PHYSICAL REVIEW C VOLUME 16, NUMBER 2 AUGUST 1977

Nucleon-nucleon interaction in nuclear matter*

Sadhan K. Adhikari and H. T. Coelho
Departamento de Física, Universidade Federal de Pernambuco, Recife PE, Brazil

F. A. Bezerra Coutinho
Instituto de Física, Universidade de São Paulo, São Paulo SP, Brazil

Y. Nogami
Department of Physics, McMaster University, Hamilton, Ontario, Canada L8S 4M1
(Received 24 January 1977)

The effective two-body *NN* potential in nuclear matter due to the two-pion-exchange three-body force can be expressed such that it arises due to a change in the pion propagator pertaining to the one-pion-exchange potential. The pion propagator in nuclear matter is examined with the *NN* correlation taken into account. It is found that the modified propagator can be approximated by $[(1-\alpha)(q^2+\mu'^2)]^{-1}$ where $\mu'=\mu(1-\alpha)^{-1/2}$, μ is the pion mass, and α is a positive constant which is roughly proportional to the nuclear matter density. For example, $\alpha\approx 0.3$ for the normal density of nuclear matter. The modified one-pion-exchange potential in nuclear matter is then obtained by scaling the one-pion-exchange potential in vacuum. It is suggested that other parts of the *NN* potential, e.g., the two-pion-exchange part, are also modified by the same mechanism, and hence the *NN* interaction in nuclear matter could be strongly density dependent.

[NUCLEAR STRUCTURE Pion propagator in nuclear matter, many-body forces.]

I. INTRODUCTION

The prime objective of studying infinite nuclear matter is to test the nucleon-nucleon (*NN*) potential and/or the technique of the many-body theory. Provided that the many-body calculation can be done accurately,[1] one hopes to be able to differentiate various "realistic" *NN* potentials which all fit the two-body data well. In recent years, however, it has become increasingly clear that none of the realistic potentials so far proposed can satisfactorily reproduce the "empirical" binding energy and density of nuclear matter.[2] It was hoped some time ago that the inclusion of $\Delta(1236)$ admixture might alleviate this difficulty,[3] but the recent analysis by Day and Coester[2] indicates that we are still missing something. It is our feeling that what we are missing most is probably a better understanding of three-body and/or many-body forces.

There are many mechanisms which give rise to three-body (3*N*) forces, but let us confine ourselves in this paper to the simplest one, the 2π-exchange 3*N* force which arises due to the process shown in Fig. 1. For the 3*N* force due to this mechanism, the potential derived by Fujita and Miyazawa[4] is commonly used. Several calculations have been done for the contribution of this 2π-exchange 3*N* force to the nuclear matter binding, and the results have recently been summarized by McKellar.[5] Unfortunately the force itself and the calculation of the effect in nuclear matter are both beset with some ambiguities which are reflected in the spread of the values obtained (Table 1 of Ref. 5). However, it is probably safe to say that the 2π-exchange 3*N* force yields from zero to about 2 MeV *attraction* per particle at the normal density of nuclear matter. One may recall on the other hand that the effect of Δ was found to be *repulsive* in the calculations of Refs. 2 and 3. This is contrary to the results of the 3*N* force calculations in the sense that both calculations deal with the effect of Δ in nuclear matter. We suspect that this is mainly because the former calculations[2,3] do not explicitly take account of three-body *correlations* to which the Δ effect is very sensitive.[6]

The purpose of this paper is to point out that there is a mechanism involving more pions which

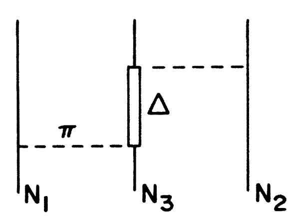

FIG. 1. The 2π-exchange process which gives rise to the 3*N* force. The double line stands for $\Delta(1236)$.

is probably comparable with or more important than those examined so far. Let us start by recalling how the calculations for the $3N$ force have been done. Rather than dealing with the $3N$ force directly in the many-body calculation, one defines an effective NN force δV which is due to the $3N$ force $W(\vec{r}_1, \vec{r}_2; \vec{r}_3)$ by[7-12]

$$\delta V(\mathrm{r}_{12}) = \rho \,\mathrm{Tr} \int d\vec{r}_3 W(\vec{r}_1, \vec{r}_2; \vec{r}_3) g(r_{13}) g(r_{23}) . \quad (1.1)$$

