Effective interactions: Qualitative discussion of Hartree-Fock effects*

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We calculate the effective interaction for ^{18}O in a simulated Hartree-Fock basis by taking a fixed oscillator parameter of 1.7 fm for the s and p shells and a variable parameter b_u for the sd and pf shells. A central interaction is used and diagrams of first, second, and third order in V are calculated. The dominant effect is one of scaling with diagrams of second and third order all decreasing in the same exponential fashion as a function of b_u. Comparing pure oscillator ($b_u = 1.7$ fm) and simulated Hartree-Fock ($b_u = 2.0$ fm) results, we find a reduction factor of roughly 0.7, 0.7^2, and 0.7^3 for first, second, and third order, respectively. Diagrams containing a particle-particle ladder usually fall off somewhat less rapidly than the others. The implications of these model results are discussed.

[NUCLEAR STRUCTURE Effective interaction theory; model simulation of a Hartree-Fock basis.]

I. INTRODUCTION AND CALCULATION

There is at the present no established theory of effective interactions.[1] It is known from the work of Kuo and Brown[2] that the bare G-matrix elements plus corrections of second order in G produce decent results in many shell model calculations. The reasons for this, however, remain unclear, since additional effects are known to be of numerical importance. In this situation it seems useful to try to establish qualitative trends and here we shall focus on Hartree-Fock (HF) effects, i.e., the effects of using a HF unperturbed Hamiltonian rather than the commonly chosen harmonic oscillator.

Previous calculations in a HF basis[3,4] show that while the wave functions of the closed shell core can be reasonably well described as harmonic oscillators with a fixed parameter b_0, this is not satisfactory for the unoccupied orbitals. There a larger value of the oscillator parameter b_u is needed to give good overlap with the HF wave functions. This is very reasonable since these orbitals are only weakly bound and the infinite nature of the oscillator well would be expected to pull in the wave functions too much. Indeed, in many cases we are dealing with orbitals which are unbound so that we hope to approximate the important interior region of the resonant continuum wave functions. These aforementioned changes in the wave functions weaken the relevant matrix elements and thereby weaken the calculated effective interaction rather substantially.

It is thus of some interest to explore the sensitivity of the results to the wave functions employed, since the best wave functions are not *a priori* known and HF calculations with different interactions will give differing results. An example of the latter is to be found in the work of Malta and Sanderson,[4] who obtain smaller effects than those originally observed in the calculations of Ellis and Mavromatis.[3] We may also remark that smaller effects have been found by Pradhan and Shakin,[5] who employed a Woods-Saxon basis, although we must point out that their pure oscillator results through $8\hbar\omega$ are rather different from those of Vary, Sauer, and Wong.[6]

We shall study the first, second, and third order diagrams of the effective interaction in a simulated HF basis. Although this represents the first attempt to calculate the diagrams of third order in anything other than an oscillator basis, our primary interest is not in order-by-order calculations *per se*. Rather, since the HF basis is surely to be preferred over the commonly employed oscillator basis both on physical grounds and on the basis of model calculations,[7,8] we hope to obtain a more realistic view of the important physical processes at work. This may suggest the partial summation of a different class of diagrams than has hitherto been studied. In addition, the numerical estimates obtained here can be used to estimate quickly and roughly the effect of switching to a HF basis. Such a qualitative estimate may be all that is justified at the present time, since an accurate calculation would be a major undertaking.

For the purposes of a qualitative discussion of the effective interaction in ^{18}O we shall employ a simple model. For the $0s$ and $0p$ orbitals which are

occupied in the ^{16}O core we shall employ oscillator wave functions with a parameter b_0 which is fixed at 1.7 fm except as noted; this gives a reasonable radius for ^{16}O and is the usual choice made. For the ($1s0d$) and ($1p0f$) shells we shall also choose oscillator wave functions but we shall allow the parameter b_u to vary and display our results as a function of this parameter. Of course the wave functions thus defined are only orthogonal at $b_0 = b_u$. We therefore Schmidt-orthogonalize by admixing into the $1s$ wave function a small $0s$ component; this increases $\langle r^2 \rangle$ by a moderate amount (10% at $b_u = 2.0$ fm). Similarly for the $1p$ orbitals. Oscillator wave functions with $b_u = 2.0$ fm are found to overlap quite well with their HF counterparts.[3,4] Thus the comparison of results obtained with $b_u = 1.7$ and 2.0 fm will give an indication of the effect of changing from an oscillator to a HF basis. Of course this simulation of a HF basis is only rough, but it should be adequate for a qualitative discussion.

