# Beta decay and muon capture in the $A = 12$ nuclei: Second-class currents and conserved-vector current*

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A fairly detailed analysis is given of the available experimental data on $^{12}B \rightarrow {}^{12}Ce^{-}\bar{\nu}_e$, $^{12}N \rightarrow {}^{12}Ce^{+}\nu_e$, and $\mu^{-}\,{}^{12}C \rightarrow \nu_\mu\,{}^{12}B$ making use of the "elementary-particle" treatment. Our results indicate that the variation with energy of the asymmetry coefficients obtained by Sugimoto *et al.* cannot be reconciled with the observed value of the muon capture rate on the hypothesis (A) of gross violation of ("strong") conserved vector current and absence of second-class axial currents and can be reconciled only with great difficulty on the alternative hypothesis (B) of validity of (strong) conserved vector current and presence of relatively large second-class axial currents. On the other hand, a good fit to the observed muon capture rate can be obtained on the hypothesis (C) of (strong) conserved vector current, ("suitably corrected") partially conserved axial vector current, and absence of second-class axial currents. This last hypothesis leads, of course, to a prediction for the variation with energy of the asymmetry coefficients in contradiction to the results of Sugimoto *et al.* We also find that the magnitude of the recoil $^{12}B$ polarization obtained by Possoz *et al.* agrees with that calculated on hypothesis (C) but not with those calculated on hypotheses (A) or (B).

[RADIOACTIVITY $^{12}B \rightarrow {}^{12}Ce^{-}\bar{\nu}_e$, $^{12}N \rightarrow {}^{12}Ce^{+}\nu_e$, $\mu^{-}\,{}^{12}C \rightarrow \nu_\mu\,{}^{12}B$; theoretical analysis of available experimental data. Are there second-class currents and is CVC valid?]

## INTRODUCTION

Recently, some striking new results have been obtained in the study of the nuclear $\beta$ decays: $^{12}B \rightarrow {}^{12}Ce^{-}\bar{\nu}_e$ and $^{12}N \rightarrow {}^{12}Ce^{+}\nu_e$. Assuming "strong" conserved vector current (CVC), i.e., assuming that the polar weak current is identified with the isospin current and so is first class and conserved, the experiment of Sugimoto, Tanihata, and Göring[1] requires the existence of a relatively large second-class axial current $A_\lambda^{(\mathrm{II})}$ in order to interpret the observed variation with energy of the asymmetry coefficients.[2] However, Calaprice and Holstein[3] (C-H) have questioned the original analysis of the experiment of Lee, Mo, and Wu on the $e^-$ and $e^+$ energy spectra in the $^{12}B$ and $^{12}N$ decays[4] and, as a result, thrown some doubt on the validity of strong CVC; the C-H critique is discussed in a new analysis by Wu, Lee, and Mo[4(a)] of the data of the experiment with the conclusion that these data still support strong CVC. Furthermore, according to Holstein's analysis[5] of the muon capture of $^{12}C$ (which assumes the validity of the Sugimoto *et al.* experiment[1]), the recoil $^{12}B$ polarization measured by Possoz *et al.*[6] requires a value for the nuclear pseudoscalar form factor $F_P(q^2)$ which is inconsistent with that estimated on the basis of ("suitably corrected") partial conservation of axial-vector current (PCAC).

