Effects of bound-state approximations on distorted-wave Born approximation and multistep proccesses in the channel-coupling-array theory

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The effects of approximating the equations of the channel-coupling-array theory by retaining only bound internal states of the clusters in two-body channels are studied. This approximation corresponds to those used in standard direct nuclear reaction calculations. By use of Lippmann's identity, the exact channel-coupling-array equations may be written in two alternate forms (different off-shell extensions) which respond differently to the bound-state approximation. When the bound state approximation is made on the equations in "folded" form, its main effect is that the distorted-wave Born approximation occurs in only one of the two-body channels, a modified distorted-wave Born approximation occurs in another, and no first order amplitudes of any kind occur in the remaining channels. If Lippmann's identity is first used to unfold the *exact* channel-coupling-array equations (thereby yielding a different off-shell extension) and then the bound state approximation is applied, distorted-wave Born approximation amplitudes are found in all channels. The need for accurate, nonperturbative solutions to the approximate equations so as to assess the effects of possible low-order approximations is stressed.

[NUCLEAR REACTIONS Bound-state approximations, channel-coupling-array theory, DWBA and multistep operators.]

I. INTRODUCTION

The channel-coupling-array theory[1] is a general scheme for treating multichannel collision processes, in particular nuclear reactions, and some discussion has already been given on its possible use in describing elastic scattering,[2,3] direct nuclear reactions,[3,4] and resonance reactions.[5] Our purpose in this paper is to extend the analyses already given for the second class of processes, focusing mainly on the role of Lippmann's identity in the theory and the way in which distorted-wave Born approximation (DWBA) and multistep amplitudes occur in approximate (i.e., realistic) calculations.[1-3]

By use of Lippmann's identity, it is possible to write the channel coupling array equations in either "folded" or "unfolded" form.[1,3] It is shown that the two forms respond differently to bound state approximations (which are the commonly used approximations). In the folded form, we establish that the standard DWBA occurs in one arrangement channel only, a modified DWBA appears in another channel, and no other arrangement channel transition operators contain the DWBA even in modified form. However, in the unfolded form of the channel-coupling-array equations, a DWBA amplitude occurs in every arrangement. Some consequences of this are discussed, in particular, the need for coupled-channel rather than low-order, perturbation-like calculations. This is illustrated by examination of the Kunz-Rost[6] estimate of the relative strength of the two-step amplitude for (p, t) reactions.

II. BRIEF REVIEW OF THE THEORY

Since the derivation of the equations of the channel-coupling-array method have been given in various published works,[1,3,4] we only summarize the results we shall use in the next section. Our emphasis will be on the equations for the prior transition operators $\hat{T}_{jk}$ and their associated channel component states $|\psi_m\rangle$, rather than on the post operators T_{jk} mainly used in previous work. They are related to the $\hat{T}_{jk}$ by $(\hat{T}^T)_{jk} = T_{jk}$ where the superscript T means transpose. Exact, on-shell matrix elements of $\hat{T}_{jk}$ and T_{jk} are equal,[1] and the proof[3] that the T_{jk} obtained from solving the relevant coupled equations gives the same on-shell matrix elements as $T_{jk}(+) = V_j + V_j G^+ V_k$ also applies to $\hat{T}_{jk}(+)$ and $V_k + V_j G^+ V_k$ where the notation used is discussed in Refs. 1 and 3. We note here that in the remainder of this article we shall choose channel permuting arrays for the W's appearing in the equations below. Only two-body channels are considered, as in Ref. 3

A. Transition operators

We assume a system of n distinguishable particles governed by a Hamiltonian H. Corresponding to the asymptotic states of the system[1,3,6] are partitions of H into channel Hamiltonians $\{H_j\}$ and channel perturbations $\{Y_j\}$:

$$H = H_1 + Y_1 = H_2 + Y_2 = \cdots . \tag{1}$$

The Y_j are assumed to go to zero sufficiently rapidly when the bound clusters of particles forming each two-body channel j are well separated. We shall also assume that Y_j is the difference of the usual[1,3] channel interaction V_j and a one-body channel distortion potential U_j; $Y_j = V_j - U_j$. This latter potential will also appear in H_j, so that in the eigenstates $|\chi_j(E_{\alpha\vec{q}}(j))\rangle\rangle$ of H_j, distorted waves rather than plane waves describe the relative motion. These states obey

$$[E_{\alpha\vec{q}}(j) - H_j]\,|\chi_j(E_{\alpha\vec{q}}(j))\rangle\rangle = 0 , \tag{2}$$

where α denotes the internal state quantum numbers of the clusters and $\vec{q}$ is the asympotic relative momentum. In an obvious notation $E_{\alpha\vec{q}}(j) = E_\alpha(j) + E_{\vec{q}}(j)$, where $E_{\vec{q}}(j) = \hbar^2 q^2/2\mu_j$ and μ_j is the reduced mass for channel j. We shall use the symbol α_b to denote bound internal states of the clusters; the $\{|\chi_j(E\alpha_b\vec{q}\,j)\rangle\rangle\}$ are then the asymptotic states[7] of the system.

Corresponding to H_j is the jth channel resolvent $G_j(z)$:

$$G_j(z) = (z - H_j)^{-1} , \tag{3}$$

with the full resolvent $G(z)$ defined by

$$G(z) = (z - H)^{-1} . \tag{4}$$

When $z \to E \pm i0 \equiv \pm$, then $G_j(z)$ becomes the usual outgoing (+) or ingoing (−) wave Green's function in channel j, $G_j^{\pm}$.

