6	···	4.006
^{46}Ti	−17.1	−17.3	−17.2	−17.3	−17.1	−17.2	−19 ± 10	4.006
^{48}Ti	−9.9	−9.8	−9.7	−9.9	−9.8	−9.7	−22 ± 8	4.006
^{50}Ti	−8.2	−8.2	−8.1	−8.3	−8.2	−8.1	−2 ± 9	4.006
^{48}Cr	−26.7	−26.8	−26.8	−26.9	−26.7	−26.9	···	4.108
^{50}Cr	−24.6	−24.4	−24.5	−24.7	−24.5	−24.4	···	4.108
^{52}Cr	−16.7	−16.8	−16.7	−16.9	−16.8	−16.9	···	4.108
^{54}Cr	−18.8	−16.6	−16.6	−16.8	−16.7	−18.7	···	4.108
^{52}Fe	−22.9	−22.7	−22.7	−22.8	−22.8	−22.9	···	4.232
^{54}Fe	−7.4	−7.3	−7.2	−7.4	−7.3	−7.2	···	4.232
^{56}Fe	−15.7	−15.8	−15.7	−15.9	−15.8	−15.9	···	4.232
^{58}Fe	−17.6	−17.4	−17.5	−17.4	−17.5	−17.6	···	4.232
^{58}Ni	−4.6	−4.4	−4.4	−4.6	−4.5	−4.5	−12 ± 13	4.314
^{60}Ni	−1.7	−1.9	−1.9	−1.8	−1.8	−1.7	0 ± 13	4.314
^{62}Ni	−8.1	−8.2	−8.1	−8.3	−8.2	−8.3	−8 ± 17	4.314
^{64}Ni	−4.3	−4.1	−4.3	−4.1	−4.2	−4.2	···	4.314

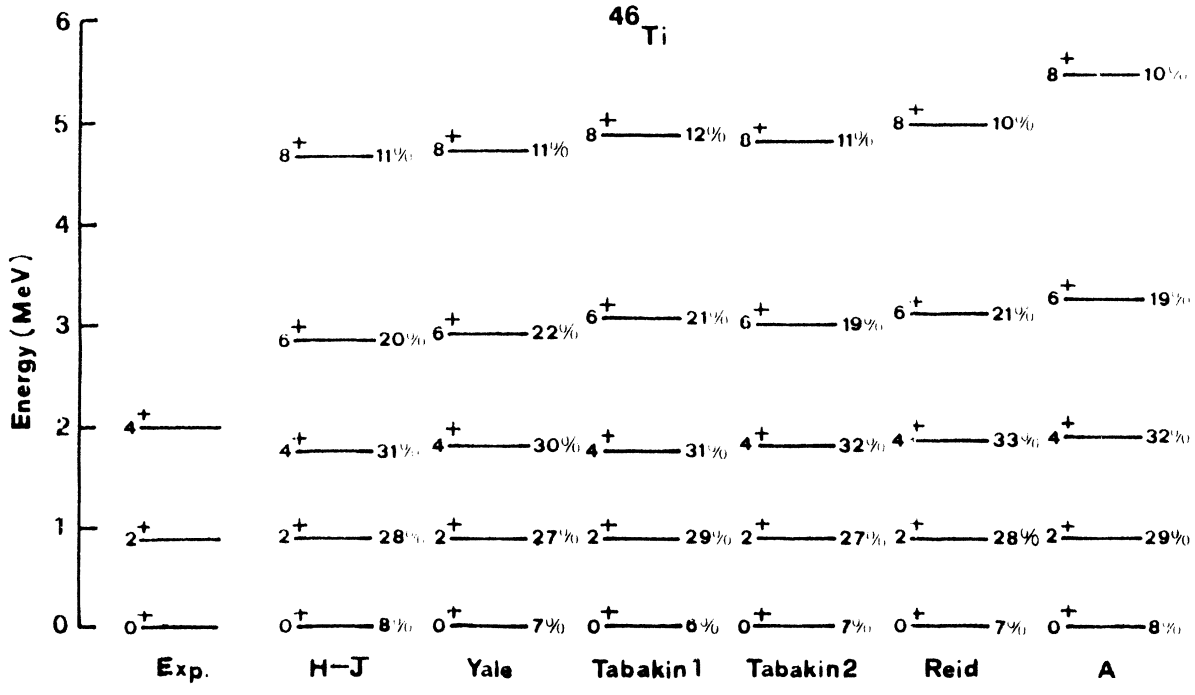

FIG. 1. The experimental energy levels of ^{46}Ti and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

