PHYSICAL REVIEW C VOLUME 16, NUMBER 2 AUGUST 1977

Final-state interactions in the decay of $^{12}C^*$ into three α particles

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(Received 9 May 1977)

We apply the Faddeev equations with separable potentials for the decay of $^{12}C^*$ ($J^{\pi} = 1^+$; 12.71 MeV) [formed in the reaction $^{13}C(^3He,\alpha)^{12}C^*$] via ^8Be (2^+; 2.9 MeV) into three α particles. The α-particle energy spectrum is calculated. No effect of unitarity constraints (rescattering) is observed in the energy spectrum.

[NUCLEAR REACTIONS $^{13}C(^3He,\alpha)^{12}C^* \to 3\alpha$, calculated α-particle energy spectrum.]

I. INTRODUCTION

In our study of the "minimal" three-body equation,[1] we have demonstrated the importance of unitarity constraints on the quasi-two-body amplitudes (the amplitudes for the creation of the spectator particle and correlated pair in the reaction) that appear in the sequential theory of three-body final states. The conventional phenomenology is to assume the quasi-two-body amplitude is slowly varying over the three-body phase space. We have seen the shaky validity of this assumption for s-wave resonant pair interactions.

The question arises whether we can detect the rescattering effect in the energy spectrum of an actual system, and draw the conclusion that any disagreement between the Watson approximation and experiment is due to the neglect of the $(E-\frac{3}{4}p^2)^{1/2}$ singularity or the rescattering effect. The practical system we choose to attack is the decay of $^{12}C^*(J^{\pi}=1^+) \to 3\alpha$. Experimentally the 12.71 MeV, 1^+ state is prepared by the reaction $^{13}C(^3He, \alpha)^{12}C^*$. Then the compound nucleus $^{12}C^*$ decays either by way of the first excited state of ^8Be,

$$^{12}C^* \to \alpha_1 + {}^8Be^{2^+}, \quad {}^8Be^{2^+} \to \alpha_2 + \alpha_3\,,$$

or directly,

$$^{12}C^* \to \alpha_1 + \alpha_2 + \alpha_3\,.$$

The decay to the ^8Be ground state ($J^{\pi}=0^+$) is forbidden since the conservation of parity forces the orbital angular momentum between α_1 and ^8Be to an even value. Furthermore it must be 2, to add up to 1 with the ^8Be's angular momentum value of 2. Thus, this system provides a transparent example for the study of final state interactions. Detailed experimental results have been obtained by Balamuth, Zurmühle, and Tabor.[2]

Since all the angular momenta are fixed, instead of using the "minimal" equations derived from unitarity and analyticity, we start from the Faddeev equations with separable two-body interactions. With our approach, though detailed dynamics assumptions enter (such as the two-body interaction on and off shell, primary decay vertex, etc.), the quasi-two-body amplitude of course satisfies the unitarity constraint. By comparing with the usual phenomenology (which violates unitarity), employed for the analysis of three-body final states we can learn whether the violation of unitarity is serious or not.

A similar treatment to ours has been given by McMahan and Duck[3] for a different ^{12}C state and the decay scheme is different:

$$\begin{aligned} ^{12}C^*(18.37\ \text{MeV};\ 2^+) &\to \alpha_1 + {}^8Be^{0^+}_{g.s.}, \quad {}^8Be^{0^+}_{g.s.} \to \alpha_2 + \alpha_3 \\ &\to \alpha_1 + {}^8Be^{2^+}, \quad {}^8Be^{2^+} \to \alpha_2 + \alpha_3\,. \end{aligned}$$

Their results seem to contradict ours in two respects. First, they show a sizable effect of the primary decay vertex on the energy spectrum, and secondly they show in an approximate way, the necessity of including the Coulomb phase $\delta_{C,l}$ to fit the experimental data.

In Sec. II the details of the separable T-matrix approximation are given. In Sec. III we discuss the integral equation for the decay amplitude with our particular angular momentum assignment, using the separable two-body interaction given in the previous section. In Sec. IV, we have calculated the energy spectrum. In Sec. IV A, we choose the simple form for the inhomogeneous term of the integral equation and in Sec. IV B we attempt to improve the results first by including the Coulomb effect and secondly with a different choice of the inhomogeneous term of the integral equation. Finally, in Sec. V we present our conclusions.

II. TWO-BODY INTERACTION

Since we need an expression for the off-shell two-body scattering T matrix in order to deal with the three-particle problem, we discuss in this section the parametrization of a separable potential to fit the α-α scattering experimental data. The formalism for a short-range separable potential together with a Coulomb potential has been given by Harrington.[4] This is appropriate for the α-α interaction. We can divide the T matrix into two parts, the pure Coulomb scattering amplitude T_C, and an amplitude T_{SC} due to the separable potential under the influence of a Coulomb field:

$$T(\vec{k}', \vec{k}) = T_C(\vec{k}', \vec{k}) + T_{SC}(E; \vec{k}', \vec{k})\,. \tag{1}$$