Here ρ is the nuclear matter density, while $g(r)$ is an appropriate NN correlation function. The trace is taken for the spin and isospin of nucleon 3. This δV is added to the original NN potential V (e.g., the Reid soft-core potential);

$$U = V + \delta V \quad (1.2)$$

and the nuclear matter calculation is done with this U.[10]

If we ignore the NN correlation, i.e., if we put $g(r) = 1$ in Eq. (1.1), δV can easily be put into the form such that δV results from the change of the one-pion-exchange potential (OPEP) due to the change of the pion propagator in nuclear matter.[13,14] That is, the free propagator $(q^2 + \mu^2)^{-1}$ is modified into

$$\frac{1}{q^2 + \mu^2} + \frac{\alpha q^2}{(q^2 + \mu^2)^2} , \quad (1.3)$$

where q is the pion momentum, μ the pion mass, and α is a positive constant which turns out to be proportional to the nuclear matter density. If α is very small Eq. (1.3) can be rewritten as

$$\frac{1}{q^2 + \mu^2 - \alpha q^2} = \frac{1}{1 - \alpha} \frac{1}{q^2 + \mu'^2} , \quad (1.4)$$

where $\mu' = \mu/(1 - \alpha)^{1/2}$. Equation (1.4) has a simple consequence on the OPEP; the range of the OPEP is reduced from $1/\mu$ to $1/\mu'$ while the strength is increased by the factor $1/(1 - \alpha)$. If we denote the OPEP with the pion mass μ by $V_\pi(r, \mu)$, the modified OPEP U_π is given by[15]

$$U_\pi(r) = V_\pi(r, \mu')/(1 - \alpha) . \quad (1.5)$$

Even if α is not very small, Eq. (1.4) can be restored by including multiple scattering processes shown in Fig. 2.[16]

Now, if the OPEP is modified because the pion propagator is modified in nuclear matter, it is only logical to imagine that the 2π-exchange NN force etc. are also modified due to the same mechanism. Since there are two pion propagators involved, the 2π-exchange NN force $V_{2\pi}(r, \mu)$ will be modified into

$$U_{2\pi}(r) = V_{2\pi}(r, \mu')/(1 - \alpha)^2 . \quad (1.6)$$

As is well known, in the medium range ($r < 2$ fm), $V_{2\pi}$ is more important than V_π and probably the same situation will obtain for $U_{2\pi}$ and U_π. Let us emphasize here that only U_π has been considered in the calculations so far done.[7-14]

The idea of relating the $3N$ force to a modified pion propagator is not new,[13] but so far it has been discussed without incorporating the NN correlation. In the presence of the NN correlation, i.e., $g(r) \neq 1$, the above prescription does not hold rigorously. This is because the pion momenta before and after the process of $\pi N \to \Delta \to \pi N$ become different. (Or, as we will see in Sec. II, $f_C \neq f_T$ in general.) In Sec. II, however, we will show that Eq. (1.4) and hence Eq. (1.5) are reasonable approximations even if $g(r) \neq 1$. Implications of the change in the NN force, in nuclear matter, are discussed in Sec. III. All formulas in this paper are given in units such that $c = \hbar = 1$, and our notations are mostly the same as those of Refs. 7–12 and 14 except for those which will be specified otherwise.

II. PION PROPAGATOR IN NUCLEAR MATTER

The matrix element for the diagram of Fig. 1 contains two pion propagators, $(q_1{}^2 + \mu^2)^{-1}$ and $(q_2{}^2 + \mu^2)^{-1}$. If we disregard the NN correlation, i.e., if we put $g(r) = 1$, the matrix element depends on $\vec{r}_3$ only through the factor $\exp[i(\vec{q}_1 - \vec{q}_2)\cdot\vec{r}_3]$. Hence the $\vec{r}_3$ integration is trivially done giving rise to $\delta(\vec{q}_1 - \vec{q}_2)$, and the matrix element is immediately reduced to the form of $\alpha_0 q^2/(q^2 + \mu^2)^2$. We denote α of this simplest case by α_0. Here we have not considered the pionic form factor $H(q^2)$.[13,14] With $H(q^2)$, Eqs. (1.3) and (1.4) become

$$\frac{H(q^2)}{q^2 + \mu^2} + \frac{\alpha_0 q^2 H^2(q^2)}{(q^2 + \mu^2)^2} \approx \frac{H(q^2)}{q^2 + \mu^2 - \alpha_0 H(q^2) q^2} . \quad (2.1)$$

Since $H(q^2)$ (≤ 1) does not vary substantially for $q^2 \leq 10\mu^2$ we can put $\alpha_0 H(q^2) q^2 \approx \alpha q^2$. With this the scaling formulas (1.5) and (1.6) should work reasonably well at large and medium distances.