considered nuclei are given in Table II for $e = 0.6$ and in Table III for $e = 0.7$. The experimental values of $B(E2)$ $(0^+ \rightarrow 2^+)$ are also given in Tables II and III for the purpose of comparison and are taken from Ref. 16. The quadrupole moments for the first excited states Q_{2^+} for these nuclei have also been calculated and the results are tabulated in Tables IV and V for $e = 0.6$ and for $e = 0.7$, respectively. The experimental values for Q_{2^+} given in Tables IV and V are taken from Ref. 22 for the Ti isotopes and from Ref. 23 for the Ni isotopes.

From the HFB results we set up the f matrix present in Eq. (11). With this f matrix we evaluate F, K, and $1/(1+K)$ for each θ. Then, the projected energies E_J are computed from Eqs. (6)–(11). The single-particle energies ϵ_α were taken from the ^{41}Ca spectrum.[24] The projected HFB spectra for the ^{46,48,50}Ti, ^{48,50,52,54}Cr, and ^{52,54,56,58}Fe isotopes are shown in Figs. 1–11, respectively. In these figures also the experimental spectra are shown for the purpose of comparison. The experimental spectra are taken from Ref. 25. The percentage distributions a_J^2 of the various angular momenta in the deformed HFB intrinsic states are given for each state in each case and are shown in the figures.

It has been shown by Sharma and Bhatt[26] that the tendency of the Kuo-Brown (KB) effective interaction is to give rise to large deformations, whenever N or $Z = 28$, which makes it inconsistent with the observed $B(E2)$ $(0^+ \rightarrow 2^+)$ systematics in the $2p$-$1f$ shell. Then the improved versions of the KB interactions also give rise to an excited deformed HF solution which lies at about 6 MeV excitations relative to the spherical solution.[27]

From the present calculations we notice that the $B(E2)$ $(0^+ \rightarrow 2^+)$ values for the considered isotopes exhibit a sudden drop when either N or $Z = 28$, which is in agreement with the experimental results. Also, the $B(E2)$ $(0^+ \rightarrow 2^+)$ values calculated with an effective charge $e = 0.7$ are in very good agreement with experiment.