In view of the uncertainty at present with regard to the reliability or the interpretation of one or more of the above mentioned experiments,[1,4,6] we shall analyze the $A=12$ muon capture and $\beta$ decays in some detail in the hope of obtaining a consistent picture regarding the validity of strong CVC and the presence of second-class axial currents. As will be shown below, any fit of theory to the Sugimoto *et al.* experiment[1] without the introduction of second-class axial currents requires nuclear weak magnetism form factors $F_M^{\mp}(q^2)$ about twice those deduced from strong CVC; this, in turn, with a reasonable $q^2$ dependence for the nuclear axial form factors $F_A^{\mp}(q^2)$, predicts a muon capture rate unacceptably greater than that observed —alternatively, $F_M^{\mp}(q^2) \cong 2[F_M^{\mp}(q^2)]_{\mathrm{CVC}}$ predicts a muon capture rate in agreement with observation only if the $F_A^{\mp}(q^2)$ decrease unreasonably rapidly with $q^2$. On the other hand, the introduction of second-class axial currents to interpret the Sugimoto *et al.* experiment[1] encounters the same, somewhat less serious but still significant, difficulty—either the predicted muon capture rate is appreciably greater than that observed or the $F_A^{\mp}(q^2)$ decrease too rapidly with $q^2$. Similarly, the experiment of Calaprice *et al.*[7] on the variation with energy of the asymmetry coefficient in $^{19}Ne \rightarrow {}^{19}Fe^{+}\nu_e$, implemented by means of impulse approximation (IA) plus nucleon-off-mass-shell and meson-exchange corrections to yield the nucleon form factors $f_E^{(\mathrm{II})}$ and $f_M^{(\mathrm{I})}$, requires either $f_E^{(\mathrm{II})} \cong (1.7 \pm 0.5)(f_M^{(\mathrm{I})})_{\mathrm{CVC}}$ and $f_M^{(\mathrm{I})} = (f_M^{(\mathrm{I})})_{\mathrm{CVC}}$ (Ref. 8) or $f_E^{(\mathrm{II})} = 0$ and $f_M^{(\mathrm{I})} \cong (1.9 \pm 0.4)(f_M^{(\mathrm{I})})_{\mathrm{CVC}}$ (Ref. 7); however, both of these sets of values for the form factors yield (analogous to the situation in the $\mu^-{}^{12}\mathrm{C}\to\nu_\mu{}^{12}\mathrm{B}$ case) rates for $\mu^- p\to\nu_\mu n$ which are appreciably greater than the one observed [see Eq. (51) below].

## CALCULATIONS

We perform our analysis in a model-independent fashion by making use of the "elementary-particle" treatment (EPT).[9] A rough idea of the relative magnitudes of the various form factors can be obtained on the basis of the connection[10] between EPT and IA, the connection being sufficiently trustworthy, at least in the case of the first-class form factors, for a treatment of allowed uninhibited $\beta$ decay transitions such as ${}^{12}\mathrm{B}\to{}^{12}\mathrm{C}e^-\bar{\nu}_e$ and ${}^{12}\mathrm{N}\to{}^{12}\mathrm{C}e^+\nu_e$. Furthermore, in the treatment of these $\beta$ decay transitions, we include both the final-state Coulomb interaction and the $q^2$ dependence of the various form factors in a suitably defined Fermi function [see Eq. (10) below].

We define the nuclear form factors in the case of the $A=12$ nuclei by the expressions immediately below.[11] We note that all these form factors are relatively real if time-reversal invariance holds. Thus

$$\langle{}^{12}\mathrm{C}(p_1)|V_\lambda(0)|{}^{12}\mathrm{B}(p_2,\xi)\rangle=\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa\frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}F_M^-(q^2)\,, \tag{1a}$$

$$\langle{}^{12}\mathrm{C}(p_1)|A_\lambda(0)|{}^{12}\mathrm{B}(p_2,\xi)\rangle=\sqrt{2}\left(\xi_\lambda F_A^-(q^2)+q_\lambda\frac{q\cdot\xi}{m_\pi^2}F_P^-(q^2)-\frac{Q_\lambda}{2M}\frac{q\cdot\xi}{2m_p}F_E^-(q^2)\right)\,, \tag{1b}$$

$$\langle{}^{12}\mathrm{C}(p_1)|\partial_\lambda A_\lambda(0)|{}^{12}\mathrm{B}(p_2,\xi)\rangle=i\sqrt{2}\,q\cdot\xi\left(\frac{F_D^{(\mathrm{I})-}(q^2)}{1+q^2/m_\pi^2}-\frac{F_D^{(\mathrm{II})-}(q^2)}{1+q^2/m_{\pi'}^2}\right)\,, \tag{1c}$$