The physical reason for the momentum conservation $\vec{q}_1 = \vec{q}_2$ obtained above is that the medium in which the pion propagates is homogeneous in the absence of NN correlations. If $g(r) \neq 1$, this is no

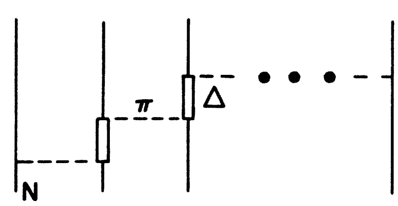

FIG. 2. The multiple scattering process which is included in Eq. (1.4).

longer the case and the $\vec{r}_3$ integration becomes quite involved. For this reason it was previously thought that the idea of the modified pion propagator or the effective pion mass would not be useful.[14] We will see, however, that the modified pion propagator is in fact a useful concept even when $g(r) \neq 1$. We prefer not to use the word "effective pion mass" because $\mu^2 - \alpha q^2$ can become negative for large values of q^2. Note that Eq. (2.1) is valid as long as $\alpha_0 q^2 \ll q^2 + \mu^2$.

Let us start with δV of Eq. (1.1) which we now denote by δV_π and is given by

$$\delta V_\pi(\vec{r}) = -(\rho C_p/2\pi^4\mu^6)\vec{\tau}_1\cdot\vec{\tau}_2\int d\vec{r}_3 g(r_{13})g(r_{23}) \times\int d\vec{q}_1 d\vec{q}_2 H(q_1^2)H(q_2^2)\frac{(\vec{q}_1\cdot\vec{q}_2)(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_2\cdot\vec{q}_2)}{(q_1^2+\mu^2)(q_2^2+\mu^2)}\exp[i(-\vec{q}_1\cdot\vec{r}_{13}+\vec{q}_2\cdot\vec{r}_{23})], \tag{2.2}$$

where $\vec{r}=\vec{r}_{12}=\vec{r}_1-\vec{r}_2$ and C_p is a constant which is related to the p-wave πN scattering. We will do the $\vec{r}_3$ integration in the same manner as in Kasahara *et al.*[12] (KAT). Note that our δV_π corresponds to their U except that they did not include $H(q^2)$, and also our $g(r)$ is their $\phi^2(r)$. The only difference between our calculation and theirs is due to $H(q^2)$.[17] We introduce $h(q)$ by

$$g(r)=\int d\vec{q}\, h(q)e^{-i\vec{q}\cdot\vec{r}} \tag{2.3}$$

and then

$$S_\alpha(q)=\int d\vec{p}\frac{p_\alpha^2 h(|\vec{q}-\vec{p}|)H(p^2)}{p^2+\mu^2}, \tag{2.4}$$

where $\alpha = x', y', z'$. The z' axis is along $\vec{q}$. KAT's $S_{\alpha\beta}$ is related to our S_α with $H=1$ by $S_{\alpha\beta}=S_\alpha\delta_{\alpha\beta}$. After a somewhat lengthy manipulation we arrive at

$$\delta V_\pi(\vec{r})=\frac{2f^2}{3\pi\mu^2}\vec{\tau}_1\cdot\vec{\tau}_2\int_0^\infty q^2dq\frac{q^2H(q^2)}{(q^2+\mu^2)^2}\left[\vec{\sigma}_1\cdot\vec{\sigma}_2 j_0(qr)f_C(q^2)+S_{12}\left(j_0(qr)-\frac{3j_1(qr)}{qr}\right)f_T(q^2)\right]. \tag{2.5}$$

Here, $S_{12}=3(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})/r^2-\vec{\sigma}_1\cdot\vec{\sigma}_2$ and

$$f_C(q^2)=\frac{8\pi\rho C_p}{\mu^4f^2}\frac{(q^2+\mu^2)^2}{q^2H(q^2)}[S_{z'}^2(q)+2S_{x'}^2(q)], \tag{2.6}$$

$$f_T(q^2)=\frac{8\pi\rho C_p}{\mu^4f^2}\frac{(q^2+\mu^2)^2}{q^2H(q^2)}[S_{z'}^2(q)-S_{x'}^2(q)]. \tag{2.7}$$