By assumption, the one-body potential U_j cannot cause transitions out of channel j. Hence, if the initial state is $|\chi_k(E_{\alpha_b\vec{p}}(k))\rangle\rangle$, then U_j can only give rise to a transition amplitude of the form $a_{jk}(\beta\vec{q}, \alpha_b\vec{p})\delta_{jk}$, where β and $\vec{q}$ label the final state of the system and we assume $E_{\beta\vec{q}}(j) = E_{\alpha_b\vec{p}}(k) \equiv E$.

The total, exact amplitude $A_{jk}(\beta_b\vec{q}, \alpha_b\vec{p})$ for a transition from an initial state labeled by $\alpha_b\vec{p}$ in channel k to a final state labeled by $\beta_b\vec{q}$ in channel j is given by[8]

$$A_{jk}(\beta_b\vec{q}, \alpha_b\vec{p}) = a_{jk}(\beta_b\vec{q}, \alpha_b\vec{p})\delta_{jk} + \langle\chi_j(E_{\beta_b\vec{q}}(j))|\hat{T}_{jk}(+)|\chi_k(E_{\alpha_b\vec{p}}(k))\rangle\rangle , \tag{5}$$

where $E_{\beta_b\vec{p}}(j) = E_{\alpha_b\vec{p}}(k) = E$ and $\hat{T}_{jk}(+) = \hat{T}_{jk}(E + i0)$ is the transition operator obeying[1-3]

$$\hat{T}_{jk}(z) = W_{jl}Y_k + \sum_m \hat{T}_{jm}(z)G_m(z)W_{ml}V_k . \tag{6}$$

Equation (6) has been derived using Lippmann's identity[1-3] which takes either of two equivalent forms:

$$(Y_j - Y_m)G_m^{\pm} = \delta_{jm} - 1 \tag{7}$$

or

$$[G_j^{\pm}]^{-1}G_m^{\pm} = \delta_{jm} . \tag{8}$$

Both of these are valid as operator identities only when acting to the left on two-body states of total energy E in channel j. Hence, $\hat{T}_{jk}(+)$ yields the correct amplitudes in (5) only when j is a two-body channel. In particular, as noted above, the solution $\hat{T}_{jk}(+)$ to (6), when W is chosen to be a channel permuting array (CPA), is identical to $V_k + V_jG^+V_k$ when acting to the left on two-body states of energy E in channel j.

B. Scattering states

The scattering states to be used in conjunction with the $\hat{T}_{jk}$ are *channel component* states, as discussed, e.g., by Hahn, Kouri, and Levin[9] and recently considered in more detail and also from a time-dependent standpoint by Kouri, Krüger, and Levin.[10] We denote these states by $|\psi_m\rangle$; they are related to the Schrödinger state $|\Psi\rangle$ by

$$|\Psi\rangle = \sum_n |\psi_n\rangle , \tag{9}$$

where n runs over the two-body channels. As shown in Ref. 9, the $|\psi_n\rangle$ obey

$$(E - H_n)\,|\psi_n\rangle = \sum_m W_{nl}Y_m\,|\psi_m\rangle . \tag{10}$$

When W is chosen to be a CPA, so that l is set equal to m, then (10) reduces to the equations for $|\psi_n\rangle$ given in Ref. 9:

$$(E - H_n)\,|\psi_n\rangle = \sum_m W_{nm}Y_m\,|\psi_m\rangle . \tag{11}$$

The boundary conditions obeyed by the $|\psi_n\rangle$ are made manifest if we recast (11) as a set of integral equations[9,10]:

$$|\psi_n\rangle = |\chi_k(E_{\alpha_b\vec{p}}(k))\rangle\rangle\,\delta_{nk} + G_n^+ \sum_m W_{nm}Y_m\,|\psi_m\rangle . \tag{12}$$

The set of channel component states has not been stressed in earlier work on the channel-coupling-array method.[1-4] Instead, the *channel scattering* states $|X_{mk}\rangle$, where k denotes the incident channel, have been used. These states are related to $|\Psi\rangle$ by[1,3,4,10,11]

$$|\Psi\rangle = \sum_m W_{lm}\,|X_{mk}\rangle . \tag{13}$$

Various sets of coupled equations obeyed by the

$|X_{mk}\rangle$ have been given in these latter references, but we need not display them here, as we shall not use them. The reason for this is that $|X_{mk}\rangle$ do not display any special asymptotic properties while the $|\psi_m\rangle$ do. In fact, it has been shown,[3,10] that for a CPA choice of W, each $|X_{mk}\rangle$ is just the full scattering state $|\Psi\rangle$:

$$|X_{mk}\rangle \equiv |\Psi\rangle, \quad \text{all } m, \quad W = \text{CPA}. \tag{14}$$

The state $|\Psi\rangle \equiv |X_{mk}\rangle$ clearly contains outgoing waves in all open channels. On the other hand, the $|\psi_m\rangle$ have the remarkable property proved in Ref. 10 that *only* $|\psi_j\rangle$ yields outgoing waves in two-body channel j. That is, $|\psi_n\rangle$, $n \neq j$, does *not* lead to outgoing waves in channel j. Hence $|\psi_j\rangle$ is, in effect, that portion of $|\Psi\rangle$ in channel j. This immediately shows us that (9) is a representation of $|\Psi\rangle$ in which the contribution from each channel is separately distinguished, in contrast to Eqs. (13) and (14).