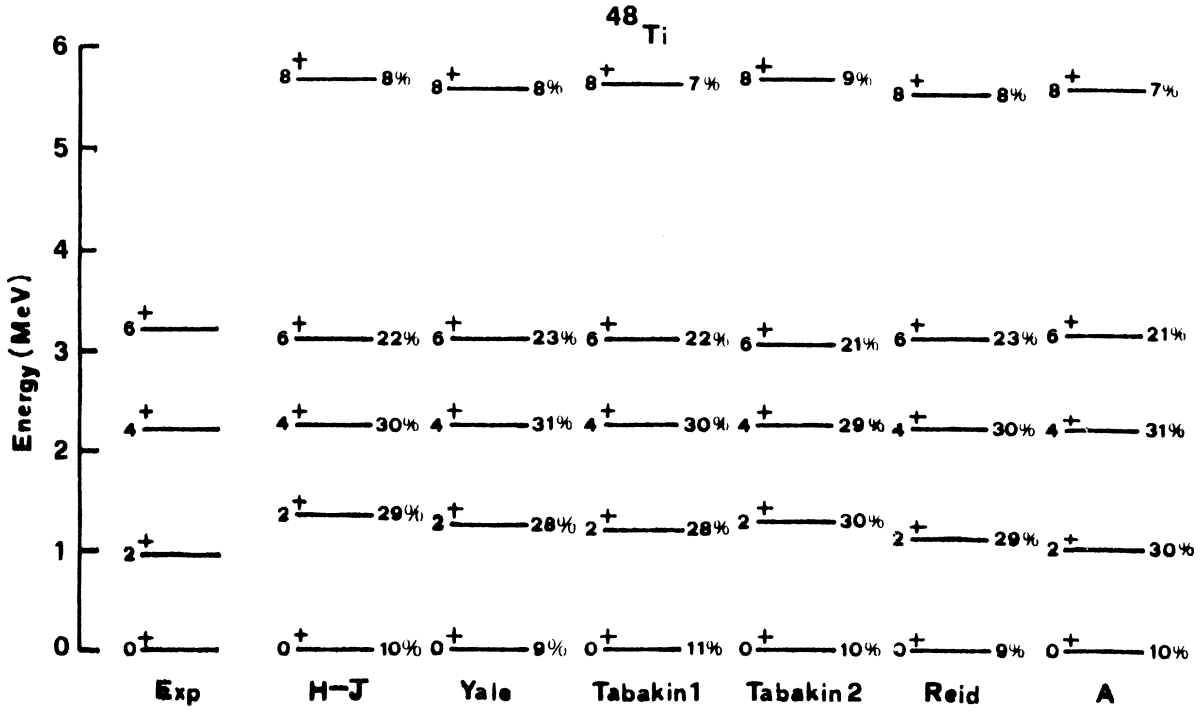

FIG. 2. The experimental energy levels of ^{48}Ti and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

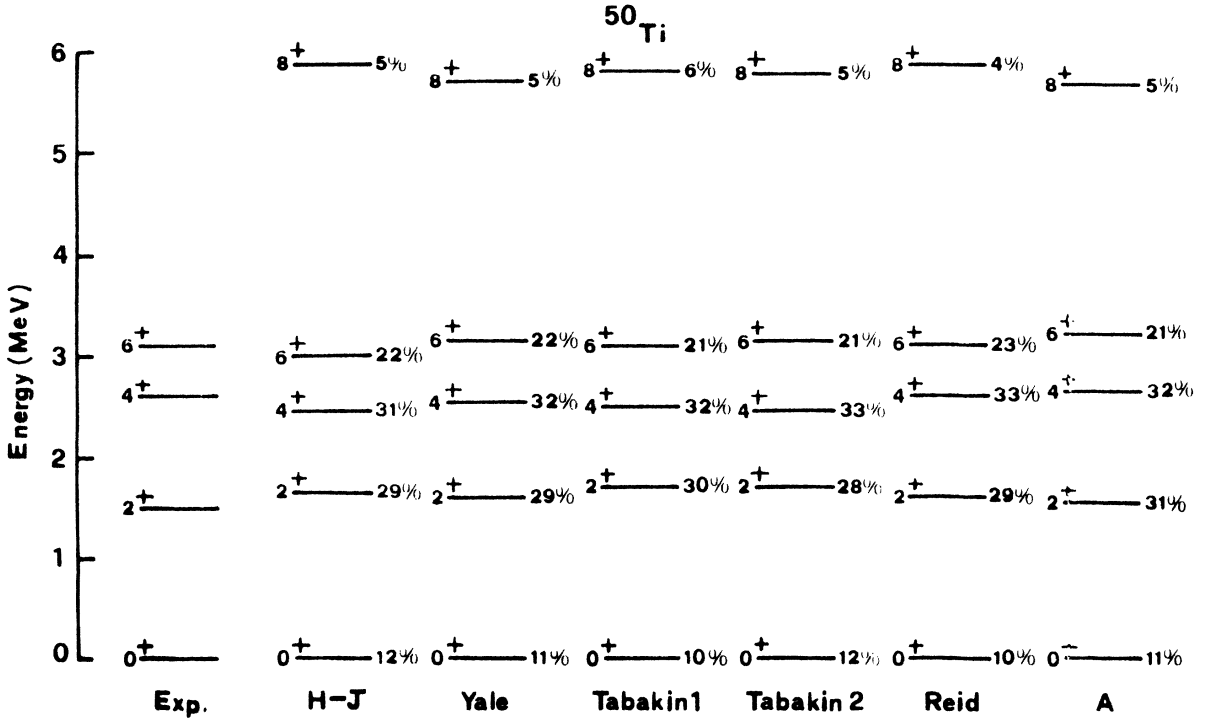

FIG. 3. The experimental energy levels of ^{50}Ti and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

