Magnetic moment of the $J^{\pi} = 6^{+}$ isomeric level in ^{134}Te

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The g factor for the $J^{\pi} = 6^{+}$ isomeric level in ^{134}Te can be reproduced within the experimental error by taking into account core polarization as well as velocity dependence corrections to the magnetic dipole operator. The wave function describing this isomeric state is obtained by diagonalizing a residual Gaussian interaction in a two-particle configuration space.

[NUCLEAR STRUCTURE ^{134}Te; calculated levels, J, π, g factor for isomeric $J^{\pi}=6^{+}$ level. Corrected $M1$ operator: core polarization, velocity dependence.]

In a recent article of Wolf and Cheifetz,[1] the magnetic dipole moment of the $J^{\pi}=6^{+}$ isomeric state in ^{134}Te has been measured. The value of $g_{\text{exp}}(6_1^+)=0.846\pm0.025$ can be reproduced with an effective spin gyromagnetic factor for the $1g_{7/2}$ proton single-particle orbit $g_s(1g_{7/2})=2.5$, a value close to effective g_s factors used for the $1g_{7/2}$ proton orbital in describing the $N=82$ isotones.[2-7] Wolf and Cheifetz point out that in order to understand the quenching from the free nucleon value $g_{\text{s.p.}}(1g_{7/2})$, core-polarization corrections and configuration admixtures still leave a discrepancy of 42% (for a 2.3% $|2d_{5/2}1g_{7/2};6^+\rangle$ admixture[5]) or 22% (for 13.7% $|2d_{5/2}1g_{7/2};6^+\rangle$ admixture[8]) in explaining the difference $g_{\text{exp}}(6_1^+)-g_{\text{s.p.}}(1g_{7/2})$.

Here, we would like to point out that by taking into account (i) core-polarization effects, (ii) influence from velocity dependent two-nucleon interactions on the expression for the magnetic dipole operator, and (iii) the complete wave function describing the $J_i^{\pi}=6_1^+$ level in a two-particle calculation, one is able to reproduce $g_{\text{exp}}(6_1^+)$ within the experimental error. The important first-order perturbation theory core-polarization diagrams have been calculated[9,10] by using the Tabakin residual interaction which was also used earlier in describing the $N=82$ isotones.[2-4] The important corrections originate from the proton $1g_{9/2}^{-1}1g_{7/2}$ and neutron $1h_{11/2}^{-1}1h_{9/2}$ excitations for which the energy denominators have been taken, as in Ref. 10. Thus, corrected values for the magnetic dipole moment in the proton single-particle orbits considered ($1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, $1h_{11/2}$) result. In the particular case of the most important single-particle configuration, a correction $\Delta g(1g_{7/2})_{\text{core pol.}}=0.223$ results.

Also, the nondiagonal reduced magnetic dipole matrix element results from first-order perturbation theory as

$$\langle j'\|(\tfrac{4}{3}\pi)^{1/2}\delta\mathfrak{M}(M1)_{\text{core pol.}}\|j\rangle=(-1)^{j'-j}(g_l-g_s)\left[\frac{(2j_h+1)l}{2l+1}\right]^{1/2} \times\left[\frac{1}{\epsilon_{\text{p}}-\epsilon_{\text{h}}+\epsilon_{j'}-\epsilon_j}\sum_J(2J+1)W(j_{\text{h}}1Jj';j_{\text{p}}j)\langle j'j_{\text{p}};J|V|jj_{\text{h}};J\rangle_{\text{a.s.}}+(\text{h}\rightleftharpoons\text{p})\right]. \quad (1)$$

Here, l denotes the orbital angular momentum of the p-h excitation, and $(l,s)j$ coupling is used for defining the single-particle states; ϵ_a denotes the unperturbed single-particle energy, whereas $(\text{h}\rightleftharpoons\text{p})$ means the same expression interchanging the indices p and h. In the particular case of $j'\equiv 2d_{5/2}$ and $j\equiv 1g_{7/2}$, the reduced matrix element becomes $0.944\mu_N$.