If $g(r)=1$, then $h(q)=\delta(\vec{q})$ and $S_{z'}(q)=q^2H(q^2)/(q^2+\mu^2)$, $S_{x'}(q)=0$, and we find that

$$f_C(q^2)=f_T(q^2)=f_0(q^2)\equiv\alpha_0 H(q^2)q^2, \tag{2.8}$$

where

$$\alpha_0=8\pi\rho C_p/\mu^4f^2=16k_F^3C_p/3\pi\mu^4f^2. \tag{2.9}$$

Here k_F is the Fermi momentum. We relegate further details to the Appendix.

The point to be noticed here is that, unless $f_C(q^2)=f_T(q^2)$, δV_π of (2.5) cannot be derived from the OPEP

$$V_\pi(\vec{r})=\frac{2f^2}{3\pi\mu^2}\vec{\tau}_1\cdot\vec{\tau}_2\int_0^\infty q^2dq\frac{q^2H(q^2)}{q^2+\mu^2}\left[\vec{\sigma}_1\cdot\vec{\sigma}_2 j_0(qr)+S_{12}\left(j_0(qr)-\frac{3j_1(qr)}{qr}\right)\right] \tag{2.10}$$

by simply modifying the propagator. It is clear from Eqs. (2.6) and (2.7) that $f_C\approx f_T$ only if $S_{x'}^2\ll S_{z'}^2$. By examining the structures of the integrals for $S_{x'}(q)$ and $S_{z'}(q)$ one can expect that $S_{x'}^2(q)$ should be smaller than $S_{z'}^2(q)$ (see Appendix), but the problem is *how much smaller*. So let us work it out.

For the strength factor C_p, let us adopt the same value as that used in Refs. 7–11. That is

$$C_p=0.61\text{ MeV}. \tag{2.11}$$

This was criticized by KAT, who determined it to be $C_p=0.45$ MeV. On the other hand a larger value $C_p\approx 1$ MeV has been suggested by a more recent analysis.[5] In view of this unsettled situation we stick to the old value of Eq. (2.11), which facili-

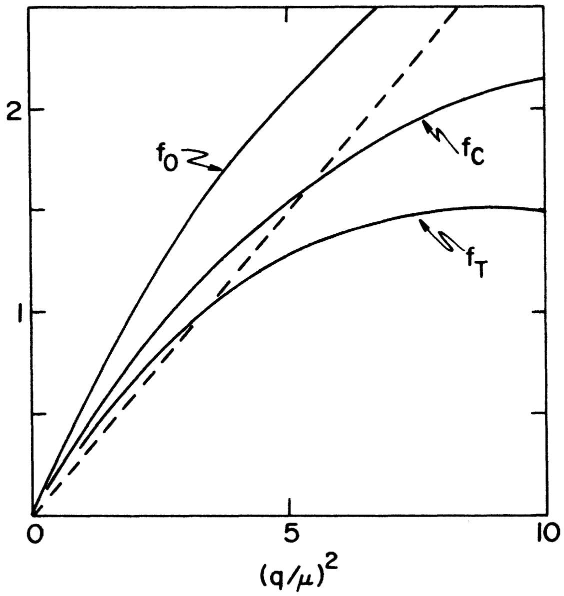

FIG. 3. The functions f_0, f_C, and f_T in units of μ^{-2} versus $(q/\mu)^2$, for the normal density of nuclear matter. The dashed line shows αq^2 with $\alpha = 0.3$.