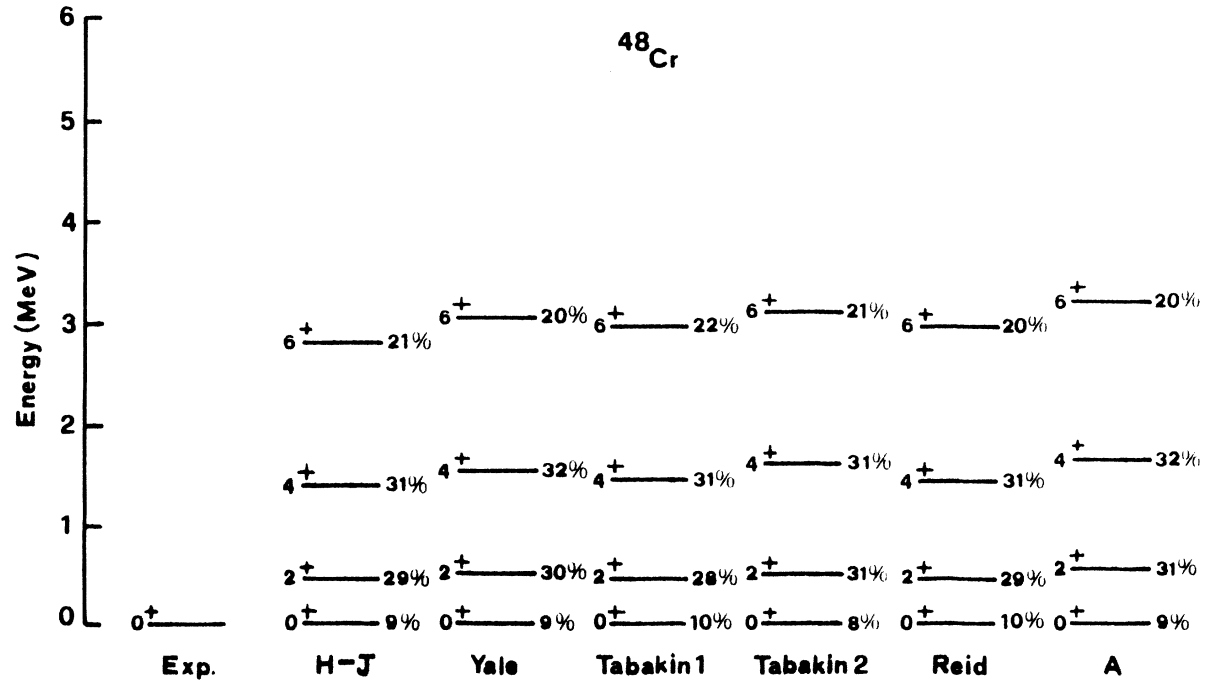

FIG. 4. The experimental energy levels of ^{48}Cr and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

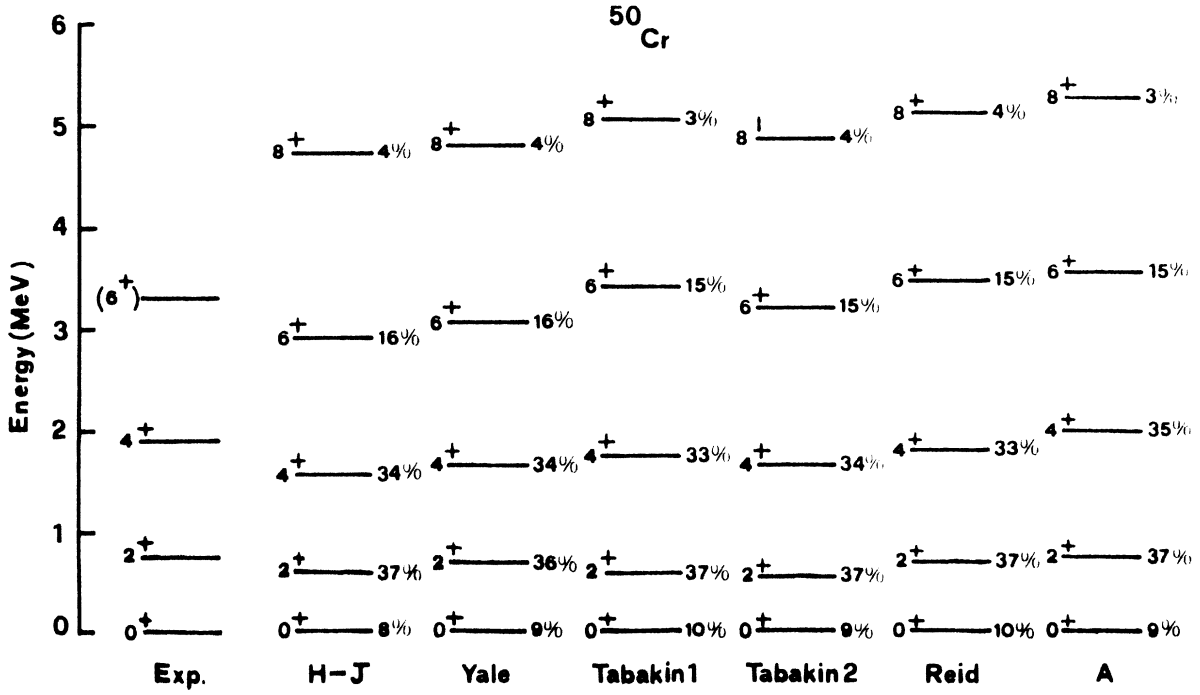

FIG. 5. The experimental energy levels of ^{50}Cr and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

