Effective charges and $E2$ transitions in $1f$-$2p$ shell nuclei

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The proton and neutron effective charges needed to reproduce the observed $E2$ data on the transitions between the ground state bands of the isotopes of Ti(A = 44–50), V(A = 47–51), Cr(A = 48–52), and Fe(A = 52,54) are found to be $e_p = (1.33 \pm 0.09)e$, $e_n = (0.64 \pm 0.10)e$. These charges were obtained by a least-squares fit between the deformed configuration mixing shell model calculated and the observed $B(E2)$ values for the transitions in these nuclei. These values support the charges $e_p = 1.25e$, $e_n = 0.47e$ obtained recently by Kuo and Osnes in a microscopic calculation and the charges $e_p = 1.21e$, $e_n = 0.79e$ obtained on the basis of macroscopic estimates of Bohr and Mottelson.

NUCLEAR STRUCTURE Semiempirical effective charges, $B(E2)$ values for transitions in Ti(A=44–51), V(A=47–49, 51), Cr(A=48–50, 52), and Fe(A=52, 54) nuclei calculated within deformed configuration mixed calculations based on projected Hartree-Fock theory within $(fp)^n$ space.

A considerable amount of experimental data on electric quadrupole moments and $E2$ transitions in $1f$-$2p$ shell nuclei has now become available. We have used this data in conjunction with our deformed configuration mixing (DCM) shell model calculation[1] to deduce the semiempirical effective charges appropriate for the $(fp)^n$ shell space.

Analogous to the microscopic derivation of effective interactions various attempts[1-7] have been made for obtaining the effective charges for the description of $E2$ rates. There is a large uncertainty in the values of effective charges resulting from these microscopic calculations. However, the recent calculations of Kuo and Osnes[6] for fp shell nuclei yielded average proton and neutron effective charges $e_p = 1.25e$, $e_n = 0.47e$. The effective charges e_p and e_n resulting from the macroscopic calculations of Bohr and Mottelson[7] are $1.21e$ and $0.79e$, respectively.

One of the ways to check the adequacy (or inadequacy) of these effective charges in reproducing the $E2$ transition data would mean doing *exact* shell model calculations within the chosen model space. Owing to the unmanageably large matrix dimensionalities, the exact shell model calculations in $(fp)^n$ space have thus far been limited[8-10] to the nuclei with $A \leq 44$. Two sets of effective charges $e_p = 1.2e$, $e_n = 0.5e$ and $e_p = 1.5e$, $e_n = 0.5e$ were used in these calculations. However, the agreement of these calculations with the experiment was not very good mainly because the low-lying states of fp shell nuclei in the neighborhood of ^{40}Ca contain large admixtures of the highly deformed "core" excited states. Earlier we showed[11] however, that for the transitions between the states projected from the lowest energy Hartree-Fock (HF) states of the even-even isotopes of Ti, Cr, and Fe the charges $e_p = 1.5e$, $e_n = 0.5e$ provide a reasonable agreement with the experimental data. A least-squares fit between the experimental and calculated $B(E2, 2 \to 0)$ values in these nuclei yielded[12] effective charges $e_p = (1.32 \pm 0.16)e$, $e_n = (0.89 \pm 0.18)e$. These charges also provide a reasonable description for some of the transitions[12-17] in odd-A isotopes of Ti, V, and Cr.

Recent shell model calculations[18] within the $(f_{7/2})^n$ configuration space show that effective charges as large as $e_p = 1.9e$, $e_n = 0.9e$ are required for the description of $E2$ rates in $f_{7/2}$ shell nuclei. The need for such large effective charges in the $(f_{7/2})^n$ configuration model calculations clearly indicates the importance of deformation in these nuclei.

The deformed configuration mixing calculations based on project Hartree-Fock (PHF) theory[19-23] within the $(fp)^n$ space take into account the effects of the deformation of the valence nucleons in the fp shell space. These calculations are also found to be quite successful[12-17,24-26] in describing the energy spectra of the low-lying states of fp shell nuclei. In all these calculations the modified Kuo-Brown effective interaction[8,27] labeled MWH2 was employed.