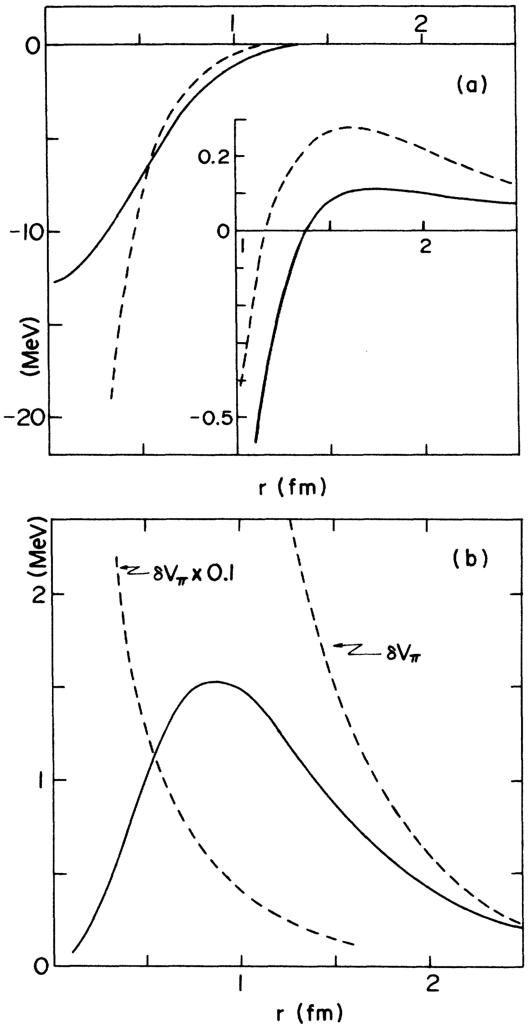

FIG. 4. Comparison between the "exact" $\delta V_\pi(r)$ and that obtained by scaling, for the normal density of nuclear matter. The central and tensor parts are the coefficients of $(\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$ and $(\vec{\tau}_1 \cdot \vec{\tau}_2)S_{12}$, respectively. The solid line is for the "exact" result while the dashed one for the scaling approximation. Figs. 3(a) and 3(b) show the central and tensor parts, respectively.

tates the comparison between the present and previous calculations. For other parameters we take $f^2 = 0.08$ and $\mu = 0.7\ \text{fm}^{-1}$. With C_p of Eq. (2.11), we find $\alpha_0 = 0.69\ \text{fm}^{-1}$ for the normal density, i.e., $\rho = 0.170\ \text{fm}^{-3}$ or $k_F = 1.36\ \text{fm}^{-1}$.

For the form factor $H(q^2)$,

$$H(q^2) = 1 - \zeta + \zeta(\eta^2 - \mu^2)/(\eta^2 + q^2) \qquad (2.12)$$

has been used in most of the $3N$ force calculations including those of Refs. 7–11. In the present calculation we use "form factor III" ($\zeta = 1$, $\eta^2 = 10\mu^2$) which has been favored by recent analyses.[18] For the NN correlation function $g(r)$ we use the same one as in Ref. 7, namely, the one obtained by a nuclear matter calculation using the Reid soft-core potential.

Figure 3 shows f_0, f_C, and f_T versus $(q/\mu)^2$ at the normal density $\rho = 0.170\ \text{fm}^{-3}$. The linear approximation

$$f_C(q^2) = f_T(q^2) = \alpha q^2 \qquad (2.13)$$

with $\alpha = 0.3$ is a fairly good approximation for $q^2 \lesssim 5\mu^2$, although its accuracy deteriorates for $q^2 \gtrsim 10\mu^2$. In deriving the long and medium range part of the force, Eq. (2.13) will be acceptable. Figure 4 compares $\delta V_\pi = U_\pi - V_\pi$ obtained in the scaling approximation (1.5) (Ref. 15) with the "exact" δV_π of Ref. 7. The latter corresponds to Eq. (2.2). The difference between the "scaling" and "exact" results derives from two reasons; the scaling is based on the approximation (2.13), but also it includes the effect of multiple scattering process (Fig. 2). This multiple scattering effect may have been overestimated because correlations among the nucleons have not properly been taken care of.[16] Also the scaling result should not be taken seriously, say for $r \lesssim 1$ fm. In the approximation (2.13) we are ignoring $H(q^2)$ in the amplitude for $\pi N \to \Delta \to \pi N$. This is a partial reason why δV obtained by scaling is singular at short distances.

The pion propagator which we have examined above is actually a special case of $(-q_0^2 + q^2 + \mu^2)^{-1}$ with $q_0 = 0$ where q_0 is the "time-component" of the pion four-momentum. Only the $q_0 = 0$ part appears in the OPEP, but in the 2π-exchange NN force, propagators with $q_0 \neq 0$ are involved. If $g(r)=1$, it is easy to show that the propagator for $q_0 \neq 0$ is modified in the same manner as that for $q_0 = 0$, that is, the same correction term $-\alpha q^2$ appears in the denominator. If $g(r) \neq 1$, the functions f_C and f_T can and will depend on q_0. However, since $g(r)$ did not substantially change the behaviors of f_C and f_T when $q_0=0$, let us *assume* that the situation will remain the same even if $q_0 \neq 0$. Then the scaling formula (1.6) for the 2π-exchange NN potential $V_{2\pi}$ follows.

