Angular-momentum-projected yrast levels from a coherent phonon state

G. J. Borse
Physics Department, Lehigh University, Bethlehem, Pennsylvania 18015
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Expressions for the energies of ground state yrast bands of even-even nuclei are obtained by angular-momentum projection on coherent phonon intrinsic states starting from a pairing plus quadrupole interaction. The results of the theory for a local minimum corresponding to zero deformation are investigated numerically and a good fit is found to the spectra of the xenon isotopes. Quadrupole moments and $B(E2)$'s are also discussed.

[NUCLEAR STRUCTURE Angular-momentum projection, even-even nuclei, yrast states, variational principle, xenon isotopes.]

I. INTRODUCTION

Numerous phonon expansion models have been used in the past several years in attempts to understand various anomalies associated with departures from harmonic vibrational nuclei and nuclei in the transition to spherical region. Some of these models have been microscopic in nature[1] and some phenomenological[2]; however, both types usually require several parameters to fit comparatively few experimental quantities. Haapakoski, Honkaranta, and Lipas[3] have proposed a phenomenological model with only two parameters to explain the ground state bands of even-even nuclei. Their model employed the so-called coherent phonon, i.e., the ground state or deformed vacuum was written as

$$|\varphi_0\rangle = \mathfrak{N} \exp[fB_0^\dagger]|0\rangle , \tag{1}$$

where $B_0^\dagger$ is the harmonic quadrupole phonon, $\mathfrak{N}$ is a normalization constant, and f can be interpreted as a deformation. The states of the system with good angular momentum are then obtained through angular-momentum projection via the Peierls-Yoccoz prescription.[4] These states are then used to diagonalize a phenomenological phonon Hamiltonian. It is the purpose of this paper to develop similar ideas from a microscopic starting point. The procedure is, in essence, to start with a pairing plus quadrupole interaction which is then rewritten in terms of phonons. The expressions for the energy of each angular-momentum state are evaluated using angular-momentum-projected states based on the ground state of Eq. (1). These are then minimized to obtain the energy. To illustrate the theory, a numerical example is discussed for the case of small f. In this limit the equations are very similar to the schematic random-phase approximation (RPA) model and yield the evenly spaced levels of an harmonic vibrational yrast band. However, a slight modification of the number equation of pairing theory alters the spectra considerably and results in a very good fit to the spectra of the xenon isotopes.

II. THEORY

We begin by assuming the intrinsic state of an even-even nucleus to be of the form given in Eq. (1) with $f=1$. The restriction of the phonon's $\langle J_z\rangle$ to zero will limit the discussion to $K=0$ rotational bands. The vacuum state $|0\rangle$ is the phonon vacuum. The phonons are expressed in terms of quasiparticle pair operators in the usual manner:

$$B_\mu^{\xi\dagger} = \frac{1}{2}\sum_i \Delta_i [X_i^\xi A_\mu^\dagger(i) - (-)^\mu Y_i^\xi A_{-\mu}(i)] , \tag{2a}$$

$$A_\mu^\dagger(i) = \sum [X_i^\xi B_\mu^\dagger + (-)^\mu Y_i^\xi B_{-\mu}^\xi] , \tag{2b}$$

$$A_\mu^\dagger(i) = [a_{\alpha i}^\dagger a_{\beta i}^\dagger]_{2\mu} , \tag{2c}$$

$$[A_\mu(i), A_\mu^\dagger(i)] = \frac{2\delta_{ij}}{\Delta_i} , \tag{3a}$$

$$[B_\mu^\xi, B_\mu^{\xi'\dagger}] = \frac{1}{2}\sum_i \Delta_i (X_i^\xi X_i^{\xi'} - Y_i^\xi Y_i^{\xi'}) = f_\xi^2 \delta_{\xi\xi'} . \tag{3b}$$

In the above equations the index i labels a two-quasiparticle state (a_i, b_i) and $\Delta_i = 1$ if $a_i = b_i$; $\Delta_i = 2$ if $a_i \neq b_i$. The index ξ labels the various modes of collective phonons. The lowest collective mode, the harmonic phonon, will be written without the superscript ξ. The expansion parameters X and Y are assumed to be real.