$$\langle{}^{12}\mathrm{C}(p_1)|V_\lambda^\dagger(0)|{}^{12}\mathrm{N}(p_2,\xi)\rangle=\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa\frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}F_M^+(q^2)\,, \tag{1d}$$

$$\langle{}^{12}\mathrm{C}(p_1)|A_\lambda^\dagger(0)|{}^{12}\mathrm{N}(p_2,\xi)\rangle=\sqrt{2}\left(\xi_\lambda F_A^+(q^2)+q_\lambda\frac{q\cdot\xi}{m_\pi^2}F_P^+(q^2)-\frac{Q_\lambda}{2M}\frac{q\cdot\xi}{2m_p}F_E^+(q^2)\right)\,, \tag{1e}$$

$$\langle{}^{12}\mathrm{C}(p_1)|\partial_\lambda A_\lambda^\dagger(0)|{}^{12}\mathrm{N}(p_2,\xi)\rangle=i\sqrt{2}\,q\cdot\xi\left(\frac{F_D^{(\mathrm{I})+}(q^2)}{1+q^2/m_\pi^2}+\frac{F_D^{(\mathrm{II})+}(q^2)}{1+q^2/m_{\pi'}^2}\right)\,, \tag{1f}$$

$$\langle{}^{12}\mathrm{C}(p_1)|J_\lambda^{\mathrm{e.m.}}(0)|{}^{12}\mathrm{C}^*(p_2,\xi)\rangle=\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa\frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}\mu(q^2)\,, \tag{1g}$$

$$V_\lambda(x)=V_\lambda^{(\mathrm{I})}(x)+V_\lambda^{(\mathrm{II})}(x),\quad A_\lambda(x)=A_\lambda^{(\mathrm{I})}(x)+A_\lambda^{(\mathrm{II})}(x)\,, \tag{1h}$$

$$V_\lambda^\dagger(x)\equiv\{V_\lambda(x)\}_{\mathrm{Herm.\,conj.}}(1-2\delta_{4\lambda}),\quad A_\lambda^\dagger(x)\equiv\{A_\lambda(x)\}_{\mathrm{Herm.\,conj.}}(1-2\delta_{4\lambda}), \tag{1i}$$

$$F_{M,A,P,E}^{\mp}(q^2)=F_{M,A,P,E}^{(\mathrm{I})\mp}(q^2)\mp F_{M,A,P,E}^{(\mathrm{II})\mp}(q^2)\,, \tag{1j}$$

where $q_\lambda\equiv(p_2-p_1)_\lambda$, $Q_\lambda\equiv(p_2+p_1)_\lambda$, $M\equiv\frac{1}{2}(M_1+M_2)=\frac{1}{2}[M({}^{12}\mathrm{C})+M({}^{12}\mathrm{B},{}^{12}\mathrm{N})]$, $\Delta^\mp\equiv M({}^{12}\mathrm{B},{}^{12}\mathrm{N})-M({}^{12}\mathrm{C})$, and $m_{\pi'}$ ($\gg m_\pi$) is a mass which governs the $q^2$ dependence of the second-class axial-divergence form factors $F_D^{(\mathrm{II})\mp}(q^2)/(1+q^2/m_{\pi'}^2)$ analogous to the way in which $m_\pi$ governs the $q^2$ dependence of the first-class axial-divergence form factors $F_D^{(\mathrm{I})\mp}(q^2)/(1+q^2/m_\pi^2)$ (i.e., for a nucleon, $F_D^{(\mathrm{I})}(q^2)$ and $F_D^{(\mathrm{II})}(q^2)$ vary but little in the intervals $-m_\pi^2\le q^2\lesssim m_\pi^2$ and $-m_{\pi'}^2\le q^2\lesssim m_{\pi'}^2$, respectively—PCAC). Further, $\mu(q^2)$ is the nuclear magnetic transition form factor, $F_{M,A,P,E}^\mp(q^2)$ are, respectively, the nuclear weak magnetism, axial, pseudoscalar, weak electricity (or pseudotensor) form factors, $\xi$ is the polarization four-vector of the spin-one nuclei, and ${}^{12}\mathrm{N}$ ($\Delta M\equiv\Delta^+=16.833$ MeV), ${}^{12}\mathrm{C}^*$ ($\Delta M=15.110$ MeV), and ${}^{12}\mathrm{B}$ ($\Delta M\equiv\Delta^-=13.881$ MeV) constitute a $J^P=1^+$ isotriplet with $(\Delta^+-\Delta^-)/M$ so small ($\cong 2.6\times10^{-4}$) that the values of $p_2$, $\xi$, and mass are only negligibly different for the different isotriplet members.