For the interaction to be used we choose a Yukawa potential of range 1.37 fm, strength 55 MeV, with a Rosenfeld exchange mixture.[9] This renders the computation of the relevant matrix elements straightforward since Wegner[10] has given an analytical expression for the Slater integrals required. We shall not of course hope, nor indeed wish, to reproduce accurately the results obtained with realistic interactions. We do however expect and, in large part find, that the qualitative trends can be carried over to the much more difficult realistic calculations.

We shall restrict our attention to the effective interaction for $J = 0$ $T = 1$ states of ^{18}O. The diagrams which we calculate are shown in Fig. 1, where we have followed the notation of Barrett and Kirson,[11] except for the cases involving a particle-particle ladder. The diagram listing is schematic. Thus we show only one member of a pair of nonfolded diagrams which together give a Hermitian contribution to the effective interaction, e.g., the "upside-down" version of diagram 12 is not explicitly shown. None of the diagrams shown contain HF or single particle potential insertions, since they cancel if a HF basis is used and we hope to simulate such a basis here. In calculating the diagrams of Fig. 1 we shall restrict the intermediate states to be those corresponding to $2\hbar\omega$ excitation energy in the pure oscillator notation [we follow the common practice of excluding the ($2s1d0g$) shell from intermediate summations]. Further, we shall take fixed energy denominators of 28 MeV so as to focus on the effect of changes in the wave functions alone. These were, in fact, found to give the dominant effect in calculations in a HF basis.[3,8] As regards intermediate states of high excitation energy, we are aware that they can give significant contributions,[6] but in such cases it is probably more reasonable to use plane wave states rather than the HF wave functions we seek to investigate here.

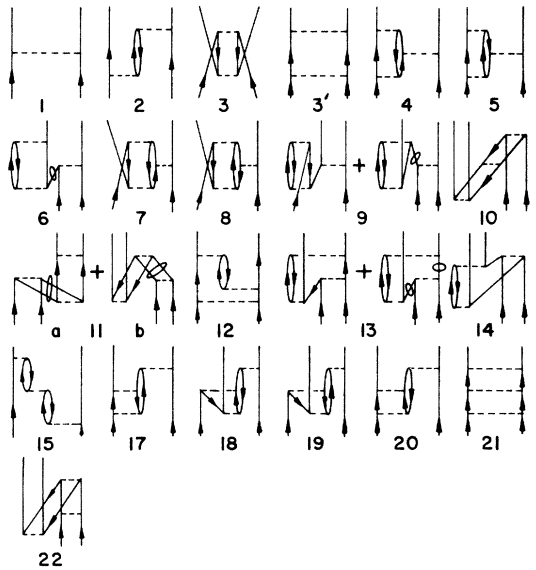

FIG. 1. Schematic listing of perturbation theory diagrams for the effective interactions through third order.

We shall ignore the question of spurious c.m. motion apart from remarking that if b_u is changed from 1.7 to 2.0 fm the value of $\langle \vec{R}^2 \rangle$ increases by about 10% for our $(sd)^2$ valence states.

In Sec. II we give a selection from the many results obtained, which is sufficient to illustrate the qualitative trends. Our conclusions are presented in Sec. III.

II. RESULTS

The two-body matrix elements needed in the calculations show a general tendency to decrease as b_u increases. This is to be expected since the wave functions are smeared out more as b_u increases and are thus less able to take advantage of the interaction. It is found that the decrease is least rapid for matrix elements which have three particles in the occupied orbitals ($0s$ or $0p$) and one in an unoccupied orbital, as would be expected. Apart from this, however, we were not able to identify any well defined trends.