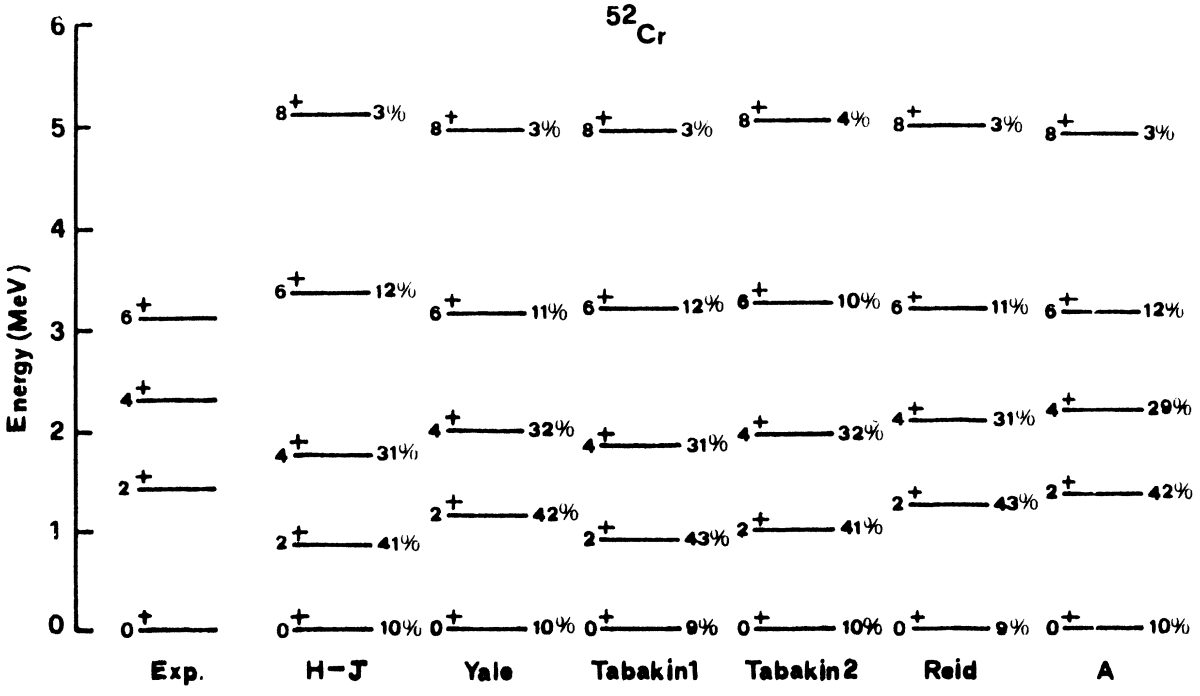

FIG. 6. The experimental energy levels of ^{52}Cr and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