We have the explicit form of the partial wave decomposition of each amplitude ($\hbar = C = 1$):

$$T_l(k, k) = -\frac{2}{\pi m k}\sin\delta_l(k)e^{i\delta_l(k)}\,, \tag{2}$$

$$T_{C,l}(k, k) = -\frac{2}{\pi m k}\sin\delta_{C,l}(k)e^{i\delta_{C,l}(k)}\,, \tag{3}$$

$$T_{C,l}(E; k', k) = e^{i\delta_{C,l}(k')}\frac{v_{C,l}(k')v_{C,l}(k)}{D_l(E)}e^{i\delta_{C,l}(k)}\,, \tag{4}$$

where

$$D_l\left(\frac{k^2}{m}\right) = -\left(\lambda_e^{-1} + m\int_0^\infty \frac{v_{C,l}^2(k')}{k^2 + i\epsilon - k'^2}k'^2 dk'\right), \tag{5}$$

m is the α-particle mass, and λ_l is a coupling strength of the separable potential. From the on-shell expression for the T matrix (1) to (4), one gets

$$\frac{v_{C,l}^2(k)}{D_l(E)} = -\frac{2}{\pi m k}\sin\delta_{SC,l}(k)e^{i\delta_{SC,l}(k)}\,, \tag{6}$$

where

$$\delta_l(k) = \delta_{C,l}(k) + \delta_{SC,l}(k)\,. \tag{7}$$

We can identify $\delta_{SC,l}$ as the commonly called "nuclear" phase shift, but it is by no means equal to the phase shift which would be obtained if the Coulomb field were switched off. Rather, it is defined by (7) under the presence of the Coulomb field.

We choose the separable potential

$$v_{C,2}(k) = \frac{k^2}{(k^2+\beta^2)^2}\,, \tag{8}$$

where β^{-1} is the range of the force. This form does not have the correct threshold behavior $e^{-\pi\eta}k^{-1}$; here $\eta = \mu e_1 e_2 k^{-1}$, $e_1 e_2$ are the respective electric charge, μ is the reduced mass, yet the potential still includes the Coulomb effect in a sense that it reproduces the observed nuclear phase shift δ_{SC2}. The form (8) clearly does have the correct angular momentum threshold for the case without the Coulomb force. Form (5) we see that

$$D_2\left(\frac{k^2}{m}\right) = -\frac{1}{\lambda_2} + \frac{\pi m}{32(k^2+\beta^2)^4}\left(\beta^5 + 5\beta^3k^2 + 15\beta k^4 - 5\frac{k^6}{\beta}\right) + i\tfrac{1}{2}(\pi m)\frac{k^5}{(k^2+\beta^2)^4}\,. \tag{9}$$

The parameters λ_2 and β are determined through (6) to fit the experimental result[5] for the nuclear phase shift at the resonance energy $E_r = 3.4$ MeV. Thus, we get $\beta^{-1}\lambda_2 = 227$ MeV, $\beta = 1.58\ \text{fm}^{-1}$ from

$$\left.\frac{d\delta_{SC2}}{dE}\right|_{E=E_r} = 48.6\ (\text{deg/MeV}) \tag{10}$$

and

$$\text{Re}[D_2(E_r)] = 0\,. \tag{11}$$

The theoretical phase shift δ_{SC2} obtained in this way is shown in Fig. 1 where it is compared with the phase shift determined from the data.

III. THREE-BODY EQUATIONS

The decay amplitude for ${}^{12}C^* \to 3\alpha$ is given by

$$R_M = \langle\psi_{3\alpha}|\mathcal{H}_{\text{weak}}|{}^{12}\text{C}\rangle\,, \tag{12}$$

where M denotes the spin projection of the ${}^{12}\text{C}^*$ state and $\langle\psi_{3\alpha}|$ denotes the final state vector of three outgoing particles defined in terms of three-particle plane wave states $\langle 3\alpha|$ as

$$\langle\psi_{3\alpha}| = \langle 3\alpha| + \langle 3\alpha|T_{3\alpha}(E)G_0(E)\,. \tag{13}$$

Here E is the total energy and G_0 is the free three-

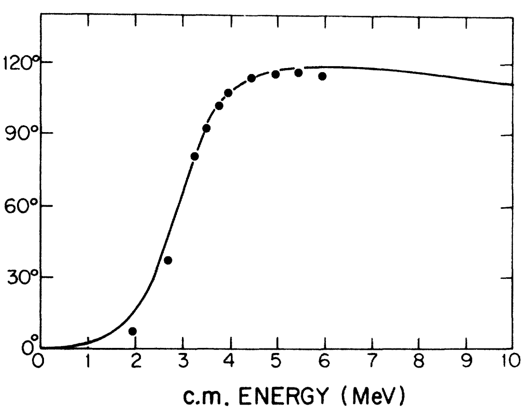

FIG. 1. The theoretical $l = 2$ phase shift as a function of the c.m. energy E. The experimental values are the points.