In presenting the results we shall employ the usual notation (a, c) for the $\langle a^2\, J = 0\ T = 1 \,|\, V_{\text{eff}} \,|\, c^2\, J = 0\ T = 1 \rangle$ component of the effective interaction where a and c run over $4 \equiv 0d_{5/2}$, $5 \equiv 1s_{1/2}$, and $6 \equiv 0d_{3/2}$. (A subscript b_u will be added if it is necessary to distinguish the oscillator parameter.) This j-j coupling basis is useful for the purposes of comparison with other work. However, L-S coupling would be more appropriate for a central force since L and S are diagonal regardless of the order of the perturbation (provided that, as here,

the energy denominators are independent of the single particle j values). Thus only one of the (4, 5) and (6, 5) effective matrix elements is independent, since they are both proportional to $\langle d^2\,{}^3S\,|\,V_{\rm eff}\,|\,s^2\,{}^3S\rangle$. Similarly, only two of the (4, 4) (4, 6), and (6, 6) effective matrix elements are independent, since they are all related to the diagonal $d^2\,{}^3S$ and $d^2\,{}^3P$ matrix elements.

A. Bare interaction

The case of the bare interaction is less interesting than the higher order results since all the oscillator parameters involved are the same namely, b_u. We show for completeness in Fig. 2 the ratio $(a, c)_{b_u}/(a, c)_{b_u=1.7}$ on a semilogarithmic plot. Early work[12] suggested a b_u^{-2} dependence which is followed quite well by the full curve labeled DD giving the (4, 4) ratio. [Similar results are obtained for the (6, 6) and (4, 6) cases, indicated respectively by boxes and crosses, here and also in higher orders where only the (4, 4) case will be shown.] The (5, 5) case (dashed curve labeled SS) shows a faster falloff since it involves a larger amount of relative d state, and the corresponding Talmi integral drops more rapidly.

The (4, 5) case (dot-dash curve labeled SD) shows only a small change between $b_u = 1.7$ and 2.0 fm, as was found in the oscillator versus HF comparison of Ellis and Mavromatis[3] (EM). Further comparison with EM in Table I shows that the present model gives a larger change in the (4, 4) results as b_u increases from 1.7 to 2.0 fm. The reason is that the $d_{5/2}$ HF orbital is closer to a $b_u = 1.7$ fm oscillator function than the $s_{1/2}$ and $d_{3/2}$ orbitals. This is to be expected, since the $d_{5/2}$ state is the most strongly bound and since the wave function is nodeless. For the (4, 6) case the EM results are in close agreement with the present model.

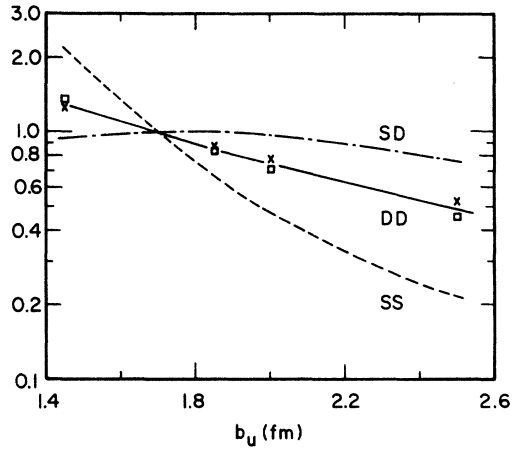

FIG. 2. The bare interaction, diagram 1 of Fig. 1, as a function of b_u; the curves are normalized to unity at $b_u = 1.7$ fm. The full curve labeled DD gives the (4, 4) matrix element and the boxes and crosses show, respectively, the (6, 6) and (4, 6) cases. The dashed curve SS gives the (5, 5) matrix elements and the dot-dashed curve SD gives the (4, 5) or (5, 6) matrix elements.