Another nonnegligible contribution to the magnetic dipole operator originates from the velocity-dependent nucleonic forces. If we consider the averaging procedure as discussed by Bohr and Mottelson,[11] and assuming a Woods-Saxon (WS) shape for the nucleon density, the correction term

$$\delta\mathfrak{M}(M1,\mu)_{\text{vel. dep.}}\equiv\left(\frac{3}{4\pi}\right)^{1/2}\left\{\frac{4m}{3\hbar^2}r^2f(r)[(2\pi)^{1/2}[\vec{Y}_2\otimes\vec{s}]^1_\mu+s_\mu]\left(t_z-\frac{N-Z}{2A}\right)-\frac{2m}{\hbar^2}\frac{2Z}{A}g(r)s_\mu\right\} \quad (2)$$

results, where we have used the abbreviations

$$f(r) \equiv -V_0\lambda \frac{\lambda_p^2}{4}\frac{1}{r}\frac{d}{dr}(V_{WS})$$

and (3)

$$g(r) \equiv V_0\lambda \frac{\lambda_p^2}{4} V_{WS} .$$

The one-body spin-orbit potential $V_{ls} = -f(r)\vec{l}\cdot\vec{\sigma}$ is used and t_z denotes the isospin projection quantum number for the type of nucleon considered. The strength of the spin-orbit potential is denoted by $V_0\lambda$ and λ_p gives the proton Compton wavelength and V_0 is the strength of the Woods-Saxon potential itself. Here, we consider the parameters as determined by Blomqvist and Wahlborn[12] ($V_0 = 58$ MeV, $\lambda = 32$) in the Pb region, not differing substantially from medium heavy-mass nuclei.[13-15] By straightforward calculation, a correction to the single-particle g factor is obtained:

$$\Delta g(j)_{\text{vel. dep.}} = \pm\frac{2j+1}{(2j+2)j}\frac{m}{\hbar^2}\langle j|r^2 f(r)|j\rangle\left(t_z - \frac{N-Z}{2A}\right)\begin{cases} -\frac{1}{j}\langle j|g(r)|j\rangle\frac{m}{\hbar^2}\frac{2Z}{A} \\ +\frac{1}{j+1}\langle j|g(r)|j\rangle\frac{m}{\hbar^2}\frac{2Z}{A} \end{cases} . \quad (4)$$

Here, the plus sign and upper part correspond with $j = l + \frac{1}{2}$, whereas the minus sign and the lower part correspond with $j = l - \frac{1}{2}$. All radial integrals are calculated with the use of Woods-Saxon wave functions.[16]

The nondiagonal term results into[17] ($l \neq l'$)

$$\langle j' \| (\tfrac{4}{3}\pi)^{1/2}\delta\mathfrak{M}(M1)_{\text{vel. dep.}} \| j\rangle = (2j+1)^{1/2} C_{j\;\,1\;\,j'}^{1/2\;0\;1/2}\{2(-1)^{j-j'+1} + (-1)^{j'-l'-1/2}[(j+\tfrac{1}{2}) + (-1)^{j'+j-1}(j'+\tfrac{1}{2})]\}$$
$$\times \frac{m}{3\hbar^2}\langle j|r^2 f(r)|j\rangle\left(t_z - \frac{N-Z}{2A}\right), \quad (5)$$

which for $j' \equiv 2d_{5/2}$, $j \equiv 1g_{7/2}$ gives a correction of $0.286\mu_N$. This contribution, together with the nondiagonal $M1$ matrix element from core-polarization effects, results in a total value of

$$\langle 2d_{5/2} \| (\delta\mathfrak{M}(M1)_{\text{core pol.}} + \delta\mathfrak{M}(M1)_{\text{vel. dep.}})(\tfrac{4}{3}\pi)^{1/2} \| 1g_{7/2}\rangle = 1.23\mu_N .$$

An estimate for this particular matrix element can be obtained starting from the experimentally determined half-life of the first excited state in the odd-mass Sb isotopes[16] ($\frac{5}{2}^+ \rightleftarrows \frac{7}{2}^+$), taking into account that the $M1$ transition is mainly $2d_{5/2} \rightleftarrows 1g_{7/2}$. When the single-particle amplitude, as deduced from the experimental one-nucleon transfer spectroscopic factors, is used,[13] the nondiagonal reduced $M1$ matrix element results. If $B(M1) \simeq 0.02\mu_N^2$ is taken as an average value for the $Z = 51$ mass region[16] as well as the average spectroscopic factors[13] $\langle S2d_{5/2}\rangle = 0.75$, $\langle S1g_{7/2}\rangle = 0.80$, a value of

$$\langle 2d_{5/2} \| (\tfrac{4}{3}\pi)^{1/2}\mathfrak{M}(M1) \| 1g_{7/2}\rangle_{\text{exp}} \simeq 1.10\mu_N$$

results.