III. DISCUSSION

We obtained the effective OPEP U_π in nuclear matter from the OPEP V_π for free nucleons by modifying the pion propagator, and showed that U_π and V_π are approximately related by scaling formula (1.5). Further, we suggested that the 2π-exchange potential is modified by the same mechanism, and $U_{2\pi}$ and $V_{2\pi}$ are approximately related by a similar scaling formula (1.6). Other parts of the NN potential due to more complicated exchange processes would also be modified by similar mechanisms. In the calculations so far done,[7-14] only the OPEP part U_π has been considered.

For $\delta V_\pi = U_\pi - V_\pi$ we compared the "exact" and scaling results in Fig. 4. As we noted then the difference between the "exact" and scaling results comes about partly from the many-body forces due to Fig. 2 which is included in the scaling case. The appreciable difference between these two results for δV_π, shown in Fig. 4, seems to suggest that the many-body force due to Fig. 2 may not be negligible.

For the 2π-exchange part, it is well known that $V_{2\pi}$ is as important or more important than V_π in the short and medium ranges ($r \lesssim 2$ fm). Hence it is likely that $\delta V_{2\pi} = U_{2\pi} - V_{2\pi}$ also plays an important role in nuclear matter. One might have a crude idea about the effect by comparing $G \equiv (\text{strength}) \times (\text{range})^2$ for V and U, where the range is proportional to $1/\mu$ or $1/\mu'$. One finds from Eqs. (1.5) and (1.6) that $G(V_\pi) = G(U_\pi)$ and $G(V_{2\pi}) = G(U_{2\pi})/(1-\alpha)$. This reinforces our feeling that $\delta V_{2\pi}$ will be significant. We are aware of only one paper which has something to do with $\delta V_{2\pi}$. Fujita, Kawi, and Tanifuji[19] derived three-body forces due to three-pion exchanges among three nucleons, and examined their effect on the triton. Such forces arise when one of the two pion-propagators in $V_{2\pi}$ is replaced by Eq. (1.3). The part of $\delta V_{2\pi}$ which Fujita *et al.* estimated appears to be comparable with or stronger than δV_π for $r \lesssim 1$ fm (see their Fig. 7).

If the effective NN interaction U in nuclear matter depends on the nuclear matter density, it is not surprising that the empirical energy and saturation density of nuclear matter cannot be reproduced by using a realistic NN force V which has been determined for $\rho = 0$. In this sense it seems to be quite futile to try to differentiate various realistic NN potentials in great details by testing them on nuclear matter unless we have a clear idea on the density dependence of U. Numerous papers have been written on effects of three-body forces on the nuclear matter binding, but the results of those calculations which deal with δV_π only should be taken with a grain of salt.

It is obviously desirable to estimate $\delta V_{2\pi}, \ldots$, and it would be worthwhile to try to construct $U = U_\pi + U_{2\pi} + \cdots$ starting with a best available potential $V = V_\pi + V_{2\pi} + \cdots$ (for example, Ref. 20) and see if the nuclear matter theory can be salvaged from the saturation difficulty emphasized by Day and Coester.[2] We note also that the pion propagator has been examined quite extensively in relation to topics such as the Lorentz-Lorenz effect and pion condensation (for example, Ref. 21). These analyses of related problems would undoubtedly be useful in clarifying the nature of the effective NN interaction U in nuclear matter.

ACKNOWLEDGMENTS

FABC and YN wish to acknowledge the warm hospitality at the Departamento de Física, Universidade Federal de Pernambuco where the work was started during their visit. YN is grateful to the National Research Council of Canada and Conselho Nacional de Pesquisas do Brasil for the support under the Canada-Brazil Scientific Exchange Program.