It is this latter property that makes the set $\{|\psi_m\rangle\}$ interesting to use. Even if the sets (11) or (12) are solved approximately, it will still be true that only $|\psi_j\rangle$ will contribute outgoing waves in channel j, although the *accuracy* of the resulting approximation to $A_{jk}(\beta_b\vec{q}, \alpha_b\vec{p})$ *will* depend on the approximations used through the set of coupled equations. To the extent that an exact solution was obtained, the amplitude would be the *same* as would be found from solving the coupled equations obeyed[1-4] by the $|X_{mk}\rangle$. In some cases, we may expect identical approximations to $|\psi_m\rangle$ and $|X_{mk}\rangle$ to yield identical (approximate) amplitudes, but this need not be the case always.[12] However, the point we wish to stress is that even in approximate calculations, the asymptotic property of the $|\psi_m\rangle$ is maintained. Hence, we may regard (9) and the concomitant sets of Eqs. (10) and (11), as an "expansion" of $|\Psi\rangle$ in a nonorthogonal set of states ($\langle\psi_n|\psi_m\rangle \neq 0$, $n \neq m$) in which the rearrangement nature of the scattering system is made evident. Notice that the equations (10) and (11) are not obtained from (9) by projection; instead one must derive them either beginning with the Schrödinger equation $(E - H)|\Psi\rangle = 0$ and subsequently introducing the W_{jk} and Eq. (9), or else by defining the $|\psi_m\rangle$ through the $\hat{T}_{jk}$ equation: Both derivations have been given elsewhere.[10,12] Use of the $\{|\psi_m\rangle\}$ thus provides one way of obtaining a set of coupled equations for rearrangement scattering states in which neither overcounting[13] nor nonorthogonality terms enter.[4,13] We shall examine ways to approximate the $|\psi_m\rangle$ in Sec. III.

C. Exact solutions

In this section we examine some properties of the exact solution to the set (6). As shown in Ref. 3, the $\hat{T}_{jk}(z)$ also obey a right hand equation of the form

$$\hat{T}_{jk}(z) = W_{jl}Y_k + \sum_n W_{jl}Y_nG_n(z)\hat{T}_{nk}(z), \tag{15}$$

where by assumption the labels j, n, k, l, etc., all refer to two-body channels. The set (15) is the most convenient for our purposes. For example, by multiplying both sides of Eq. (12) by $W_{jn}Y_n$, summing the resulting equation on n and then using the relation $\hat{T}_{jk}(+)|\chi_k(E_{\alpha_b\vec{p}}(k))\rangle = \sum_n W_{jn}Y_n|\psi_n\rangle$, we are immediately led to Eq. (15) for $\hat{T}_{jk}(+)$, specialized to the case where the free subscript l in W_{jl} is set equal to the channel index of the channel perturbation it multiplies. From this it immediately follows that the asymptotic form of each $|\psi_j\rangle$ yields as amplitudes precisely the $A_{jk}(\beta_b\vec{q}, \alpha_b\vec{p})$ of Eq. (5), where the $\hat{T}_{jk}(+)$ are the solutions to (15).

In order to examine the structure of the exact solutions, we select W to be a CPA, choosing $W_{jl}Y_n = W_{jn}Y_n$; $k = 1$ (without loss of generality[1,3]); and

$$\begin{aligned} &W_{1N} = 1, \\ &W_{jm} = \delta_{j,m+1}, \quad 1 \leq m \leq N-1, \\ &\text{all other } W_{jm} = 0. \end{aligned} \tag{16}$$

This choice is the transpose of one introduced for the N-arrangement problem by Tobocman[1] and is a generalization of one used by us initially[1-3] for the three-body problem. Also, N is the total number of two-body channels under consideration.

Use of Eq. (16) in Eq. (15) yields

$$\begin{aligned} &\hat{T}_{11}(+) = Y_nG_N^+\hat{T}_{N1}(+), \\ &\hat{T}_{21}(+) = Y_1 + Y_1G_1^+\hat{T}_{11}(+), \\ &\hat{T}_{31}(+) = Y_2G_2^+\hat{T}_{21}(+), \\ &\qquad\vdots \\ &\hat{T}_{N1}(+) = Y_{N-1}G_{N-1}^+\hat{T}_{N-1,1}(+), \end{aligned} \tag{17}$$

as the set of coupled equations of interest. The kernel of this set is such that its $(N-1)$st iterate is connected.[1,3] Although we have used a specific choice for W, (17) is completely general. The reason, of course,[1,3] is that no correspondence between channel labels $(1, 2, \ldots)$ and the actual physically observable channels of the system has yet been established, so that assignment of channels to labels remains arbitrary.