We have used the wave functions obtained in our DCM calculations to calculate the $B(E2)$ values for a number of transitions in the various fp shell nuclei.

The operator corresponding to $E2$ transitions is

$$Q_M{}^2 = e_p\left(\sum_{i=1}^{Z} r_i{}^2 Y_M{}^2(\theta_i, \phi_i)\right)_p + e_n\left(\sum_{j=1}^{N} r_j{}^2 Y_M{}^2(\theta_j, \phi_j)\right)_n,$$

where $Y_M{}^2(\theta, \phi)$ are the usual spherical harmonics of rank 2, p and n stand for proton and neutron,

TABLE I. The contributions M_p and M_n due to protons and neutrons to the reduced matrix element of the electric quadrupole operator and the $B(E2)$ values for the transition between the members of the ground state bands of the states projected from the lowest energy Hartree-Fock intrinsic states of the even-even fp shell nuclei. The values of the transitions marked by asterisks are used in the least-squares fitting calculations. The symbol @ indicates sharp deviations of our calculated $B(E2)$ values from the observed values.

Nucleus	Transition $J' \to J$		M_p (fm^2)	M_n	$B(E2)$ (e^2fm^4) Present[b]	$(f_{7/2})^{n}$[a]	Expt.	Refs.[c]
^{44}Ti	2	0	12.0	12.0	117	107	120 ±30*	30
							117 ±25	32[d]
							157 ±22	31
	4	2	18.9	18.9	154	135	252 ±75	31,32
							280 ±60	30,32
	6	4	22.7	22.7	154	59	157 ±22*	31,32
	8	6	24.6	24.6	137	61		
^{46}Ti	2	0	11.8	16.5	138	116	160 ±34*	36[e]
							171 ±8	37
							209 ±12	32
							214 ±20	34
							217 ±17	33
	4	2	18.6	26.2	191	128	177 ±20*	32,33
	6	4	22.5	32.5	198	110	150 ±80*	32,33
	8	6	24.2	37.6	186	122		
^{48}Ti	2	0	10.6	14.5	109	101	140 ±28*	36
							146 ±24	38
							138 ±12	34,39
							142 ±8	32
							151 ±18	40
	4	2	17.0	22.6	153@	126	95 ±22*	32
	6	4	20.1	28.6	156@	76	53 ±5*	31,32
	8	6	19.7	34.4	137	80		
^{50}Ti	2	0	−10.1	−4.8	55	77	66 ±8*	31,32,34
							48 ±4	41
							49 ±8	40,42
							63 ±6	37
	4	2	−13.5	−10.2	66	77	60 ±12*	31,32
	6	4	−12.0	−18.9	61	35	34.2 ±1.2*	31,32
^{48}Cr	2	0	17.0	17.0	225	152	207 ±27*	43
							350 ±100	32,40[d]
	4	2	27.1	27.1	316	184	317 ±176	43
							330 ±190	40
							210 ±120	32
	6	4	33.5	33.5	335	187		
	8	6	38.1	38.1	331	196	>320*	43
^{50}Cr	2	0	15.9	15.1	190	136	208 ±23*	33,44–46
							204 ±8	37
							213 ±12	32
							227 ±20	31
							229 ±12	33
	4	2	25.1	24.1	264@	176	160 ±20*	32,33
	6	4	30.6	30.4	278@	142	130 ±30*	32,33
	8	6	34.1	35.4	272	151		
^{52}Cr	2	0	14.3	4.4	95	105	96 ±4*	41
							113 ±10	40,47
							115 ±7	32
							119 ±7	46
							132 ±6	37
							86	42
	4	2	21.2	7.6	121@	117	83 ±17*	31

TABLE I. (*Continued*)