Clearly there is a fairly wide scatter in the model results, but a reduction factor of 0.7 gives a rough measure of the average behavior and is consistent with a b_u^{-2} dependence.

B. Second order diagrams

In Fig. 3 we display results for the core polarization diagram (number 2 of Fig. 1). The general pattern observed here is fairly typical for diagrams

TABLE I. Comparison of results obtained for the first and second order diagrams of Fig. 1 in the present model and in realistic calculations.

	Present model			Sussex matrix elements EM [a]		Kuo-Brown matrix elements [b]
Diagram	$b_u = 1.7$	$b_u = 2.0$	$\frac{b_u = 2.0}{b_u = 1.7}$	Oscillator	HF	
			(4,4)			
1	−2.52	−1.88	0.75	−1.38	−1.22	−1.24
2	−0.94	−0.45	0.48	−0.60	−0.12	−0.76
				(−0.53) [c]	(−0.32) [c]	
3	−0.35	−0.17	0.48	−0.31	−0.20	−0.25
3′	−0.36	−0.21	0.59			−0.22
			(5,5)			
1	−2.90	−1.39	0.48	−2.37	−1.36	−2.05
2	−0.22	−0.05	0.23	0.15	0.03	0.05
3	−0.04	−0.02	0.44	−0.05	−0.01	−0.04
3′	−0.12	−0.08	0.63			−0.002

[a] Reference 3.
[b] Reference 11.
[c] Reference 4.

of second and third order. Except at the extremes of the range the full curve (DD) falls off linearly, i.e., the diagram decreases exponentially as a function of b_u, rather than as b_u^{-4}. The SD results (dot-dash curve) are comparable in magnitude to the DD values although they show more curvature. However, the SS results usually show a faster fall-off than both the DD and SD cases and, as in Fig. 3, may lie on quite a different curve. This behavior does not arise from the Schmidt orthogonalization necessary for the 1s orbital and we suspect that it is due to the presence of a node in the 1s wave function.

Turning to Table I we see that by changing b_u from 1.7 to 2.0 fm the (4, 4) result is reduced by a factor of 0.5. This is comparable to the oscillator versus HF comparison of Malta and Sanderson[4]; the remaining difference may well be due to the fact that in their HF calculation pure $b_u = 1.7$ fm oscillator wave functions were used for the 0f orbitals. The EM results show a significantly larger reduction and we need to go into more detail here. Firstly we should take the HF results of EM calculated with fixed 28 MeV energy denominators for comparison with the present results. Then, taking the ratio to the pure oscillator results and averaging over all $J = 0$ $T = 1$ matrix elements, a reduction factor of 0.27 is obtained. Now in the HF calculation of EM the wave functions of the occupied states in the ^{16}O core correspond best to oscillator functions with $b \approx 1.5$ fm. The radius of the ^{16}O nucleus was thus too small, a difficulty which was remedied in the later work of Malta and Sanderson[4] by modification of the Sussex interaction. Thus for comparison with EM we should calculate in our model

$$\frac{(a,c)_{b_0=1.5,\,b_u=2.0}}{(a,c)_{b_u=1.5,\,b_u=1.5}} \quad \frac{(a,c)_{b_0=1.5\;\,b_u=1.5}}{(a,c)_{b_0=1.7,\,b_u=1.7}} .$$

Now the first factor can be determined from Fig. 3 if we plot on the abscissa $(b_u - b_0)$ rather than b_u. The point is that the ratio $(a,c)_{b_0 b_u}/(a,c)_{b_0 b_u = b_0}$ considered as a function of $(b_u - b_0)$ is practically the same whether $b_0 = 1.5$ or 1.7 fm. Thus, a typical value for the first factor of Eq. (1) is 0.24. For the second factor of Eq. (1) a typical value is 1.5. The average value of Eq. (1) is found to be 0.36, in qualitative agreement with the EM value of 0.27 quoted above. The difference is probably due to the fact that the HF calculation indicates a somewhat larger value of b_u, ≈2.3 fm, for the *fp* orbitals.