particle Green function $(E+i\epsilon=H_0)^{-1}$. The T matrix may be decomposed, following Faddeev, into the sum of three terms

$$T_{3\alpha}(E)=\sum_{i=1}^{3} X_i(E). \tag{14}$$

The X_i satisfy the Faddeev equations

$$X_i(E)=t_i(E)+t_i(E)G_0(E)\sum_{j\neq i} X_j(E), \tag{15}$$

where, for example, $t_1(E)$ is the fully off-shell two-body T matrix for particles 2 and 3. From (12) and (13) we obtain an equation for R_M:

$$R_M=\langle 3\alpha|\mathcal{H}_{\rm weak}|^{12}\mathrm{C}\rangle+\langle 3\alpha|T_{3\alpha}(E)G_0(E)\mathcal{H}_{\rm weak}|^{12}\mathrm{C}\rangle. \tag{16}$$

Thus we need to know the following matrix element:

$$\langle \vec{\mathrm{P}}_i, \vec{\mathrm{k}}_i|X_i(E)G_0(E)\mathcal{H}_{\rm weak}|^{12}\mathrm{C}\rangle, \tag{17}$$

where P_i is the momentum of particle i and $k_i=\frac{1}{2}(P_j-P_k)$ $(i,j,k$ cyclic) is the momentum conjugate to the relative coordinate of the (j,k) pair.

The fully off-shell two-body T matrix appearing in Eq. (15) is taken to have the separable form T_{SC}, just discussed in the previous section. We neglect the Coulomb amplitude T_C on the grounds that it acts on less strongly correlated pairs than T_{SC}. We have

$$\begin{aligned}\langle \vec{\mathrm{P}}_i, \vec{\mathrm{k}}_i|t_i(E)|\vec{\mathrm{P}}_i', \vec{\mathrm{k}}_i'\rangle &=\delta(\vec{\mathrm{P}}_i-\vec{\mathrm{P}}_i')T_{SC}(E-\tfrac{3}{4}\vec{\mathrm{P}}_i^{\,2};\vec{\mathrm{k}}_i,\vec{\mathrm{k}}_i')\\ &=\delta(\vec{\mathrm{P}}_i-\vec{\mathrm{P}}_i')\sum_{lm} Y_{lm}^*(\hat{\vec{\mathrm{k}}}_i)\frac{v_{Cl}(k_i)v_{Cl}(k_i')}{D_l(E-\frac{3}{4}\vec{\mathrm{P}}_i^{\,2})}Y_{lm}(\hat{\vec{\mathrm{k}}}_i').\end{aligned} \tag{18}$$

We put $\delta_{C,l}=0$ for simplicity, because we do not know the exact nature of the shielding. Equation (18) allows us to write

$$\begin{aligned}\langle \vec{\mathrm{P}}, \vec{\mathrm{k}}|X(E)G_0(E)\mathcal{H}_{\rm weak}|^{12}\mathrm{C}\rangle &=\sum_{\substack{lL\\ mM_2}} Y_{lm}^*(\hat{\vec{\mathrm{P}}})Y_{LM_L}^*(\hat{\vec{\mathrm{k}}})(lLJM|lm, LM_L)\\ &\quad\times v_{C,L}(k)\frac{1}{D_L(E-\frac{3}{4}p^2)}F_{L,l}^J(p^2,E),\end{aligned} \tag{19}$$

and for the primitive decay vertex

$$\langle \vec{\mathrm{P}}, \vec{\mathrm{k}}|\mathcal{H}_{\rm weak}|^{12}\mathrm{C}\rangle =\sum_{\substack{lL\\ mM_L}} Y_{lm}^*(\hat{\vec{\mathrm{P}}})Y_{LM_L}^*(\hat{\vec{\mathrm{k}}})(lLJM|lm, LM_L)G_{L,l}^J(p^2,k^2;E). \tag{20}$$

From these we have the reduced Faddeev equation

$$F_{L',l'}^J(p^2,E) =g_{L',l'}^J(p^2,E)+\sum_{L,l}\int_0^\infty P^{12}dP'K(p,p';E)F_{L,l}^J(p^{12},E), \tag{21}$$

$$g_{L',l'}^J(p^2,E) =\int_0^\infty k^2dk\, v_{C,L'}(k)G_0(p^2,k^2;E)G_{L',l'}^J(p^2,k^2;E), \tag{22}$$

where

$$\begin{aligned}K(p,p';E) &=[(-1)^{L'}+(-1)^L]\int d\Omega_{\hat{p}}d_{\hat{p}'}\sum_{\substack{mM_L\\ m'M_L'}}(l'L'JM|l'm', L'M_{L'})Y_{l'm'}(\hat{\vec{\mathrm{P}}})Y_{L'M_L'}((\vec{\mathrm{P}}'+\tfrac{1}{2}\vec{\mathrm{P}})/|\vec{\mathrm{P}}'+\tfrac{1}{2}\vec{\mathrm{P}}|)v_{CL'}(|\vec{\mathrm{P}}'+\tfrac{1}{2}\vec{\mathrm{P}}|)\\ &\quad\times G_0(p^2,(\vec{\mathrm{P}}'+\tfrac{1}{2}\vec{\mathrm{P}})^2;E)(lLJM|lm, LM_L)Y_{lm}^*(\hat{\vec{\mathrm{P}}}')Y_{LM_L}^*((\vec{\mathrm{P}}+\tfrac{1}{2}\vec{\mathrm{P}}')/|\vec{\mathrm{P}}+\tfrac{1}{2}\vec{\mathrm{P}}|)v_{CL}(|\vec{\mathrm{P}}+\tfrac{1}{2}\vec{\mathrm{P}}'|)\frac{1}{D_L(E-\frac{3}{4}p'^2)},\end{aligned} \tag{23}$$

and the free three-particle Green function $G_0(p^2,k^2;E)=(E+i\epsilon-\frac{3}{4}p^2-k^2)^{-1}$ $(\hbar=m=1)$. The sequence of above equations is represented diagrammatically in Fig. 2.