Of course Eqs. (2), which express bosonlike operators in terms of fermion operators and vice versa, correspond to a lowest order Beliaev-Zelevinsky transformation and such a truncation may not even be approximately valid in a transitional nuclear system[5] such as the xenon isotopes, which are to be considered below. Before one ap-

plies any theory based on these equations, the contributions from higher order contributions should be determined.

Note that we are not requiring the phonon commutator to be normalized. This means that we can write Eq. (1) in the form

$$|\varphi_0\rangle = \mathfrak{N}\exp(B_0^\dagger)|0\rangle\,, \tag{4}$$

and the normalization of the commutator will play the role of the "deformation" f.

The states of the system with good angular momentum are obtained through projection:

$$|J,M\rangle \equiv |J,M,K=0\rangle = N_{JM}^{1/2}\int d\Omega\, D^{J*}_{M0}(\Omega)|\varphi_0\rangle\,. \tag{5}$$

The properties of each state are determined by the values of the parameters X and Y, which are in turn calculated by minimizing the quantity

$$E_J = \frac{\langle J,M|H|J,M\rangle}{\langle J,M|J,M\rangle} = \frac{\int_0^\pi d(\cos\theta)P_J(\cos\theta)H(\theta)}{\int_0^\pi d(\cos\theta)P_J(\cos\theta)n(\theta)} \tag{6}$$

after projecting. In Eq. (6), P_J is a Legendre polynomial and

$$H(\theta) = \langle\varphi_0|He^{-iJ_y\theta}|\varphi_0\rangle \equiv n(\theta)h(\theta), \tag{7a}$$

$$n(\theta) = \langle\varphi_0|e^{-iJ_y\theta}|\varphi_0\rangle, \tag{7b}$$

where H is the Hamiltonian, which we choose for simplicity to be of the pairing plus quadrupole type;

$$H = H_{s.p.} - \frac{G}{4}\sum_i \mathcal{A}_0^\dagger(i)\mathcal{A}_0(i) - \frac{\chi}{2}\sum_\mu Q_\mu Q_{-\mu}(-)^\mu, \tag{8}$$

where

$$H_{s.p} = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha, \tag{9a}$$

$$\mathcal{A}_0^\dagger(i) = [c_{ai}^\dagger c_{bi}^\dagger]_{J=0}, \tag{9b}$$

$$Q_\mu = \sum_{\alpha\beta}\langle j_\alpha m_\alpha|r^2Y_{2\mu}|j_\beta m_\beta\rangle[c_\alpha^\dagger c_\beta]_{2\mu}. \tag{9c}$$

The $c_\alpha^\dagger$ operators are particle creation operators and the index α means the set of quantum numbers $(n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ plus the charge. The index a is the set α less m_α, and $-\alpha$ means the set with $-m_\alpha$. Also, G and χ are the pairing and quadruple strength parameters.

Quasiparticles are next introduced by the usual Bogolyubov-Valatin transformation to approximately diagonalize the pairing part of the Hamiltonian

$$a_\alpha^\dagger = u_a c_\alpha^\dagger - (-)^{j_a-m_\alpha}v_a c_{-\alpha}. \tag{10}$$

The occupation probability amplitudes u_a and v_a are determined by the gap and number equations. The form of the number equation will, however, have a very important influence on the results and will be discussed in more detail below.