Note that since the core polarization diagram involves a summation over contributions of various sign from particle-hole pairs coupled to given J'' and T'', it is possible for the model and realistic calculations to yield different signs. This only occurs in the (5, 5) case shown in Table I; we do not believe that this represents a serious defect of the model.

Results for the four particle-two hole diagram (number 3 of Fig. 1) are shown in Table I. About a factor of 0.5 reduction is found between $b_u = 1.7$ and 2.0 fm. Making the comparison between our model and the EM results, which was described in detail for the core polarization diagram, we find reduction factors of 0.31 and 0.44, respectively. If we omit all cases involving a $d_{5/2}^2$ valence state the corresponding figures are much closer—0.29 and 0.31, respectively. Thus the realistic calculations show the feature, remarked on previously, that for the $d_{5/2}$ valence state the wave function is closer to a $b_u = 1.7$ fm oscillator function than for the $s_{1/2}$ and $d_{3/2}$ valence states.

Finally, we give results for the ladder diagram (number 3′ of Fig. 1) in Fig. 4 and Table I. The dominant feature here is that this diagram falls off somewhat less rapidly than in the previous cases. Between $b_u = 1.7$ and 2.0 fm a factor of about 0.6 is obtained compared with a factor of about 0.5 for the core polarization and four particle-two hole diagrams. Now the ladder diagram only contains *sd* and *fp* states so that all the oscillator parameters involved are the same, namely b_u. On the other hand, the core polarization and four particle-two hole diagrams involve not only states with parameter b_u, but also at least one hole with parameter b_0 which we have kept fixed. For the purposes of comparison we have evaluated

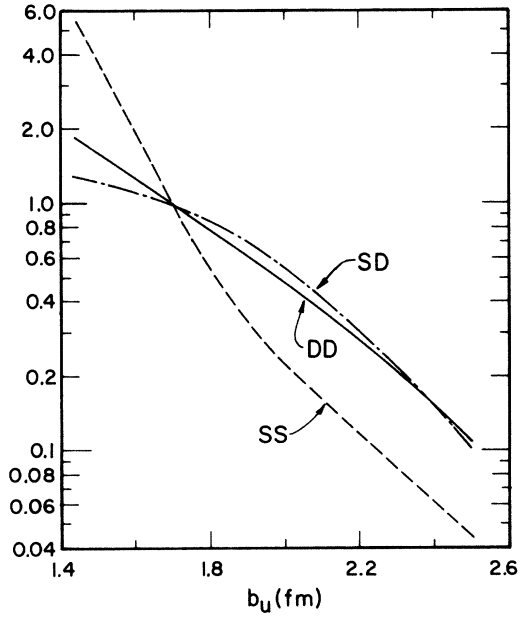

FIG. 3. The core polarization diagram (No. 2 of Fig. 1) as a function of b_u. See caption to Fig. 2.

these diagrams with $b_u = b_0 = 2.0$ fm and compared with the $b_u = b_0 = 1.7$ fm case. The appropriate ratio is found to be 0.6, very close to the value found for the ladder diagram. Thus the faster falloff of the core polarization and four particle-two hole diagrams exhibited in Table I is due to the presence of holes whose wave functions are fixed. We are surprised that the effect is not larger.

These second order results for $J = 0$ $T = 1$ are quite representative of results we have obtained for other J and T.

C. Third order diagrams

In Fig. 5 we show the third order particle-particle and particle-hole vertex correction diagrams (numbers 20 and 17, respectively, of Fig. 1) as a function of b_u. These results are fairly typical although, as in second order, the SS results fluctuate with respect to the DD and SD results, sometimes falling off at about the same rate and sometimes much more rapidly. An obvious general point to make regarding Fig. 5 is that third order diagrams fall off more rapidly than second order because we are dealing with three matrix elements rather than with two. The rate of decrease is indicated in Table II for all the diagrams of Fig. 1 by giving results at $b_u = 1.7$ and 2.0 fm and the ratio thereof; the (4, 4) matrix element is tabulated. We should first compare the $b_u = 1.7$ fm results of the present model with those obtained by Barrett and Kirson and by Barrett[11] using the realistic Kuo-Brown matrix elements. Clearly the agreement is only qualitative, but is sufficient to make