Not only is (17) a connected kernel equation, it can be decoupled.[1,3] This latter feature is particularly useful for examining the nature of the solutions. Repeated substitution leads to the following set of uncoupled, connected kernel equations for

the $\hat{T}_{j1}(+)$:

$$\begin{aligned}
\hat{T}_{11}(+) &= Y_N G_N^+ Y_{N-1} G_{N-1}^+ \cdots Y_2 G_2^+ Y_1[1 + G_1^+ \hat{T}_{11}(+)], \\
\hat{T}_{21}(+) &= Y_1[1 + (G_1^+ Y_N G_N^+ Y_{N-1} \cdots Y_3 G_3^+ Y_2) G_2^+ \hat{T}_{21}(+)], \\
\hat{T}_{31}(+) &= Y_2 G_2^+ Y_1[1 + (G_1^+ Y_N \cdots Y_4 G_4^+ Y_3) G_3^+ \hat{T}_{31}(+)], \\
&\cdots \\
\hat{T}_{N1}(+) &= Y_{N-1} G_{N-1}^+ \cdots Y_1[1 + (G_1^+ Y_N) G_N^+ \hat{T}_{N1}(+)].
\end{aligned} \tag{18}$$

We shall refer to Eqs. (18) as the folded form of the channel coupling array equations.

Our purpose in this paper is to consider some aspects of multistep processes in direct reactions. These are generally assumed to be associated with higher-order corrections to the DWBA, the amplitude for which, $A_{j1}^{\mathrm{DW}}(\beta_b \vec{q}, \alpha_b \vec{p})$, is given in the prior form by[8]

$$A_{j1}^{\mathrm{DW}}(\beta_b \vec{q}, \alpha_b \vec{p}) = \langle \chi_j(E_{\beta_b \vec{q}}(j)) | Y_1 | \chi_1(E_{\alpha_b \vec{p}}(1)) \rangle\!\rangle. \tag{19}$$

That is, the DWBA amplitude is an on-shell matrix element of Y_1. There is as yet no unambiguous way to determine the higher-order corrections corresponding to multistep processes as long as approximations are introduced. However, in order to consider *any* form of such corrections to DWBA from the present formulation, one must first have an equation for $\hat{T}_{j1}(+)$ which has a single Y_1 as the leading term. Inspection of (18) indicates that this is the case only when $j=2$. But, as shown in Ref. 3, such a lack in the equations for the $\hat{T}_{j1}(+)$, $j \neq 2$, is only apparent, since the inhomogeneous term in each of these equations can be unfolded using Lippmann's identity to yield a sum of terms, one of whose members is just the correct Born term Y_j. This is easily demonstrated, as follows. The typical equation in (18) has the form

$$\begin{aligned}
\hat{T}_{j1}(+) &= Y_{j-1} G_{j-1}^+ Y_{j-2} \cdots Y_2 G_2^+ Y_1 \\
&\quad \times [1 + (G_1^+ Y_N G_N^+ \cdots G_{j+1}^+ Y_j) G_j^+ \hat{T}_{j1}(+)] \\
&\equiv I_j[1 + (G_1^+ Y_N G_N^+ \cdots G_{j+1}^+ Y_j) G_j^+ \hat{T}_{j1}(+)].
\end{aligned} \tag{20}$$

Since we are only interested in on-shell matrix elements of (20), we may use Lippmann's identity, Eq. (7), $j-1$ times in the leading term I_j, which contains the $j-1$ factors Y_n, $1 \leq n \leq j-1$. Thus applying (7) once gives

$$I_j = Y_{j-2} G_{j-2}^+ Y_{j-3} \cdots G_2^+ Y_1 + Y_j G_{j-1}^+ Y_{j-2} \cdots G_2^+ Y_1,$$

where $Y_0 G_0^+$ means $Y_N G_N^+$. Continuing this reduction $j-1$ times finally gives[3]

$$\begin{aligned}
I_j &= Y_1 + Y_j G_2^+ Y_1 + Y_j G_3^+ Y_3 G_2^+ Y_1 \\
&\quad + \cdots + Y_j G_{j-1}^+ Y_{j-2} G_{j-2}^+ \cdots G_2^+ Y_1.
\end{aligned} \tag{21}$$

The use of Eq. (21) in Eq. (20) above yields the unfolded form of the channel-coupling-array equations.

Equation (21) clearly establishes that the exact I_j, and thus the exact $\hat{T}_{j1}$, contains the usual first-order Born term. Furthermore, in (20), every term I_j except for $j=2$ will be of the form

$$I_j = Y_1 + Y_j G_2^+ Y_1 + \cdots.$$

Hence, for every term $\hat{T}_{j1}(+)$ other than $\hat{T}_{21}$, the leading term I_j can be decomposed into a Born term (one-step) plus a two-step term $Y_j G_2^+ Y_1$. And, once one iteration is considered for the $\hat{T}_{21}(+)$ equation, it will also reduce to the form

$$\hat{T}_{21}(+) = Y_1 + Y_2 G_2^+ Y_1 + \cdots$$

Clearly, every $\hat{T}_{j1}(+)$ of (18) can be reexpressed as an infinite series of terms, each corresponding to a different multistep process. This is similar to the corresponding result one finds[13] from use of Lippmann-Schwinger (L-S) equations to express $\hat{T}_{jk}(+)$ as a series. One important difference is that the equations of the set (17) are each connected kernel equations, unlike the L-S equations. Since the solutions to (17) are given by (20), we immediately see that solving (17) [or (12)] will lead to amplitudes in which multistep processes are taken into account in a compact way. While these comments are valid for the exact solutions, only approximate solutions can be determined in practice. We examine aspects of approximate solutions in the next section. We also note here that Tobocman[4] has compared the series solutions for the CRC[13] and the T_{jk} amplitudes. His analysis also holds for the amplitudes obtained using the $\hat{T}_{jk}$. Since we study approximations, our analysis essentially begins where that of Tobocman ends.