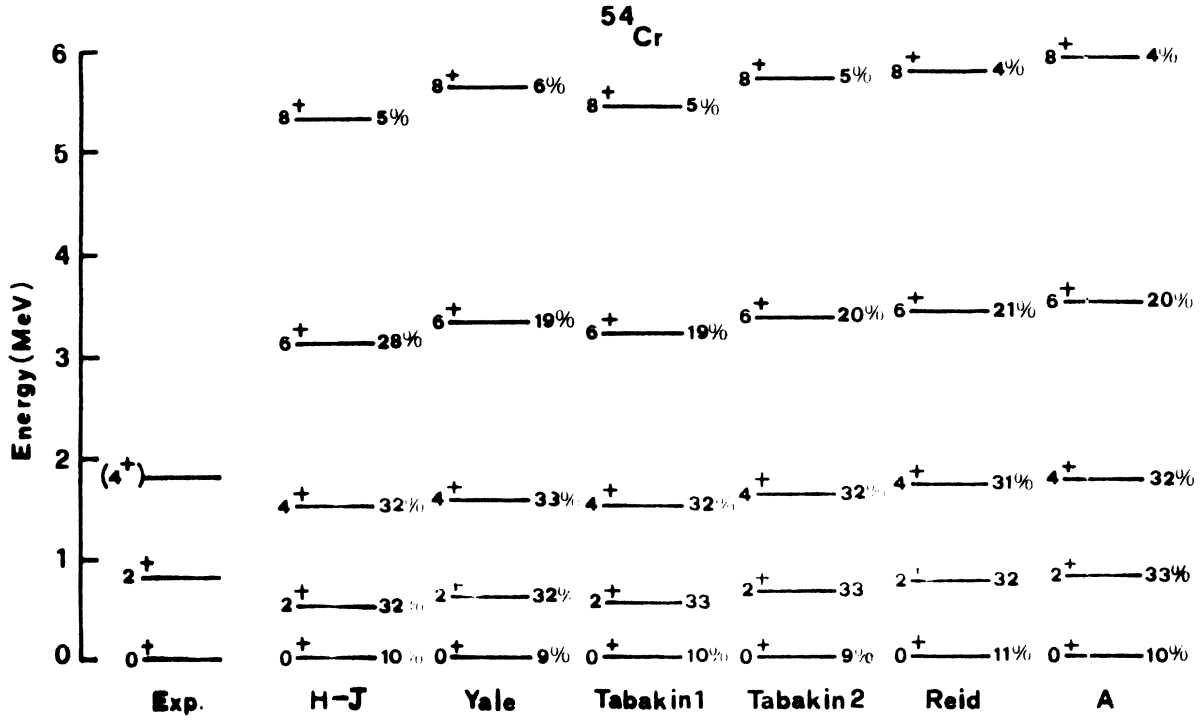

FIG. 7. The experimental energy levels of ^{54}Cr and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

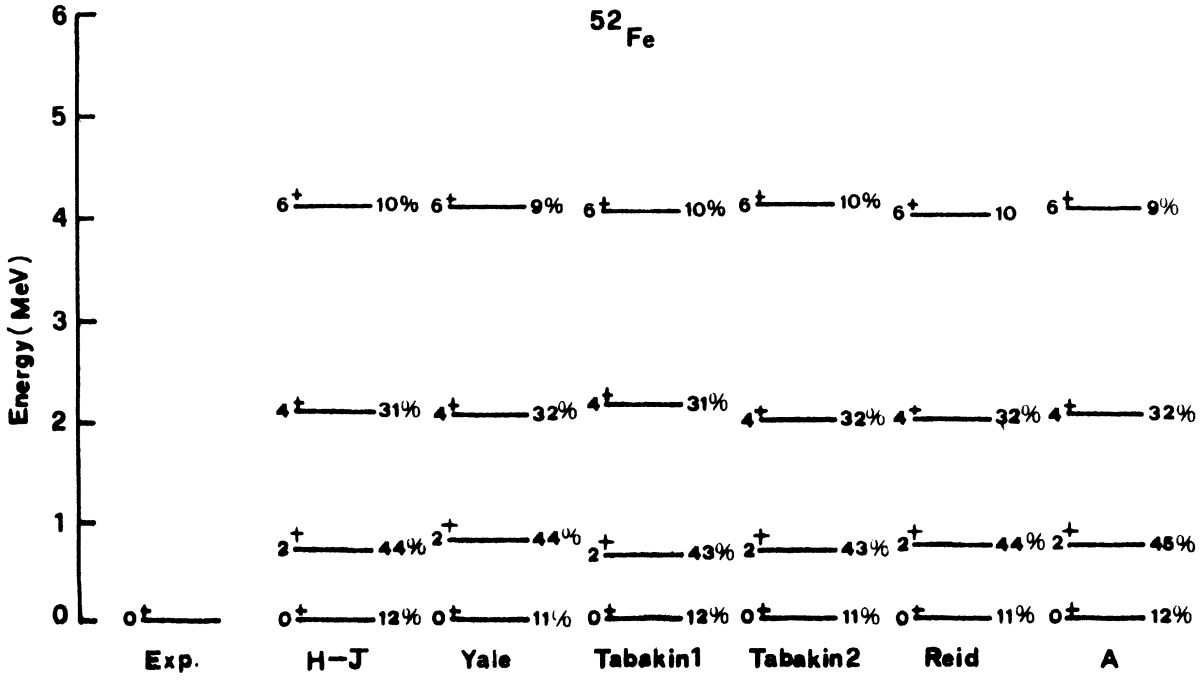

FIG. 8. The experimental energy levels of ^{52}Fe and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