The Hamiltonian, rewritten in terms of quasiparticle operators, is:

$$H = H_0 + \sum_i e_i\eta_0^\dagger(i) - \frac{\chi}{10}\sum_\mu \hat{Q}_\mu\hat{Q}_\mu^\dagger, \tag{11}$$

where $\eta_0^\dagger(i) = [a_{a_i}^\dagger a_{b_i}^\dagger]_{J=0}$, (12)

$$e_i = (e_{a_i} + e_{b_i}), \tag{13}$$

$$\hat{Q}_\mu = \frac{1}{2}\sum_i \Delta_i\{Q_i^+[A_\mu^\dagger(i) + (-)^\mu A_{-\mu}^{(i)}] + 2Q_i^-\eta_{2\mu}^\dagger(i)\}, \tag{14}$$

$$Q_i^+ = (u_a v_b + u_b v_a)\langle j_a\|r^2Y_2\|j_b\rangle, \tag{15a}$$

$$Q_i^- = (u_a u_b - v_a v_b)\langle j_a\|r^2Y_2\|j_b\rangle. \tag{15b}$$

$$A_\mu^\dagger(i) = [a_{a_i}^\dagger a_{b_i}^\dagger]_{2\mu}. \tag{15c}$$

The task now is to evaluate $H(\theta)$, Eq. (7a), using Eq. (11) for H. The procedure is to express the Hamiltonian in terms of phonons via Eq. (2) and to use the identities

$$B_\mu^\xi e^{-iJ_y\theta}|\varphi_0\rangle \equiv B_\mu^\xi|\varphi_\mu(\theta)\rangle = f_\xi d^2_{\mu 0}(\theta)|\varphi_0(\theta)\rangle\delta_{\xi 0}, \tag{16a}$$

$$\langle\varphi_0|B_\mu^{\xi\dagger} = f_\xi\langle\varphi_0|\delta_{\mu 0}\delta_{\xi 0}. \tag{16b}$$

However, before this can be accomplished we require an approximate expansion of the quasiparticle scattering operator $\eta_{2\mu}^\dagger(i)$ in terms of phonons. This expansion is usually expressed in the form

$$\sum_i \Delta_i\, Q_i^-\,\eta_{2m}^\dagger(i) \cong 5\sum_{ij\mu}\Delta_i\,\Delta_j\,W_{ij}\,C^{2\,2\,2}_{\mu m}A_{m+\mu}^\dagger(i)A_\mu(j), \tag{17}$$

where

$$W_{ij} = W(a_ib_i22;2a_j)\delta_{b_ib_j}Q^-_{a_ia_j}, \tag{18}$$

and $W(\,)$ is a Racah coefficient. The validity of such an expansion is the same as that of Eqs. (2) and (3). The presence of the quasiparticle scattering operator in the Hamiltonian via Eq. (14) prohibits diagonalization by a canonical transformation to phonons and thus is responsible for anharmonicities in the theory.

After a bit of algebra the result for $h(\theta)$ is

$$\begin{aligned}2h(\theta) = {} & P_2(\cos\theta)\{C_1 - \tfrac{1}{10}\chi \\ & \times [D^2 - 10B_1 - 10\sqrt{\tfrac{2}{7}}D(2A_1 + A_2)]\} \\ & + P_0[C_2 - \tfrac{1}{10}\chi(D^2 - 10B_2 - 10\sqrt{\tfrac{2}{7}})DA_2],\end{aligned} \tag{19}$$

where

$$\begin{aligned}D &= \sum_j \Delta_i Q_i^+(X_i + Y_i),\\ C_1 &= \sum_i \Delta_i e_i(X_i^2 + Y_i^2),\\ C_2 &= 2\sum_i \Delta_i e_i X_i Y_i,\end{aligned} \tag{20a}$$

$$B_1=\sum_{ij}\Delta_i\Delta_j B_{ij}(X_iX_j+Y_iY_j),$$

$$B_2=\sum_{ij}\Delta_i\Delta_j B_{ij}(X_iY_j+X_jY_i), \tag{20b}$$

$$A_1=\sum_{ij}\Delta_i\Delta_j A_{ij}(X_iX_j+Y_iY_j),$$

$$A_2=\sum_{ij}\Delta_i\Delta_j A_{ij}(X_iY_j+X_jY_i), \tag{20c}$$

and

$$A_{ij}=Q^-_{a_ia_j}\delta_{b_ib_j}W(a_ib_j22;2a_j), \tag{21a}$$

$$B_{ij}=Q^-_{a_ia_j}Q^-_{b_ib_j}W(a_ia_jb_ib_j;22)(-)^{a_i-b_i}. \tag{21b}$$

All of the terms involving A or B represent anharmonic effects and are traceable to the inclusion of the quasiparticle scattering term.