The above expressions are general; now we assign the angular momenta of our case, $L=l=L'=l'=2$, $J=1$. The evaluation of the kernel $K(p,p';E)$ is tedious. We do the following partial wave decomposition

$$\begin{aligned}f(\vec{\mathrm{P}},\vec{\mathrm{P}}')&\equiv\frac{u(|\vec{\mathrm{P}}+\frac{1}{2}\vec{\mathrm{P}}'|)u(|\vec{\mathrm{P}}'+\frac{1}{2}\vec{\mathrm{P}}|)}{E-p^2-p'^2-\vec{\mathrm{P}}\cdot\vec{\mathrm{P}}'}\\ &=\sum_{\lambda\mu}f_\lambda(p,p')Y_{\lambda\mu}(\vec{\mathrm{P}})Y_{\lambda\mu}^*(\hat{\vec{\mathrm{P}}}'),\end{aligned} \tag{24}$$

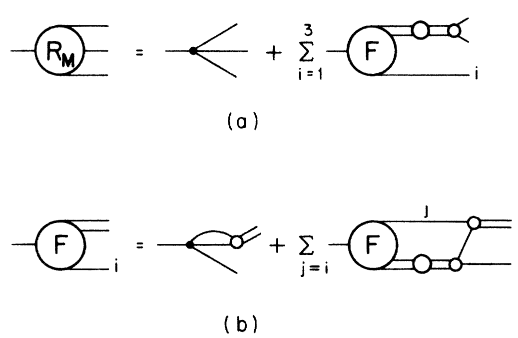

FIG. 2. Diagrammatic representation of (a) Eq. (16) and (b) Eq. (21).

where $v_{C2}(k) = k^2u(k)$. Further, we use the formulas

$$Y_{2\nu}(\hat{x}) = 5\left(\frac{2\pi}{3}\right)^{1/2}\sum_{tt'}(-1)^{\nu}\begin{pmatrix}1 & 1 & 2\\ t & t' & -\nu\end{pmatrix} Y_{1t}(\hat{x})Y_{1t'}(\hat{x}), \tag{25}$$

$$Y_{1t}(\vec{P}+\tfrac{1}{2}\vec{P}')/|\vec{P}+\tfrac{1}{2}\vec{P}'| = \frac{1}{|\vec{P}+\frac{1}{2}\vec{P}'|}[PY_{1t}(\hat{\vec{P}})+\tfrac{1}{2}P'Y_{1t}(\hat{\vec{P}}')]. \tag{26}$$

Then the angular integrals reduce to the evaluation of $6j$, $9j$, and $12j$ symbols. Finally we have

$$K(p', p; E) = \left[-\frac{5}{2^5\pi}p^2p'^2\left(3f_0 - \frac{29}{7}f_2 + \frac{2^2}{3\cdot 7}f_4\right) - \frac{3}{2^4\pi}(pp'^3 + p'p^3)(f_1 - f_3)\right]\Big/ D_2(E - \tfrac{3}{4}p'^2). \tag{27}$$

To make life much simpler, we made the detailed dynamics assumption that the primitive decay vertex ($^{12}C^* \to \alpha_1 + \alpha_2 + \alpha_3$) is negligible compared to the amplitude for decay via $^8Be^{2+}$. Furthermore we assume for the form of the driving term

$$g^1_{22} = p^2/(\mu^2 + p^2), \tag{28}$$

where μ is some parameter and the factor p^2 expresses the angular momentum barrier effect of the free particles.

We can have an alternate expression for the driving term, by assuming the primary decay vertex $G^1_{2,2} = \gamma_0 p^2k^2 \exp[-\frac{1}{2}R_0^2(\frac{3}{4}p^2 + k^2)]$, where p^2k^2 is again the angular momentum threshold and R_0 expresses the range of the decay vertex in the real space. The strength of the decay γ_0 is immaterial since it enters linearly in the integral equation and only the relative rates are compared with experiment. This choice of the primary decay, in (22), gives

$$g^1_{2,2}(p^2, E) = \tfrac{1}{2}\pi\gamma_0 p^2 e^{-3/4Kp^2} \times\left(\frac{-ia^{5/2}}{(a+b)^2}e^{-Ka} + \frac{b^{3/2}}{2(a+b)^2}e^{Kb}[5a + 3b + 2Kb(a+b)]\right), \tag{29}$$

where

$$a = E - \tfrac{3}{4}p^2 + i\epsilon,$$

$$b = \beta^2,$$

$$K = \tfrac{1}{2}R_0^2.$$

IV. RESULTS

A. Energy spectrum

With the choice of driving term (28) and $\mu = 1$ fm^{-1}, we have the quasi-two-body amplitude $F^1_{2,2}$ as shown in Fig. 3. The integral equation (21) is solved numerically by rotating the contours of integration away from the singularities.[6] We can parametrize the result in a form