We now note that strong CVC, i.e., $V_\lambda=V_\lambda^{(\mathrm{I})}=I_\lambda^{(-)}$ (so that $\partial_\lambda V_\lambda=\partial_\lambda V_\lambda^{(\mathrm{I})}=\partial_\lambda I_\lambda^{(-)}=0$),[12] implies

$$F_M^{(\mathrm{I})\mp}(q^2)=\sqrt{2}\,\mu(q^2),\quad F_M^{(\mathrm{II})\mp}(q^2)=0;\quad F_M^\mp(q^2)=\sqrt{2}\,\mu(q^2), \tag{2a}$$

or, if we include the corrections associated with the fact that the electromagnetic interaction breaks isospin symmetry

$$F_M^{(\mathrm{I})\mp}(q^2)=\sqrt{2}\,\mu(q^2)(1\mp\tfrac{1}{2}\delta^{(\mathrm{I})\mp}),\quad F_M^{(\mathrm{II})\mp}(q^2)=\sqrt{2}\,\mu(q^2)\tfrac{1}{2}\delta^{(\mathrm{II})\mp};\quad F_M^\mp(q^2)=\sqrt{2}\,\mu(q^2)(1\mp\tfrac{1}{2}\delta^\mp)\,, \tag{2b}$$

where $|\delta^{\mp}| \equiv |\delta^{(\mathrm{I})\mp} + \delta^{(\mathrm{II})\mp}|$ is expected to be $\lesssim Z\alpha \ll 1$. Since the isospin-symmetry breaking parameter $\delta^{\mp}$ is small, we consistently neglect it in what follows. Further, we obtain from Eqs. (1b), (1c), (1e), and (1f),

$$F_A^{\mp}(q^2) + \frac{q^2}{m_\pi^2} F_P^{\mp}(q^2) + \frac{\Delta^{\mp}}{2m_p} F_E^{\mp}(q^2) = \frac{1}{1+q^2/m_\pi^2}\left[F_D^{(\mathrm{I})\mp}(q^2) \mp \frac{1+q^2/m_\pi^2}{1+q^2/m_{\pi'}^2} F_D^{(\mathrm{II})\mp}(q^2)\right] \tag{3}$$

which implies

$$F_A^{\mp}(0) + \frac{\Delta^{\mp}}{2m_p} F_E^{\mp}(0) = F_D^{(\mathrm{I})\mp}(0) \mp F_D^{(\mathrm{II})\mp}(0) \tag{3a}$$

and

$$F_P^{\mp}(q^2) = -\left[F_A^{\mp}(q^2) + \frac{\Delta^{\mp}}{2m_p} F_E^{\mp}(q^2)\right] \frac{1+\epsilon^{\mp}(q^2)}{1+q^2/m_\pi^2} \tag{3b}$$

with

$$\epsilon^{\mp}(q^2) \equiv \frac{m_\pi^2}{q^2}\left[1 - \frac{F_D^{(\mathrm{I})\mp}(q^2) \mp [(1+q^2/m_\pi^2)/(1+q^2/m_{\pi'}^2)] F_D^{(\mathrm{II})\mp}(q^2)}{F_A^{\mp}(q^2) + (\Delta^{\mp}/2m_p) F_E^{\mp}(q^2)}\right]. \tag{3c}$$