The integration over θ to obtain the energy, Eq. (6), involves two integrals which must be done numerically:

$$N_0^J=\int_0^\pi d(\cos\theta)P_J(\cos\theta)n(\theta), \tag{22a}$$

$$N_2^J=\int_0^\pi d(\cos\theta)P_2P_Jn(\theta), \tag{22b}$$

with

$$n(\theta)=\exp[f^2P_2(\cos\theta)]. \tag{23}$$

Using standard computer codes, one then minimizes the energy expression as a function of the parameters X and Y. It should be noted that since f^2 depends on X and Y through Eq. (36), each stage in the minimization requires a reevaluation of the integrals in Eq. (22). In the present situation it is more economical to fit these integrals to polynomials in f^2 to some prespecified accuracy than to recalculate at each step. Alternatively, one could apply the variational principle to the expression for the energy which would result in a set of algebraic equations in the parameters X and Y. These equations are nonlinear [$h(\theta)$ is cubic in X and Y] and their solution then requires a linearization algorithm plus solution by iteration. Both of these procedures have been applied, but because of the complexity of solution, a variation of the Hamiltonian strength parameters and the single particle parameters to obtain a best fit to the data was not feasible and these results will not be presented here. However, for all reasonable input parameters, the energy expression has a local minimum for $f^2\to 0$. Also in this limit, the abrupt truncation of the Beliaev-Zelevinsky expansion is less disturbing since the elimination of anharmonic complications occurs naturally and results in a purely harmonic theory, as we shall see. The solution of the problem for an absolute minimum where $f^2\neq 0$, besides being much more complicated, also requires a justification of the truncation of the Beliaev-Zelevinsky expansion and the elimination of the nonphysical many-fermion states contained in the coherent phonon basis state. These problems, although soluble in principle,[5] have not as yet been solved for the formulation of the problem discussed here. A more detailed analysis of the nonharmonic solutions of Eqs. (19) is in progress. For the present we shall investigate only the properties of the solution in the vicinity of the local minimum, $f^2\to 0$, for which the theory becomes very simple indeed.

In the limit of small f^2 the terms in $h(\theta)$ involving A_{ij} (the cubic terms) can be ignored. Also in this limit,

$$\frac{N_0^J}{N_2^J}\equiv\beta(f^2)\Rightarrow\begin{cases}5/f^2 & \text{if } J=0\\ \dfrac{2f^2}{J} & \text{if } J=2,4,\cdots,\end{cases} \tag{24}$$

so that the expression for the energy in the limit of small f^2 becomes

$$E_J=\frac{\langle|hn|\rangle}{\langle|n|\rangle}=\frac{1}{2\beta}\left\{\left[C_1-\frac{\chi}{10}(D^2-10B_1)\right]+\beta\left[C_2-\frac{\chi}{10}(D^2-10B_2)\right]\right\}. \tag{25}$$

All terms are now quadratic and thus proportional to f^2. Most importantly, the $J=0$ state has zero energy in the small f^2 limit. For $J\neq 0$ states the terms multiplying β can be ignored and thus to determine the energies we must only minimize the expression

$$E_J(J\neq 0)=\frac{1}{2\beta}\left[C_1-\frac{\chi}{10}(D^2-10B_1)\right]. \tag{26}$$