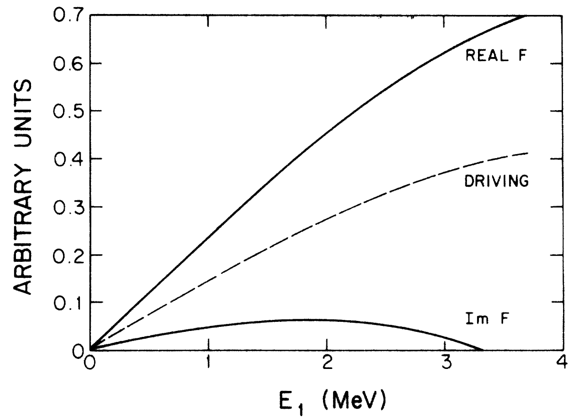

FIG. 3. The quasi-two-body amplitude solution $F^1_{2,2}(p^2)$ for the driving term Eq. (28). Driving is shown with dashed line.

$$F^1_{2,2}(p^2) = Ae^{i\phi}p^2\left[1 + Be^{i\phi'}\left(\frac{P}{P_{\max}}\right)^2\right], \tag{30}$$

where $e^{i\phi}$ is an observable phase factor, because it is common for the three terms in (14), and hence there is no interference effect between them due to this phase factor. We can interpret the dimensionless parameter B as a measure of the rescattering effect. We find $B = 0.409$; $P_{\max}$ is the physically allowed maximum momentum value $\sqrt{\frac{4}{3}E}$. Whereas $Ae^{i\phi}p^2$ merely expresses the angular momentum barrier effect. The total decay rate is proportional to

$$|R|^2 = \tfrac{1}{4}|R_1|^2 + \tfrac{1}{2}|R_0|^2 + \tfrac{1}{4}|R_{-1}|^2. \tag{31}$$

The alignment of the initial spin state in $^{12}C^*$ was determined by a measurement of the angular correlation of 12.71 MeV γ rays and α particles from $^{13}C(^3He, \alpha)^{12}C_{12.71\,MeV} \to \gamma + {}^{12}C_{g.s.}$ at the same bombarding energy as the experiment $^{13}C(^3He, \alpha)^{12}C_{12.71\,MeV} \to 3\alpha$. The total decay rate is then calculated with the experimentally given alignment as

$$\sigma \propto \int d\vec{P}_1 d\vec{P}_2 d\vec{P}_3 \delta(\vec{P}_1 + \vec{P}_2 + \vec{P}_3) \times \delta(E - \tfrac{1}{2}p_1^2 - \tfrac{1}{2}p_2^2 - \tfrac{1}{2}p_3^2)|R|^2. \tag{32}$$

We choose a coordinate system in which the initial $^{12}C^*$ momentum P_0 is in the positive z direction and the final α-particle momentum P is in the x-z plane (see Fig. 4). The energy spectrum in the center of mass system is

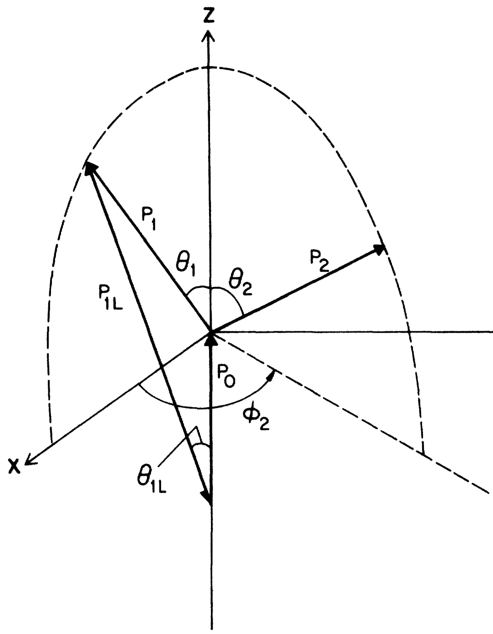

FIG. 4. The coordinate system used in Eq. (42).

$$\frac{d^2\sigma}{d\Omega_1 dE_1} \propto P_1 \int_{-1}^{1} d(\cos\theta_2) \int_0^{2\pi} d\phi_2 \frac{p_2^2}{|2p_2 + p_1 \cos\Delta_{12}|} |R|^2 , \tag{33}$$

where $\cos\Delta_{12} = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos\phi_2$, Δ_{12} is the angle between the direction of particle 1 and 2. In order to make the best of experimental results, we go to the lab system:

$$\frac{d^2\sigma}{d\Omega_{1L} dE_{1L}} = J \frac{d^2\sigma}{d\Omega_1 dE_1} , \tag{34}$$

where the Jacobian $J = P_{1L}/P_1$, the subscript L refers to laboratory quantities. We evaluate the above double integral on a computer, carefully treating the limitations imposed by the δ functions.