III. APPROXIMATE SOLUTIONS

The work of the last subsection shows that the exact folded equations for the $\hat{T}_{j1}$ can be unfolded to display explicitly a DWBA term Y_1 plus higher-order corrections, which include the effects of multistep processes. It is to be expected that these two (folded and unfolded) forms of the equations for $\hat{T}_{j1}$ will respond differently to approximations such as a bound-state truncation in the spectral expansion of Green's functions, and we illustrate this in detail, the essential difference being the absence of DWBA amplitudes when the folded set of equations is approximated.

The channel Hamiltonian H_m may be decomposed as follows:

$$H_m = (K_m + U_m) + h_m, \tag{22}$$

where K_m is the kinetic energy operator for relative motion of the two clusters forming channel m,

and h_m is the Hamiltonian describing their internal states. The eigenstates $|\varphi_m\rangle$ and $|\zeta_m\rangle$ obey

$$[E_\alpha(m) - h_m]|\varphi_m(E_\alpha(m))\rangle\rangle = 0$$

and

$$[K_m + U_m - E_q^+(m)]|\zeta_m(E_q^+(m))\rangle\rangle = 0 .$$

The internal states $\{|\varphi_m\rangle\}$ are usually assumed to be complete:

$$G_m^+ = \sum_\alpha \frac{|\varphi_m(E_\alpha(m))\rangle\rangle\langle\langle\varphi_m(E_\alpha(m))|}{E - E_\alpha(m) + i0 - K_m - U_m}$$

and

$$|\psi_m\rangle = \sum_\alpha |\varphi_m(E_\alpha(m))\rangle\rangle\, |\eta_m(E_\alpha(m))\rangle\rangle . \tag{23}$$

Notice that the structure of (11) or (12) allows (23) to be used for each $|\psi_m\rangle$ without overcounting problems. This is one of the advantages of using the channel component states.

The standard bound-state approximation is to limit the set $\{|\varphi_m(E_\alpha(m))\rangle\rangle\}$ for each m to some or all of the set of bound states $\{|\varphi_m(E_{\alpha_b}(m))\rangle\rangle\}$. This is the procedure we shall follow in introducing approximations in the present case. (It has been remarked on previously,[3] though not studied in detail.) We therefore define a projection operator P_m and its complement Q_m by

$$P_m = \sum_{\alpha_b} |\varphi_m(E_{\alpha_b}(m))\rangle\rangle\langle\langle\varphi_m(E_{\alpha_b}(m))|,$$

$$Q_m = \sum_\alpha{}' |\varphi_m(E_\alpha(m))\rangle\rangle\langle\langle\varphi_m(E_\alpha(m))|,$$

where the sum over α_b in P_m includes at least one but not more than all of the bound states in channel m and Q_m contains the rest of the complete set of internal states (bound and/or continuum) in channel m. Clearly,

$$P_m + Q_m = 1 ,$$

where the resolution of the identity is then given by

$$1 = \sum_\alpha |\varphi_m(E_\alpha(m))\rangle\rangle\langle\langle\varphi_m(E_\alpha(m))|,$$

and now the sum and integral is over *all* internal states in channel m (bound plus continuum). Including all bound states in P_m is presumably the best approximation but is usually not feasible numerically. Notice of course that $0 \neq P_m P_n \neq P_n P_m \neq 0$, $m \neq n$. This is due to the nonorthogonality of bound states in different arrangements.

The approximations we wish to consider are obtained by the replacements

$$|\psi_m\rangle \to |\tilde{\psi}_m\rangle \equiv P_m|\psi_m\rangle \tag{24}$$

and

$$G_m^+ \to \tilde{G}_m^+ \equiv P_m G_m^+ = G_m^+ P_m . \tag{25}$$

If these replacements are made in Eq. (17), we will have new operators $\tilde{T}_{j1}(+)$ and (approximate) amplitudes $\tilde{A}_{j1}(\beta_b\vec{q}, \alpha_b\vec{p})$. Choosing (16) for W, we find

$$|\tilde{\psi}_j\rangle = |\chi_1(E_{\alpha_b}\vec{p}(1))\rangle\rangle\delta_{j1} + \tilde{G}_j^+ Y_{j-1}|\tilde{\psi}_{j-1}\rangle \tag{26}$$

and

$$\tilde{T}_{j1}(+) = Y_{j-1}[\delta_{j2} + \tilde{G}_{j-1}^+ \tilde{T}_{j-1,1}(+)] , \tag{27}$$

where the subscript 0 (i.e., $j=1$ in $j-1$) is to be replaced by the channel label N. The solution to (27) is, from (20),

$$\tilde{T}_{j1}(+) = \tilde{I}_j[1 + (\tilde{G}_1^+ Y_N \cdots \tilde{G}_{j+1} Y_j)\tilde{G}_j^+ \tilde{T}_{j1}(+)] , \tag{28}$$

where

$$\tilde{I}_j = Y_{j-1}\tilde{G}_{j-1}\cdots Y_2\tilde{G}_2^+ Y_1 . \tag{29}$$

The approximate amplitude $\tilde{A}_{jk}$ is given in Eq. (5) with $\tilde{T}_{jk}(+)$ replaced by $\tilde{T}_{j1}(+)$.