Nucleus	Transition $J'\to J$		M_p (fm^2)	M_n	$B(E2)$ (e^2 fm^4) Present[b]	$(f_{7/2})^n$ [a]	Expt.	Refs.[c]
							79 ± 17	32
	6	4	24.1	12.1	122@	97	59.5 ± 3.4	31, 32
	8	6	25.7	16.1	116	75		
^{52}Fe	2	0	14.0	14.0	153	119		
	4	2	22.1	22.1	210	151		
	6	4	35.6	17.1	214	66		
	8	6	29.5	29.5	198	68		
^{54}Fe	2	0	13.1	3.3	77	81	102 ± 4*	41
							108 ± 10	40
							122	42
	4	2	18.8	6.8	96	81	78 ± 16*	31, 32
							≤147	42
	6	4	18.3	12.5	80@	37	40 ± 0.5	31, 32

[a] Effective charges $e_p = 1.9e$, $e_n = 0.9e$. See Ref. 18.
[b] $B(E2)$ values calculated between the members of the ground state bands obtained by projected Hartree-Fock calculation. The least-squares fitted effective charges $e_p = 1.33e$, $e_n = 0.64e$ are used in these calculations.
[c] References to the experimental $B(E2)$ values.
[d] Also see Ref. 29.
[e] Also see Ref. 35.

and e's are the corresponding effective charges.

The $B(E2, J'\to J)$ value for the transition from an eigenstate J' to the state J can be expressed as:

$$B(E2, J'\to J) = \frac{1}{2J'+1}\,|\langle J \| Q^2 \| J'\rangle|^2 = \frac{1}{2J'+1}\,|M_p e_p + M_n e_n|^2,$$

where M_p and M_n are the contributions from the protons and neutrons to the reduced matrix element of the quadrupole operator. The matrix elements of r^2 have been evaluated by calculating the oscillator length parameter from the relation $\hbar\omega = 41A^{-1/3}$ MeV.

In Tables I and II are given the values of the contributions M_p and M_n for the $E2$ transitions in some of the even-even and odd isotopes of the fp shell nuclei. For the even-even nuclei the wave functions used for the states J' and J were the ones projected from the HF state of a nucleus and the mixing of configurations was ignored. Full deformed configuration mixed wave functions were used for the odd isotopes.

We can now determine the semiempirical effective charges e_p and e_n by making least-squares fits between calculated and about 38 well determined experimental $B(E2)$ values. The $B(E2)$ values included in the fit are indicated by an asterisk in the tables. The best fit values of the charges turned out to be:

$$e_p = (1.33 \pm 0.09)e\,,$$

$$e_n = (0.64 \pm 0.10)e\,.$$

These are quite close to Bohr-Mottelson estimates but slightly larger than the Kuo-Osnes charges.

The $B(E2)$ values calculated using the best fitted charges are compared in Tables I and II with the experimental values and those obtained in the $(f_{7/2})^n$ configuration model[18] using $e_p = 1.9e$, $e_n = 0.9e$. A similar comparison is also made in Table III for the electric quadrupole moments of the first 2^+ states in even-even nuclei and of the first few excited states in even-odd nuclei of the fp shell. The maximum errors in our calculated $B(E2)$ values and quadrupole moments due to the errors associated with our effective charges would be less than 20% and 10%, respectively.

The agreement with the experiment of our calculated $B(E2)$ values and quadrupole moments is quite satisfactory. However, it is seen that the experimental trend of the reduction in the $B(E2)$ values for the transitions (indicated by the symbol @ in the tables) between some of the higher members of the ground state bands, particularly in even-even nuclei (^{50,52}Cr, ^{54}Fe), is not reproduced. It is likely that the inclusion of two particle-two hole excited states in the basis space or the variation-after-projection calculation might help in reducing the deformation with increasing angular momenta in the yrast bands of these nuclei.

TABLE II. The contributions M_p and M_n due to protons and neutrons to the reduced matrix element of the electric quadrupole operator and the $B(E2)$ values for transitions between the ground state bands and some of the other low-lying states of the odd isotopes of Ti, V, and Cr. The values marked with asterisk are used in the least-squares fitting calculations.