The comparison with the experiment is made for laboratory angles $\theta_{1L} = 30$, 45, 55, and 70° in Fig. 5. The amplitude $\langle \vec{P}_1 \vec{k}_1 | X_1(E) G_0(E) \mathcal{H}_{\text{weak}} | ^{12}\text{C} \rangle$ is responsible for the middle peak, while the other two amplitudes (with suffix $i = 2, 3$ instead of 1) each contribute to the two side peaks (Fig. 6). In other words, the other two terms are identical functions of P_1 after the integration of P_2 and P_3.

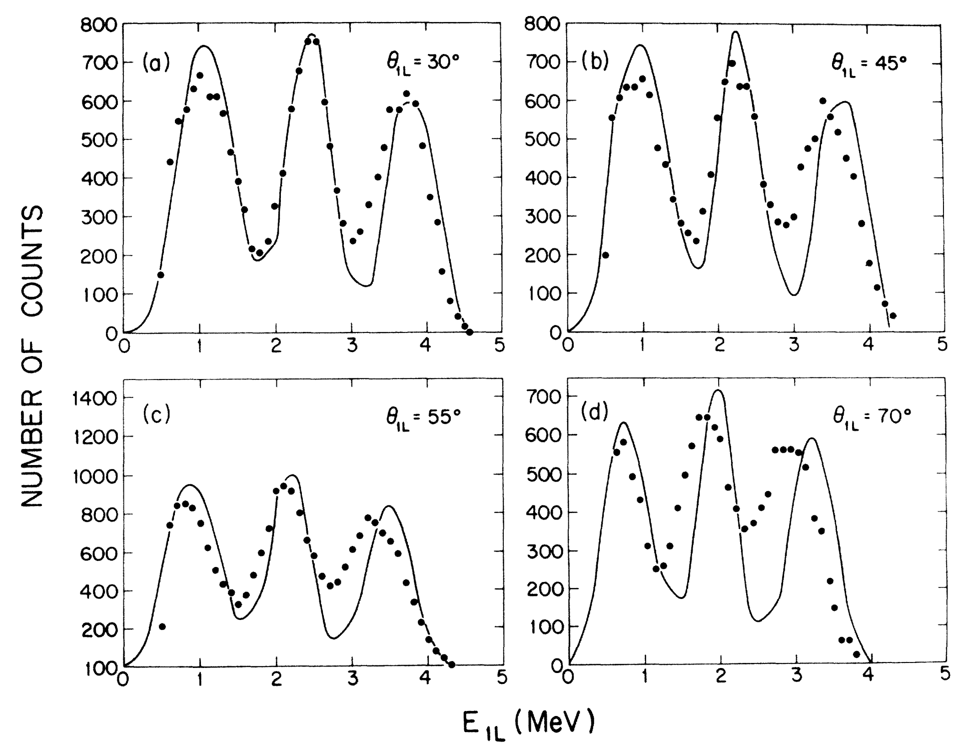

FIG. 5. A comparison of the calculated and experimental (Ref. 2) α-particle spectra with the movable detector at the angles indicated.

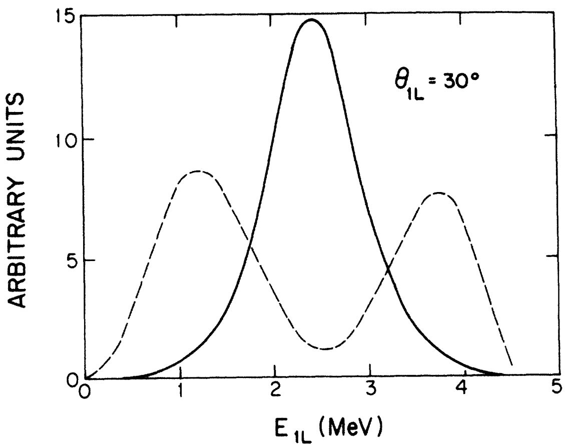

FIG. 6. The solid line indicates the energy spectrum obtained only from the amplitude $\langle \vec{P}_1, \vec{k}_1 | X_1(E) \times G_0(E)\mathcal{H}_{\rm weak} | {}^{12}{\rm C}\rangle$ (without interference with other amplitudes) and the dashed line is the energy spectrum obtained only from the amplitude $\langle \vec{P}_i, \vec{k}_i | X_i(E) \times G_0(E)\mathcal{H}_{\rm weak} | {}^{12}{\rm C}\rangle$, $i=2,3$.

We also evaluated the spectrum with the quasi-two-body amplitude $F^1_{2,2} = p^2$, namely just the angular momentum barrier effect. This corresponds to putting $B=0$, no rescattering effect in (30). We find no difference in the energy spectrum. From this fact, we may conclude that the $(E - \frac{3}{4}p^2)^{1/2}$ singularity or the rescattering effect is not important in the energy spectrum with the choice of the driving term (28).