APPENDIX

Let us show some details of the integrations for $S_\alpha(q)$ of Eq. (2.4). We start with $S_{z'}(q)$. In an arbitrary (x, y, z), coordinate system, $p_z{}^2 = (\vec{p}\cdot\vec{q})^2/q^2$ can be replaced by $\sum_i (p_i q_i)^2/q^2$ because of the symmetries of the functions h and H. Hence we obtain

$$S_{z'}(q) = (2\pi)^{-3} \int d\vec{r}\, g(r) e^{i\vec{q}\cdot\vec{r}} q^{-2} \times [(q_x{}^2 + q_y{}^2) I_\perp(r) + q_z{}^2 I_z(r)], \quad \text{(A1)}$$

where

$$I_z(r) = \int d\vec{p}\, \frac{p_z{}^2 H(p^2) e^{-i\vec{p}\cdot\vec{r}}}{p^2 + \mu^2}, \quad \text{(A2)}$$

$$I_\perp(r) = \tfrac{1}{2}[I(r) - I_z(r)],$$
$$I(r) = \int d\vec{p}\, \frac{p^2 H(p^2) e^{-i\vec{p}\cdot\vec{r}}}{p^2 + \mu^2}. \quad \text{(A3)}$$

Doing the angular integration for $\vec{r}$, $S_{z'}(q)$ is further reduced to

$$S_{z'}(q) = (2\pi^2)^{-1}\int_0^\infty r^2 dr\, g(r)\,\{j_0(qr)I_z(r) - (2/qr)j_1(qr) \times [I_z(r) - I_\perp(r)]\}\,. \quad \text{(A4)}$$

$S_{x'}(q)$ is obtained from

$$S_x(q) = \tfrac{1}{2}[S(q) - S_{z'}(q)]\,, \quad \text{(A5)}$$

where

$$S(q) = (2\pi^2)^{-1}\int_0^\infty r^2 dr\, g(r) j_0(qr) I(r)\,. \quad \text{(A6)}$$

This is as far as we can go without specifying $H(q^2)$.

In order to obtain the limit $H(q^2)\to 1$ unambiguously, let us first put $H(q^2)=\exp(-q^2/\Lambda^2)$. Then we find[22]

$$I(r) = (\pi^{1/2}\Lambda)^3 e^{-(\Lambda r/2)^2} - \pi^2\mu^2 e^{(\mu/\Lambda)^2} \times \frac{1}{r}\left[e^{-\mu r}\mathrm{Erfc}\left(\frac{\mu}{\Lambda}-\frac{\Lambda r}{2}\right) - e^{\mu r}\mathrm{Erfc}\left(\frac{\mu}{\Lambda}+\frac{\Lambda r}{2}\right)\right] \quad \text{(A7)}$$

and

$$I_z(r) = -(d^2/dr^2)I(r)\,. \quad \text{(A8)}$$

If we put $\Lambda\to\infty$, we obtain $I(r)\to(2\pi)^3\delta(\vec{r}) - 2\pi^2 e^{-\mu r}/r$ etc. and hence

$$S_{z'}(q) = \tfrac{1}{3}g(0) - \int_0^\infty r^2 dr\, g(r)(e^{-\mu r}/r) \times\left[j_0(qr)\left(1+\frac{2}{\mu r}+\frac{2}{(\mu r)^2}\right) + \frac{2j_1(qr)}{qr}\left(1+\frac{3}{\mu r}+\frac{3}{(\mu r)^2}\right)\right] \quad \text{(A9)}$$

and

$$S(q) = g(0) - \mu^2\int_0^\infty r^2 dr\, g(r)(e^{-\mu r}/r) j_0(qr)\,. \quad \text{(A10)}$$

From Eqs. (A9) and (A10), one can see that the integrand of $S_{x'}(q)$ decays much faster than that of $S_{z'}(q)$ for $\mu r \gg 1$. This is a reason why $S_{x'}{}^2(q)$ is much smaller than $S_{z'}{}^2(q)$.

Finally, let us point out that for $H(q^2)$ of Eq. (2.12)

$$\frac{H(q^2)}{q^2+\mu^2} \to \frac{1}{q^2+\mu^2} - \frac{\zeta}{q^2+\eta^2}\,. \quad \text{(A11)}$$

$S_{z'}(q)$ and $S_{x'}(q)$ are obtained from those of Eqs. (A9) and (A10) minus ζ times the same in which μ is replaced by η.

Note added in proof. A recent theoretical analysis of the πNN vertex factor [J. W. Durso, A. D. Jackson, and B. J. Verwest, Nucl. Phys. A282, 404 (1977)] suggests a much larger "regulator mass" η in $H(q^2)$ of Eq. (2.12), i.e., $\eta^2\approx 72\mu^2$. Since the force due to the exchange of a ρ tends to cancel the OPEP at short distances, form factor III with $\eta^2=10\mu^2$ which we have used could be interpreted as an effective one which approximately includes the effect of the ρ exchange.