Nucleus	Transition $J' \to J$	M_p (fm^2)	M_n	$B(E2)$ (e^2 fm^4) DCM [a]	$(f_{7/2})^n$ [b]	Expt.	Refs. [c]
^{45}Ti	$\frac{3}{2}$ $\frac{7}{2}$	10.3	13.6	126	71		
	$\frac{5}{2}$ $\frac{3}{2}$	−14.9	−19.0	171	6.5		
	$\frac{5}{2}$ $\frac{7}{2}$	15.2	19.0	175	211		
	$\frac{9}{2}$ $\frac{5}{2}$	15.4	17.7	101	53		
	$\frac{9}{2}$ $\frac{7}{2}$	−13.5	−13.6	71	98		
	$\frac{11}{2}$ $\frac{7}{2}$	18.2	21.9	122	115		
	$\frac{11}{2}$ $\frac{9}{2}$	−11.9	−13.8	51	69		
	$\frac{13}{2}$ $\frac{9}{2}$	21.3	25.9	144	89		
	$\frac{13}{2}$ $\frac{11}{2}$	−12.7	−10.7	40	34		
	$\frac{15}{2}$ $\frac{11}{2}$	21.9	25.0	127	130		
	$\frac{15}{2}$ $\frac{13}{2}$	−9.8	−11.5	26	17		
	$\frac{17}{2}$ $\frac{13}{2}$	22.0	27.5	122	5		
	$\frac{17}{2}$ $\frac{15}{2}$	−11.0	−8.9	23	2		
^{47}Ti	$\frac{7}{2}$ $\frac{5}{2}$	15.4	24.7	167	115	232 ±33*	48
						252 ±44	49
	$\frac{9}{2}$ $\frac{5}{2}$	10.8	14.3	55	73	126 ±64	48
	$\frac{9}{2}$ $\frac{7}{2}$	16.6	22.5	133	31	51^{+46}_{-20}	48
	$\frac{11}{2}$ $\frac{7}{2}$	15.5	20.8	96	127	149^{+40}_{-142}*	48 49
	$\frac{11}{2}$ $\frac{9}{2}$	−14.7	−22.3	95	39		
	$\frac{13}{2}$ $\frac{9}{2}$	18.9	24.6	119	0.4		
	$\frac{13}{2}$ $\frac{11}{2}$	−15.2	−18.0	72	23		
	$\frac{15}{2}$ $\frac{11}{2}$	−19.9	−26.0	116	109	70	50
	$\frac{15}{2}$ $\frac{13}{2}$	11.2	17.0	42	9		
	$\frac{17}{2}$ $\frac{13}{2}$	20.3	31.8	125	54		
	$\frac{17}{2}$ $\frac{15}{2}$	−13.6	−14.5	42	29		
^{49}Ti	$\frac{11}{2}$ $\frac{7}{2}$	−16.3	−7.3	58	90		
	$\frac{3}{2}$ $\frac{7}{2}$	9.1	6.3	65	78	>32, <60	42,49
	$\frac{9}{2}$ $\frac{7}{2}$	16.0	12.5	86	58		
	$\frac{9}{2}$ $\frac{11}{2}$	3.7	11.5	15	0.4		
	$\frac{5}{2}$ $\frac{7}{2}$	10.8	1.3	38	57	>7.5	49
	$\frac{5}{2}$ $\frac{3}{2}$	4.8	9.8	26	45		
	$\frac{15}{2}$ $\frac{11}{2}$	−13.3	−7.9	32	47		
^{51}Ti	$\frac{7}{2}$ $\frac{3}{2}$	−14.6	−11.0	88			
	$\frac{5}{2}$ $\frac{3}{2}$	11.3	6.9	63			
	$\frac{5}{2}$ $\frac{1}{2}$	5.3	5.3	18			
	$\frac{11}{2}$ $\frac{7}{2}$	−18.2	−16.7	101		95 ±16	56
	$\frac{9}{2}$ $\frac{7}{2}$	4.2	4.3	7			