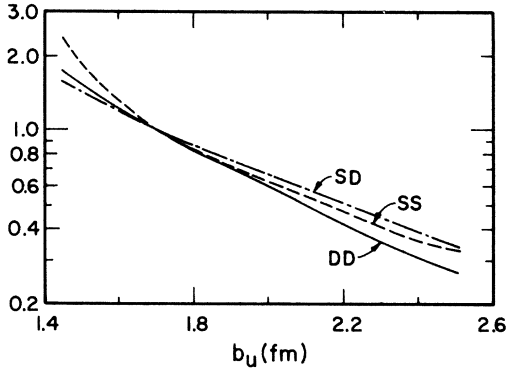

FIG. 4. The ladder diagram (No. 3′ of Fig. 1) as a function of b_u. See caption to Fig. 2.

further discussion of the model results worthwhile. Returning to the $(4,4)_{b_u=2.0}/(4,4)_{b_u=1.7}$ ratio given in Table II, and excluding for the moment diagrams 10, 11, 20, and 21, we see that in each of the remaining cases the ratio is close to the average value of 0.33. The corresponding value for the (4, 5) case is 0.37, while the (5, 5) case, as we have remarked, shows more fluctuations. Nevertheless, there is a definite trend for diagrams 10, 11, 20, and 21 to show a slower falloff than the other diagrams. First consider diagram 10. This is in a special category since it is the only third order diagram with a hole-hole interaction. This is independent of b_u, so we would expect this diagram to scale like a second order diagram and indeed it does. The remaining three excluded diagrams, 11, 20, and 21, all involve a particle-particle

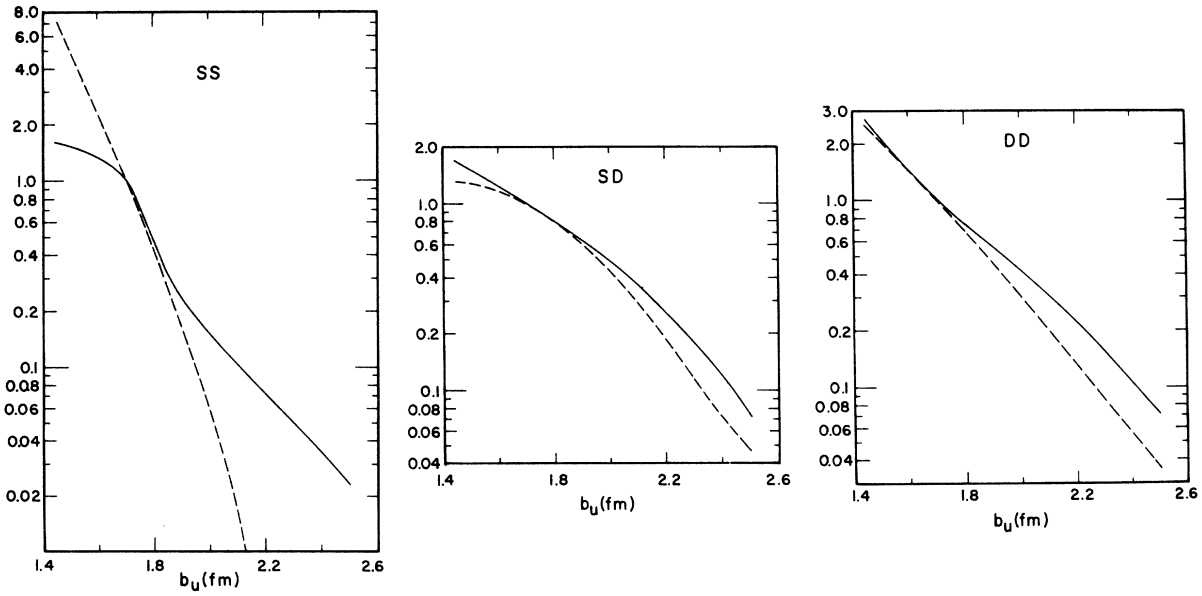

FIG. 5. The third order particle-particle (solid curve) and particle-hole (dashed curve) vertex correction diagrams as a function of b_u. The diagrams are numbered 20 and 17, respectively, in Fig. 1.