The minimization can be accomplished by differentiating with respect to the X_i and Y_i parameters, obtaining $2n$ linear equations

$$\frac{1}{\beta}\left[(e_a-f^2\gamma E_J)X_a-\frac{\chi}{10}\left(DQ_a^+-10\sum_i\Delta_iB_{ai}X_i\right)\right]=0, \tag{27a}$$

$$\frac{1}{\beta}\left[(e_a+f^2\gamma E_J)Y_a-\frac{\chi}{10}\left(DQ_a^+-10\sum_i\Delta_iB_{ai}Y_i\right)\right]=0, \tag{27b}$$

where

$$\gamma=\frac{N_2}{N_0}-\frac{N_4}{N_2}. \tag{28}$$

The terms involving B_{ij} are invariably less than 5% of the magnitude of the other terms in the equations and the properties of the solutions can be

most clearly seen by ignoring these terms, although this is not necessary for the solution of the set of equations. Without the B_{ij} terms and defining $\omega_J = f^2\gamma E_J$, the equations become identical to the schematic model RPA equation[6] which yield a single dispersion equation for ω_J,

$$1 = \frac{\chi}{5}\sum_a \frac{Q_a^{+2} e_a}{(e_a^2 - \omega_J^2)}. \tag{29}$$

The solution of this equation will yield a value of ω that is independent of J and the energies of the yrast states are then determined by

$$E_J = \frac{1}{f^2\gamma}\,\omega. \tag{30}$$

In the limit of small f^2 it is easily determined that

$$\frac{1}{f^2\gamma} \to \frac{J}{2}, \tag{31}$$

and thus the theory predicts equally spaced or harmonic levels. This result has been obtained several times before.[7]

However, a flaw in the argument has been in the transformation to quasiparticles. This occurred in the solving of the gap and number equations:

$$1 = \frac{G}{2}\sum_i \frac{\hat{j}_i^{\,2}}{e_i} = \frac{G}{2}\sum_i \hat{j}_i^{\,2}((\epsilon_i - \lambda)^2 + \Delta^2)^{-1/2}, \tag{32a}$$

$$\langle \mathrm{BCS}|\hat{N}|\mathrm{BCS}\rangle = N = \sum_i \frac{1}{2}\,\hat{j}_i^{\,2}\left(1 - \frac{\epsilon_i - \lambda}{e_i}\right). \tag{32b}$$

That is, the average number of quasiparticles undergoing pairing was constrained to equal the number of particles in the shell. This average was taken over the BCS vacuum, whereas it would be more correct to calculate the average for each projected state individually.

The number operator, written in terms of quasiparticles, is

$$\hat{N} = \sum_a \left[\frac{\epsilon_a - \lambda}{e_a}\,\eta_0^+(aa) + \tfrac{1}{2}\hat{j}_a^{\,2}\left(1 - \frac{\epsilon_a - \lambda}{e_a}\right)\right]. \tag{33}$$

The term in the quasiparticle scattering operator (in this case, for $J=0$, the quasiparticle number operator) is of course zero when evaluated in the BCS vacuum. It is also zero in anharmonic phonon theory in which the vacuum is both the phonon and the quasiparticle vacuum and where the phonon-quasiparticle commutator is zero. In an anharmonic theory in which the vacuum is not a quasiparticle vacuum and in which the phonon-quasiparticle commutator is not zero, the evaluation of the matrix elements of $\eta_0^+(aa)$ in a phonon basis of the form

$$|JM\rangle = \sum_i A_i^J \underbrace{[B^+B^+\cdots B^+]_{JM}}_{i}|0\rangle$$

can be achieved. See, for example, the appendix to Ref. 9. The number equation will then have a dependence on J that it did not have previously. It will also depend on the parameters X and Y and on the expansion parameters A_i^J. We also would face all of the difficulties involving in eliminating nonphysical states from the multiphonon basis. However, if the number equation is evaluated in the basis employed here, viz, angular-momentum projections of the coherent phonon state, the only external parameter dependence is on f^2, which is to be determined by minimization. Further, if we restrict our attention to the local $f^2 \to 0$ minimum, the problem is self-contained and parameter free, except for Hamiltonian strength constants and single particle parameters. Thus, the lowest order anharmonic corrections are found in the number equation and the simple form of the angular-momentum-projected coherent phonon state makes the inclusion of this correction rather easy. The number equation in this basis becomes

$$\langle JM|N|JM\rangle = N = \sum_i \tfrac{1}{2}\hat{j}_i^{\,2}\left(1 - \frac{\epsilon_i - \lambda}{e_i}\right) + \sum_i \frac{\epsilon_i - \lambda}{e_i}\,C_i^J, \tag{34a}$$

where

$$C_i^J = \frac{1}{2\beta_J}\sum[(X_{ij}^{\,2} + Y_{ij}^{\,2}) + 2\beta X_{ij}Y_{ij}]. \tag{34b}$$