TABLE II. (*Continued*)

Nucleus	Transition $J' \rightarrow J$		M_p (fm^2)	M_n	$B(E2)$ (e^2 fm^4) DCM [a]	$(f_{7/2})^n$ [b]	Expt.	Refs. [c]
	$\frac{15}{2}$	$\frac{11}{2}$	−17.2	−19.7	78		58 ±20	56
	$\frac{13}{2}$	$\frac{9}{2}$	16.4	20.6	88			
^{47}V	$\frac{5}{2}$	$\frac{3}{2}$	19.0	22.7	264	51		
	$\frac{7}{2}$	$\frac{3}{2}$	14.7	16.7	114	50	>25	49
							312 ±121	51, 52
	$\frac{7}{2}$	$\frac{5}{2}$	19.7	21.3	195	207		
	$\frac{9}{2}$	$\frac{5}{2}$	18.5	23.3	156	45	161 ±91	51, 52
	$\frac{9}{2}$	$\frac{7}{2}$	14.1	18.7	94	78	141 ±81	53
							907 ±504	53
	$\frac{11}{2}$	$\frac{7}{2}$	−23.2	−27.6	196	134	200^{+100}_{-80}	53
	$\frac{11}{2}$	$\frac{9}{2}$	−16.3	−17.1	89	80		
	$\frac{13}{2}$	$\frac{9}{2}$	25.8	30.8	209	43		
	$\frac{13}{2}$	$\frac{11}{2}$	−11.4	−16.2	46	23		
	$\frac{15}{2}$	$\frac{11}{2}$	−27.9	−33.5	214	130	>110	53
	$\frac{15}{2}$	$\frac{13}{2}$	13.7	14.5	47	0.7		
^{48}V	5	4	25.2	29.0	246	0.7	120 ±45*	54
							$100.5^{+81.9}_{-49.7}$	55
	6	4	−11.7	−14.0	46	97	48 ±5*	54
							$41.4^{+9.3}_{-8.3}$	55
	6	5	−26.7	−31.1	236	111	⩽510	54
							$202.1^{+128.5}_{-86.0}$	55
	7	5	15.6	18.6	71	0.3		
	7	6	−24.4	−29.8	177	89		
	8	6	21.6	26.0	121	126		
	8	7	−18.2	−24.7	94	0.01	⩾44.04	55
	9	7	25.1	29.9	145	139	⩾68.2	55
	9	8	−20.3	−26.9	103	49	⩾9.12	55
	10	8	25.3	30.5	134	110		
	10	9	−15.5	−22.3	58	0.2		
	2	4	−3.9	−3.3	11	198	28.5 ±0.2*	54
	1	2	15.9	18.8	366	202		
	4	4	2.1	4.0	3	162	13.7 ±3.5	54
	4	5	−5.5	2.5	9	196	⩽110	
	3	4	−1.5	1.6	1	39		
	3	2	−12.8	−15.2	102	119	⩾0.06	54
	5	4	−10.5	−14.5	49	58	⩾1.6	54
	5	6	−4.7	−0.5	4	2		
	2	2	5.3	7.4	28	61	⩾0.6	54
^{49}V	$\frac{5}{2}$	$\frac{7}{2}$	−18.8	−18.2	223	245		
	$\frac{3}{2}$	$\frac{7}{2}$	13.9	14.6	193	116	197 ±30*	53
							197 ±20	57
	$\frac{3}{2}$	$\frac{5}{2}$	16.0	18.5	275	88		
	$\frac{11}{2}$	$\frac{7}{2}$	−21.1	−24.6	160	119	144 ±28*	57
							172 ±59	58
							200 ±90	53
	$\frac{9}{2}$	$\frac{7}{2}$	−9.5	−15.9	52	9	58 ±33*	57
							106 ±28	58
	$\frac{9}{2}$	$\frac{5}{2}$	−14.6	−20.4	105	29	126 ±17*	58
							83 ±44	57
	$\frac{9}{2}$	$\frac{11}{2}$	−16.0	−14.0	91	75		