ladder and as we have seen in second order ladders fall off rather more slowly. We note that diagram 22 of Fig. 1 does contain a ladder, but the presence of two-hole orbitals at the upper end is sufficient to give a reduction factor close to the average of 0.33. As a practical matter we may note that diagrams 11 and 22 are both small and tend to cancel.

It is worthwhile to comment briefly on some of the diagrams which have been of interest in the literature. Consider first the number-conserving set comprising diagrams 4, 5, and 6 of Fig. 1. It has been suggested that these diagrams would cancel to a considerable degree and numerical calculations[1,13-15] in an oscillator basis do indeed show this tendency. However, the accuracy of the cancellation is not sufficient to allow these diagrams to be neglected. In Table III we give some examples of the ratio (5+6)/4 as a function of b_u. This allows us to test the degree of cancellation, even though the magnitude of the individual diagrams decreases. It is seen that increasing b_u to 2 fm improves the cancellation, i.e., the ratio is closer to −1, and this is a general trend since diagram 4 decreases a little less rapidly than the other two. A second number-conserving set has been identified, diagrams 7, 8, and 9 of Fig. 1, and here we find no uniform trend as is evident from the examples in Table III. Finally, we discuss the vertex corrections for the core polarization diagram. The particle-hole vertex correction diagram (number 17 of Fig. 1) falls off more rapidly than the particle-particle vertex correction diagram (number 20), as is evident from Fig. 5. The point is also made in Table III by giving the ratio of the two diagrams; as b_u increases, the cancellation improves significantly. This means that the sum of the third order diagram, which starts out positive at $b_u = 1.7$ fm, becomes more negative as b_u increases. In fact, by $b_u = 2.0$ fm the third order total is, in most cases quite small and negative. Thus the convergence of the series appears improved, however,

TABLE II. Third order results for the (4,4) matrix element of the effective interaction.

Diagram	Kuo-Brown matrix elements [a]	Present model $b_u = 1.7$	Present model $b_u = 2.0$	$\frac{b_u = 2.0}{b_u = 1.7}$
4	−0.48	−0.68	−0.23	0.34
5	0.38	0.37	0.11	0.30
6	0.22	0.48	0.16	0.32
7	0.20	0.44	0.14	0.32
8	−0.28	−0.43	−0.14	0.33
9	0.05	0.26	0.09	0.35
10	−0.04	−0.07	−0.03	0.48
11	0.04	0.13	0.05	0.40
12	−0.01	−0.06	−0.02	0.35
13	0.09	0.23	0.07	0.33
14	−0.16	−0.60	−0.21	0.36
15	−0.24	−0.48	−0.15	0.32
17	0.48	0.72	0.22	0.31
18	0.29	0.49	0.15	0.32
19	−0.07	−0.09	−0.03	0.34
20	−0.18	−0.32	−0.14	0.42
21	−0.01	−0.06	−0.03	0.47
22	−0.05 [b]	−0.13	−0.05	0.36

[a] Reference 11.
[b] With Barrett, Hewitt, and McCarthy matrix elements; Barrett, Ref. 17.

it is unlikely that third order gives a reliable approximation for two main reasons. Firstly, Schucan and Weidenmüller[16] have shown that ultimately the order-by-order series must diverge due to the presence of intruder states in ^{18}O (although it has been suggested[17] that low order perturbation theory could still provide a reasonable approximation in the presence of intruder states). Secondly, Goode and Koltun[15] have shown that the average of the fourth order diagrams is large and, furthermore, this is unrelated to the presence of intruder states. The fourth order averages are found[18] to remain large in comparison to lower order if the simulated HF basis, discussed here, is employed.

TABLE III. Ratios of third order diagrams for the (4,4) and (5,5) matrix elements of the effective interaction.