B. Further attempts

In order to explain the disagreement with the experiment for larger θ_{1L} and $E_{1L} = 2$–3 MeV, we replace the last decay vertex with the vertex including the Coulomb phase shift factor and the Coulomb penetration factor, i.e., $v_{C1} \to v_{C1} e^{-\pi\eta} e^{i\delta_{C1}}$. The shielding radius was taken to be $R = 10^9$ fm. The exponential penetration factor is necessary physically and mathematically to suppress the rapid phase factor's oscillation for the low momentum. This approximate way to include the Coulomb effect was used by McMahan and Duck[3] to improve their agreement with experiment. In our case we do not see any improvement from making this change (Fig. 7). In addition to that, we included the Coulomb phase shift factor and the Coulomb penetration factor between α and ^{8}Be, in the quasi-two-body amplitude (30), which is obtained by solving the integral equation (in which the Coulomb effect is partly ignored for simplicity). That is $v_{C2}(k) \to v_{C2} e^{-\pi\eta} e^{i\delta_{C2}}$ and $F^1_{2,2}(p^2, E) \to F^1_{2,2}(p^2, E) \times e^{-\pi\eta'} e^{i\delta_{C'2}}$ in (19). The same shielding radius $R = 10^9$ fm was used to evaluate δ_{C2} and $\delta_{C'2}$. Again we do not observe any improvement (Fig. 8).

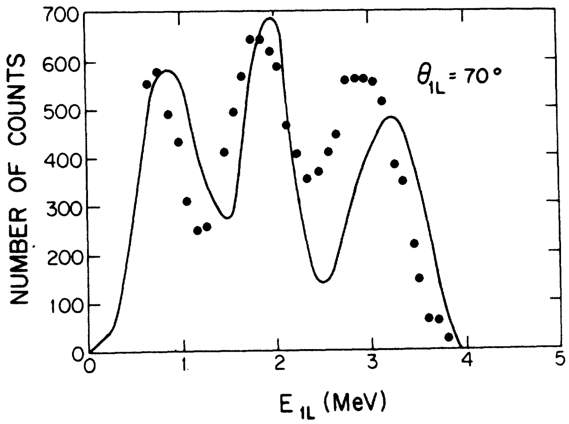

FIG. 7. The energy spectrum of α particles including the Coulomb penetrability and phase shift between α-α. $\theta_{1L} = 70°$.

The alert reader may suspect that the inclusion of effects of the primitive decay vertex will change the result greatly, especially since a little phase difference is crucial to the destructive interference producing the minimum at about $E_{1L} = 2.5$–3 MeV, but that is not the case. The primitive decay vertex is much smaller in comparison to the decay amplitude via ^{8}Be^{2+}, because of the resonance peak.

In Figs. 9 and 10 we show the solution of the reduced Faddeev equation by using the inhomogeneous term (29), with the range $R_0 = 2$ and 3 fm. For $R_0 = 2$ fm, the result is quite similar to the previous case; we can parametrize again in a form (30) with $B = 0.391$. We are not surprised to have the same spectrum. However, for $R_0 = 3$ fm, we have large rescattering effects. We can parametrize the re-

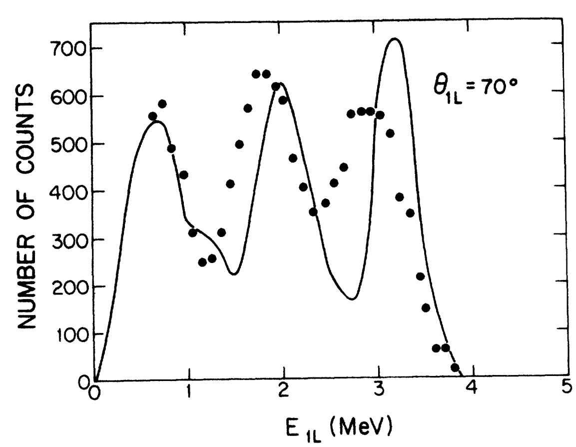

FIG. 8. The energy spectrum of α particles including the Coulomb penetrability and phase shift between α-α and α-^{8}Be. $\theta_{1L} = 70°$.

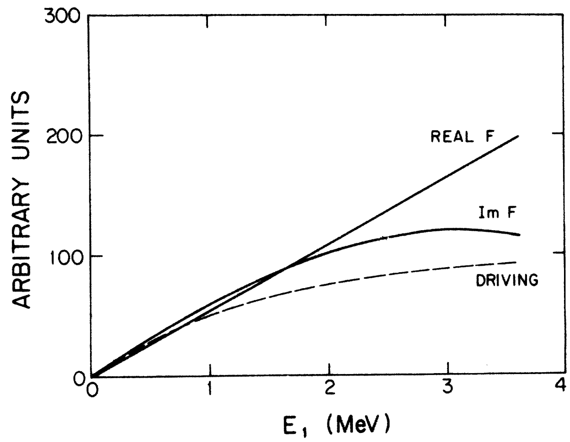

FIG. 9. The driving and the quasi-two-body amplitude for $R_0 = 2$ fm.