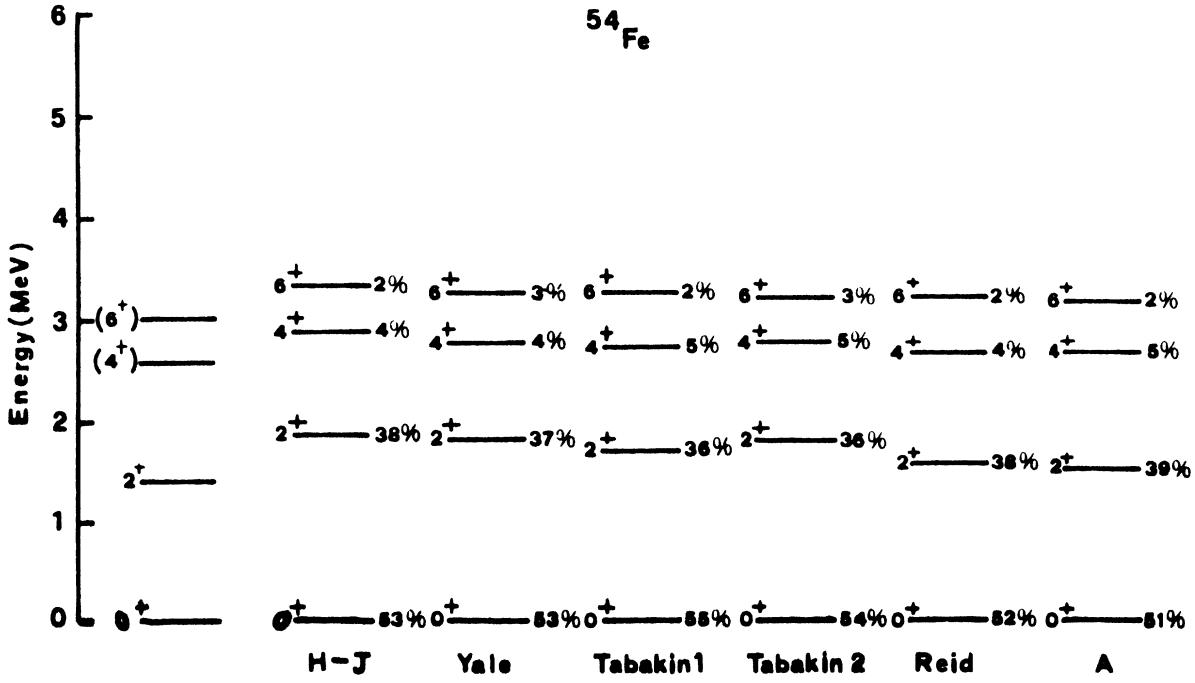

FIG. 9. The experimental energy levels of ^{54}Fe and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