TABLE II. (*Continued*)

Nucleus	Transition $J' \to J$	M_p (fm^2)	M_n	$B(E2)$ (e^2 fm^4) DCM [a]	$(f_{7/2})^{n}$ [b]	Expt.	Refs. [c]
	$\frac{15}{2}$ $\frac{11}{2}$	−25.1	−29.1	170	114	279 ±128	57
						<71	58
	$\frac{13}{2}$ $\frac{9}{2}$	21.2	25.1	140	0.3	295^{+230}_{-125}	57
	$\frac{5}{2}$ $\frac{7}{2}$	−5.2	3.9	15	0.8		
	$\frac{5}{2}$ $\frac{5}{2}$	−0.4	−7.9	5	19	>0.3, <435	57
	$\frac{5}{2}$ $\frac{3}{2}$	2.9	0.5	3	7	570^{+595}_{-260}	57
	$\frac{3}{2}$ $\frac{5}{2}$	−3.1	−1.9	7	7	<122	57
	$\frac{7}{2}$ $\frac{5}{2}$	−0.8	5.1	2	6	$4.2^{+8.8}_{-3.7}$	57
^{51}V	$\frac{5}{2}$ $\frac{7}{2}$	−19.6	−6.3	151	191	154 ±8*	59, 60
						146 ±7	49
	$\frac{3}{2}$ $\frac{5}{2}$	−11.9	−3.9	81	49	107 ±9*	59, 60
						101 ±8	49
	$\frac{3}{2}$ $\frac{7}{2}$	11.3	3.2	74	67	76 ±5*	59, 60
						72 ±13	42
	$\frac{11}{2}$ $\frac{7}{2}$	−20.2	−7.1	82	87	83 ±8*	59
						78 ±14	61
	$\frac{9}{2}$ $\frac{7}{2}$	7.8	4.1	17	32	27.5 ±6.3*	59, 61
	$\frac{9}{2}$ $\frac{5}{2}$	−9.0	−4.3	22	30	27.6 ±6.6*	61
						32.7 ±5	59
	$\frac{9}{2}$ $\frac{11}{2}$	−13.7	−4.1	43	66		
	$\frac{15}{2}$ $\frac{11}{2}$	−19.4	−7.1	58	64	66 ±5*	62
^{49}Cr	$\frac{7}{2}$ $\frac{5}{2}$	26.7	26.4	343	187	302 ±79*	63
	$\frac{9}{2}$ $\frac{5}{2}$	−16.3	−15.4	99	54	160*	63
	$\frac{9}{2}$ $\frac{7}{2}$	−27.1	26.2	279	97	310^{+260}_{-110}	63
	$\frac{11}{2}$ $\frac{7}{2}$	−24.2	−22.0	179	153	133^{+66}_{-44}	63
	$\frac{11}{2}$ $\frac{9}{2}$	24.6	25.2	199	70	505^{+620}_{-230}	63
	$\frac{13}{2}$ $\frac{9}{2}$	28.9	26.5	219	44		
	$\frac{13}{2}$ $\frac{11}{2}$	24.2	22.6	156	48		
	$\frac{15}{2}$ $\frac{11}{2}$	−26.5	−24.9	164	164		
	$\frac{15}{2}$ $\frac{13}{2}$	−15.1	−18.0	62	2		
	$\frac{17}{2}$ $\frac{13}{2}$	−33.5	−33.6	242	48		
	$\frac{17}{2}$ $\frac{15}{2}$	14.6	16.2	50	23		
	$\frac{19}{2}$ $\frac{15}{2}$	−20.1	−18.4	74	154		
	$\frac{19}{2}$ $\frac{17}{2}$	−7.4	−10.8	14	38		

[a] $B(E2)$ values calculated using deformed configuration mixed wave functions. The least-squares fitted effective charges $e_p = 1.33e$, $e_n = 0.64$ are employed.
[b] Effective charges $e_p = 1.9e$, e_n 0.93. See Ref. 18.
[c] References to experimental $B(E2)$ values.