Tobocman[4] has remarked that some of the terms I_j of (20) are of arbitrarily high order. We have shown in Ref. 3 and outlined again in Sec. II C [Eq. (21)] that I_j, through Lippmann's identity, reduces to a sum, the lowest-order term of which is just the usual DWBA operator Y_1. It is clearly of interest to determine if this latter result holds for the approximate leading term $\tilde{I}_j$ of Eq. (29). As we now show, the answer is no: Only when $j=2$ does $\tilde{I}_j$ "reduce" to Y_1. For other values of j, $\tilde{I}_j$ generally cannot even be reduced to $F(P)Y_1$, where $F(P)$ is a linear combination of the projection operators P_m. [Similarly, the various iterated (first, second, etc.) terms in each $\tilde{T}_{j1}(+)$ will not reduce to simple forms analogous to those obtained when $P_m = 1$, i.e., when $\tilde{G}_m^+$ is not approximated.] The reason for this is the presence of the factor P_m in $\tilde{G}_m^+$, which alters Lippmann's identity.

The effect of P_m is to change Eq. (7) to

$$(Y_j - Y_m)\tilde{G}_m^+ = (\delta_{jm} - 1)P_m ; \tag{30}$$

in order to reduce the various $\tilde{I}_j$ factors, (30) rather than (7) must be used. To establish the nonreducibility of the $\tilde{I}_j$ to the form (21) it is sufficient to consider the case $N=3$. Then we find

$$\tilde{I}_2 = Y_1 ,$$

$$\tilde{I}_3 = Y_2 G_2^+ Y_1 ,$$

and

$$\tilde{I}_1 = Y_3\tilde{G}_3^+ Y_2\tilde{G}_2^+ Y_1 .$$

Clearly, $\tilde{I}_2$ is in the desired form. Since $\tilde{I}_3$ will act to the left on the on-shell state $\langle\chi_3|$, we use (30) to find

$$\tilde{I}_3 = P_2 Y_1 + Y_3\tilde{G}_2^+ Y_1 . \tag{31}$$

It is obvious from (31) that the lowest-order ma-

trix element contained in $\tilde{A}_{31}$ will not be A_{31}^{DW} but $\langle\chi_3|P_2Y_1|\chi_1\rangle$. In terms of the discussion following Eq. (29), $F(P)=P_2$ when $j=3$. If one replaces P_2 by $1-Q_2$, then (31) can be rewritten as

$$\tilde{I}_3 = Y_1 - Q_2Y_1 + Y_3\tilde{G}_2^+Y_1 .$$

However, the terms Y_1 and Q_2Y_1 are of the same order in the perturbation Y_1 and, therefore, the appearance of a DWBA term is only apparent since the term Q_2Y_1 removes that portion of Y_1 that is complementary to P_2Y_1. Our conclusion is that a standard DWBA leading term no longer appears, but instead we obtain the modified interaction P_2Y_1. The term $Y_3\tilde{G}_2^+Y_1$ is clearly a two-step operator.

The term $\tilde{I}_1$ appears in the equation for $\tilde{T}_{11}(+)$, and here we shall see that there is no $F(P)$, as noted above. Applying Eq. (30) once, we have

$$\tilde{I}_1 = P_3Y_2\tilde{G}_2^+Y_1 + Y_3\tilde{G}_3^+Y_2\tilde{G}_2^+Y_1 . \tag{32}$$

Now, if $\langle\chi_1|P_3$ were an on-shell eigenstate of H_1, then we could use Lippmann's identity in the form of (30) to rewrite the first term of (32) as $P_3P_2Y_1 + P_3Y_1\tilde{G}_2^+Y_1$, in which case we would have $F(P) = P_3P_2$. But $\langle\chi_1|P_3$ is *not* an on-shell eigenstate of H_1 and hence the term $P_3Y_2\tilde{G}_2^+Y_1$ cannot be further reduced within the bound-state approximation [nor, indeed, can it be so reduced within any approximation wherein $P_m \neq 1$]. Similar comments will obviously hold for any number of channels greater than three. Returning to Tobocman's comment[4] on the presence of arbitrary high orders in the leading terms of $T_{j1}(+)$, and thus of $\hat{T}_{j1}(+)$ also, we see that though it was not intended for this present set of approximations, these approximations do define the situation for which it does hold: the presence of the projection operator P_m in $\tilde{G}_m^+$ not only prevents the DWBA operator Y_1 from appearing as the lowest-order term in $\tilde{T}_{31}(+)$, but it also means that (approximate) high-order terms will be present as the leading component in most of the $\tilde{T}_{j1}(+)$. For example, the leading term in $\tilde{I}_1$ is a modified two-step operator.

We thus reach the following conclusion. The approximate W-array formalism in folded form under consideration yields one pure DWBA amplitude and one modified DWBA amplitude involving P_2Y_1, with all remaining amplitudes having as the lowest-order transition operator a nonreducible term of higher order than P_2Y_1, namely, $P_3Y_2\tilde{G}_2^+Y_1$. This is a direct result of the off-shell transformation represented by Lippmann's identity (7) not being "reversible" when an approximation is made. Alternatively we may say that if only one nonorthogonality overlap occurs, as in P_2 of (31), then a modified DWBA results, but the presence of more than one nonorthogonality overlap as in $P_3G_3^+Y_2P_2G_2^+Y_1$ effectively eliminates any type of DWBA term from occurring in any of the other $\hat{T}_{j1}(+)$.