sult as

$$F^1_{2,2}(p^2) = Ae^{i\phi}p^2\left[1 + Be^{i\phi'}\left(\frac{P}{P_{max}}\right)^2 + Ce^{i\phi''}\left(\frac{P}{P_{max}}\right)^4\right], \tag{35}$$

with $B = 2.82$ and $C = 1.64$. Despite the fact, that the large rescattering effect changes the behavior of $\langle \vec{P}_i, \vec{k}_i | X_i(E) G_0(E) \mathcal{H}_{weak} |^{12}C\rangle$, the energy spectrum obtained only from the $i = 1$ amplitude squared and the energy spectrum obtained only from the $i = 2$ or 3 amplitude squared is shown in Fig. 11. In comparison with Fig. 6 we see the large differences. However, after allowing for interference of three

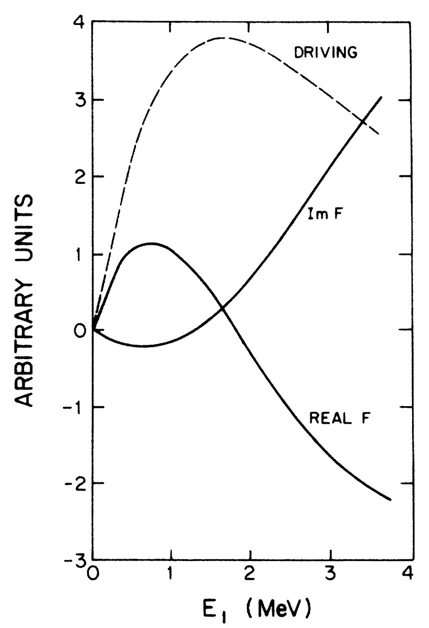

FIG. 10. The driving and the quasi-two-body amplitude for $R_0 = 3$ fm.

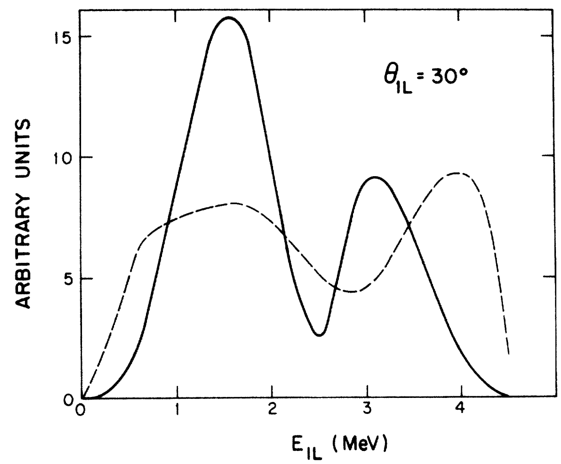

FIG. 11. Same as Fig. 6 except the quasi-two-body amplitude is obtained from the driving Eq. (29), $R_0 = 3$ fm.

amplitudes, the final spectra do not show any difference to the previous results (Fig. 5) and hence we do not show them in a separate figure.

V. CONCLUSIONS

We have solved the reduced Faddeev equations with separable potentials for the decay of $^{12}C^*(1^+)$ into three α particles in order to study the effect of the square root singularity (rescattering).

We have solved the integral equations of the quasi-two-body amplitude with the assumptions about the driving (the inhomogeneous) term: (a) $p^2/(\mu^2 + p^2)$, $\mu = 1\ \text{fm}^{-1}$, (b) the driving term was constructed from the primitive decay vertex (the direct breakup amplitude to the 3α) which is spread out in configuration space, and (c) the driving term is point-like.

For the cases (a) and (b) we had similar results, namely a small effect of the square root singularity compared with the simple quasi-two-body amplitude P^2 (Watson approximation).

We have calculated the energy spectra and compared with experiment for various detector angles θ_{1L}. We cannot observe the square root singularity effect. The spectra of the (a), (b), and Watson approximations do not show any difference. Even for the case (c), although we had a large rescattering effect (clearly the strong interaction effect, the wave function at a short range more than one spread out in momentum space), we could not detect its effect in the spectrum. In general this must not be the case. Amado and Noble[7] have studied with a similar model, the decay of a 0^+ particle into three identical 0^+ particles interacting through separable s-wave interactions. They show different single particle spectra for the different spreading size of primary decay vertex input. The reason we could not detect the square root singularity is due to the angular momentum distribution and the interference effect.

We cannot explain the disagreement of the Watson approximation theory and the experimental result for the spectrum at large detector angle θ_{1L} and the observed particle lab energy $E_{1L}=2-3$ MeV by solving our exact soluble model.

Our spectrum is kinematically incomplete; that is we integrate over the unobserved second particle direction (θ_2, ϕ_2). It is possible to do a triple coincidence spectrum, measure $d^3\sigma/dE_{1L}d\Omega_{1L}d\Omega_{2L}$, and have more detailed information about the quasi-two-body amplitude. This may help to clarify the discrepancy. More theoretical and experimental work should be done in this direction.

ACKNOWLEDGMENTS

The author wishes to express his sincere thanks to Professor R. D. Amado for suggesting this problem and for continued guidance throughout the whole course of work. He is also indebted to Professor D. P. Balamuth for many helpful discussions.

†Supported in part by the National Science Foundation.

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