Finally, we consider the wave function resulting from diagonalizing a residual Gaussian two-body force within the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ proton single-particle states.[4] The resulting wave function is then obtained as

$$|J_i^\pi M\rangle = \sum_{ab}\phi^i(ab;J)|ab;JM\rangle , \quad (6)$$

TABLE I. The separate contributions to $g(6_1^+)$. Results from our calculation are compared with Wildenthal and Larson (Ref. 8). In the last row, we distinguish between the nondiagonal $M1$ matrix element connecting the $2d_{5/2}$ and $1g_{7/2}$ single-particle states as taken from experiment (first column) or calculated from core polarization and velocity dependence (second column).

	Contributions	Wildenthal		This work	
(1)	$g(1g_{7/2})$	0.491		0.491	
(2)	$\Delta g(1g_{7/2})_{\text{core pol.}}$	0.223		0.223	
(3)	$\Delta g(1g_{7/2})_{\text{vel. dep.}}$	0.079		0.079	
	$\lvert(1g_{7/2})^2;6^+\rangle$	0.681		0.722	
	$\lvert(1h_{11/2})^2;6^+\rangle_{\text{adm.}}$				
(4)	$\Delta g(6_1^+)$	0.002		0.002	
	$\lvert 2d_{5/2}1g_{7/2};6^+\rangle_{\text{adm.}}$				
(5)	$\Delta g(6_1^+)$ (diag.)	0.151		0.096	
(6)	$\Delta g(6_1^+)$ (nondiag.)	0.025	0.027	0.021	0.022
	$g(6_1^+)$ (total)	0.859	0.861	0.841	0.842

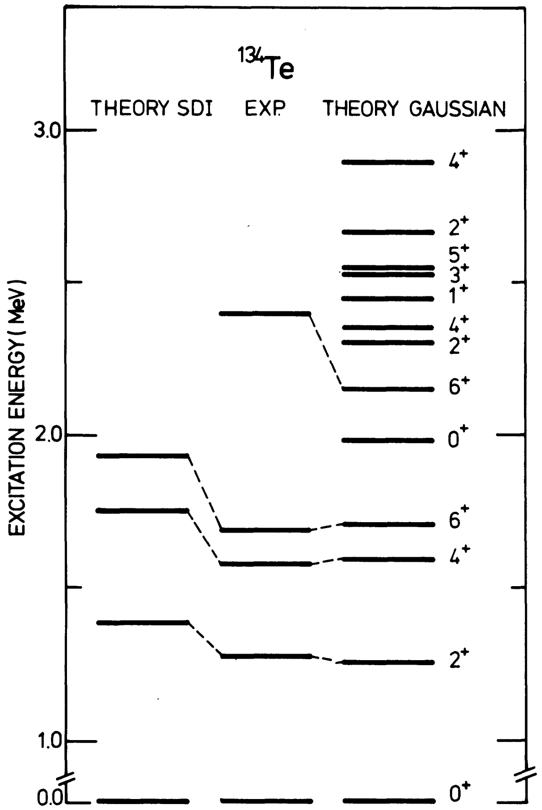

FIG. 1. The level scheme for ^{134}Te as calculated with a Gaussian residual interaction (Ref. 4) of force strength $V_0^1=-37$ MeV, $t=+0.2$, compared with experiment and with calculations from Wildenthal (Ref. 8).