The similarity between the semiempirical effective charges obtained by our microscopic DCM calculations and the ones obtained by Kuo-Osnes and Bohr-Mottelson indicate that the deformations of the nuclei in the first half of the fp shell, generated by the MWH2 effective interactions, are consistent with the ones required by the experimental $E2$ transitions. The $(f_{7/2})^n$ model calculations generally succeed well in reproducing the enhanced $B(E2)$ values by using the larger effective charges. It should be noted, however, that in many instances the symmetry[28] inherent in the $(f_{7/2})^n$ configuration space leads to vanishing or small $B(E2)$ values in contrast to the experimental

TABLE III. A comparison of the calculated and experimental quadrupole moments of the first 2^+ states in even-even nuclei and first few excited states in odd-A and odd-odd nuclei of the fp shell.

		Q ($e\,\mathrm{fm}^2$)			
Nucleus	J^π	Expt.	Present[a]	$(f_{7/2})^n$ [b]	Ref.[c]
^{44}Ti	2^+		−21.2	−18.1	
^{45}Ti	$\frac{7}{2}^-$	±1.5 ± 1.5	−10.8	−2.5	64
	$\frac{3}{2}^-$		15.7	−22.3	
	$\frac{5}{2}^-$		−3.6	12.7	
^{46}Ti	2^+	−21 ± 6, −19 ± 10	−23.7	13.2	37, 34
^{47}Ti	$\frac{5}{2}^-$	29 ± 1	24.2	13.9	65
	$\frac{7}{2}^-$		8.0	10.5	
^{48}Ti	2^+	−13.5 ± 8.8	−20.3	3.5	39
		−22 ± 8			34
^{49}Ti	$\frac{7}{2}^-$	24	22.6	21.0	66, 67
	$\frac{3}{2}^-$		7.5	8.2	
^{50}Ti	2^+	8 ± 16, −2 ± 9	12.4	−13.9	37, 34
^{51}Ti	$\frac{3}{2}^-$		−11.2		
^{47}V	$\frac{3}{2}^-$		19.3	16.1	
	$\frac{5}{2}^-$		−8.9	−23.7	
	$\frac{7}{2}^-$		−20.0	−7.9	
^{48}V	4^+		44.5	1.5	
	2^+		−8.8	−1.5	
	1^+		8.9	0.8	
	5^+		18.1	−2.6	
^{49}V	$\frac{7}{2}^-$		−12.4	−9.1	
	$\frac{5}{2}^-$		−10.8	−20.8	
	$\frac{3}{2}^-$		17.8	15.4	
^{51}V	$\frac{7}{2}^-$	−5.2 ± 1.0	−5.8	−7.1	68
	$\frac{5}{2}^-$		−14.2	−19.8	
	$\frac{3}{2}^-$		13.1	12.8	
^{48}Cr	2^+		30.4	0	
^{49}Cr	$\frac{5}{2}^-$		35.1	22.8	
	$\frac{7}{2}^-$		6.9	11.1	
^{50}Cr	2^+	−36 ± 7	−27.7	−11.4	37
^{52}Cr	2^+	−14 ± 8	−17.3	0	37
^{52}Fe	2^+		−24.8	−19.1	
^{54}Fe	2^+		−15.9	−14.3	

[a] Effective charges $e_p = 1.33e$, $e_n = 0.64e$.
[b] Effective charges $e_p = 1.9e$, $e_n = 0.9e$. See Ref. 18.
[c] References to the experimental data.

as well as DCM values. The DCM calculation includes the effects of seniority breaking. We have predicted the values of quadrupole moments of the first few states and $E2$ transitions in the yrast bands of a large number of fp shell nuclei. Experimental measurement of these values is quite desirable.

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