In Eq. (34b) an extra factor of $\frac{1}{2}$ has been included to avoid double counting.[8]

This prescription changes the nature of the solution. The solution of Eqs. (27) or Eq. (29) for the parameters X and Y depends on the values used for λ and Δ from the gap and number equations which in turn depend on the values for the parameters X and Y through Eq. (30a). Thus the problem is nonlinear and must be solved self-consistently. Since the number equation now has a dependence on J, the dispersion equation solution for ω_J will likewise, which was not the case above. The inclusion of the C_i^J terms in the number equation for $J \neq 0$ resembles a blocking effect. That is, for $J \neq 0$ the number of quasiparticles undergoing pairing is effectively reduced, thereby increasing the collective energy ω_J. It was found that the ω_J's calculated by this means increased with increasing J thus becoming more rotationlike.

Once the prescriptions indicated above have been followed to obtain ω_J and the set X_i^J, and Y_i^J for each level in the yrast band the energies are obtained from Eq. (26) and the set X_i^J and Y_i^J can be used to calculate static quadrupole moments. To

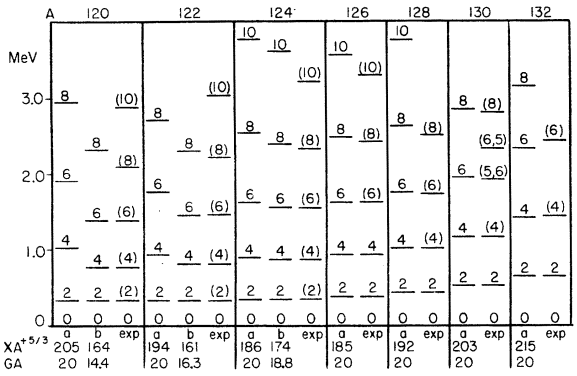

FIG. 1. A comparison of the experimental spectra of the even xenon isotopes with the theory. Spectra (a) were obtained by varying χ to fit the first 2^+ state and keeping $G = 20/A$ throughout. Spectra (b) were obtained by varying χ and G to fit the 2^+ and the 4^+ levels.

lowest order in f^2 this yields

$$\langle 0||Q_2||2\rangle = \frac{\sqrt{5}}{2}\sum_i \Delta_i Q_i^+(X_i + Y_i), \tag{35a}$$

$$\langle 2||Q||2\rangle = -\frac{1}{2}\frac{\sqrt{10}}{7} f\sum_i \Delta_i Q_i^+(X_i + Y_i) \Rightarrow 0. \tag{35b}$$

Thus in the small f^2 limit the static quadrapole moments are zero, which is to be expected as the model is then essentially harmonic.

III. NUMERICAL EXAMPLE

To illustrate these ideas we next present some results of calculations on even-even xenon isotopes. As can be seen in Fig. 1, as one progresses through the xenon isotopes, which corresponds to proceeding from about the middle of the neutron shell to a nearly closed shell, the spectra changes from rotational-like to vibrational-like and thus provides a rather good test of the range of utility of the theory. The input parameters used were as follows. For single particle neutron and proton parameters we use the values of Reehal and Sorenson,[10] which are listed in Table I. These values are quite similar to the earlier and perhaps better known values of Kisslinger and Sorenson.[11] No attempt was made to vary any of these parameters. The only other parameters are the pairing and quadrupole force constants, G and χ. For the pairing force constant we chose $G_p = G_n = 20/A$ MeV. Neutron and proton pairing were assumed equal for simplicity and this value is consistent with values used in most previous calculations. This parameter was not a variable in the calculations; however, some calculations were performed with different pairing strengths for reasons that will be noted below. The quadrupole strength parameter

TABLE I. Single particle shell model energies.