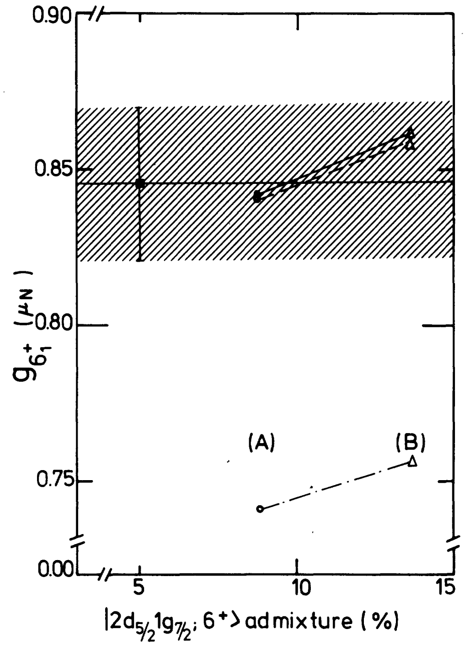

FIG. 2. The calculated g factor for the $J_i^{\pi}=6_1^{+}$ level in ^{134}Te in this study (A) compared with the calculations of Wildenthal (Ref. 8) (B) and with the experimental value (error bar). The dashed-dot, dashed, and full lines are drawn as an eye guide and indicate (i) influence of the $|2d_{5/2}\,1g_{7/2};\,6^{+}\rangle$ admixture with diagonal core-polarization effects only, (ii) all contributions with the nondiagonal reduced $M1$ matrix element connecting the $2d_{5/2}$ and $1g_{7/2}$ single-particle states taken from experiment, and (iii) same as (ii), but with the reduced $M1$ matrix element as calculated from core polarization and velocity dependence, respectively.

where $\phi^{i}(ab;J)$ describes the amplitudes for the two-particle configurations $|ab;JM\rangle$. The single-particle energies are taken from Ref. 18, whereas the best agreement with the experimental level scheme is obtained for a force strength $V_0^1=-37$ MeV. The level scheme is shown in Fig. 1 in which the calculation of Wildenthal and Larson[8] is also indicated. The $J_i^{\pi}=6_1^{+}$ wave function results in

$$|6_1^{+}\rangle=0.954|(1g_{7/2})^2;6^{+}\rangle+0.297|2d_{5/2}1g_{7/2};6^{+}\rangle-0.042|(1h_{11/2})^2;6^{+}\rangle\,.$$

The magnetic dipole moment can then be calculated with this particular wave function from

$$g(6_1^{+})=(\tfrac{4}{3}\pi)^{1/2}(\tfrac{13}{42})^{1/2}\sum_{\substack{ab\\cd}}\phi^{1}(ab;6^{+})\phi^{1}(cd;6^{+})[(1+\delta_{ab})(1+\delta_{cd})]^{-1/2}$$
$$\times(-1)^{j_a+j_d}[\delta_{bd}W(66ac;1d)\langle c\|\mathfrak{M}(M1)\|a\rangle+(d\rightleftharpoons c)+(b\rightleftharpoons a)+(d\rightleftharpoons c;b\rightleftharpoons a)]\,. \quad (7)$$

In Table I, all separate contributions to $g(6_1^{+})$ (also for the $J_i^{\pi}=6_1^{+}$ wave function of Wildenthal and Larson[8]) are given. In the last row, distinction is made between the theoretically (core polarization + velocity dependence) and experimentally determined nondiagonal reduced matrix element.

We thus observe that the contributions, as considered by Wolf and Cheifetz, add up to $g(6_1^{+})=0.741\,(0.757)$ with our wave function and Wildenthal's, respectively (see Fig. 2). The neglected contributions: velocity dependence correction to the single-particle magnetic dipole moments, the nonzero reduced $M1$ matrix element connecting the $2d_{5/2}$ and $1g_{7/2}$ single-particle states, and the $|(1h_{11/2})^2; 6^+\rangle$ admixtures, give an additional $\Delta g(6_1^+) = 0.101(0.104)$ when using our wave function and Wildenthal's, respectively. So, we can conclude that by taking into account core polarization and velocity dependence as well as the full $J_i^\pi = 6_1^+$ wave function, the experimental value of $g(6_1^+)$ can be reproduced within the error.

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[17]$C_j^{1/2}\,{}^{0}_{1}\,{}^{1/2}_{j'}$ is a notation for the Clebsch-Gordan coefficent.

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