Matrix element	Ratio of diagrams	Present model $b_u = 1.44$	$b_u = 1.7$	$b_u = 1.85$	$b_u = 2.0$	$b_u = 2.5$	Kuo-Brown matrix elements [a]
(4,4)	(5+6)/4	−1.22	−1.26	−1.21	−1.15	−0.96	−1.24
(4,5)	(5+6)/4	−1.56	−1.23	−1.04	−0.90	−0.67	−0.86
(4,4)	(7+9)/8	−1.79	−1.63	−1.62	−1.66	−2.00	−0.90
(4,5)	(7+9)/8	−4.25	−4.28	−4.62	−5.19	−10.7	−1.14
(4,4)	17/20	−2.20	−2.22	−1.94	−1.65	−1.13	−2.58
(4,5)	17/20	−1.00	−1.27	−1.24	−1.15	−0.82	−1.15

[a] Reference 11.

III. DISCUSSION AND CONCLUSIONS

We have examined a simple model for the effective interaction in ^{18}O where we take one oscillator parameter $b_0 = 1.7$ fm for the occupied orbitals of the ^{16}O core and another oscillator parameter b_u for the unoccupied orbitals. By taking $b_u = 1.7$ fm we obtain the usual oscillator basis, but by choosing $b_u = 2.0$ fm we hoped to simulate the main qualitative features of using a HF basis. This hope was fulfilled in the rather reasonable agreement between our model results with a simple central interaction and the more difficult realistic calculations.

Our model shows dominantly a scaling behavior with the various diagrams dropping off exponentially as b_u increases. Comparing b_u of 1.7 and 2.0 fm, a reduction factor of roughly 0.7, 0.7^2, and 0.7^3 is obtained for first, second, and third order, respectively. A closer inspection reveals the following four points.

(i) The (5, 5) matrix element of the effective interaction and, to some extent, the bare interaction show fluctuations from the average behavior and are more difficult to characterize. Further, it is much easier to identify trends by calculating the various diagrams rather than by studying tables of the relevant matrix elements.

(ii) The reduction found here in second order is quantitatively less marked than in the EM work,[3] where rather small radii for the occupied orbitals were obtained. This difficulty was removed in the work of Malta[4] and the few results available are similar to ours.

(iii) A hole-hole interaction is independent of b_u. Such an interaction first enters in third order and means that the diagram scales like a second order one. Thus, although hole-hole interactions usually give fairly small effects,[17] their importance could be enhanced in a HF basis.

(iv) Diagrams containing a particle-particle ladder drop off less rapidly than the others in most cases. This is clearly seen in second order where all the wave functions involved in the ladder diagram depend on b_u, whereas in the other diagrams we have hole orbitals whose wave functions are held fixed, independent of b_u, which tends to decrease the overlap. This ladder effect is particularly important in third order for the pair of diagrams which correct the vertex of the core polarization diagram. It means that by going to $b_u = 2.0$ fm the diagram with a particle-particle ladder (number 20 of Fig. 1) cancels a much larger part of the corresponding diagram (number 17) with a particle-hole interaction than is normally the case at $b_u = 1.7$ fm. This argues in favor of excluding from the G matrix in realistic calculations low-lying two-particle states so that the corresponding ladder diagrams can be included on the same footing as the other diagrams, ideally in a HF basis. This is the double-partition approach which has been advocated by Barrett.[19]

If the degenerate schematic model[20] is used, it is straightforward to combine the present work with previous oscillator results[21] in order to estimate the renormalization of the effective interaction in the Tamm-Dancoff (TDA) and random phase (RPA) approximations using a HF basis. We find that, averaging over the $(sd)^2$ $J=0$ $T=1$ renormalizations, the HFTDA gives about $\frac{3}{4}$ of the usual second order core polarization result for an oscillator basis.[2] The HFRPA gives a renormalization about equal to the second order oscillator value. This, together with the greater degree of cancellation amongst the vertex correction processes in a HF basis, might be interpreted as encouraging.

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