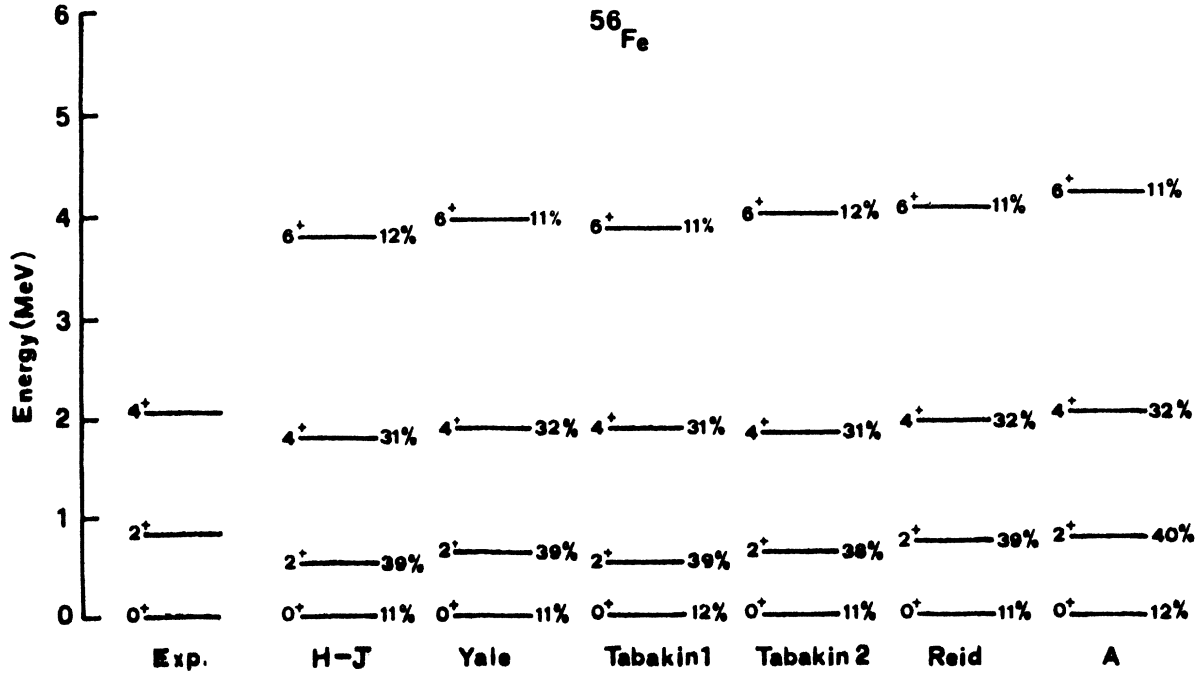

FIG. 10. The experimental energy levels of ^{56}Fe and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

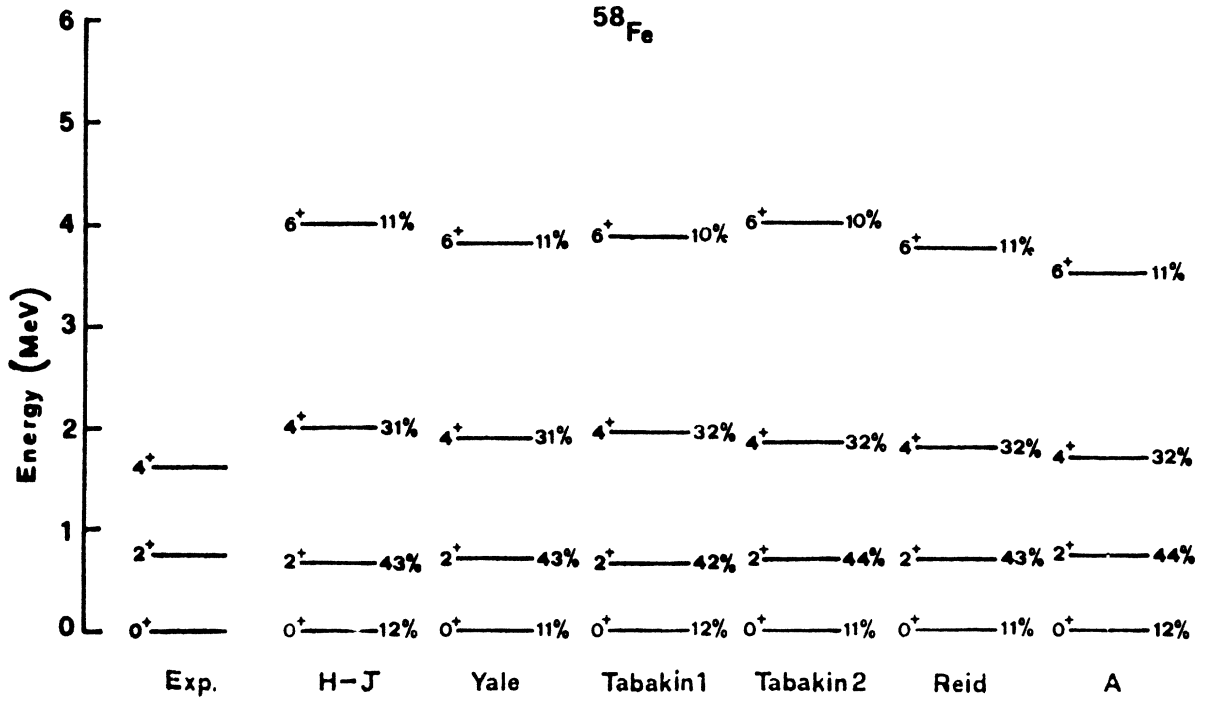

FIG. 11. The experimental energy levels of ^{58}Fe and the presently calculated projected HFB spectra with different effective interactions. The percentages give the probability a_J^2 of each angular momentum state in the HFB wave function.

Thus, in the framework of the projected HFB method employing the effective interactions in the $2p$-$1f$ shell, a reasonably good agreement between the calculated and the experimental $B(E2)$ $(0^+ \rightarrow 2^+)$ values is obtained for a large number of even-even nuclei. Also, it is possible to reproduce reasonably well the experimental levels of the Ti, Cr, and Fe isotopes.

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