We would like to thank Professor G. E. Brown for a discussion on this point.

*Work supported in part by the Conselho National de Pesquisas do Brasil and National Research Council of Canada.

[1]A doubt has recently been cast on the accuracy of the Brueckner approach; V. R. Pandharipande, R. B. Wiringa, and B. D. Day, Phys. Lett. 57B, 205 (1975); J.-P. Jekenne, A. Lejene, and C. Mahaux, Nucl. Phys. A245, 411 (1975).

[2]B. D. Day and F. Coester, Phys. Rev. C 13, 1920 (1976).

[3]A. M. Green and J. A. Niskanen, Nucl. Phys. A249, 493 (1975); A. M. Green and P. Haapakoski, *ibid.* A221, 429 (1974); T. K. Dahlblom and L. E. W. Smulter, *ibid.* A263, 445 (1976).

[4]J. Fujita and H. Miyazawa, Progr. Theor. Phys. 17, 360 (1957).

[5]B. H. J. McKellar, in *Few Body Problems,* proceedings of the Seventh International Conference on Few Body Problems in Nuclear Particle Physics, Delhi, 1976, edited by A. N. Mitra, I. Šlaus, V. S. Bhasin, and V. K. Gupta (North-Holland, Amsterdam, 1976).

[6]Y. Nogami, Rev. Brasil. Fis. (to be published).

[7]P. Grangé, M. Martzolff, Y. Nogami, D. W. L. Sprung, and C. K. Ross, Phys. Lett. 60B, 237 (1976).

[8]B. A. Loiseau, Y. Nogami, and C. K. Ross, Nucl. Phys. A165, 601 (1971); Erratum A176, 665 (1971).

[9]B. H. J. McKellar and R. Rajaraman, Phys. Rev. C 3, 1877 (1971).

[10]Strictly speaking, the effective NN force δV of Eq. (1.1) of the text describes the effect of the $3N$ force only to the extent that it can be treated in first order perturbation theory. There is no rigorous justification for the transition from Eq. (34) to Eq. (35) of Ref. 9. Calculations such as Refs. 7-14 are all subjected to the same criticism.

[11]D. W. E. Blatt and B. H. J. McKellar, Phys. Lett. 52B, 10 (1974); Phys. Rev. C 11, 614 (1975).

[12]T. Kasahara, Y. Akaishi, and H. Tanaka, Prog. Theor. Phys. Suppl. 56, 96 (1974).

[13]G. E. Brown and A. M. Green, Nucl. Phys. A137, 1 (1969).

[14]R. K. Bhaduri, Y. Nogami, and C. K. Ross, Phys. Rev. C 2, 2082 (1970).

[15]A care has to be exercised in applying the scaling formula; the pion mass μ which appears in $H(q^2)$ and also in the pion-nucleon interaction Hamiltonian [in combination with the coupling constant f as (f/μ)] should not be changed. Here, of course, we are

assuming that the coupling constant and πNN vertex factor are not modified in nuclear matter.

[16]B. H. J. McKellar and R. Rajaraman, Phys. Rev. C 10, 871 (1974); D. W. E. Blatt and B. H. J. McKellar, *ibid*. 12, 637 (1975). In the multiple scattering process we do not include those involving $\pi N \to N \to \pi N$ to avoid double counting. In this sense our pion propagator is different from the true pion propagator.

[17]Although the factor $H(q^2)$ appears in Eq. (12) of KAT their results are not valid unless $H(q^2)=1$.

[18]K. Bonyardt, H. Pilkuhn, and H. G. Schlaile, Phys. Lett. 52B, 2716 (1974); W. Nutt and B. A. Loiseau, Nucl. Phys. B104, 98 (1976).

[19]J.-I. Fujita, M. Kawai, and M. Tanifuji, Nucl. Phys. 29, 252 (1962).

[20]W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D 8, 800 (1973).

[21]G. Baym and G. E. Brown, Nucl. Phys. A247, 395 (1975).

[22]*The Bateman Manuscript Project, Table of Integral Transforms*, edited by A. Endélyz (McGraw-Hill, New York, 1954), Vol. 1, p. 74.