In order to attempt to understand this result, it is useful to keep in mind two distinct aims of some recent work in direct reaction theory. One is the desire to investigate multistep effects, as typified, e.g., by the work of Refs. 6 and 13 and the references cited therein. The assumption in this case is that the one-step DWBA amplitude provides a good, if not very good, zeroth-order approximation, and that one only needs a means for including various two-step or possibly three-step amplitudes to add to the one-step DWBA. The CRC[13,14] method does this, albeit through use of an ansatz of the form

$$|\Psi\rangle \cong \sum_m P_m|\Psi\rangle \tag{33}$$

and

$$\sum_m (E-H)P_m|\Psi\rangle = 0 , \tag{34}$$

where m runs over two-body channels and appropriate boundary conditions are assumed. The range of validity of (33) and (34) is not well established, nor is it clear how to include three-, four-, etc., body channels.

The other aim is to reformulate nuclear reaction theory in terms of connected kernel equations, as in Refs. 1, 3, 4, and 15, and then to investigate the properties of these new equations and apply them to specific reactions. The utility of these latter approaches, in terms of their application via specific approximations, can only be determined by the results they yield, exactly as with the CRC type of theory. The dynamical effect of channel coupling as it is manifested in the W-array equations is a subject for future investigation. The lack of the DWBA amplitude in particular channels in the approximated folded form of the equations is itself no indication that the theory will fail to predict correct amplitudes in those channels precisely because of the presence of explicit channel coupling.[16] Indeed, the data on relative magnitudes of cross sections leading to different final channels in the $p+{}^{12}\mathrm{C}$ reaction at 50 MeV, surveyed by Redish,[15] indicates the role that channel coupling can play. The cross sections with largest magnitude are (p,p) and (p,p'), with the next being (p,d). Below these by at least an order of magnitude are the (p,n), (p,τ), (p,t), (p,α), and $(p,{}^6\mathrm{Li})$ cross sections. Since all channels are dynamically coupled, it would seem reasonable to expect that the major contributions to all cross sections would come from the (p,p), (p,p'), and (p,d) channels, with the other channels playing a smaller, more perturbative role. In terms of the present approximate folded theory [Eqs. (26) and (27)], we would

expect in first order to be able to treat the strong elastic and inelastic channels through the U_1 potential and the strong (p,d) channel by identifying it as channel 2. The other channels would then be coupled together by the dynamical equations, with, for example, the (p, n) channel being number 3, the (p, t) or (p, τ) channels being next, etc.

This discussion is easily extended to other reactions, the general rule being to consider the incident (elastic and inelastic) channel, the strongest reaction channel, and then the other arrangement channel (or channels if computation facilities permit more than one) of interest. For example, if (d, p) processes are being studied, the simplest possibilities are elastic and inelastic, (d, τ) or (d, t), and (d, α). Similar remarks hold for heavier projectiles including heavy ions.

The preceding argument is based on the need for the DWBA in only one reaction channel, on the assumption that only one such channel appears to be strong; the explicit dynamical coupling inherent in the theory, along with the weaker (projected) coupling matrix elements, should then produce amplitudes of the desired shape and magnitude. However, because of computing restrictions, an interesting situation does arise which tends to restore some of the role played by the DWBA in conventional one-step-only calculations. It is not likely that numerically exact calculations will soon be done in which more than three arrangement channels will be present, i.e., channels 1, 2, and 3. In addition to the incident channel (number 1), only two reaction channels would be taken into account. For the $p+{}^{12}C$ case, these would presumably be (p, d) as channel 2, and then the other channel of interest, e.g., (p, τ). In channel 2 the lowest-order operator is the DWBA term Y_1, in channel 3 it is the modified DWBA operator $P_2 Y_1$. To the extent that $\langle\chi_3|P_2 Y_1$ is a good approximation to $\langle\chi_3|Y_1$, then $\langle\chi_3|P_2 Y_1|\chi_1\rangle$ is a DWBA amplitude, in the present case for the (p, τ) reaction. Hence, when $\langle\chi_3|P_2 Y_1|\chi_1\rangle \cong \langle\chi_3|Y_1|\chi_1\rangle$, there is, in effect, a DWBA amplitude on both reaction channels. This remark is limited to the case of $N=3$ and requires that the states in P_2 have extremely good overlap with the states in $\langle\chi_3|$.

To evaluate $\langle\chi_3|P_2$ we need to know two overlap functions: one for the final nuclear state relative to the nuclear state in P_2, and a similar overlap function for the ejectile and its channel 2 counterpart, normally differing from it by one or two nucleons. While both are nuclear structure problems, the latter has been examined for a variety of light nuclear projectiles or ejectiles by Rinat, Kok, and Stingl.[17] Their results give the probability amplitude $Z^{1/2}$ that an n-particle system ($n \leq 6$) looks like an $n-1$ or $n-2$ particle system in its ground state, plus a single nucleon (or deuteron in the case of ^{6}Li). In general, they find $0.6 \leq Z^{1/2} \leq 1.0$ for the amplitude $Z^{1/2}$. Thus, for light ion projectiles, the light ion part of $\langle\chi_3|P_2$ may be a reasonable approximation to the light ion internal wave function in $\langle\chi_3|$, depending on the projectile and the chosen form of its internal wave function. The nuclear overlap will depend on the target and the residual nuclei in question and, in the case of good single particle states for example, can also provide large overlaps, thereby helping to guarantee that a reasonable approximation to a DWBA amplitude is also present in channel 3. We thus find the not-so-paradoxical situation in which an approximate many-channel calculation will yield no DWBA in most amplitudes, while an approximate three- (or even two-) channel calculation (the most feasible ones in the near future) can have DWBA amplitudes. Their occurrence or nonoccurrence is not, in our opinion, a crucial factor as long as channel 2 is chosen as the reaction channel for which the observed cross sections are largest: The DWBA in this channel along with the dynamical coupling of the W-array equations should be the key to the theory being able to fit the data.