Neutrons ($50 < N < 78$)		Protons ($50 < Z < 76$)	
$2d_{5/2}$	0.0	$1g_{7/2}$	0.0
$1g_{7/2}$	0.2	$2d_{5/2}$	0.8
$3s_{1/2}$	2.0	$1h_{11/2}$	2.1
$1h_{11/2}$	1.9	$2d_{3/2}$	2.6
$2d_{3/2}$	2.6	$3s_{1/2}$	2.95

is the only parameter which was varied to fit the data and this was done to fit the first 2^+ state. A theoretical estimate of the strength constant is $\chi = 240A^{-5/3}$ MeV.[12] The values used here are represented on Fig. 1 with the calculated spectra.

Comparing the calculated with the experimental spectra, one sees a very good fit for the heavier more vibrational isotopes and the fit deteriorates as the nuclei become rotational. What appears to be happening is that as one moves toward the middle of the shell the overall strength of the residual interaction weakens and by holding the pairing force constant we require too large a quadrupole interaction to bring the 2^+ down to its experimental position and thus overstress the rotational nature of the system. If, however, we allow the pairing force to weaken as well when moving towards the center of the shell, the fit becomes much better. Of course, these latter spectra are now two-parameter fits to the 2^+ and 4^+.

IV. CONCLUSION

The expression for the energy of levels in the ground state yrast band of even-even nuclei has been obtained by angular-momentum projection on the coherent phonon state. The energies are then obtained by a minimization of this expression. Since this formulation contains a minimum of parameters as compared with ordinary phonon expansion models, more reliable tests of the model are possible. Because of the computational complexity involved in determining the absolute minimum, numerical results are presented only for a local minimum corresponding to harmonic vibrational nuclei. The resulting band is then of course evenly spaced. However, if the number equation of pairing theory is corrected by using the projected states rather than the BCS vacuum, excellent one-parameter fits to the data are obtained for the vibrational-like xenon isotopes. In addition, if the pairing strength is reduced in the rotational region, satisfactory two-parameter fits are obtained in this region as well. These results are quite helpful in the interpretation of the contribution of the pairing force in the transitional region. More complete investigations of the absolute minimum are underway.

[1]S. T. Beliaev and V. B. Zelevinsky, Nucl. Phys. 39, 582 (1962); M. Yamamura, Prog. Theo. Phys. 33, 199 (1965); S. R. Almoney and G. J. Borse, Nucl. Phys. A171, 660 (1971); F. Dönau and D. Janssen, *ibid.* A209, 109 (1973); E. R. Marshalek, *ibid.* A224, 221, 245 (1974).
[2]T. Tamura and T. Udagawa, Phys. Rev. Lett. 15, 765 (1965), Phys. Rev. 150, 783 (1966); H. Ogata and Y. Akiyama, Phys. Lett. 29B, 558 (1969).
[3]P. Haapakoski, T. Honkaranta, and P. O. Lipas, Phys. Lett. 31B, 493 (1970).
[4]R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. A70, 381 (1957).
[5]G. Holzwarth, D. Jannsen, and R. V. Jolos, Nucl. Phys. A261, 1 (1976).
[6]G. E. Brown, J. A. Evans, and D. J. Thouless, Nucl. Phys. 24, 1 (1961).
[7]S. Frauendorf, D. Janssen, and L. Munchov, Yad. Fiz. 12, 1183 (1970) [Sov. J. Nucl. Phys. 12, 649 (1971)].
[8]R. E. Johnson, R. M. Dreizler, and A. Klein, Phys. Rev. 186, 1289 (1969).
[9]G. J. Borse, Nucl. Phys. A123, 546 (1973).
[10]B. S. Reehal and R. A. Sorensen, Phys. Rev. C 2, 819 (1970).
[11]L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).
[12]D. R. Bèsand and R. A. Sorensen, Adv. Nucl. Phys. II, 129 (1969).