We also stress, as have other authors,[6,13,14] that one should not rely on a low order perturbation-like calculation to determine the approximate amplitude, as this may not give a reliable estimate: The (approximate) set of coupled-channel calculations should be performed first. Afterwards, an examination of the validity of further approximations, e.g., low-order perturbation terms, could be investigated. The work of Kunz and Rost[6] is a pertinent instance of this. They estimated, in the zero-range approximation (ZRA), the one-step amplitude arising from the matrix element of the first term of $\bar{I}_3$ of Eq. (31), viz., $P_2 Y_1$, for a (p, t) reaction. The magnitude was found to be about one-half of the "usual" DWBA term, which is the matrix element of $\bar{I}_2 = Y_1$, appearing below Eq. (30). The effect of the projection is to give (in ZRA) an amplitude similar to DWBA but not as large. (Using the estimates of Ref. 17, we have redetermined the ratio of the $P_2 Y_1$ and Y_1 matrix elements and found it to lie in the range 0.55–0.85, in rough agreement with the Kunz-Rost value.) However, it is now generally accepted that this estimate of either the full amplitude or a portion of it would not remain unchanged when the set of coupled equations for the approximate $|\bar{\psi}_p\rangle$ is solved. Calculations designed to examine this are currently being undertaken[18] and results will be reported subsequently.

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[1]M. Baer and D. J. Kouri, J. Math. Phys. 14, 1637 (1973); D. J. Kouri and F. S. Levin, Bull. Am. Phys. Soc. 19, 489 (1974); Phys. Lett. 50, 421 (1974); Phys. Rev. A 10, 1616 (1974); Nucl. Phys. A250, 127 (1975); Phys. Rev. C 11, 352 (1975); 13, 14 (1976); 13, 1825 (1976); W. Tobocman, *ibid.* 9, 2466 (1974).

[2]J. Lin and D. J. Kouri, J. Chem. Phys. 60, 303 (1974); D. J. Kouri, J. Lin, and F. S. Levin (unpublished); F. S. Levin, invited paper presented at the Institute of Physics meeting on Polarization Effects and Mechanisms of Nuclear Reactions, Birmingham, England, December, 1974 (unpublished); D. J. Kouri, in *Few-Body Dynamics*, edited by A. N. Mitra *et al.* (North-Holland, Amsterdam, 1976), p. 378.

[3]D. J. Kouri and F. S. Levin, Nucl. Phys. A253, 395 (1975).

[4]W. Tobocman, Phys. Rev. C 11, 43 (1975).

[5]W. Tobocman, Phys. Rev. C 12, 741 (1975).

[6]P. D. Kunz and E. Rost, Phys. Lett. 47B, 136 (1973).

[7]H. Ekstein, Phys. Rev. 101, 880 (1960).

[8]For example, M. Gell-Mann and M. L. Goldberger, *Collision Theory* (Wiley, New York, 1964); R. G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill, New York, 1966); J. R. Taylor, *Scattering Theory* (Wiley, New York, 1972).

[9]Y. Hahn, D. J. Kouri, and F. S. Levin, Phys. Rev. C 10, 1620 (1974).

[10]D. J. Kouri, H. Krüger, and F. S. Levin, Phys. Rev. D 15, 1156 (1977).

[11]Y. Hahn, D. J. Kouri, and F. S. Levin, Phys. Rev. C 10, 1615 (1974).

[12]Such identical pairs of approximate amplitudes have been found: D. J. Kouri and R. Goldflam, unpublished calculations.

[13]For example, T. Udagawa, H. H. Wolter, and W. R. Coker, Phys. Rev. Lett. 31, 1507 (1973); D. Robson, Phys. Rev. C 7, 1 (1973); 10, 2626 (1974); N. Austern and M. Vincent, *ibid.* 10, 2628 (1974).

[14]S. R. Cotanch and C. M. Vincent, Phys. Rev. Lett. 36, 21 (1976); Phys. Rev C 14, 1739 (1976).

[15]Gy. Bencze, Nucl. Phys. A210, 568 (1973); E. F. Redish, *ibid.* A225, 16 (1974), and references cited therein.

[16]The theory of Robson (see Ref. 13) and calculations of Managoli and Robson [Bull. Am. Phys. Soc. 19, 453 (1974); Nucl. Phys. A252, 354 (1975)], is an example of this. The theory is based on unitarity considerations and hence includes no one-step DWBA amplitudes, but only products of DWBA transition operators. Angular distributions, at least for the weak transitions studied so far, are reasonable.

[17]A. S. Rinat, L. P. Kok, and M. Stingl, Nucl. Phys. A190, 328 (1972).

[18]F. S. Levin and J. M. Greben (unpublished).