In the nucleon case, we use the form factor definitions[13]:

$$\begin{aligned}
&\langle p(p_p)|\{[V_\lambda^{(\mathrm{I})}(0) + V_\lambda^{(\mathrm{II})}(0)] + [A_\lambda^{(\mathrm{I})}(0) + A_\lambda^{(\mathrm{II})}(0)]\}|n(p_n)\rangle \\
&= \frac{i}{(2\pi)^3}\bar{u}_p(p_p)\Big\{\Big[\Big(\gamma_\lambda f_V^{(\mathrm{I})}(q_N^2) + \frac{\sigma_{\lambda\eta}(q_N)_\eta}{2m_p} f_M^{(\mathrm{I})}(q_N^2)\Big) + i\frac{(m_p+m_n)(q_N)_\lambda}{m_\pi^2} f_S^{(\mathrm{II})}(q_N^2)\Big] \\
&\quad + \Big[\Big(\gamma_\lambda\gamma_5 f_A^{(\mathrm{I})}(q_N^2) - i\frac{(m_p+m_n)(q_N)_\lambda\gamma_5}{m_\pi^2} f_P^{(\mathrm{I})}(q_N^2)\Big) - \frac{\sigma_{\lambda\eta}(q_N)_\eta\gamma_5}{2m_p} f_E^{(\mathrm{II})}(q_N^2)\Big]\Big\} u_n(p_n),
\end{aligned} \tag{4a}$$

$$\begin{aligned}
&\langle n(p_n)|\{[V_\lambda^{(\mathrm{I})\dagger}(0) + V_\lambda^{(\mathrm{II})\dagger}(0)] + [A_\lambda^{(\mathrm{I})\dagger}(0) + A_\lambda^{(\mathrm{II})\dagger}(0)]\}|p(p_p)\rangle \\
&= \frac{i}{(2\pi)^3}\bar{u}_n(p_n)\Big\{\Big[\Big(\gamma_\lambda f_V^{(\mathrm{I})}(q_N^2) - \frac{\sigma_{\lambda\eta}(q_N)_\eta}{2m_p} f_M^{(\mathrm{I})}(q_N^2)\Big) + i\frac{(m_p+m_n)(q_N)_\lambda}{m_\pi^2} f_S^{(\mathrm{II})}(q_N^2)\Big] \\
&\quad + \Big[\Big(\gamma_\lambda\gamma_5 f_A^{(\mathrm{I})}(q_N^2) + i\frac{(m_p+m_n)(q_N)_\lambda\gamma_5}{m_\pi^2} f_P^{(\mathrm{I})}(q_N^2)\Big) - \frac{\sigma_{\lambda\eta}(q_N)_\eta\gamma_5}{2m_p} f_E^{(\mathrm{II})}(q_N^2)\Big]\Big\} u_p(p_p)
\end{aligned} \tag{4b}$$

with $(q_N)_\lambda \equiv (p_n - p_p)_\lambda$, and $f_V^{(\mathrm{II})}(q_N^2) = f_M^{(\mathrm{II})}(q_N^2) = f_S^{(\mathrm{I})}(q_N^2) = f_A^{(\mathrm{II})}(q_N^2) = f_P^{(\mathrm{II})}(q_N^2) = f_E^{(\mathrm{I})}(q_N^2) = 0$ in the limit that $|n\rangle$ and $|p\rangle$ are members of the same isodoublet.

To obtain the connection between EPT and IA for the $^{12}\mathrm{B}\to{}^{12}\mathrm{C}$ case, we assume

$$q_N \equiv p_n - p_p = p_2 - p_1 \equiv q \tag{5}$$

and define

$$\mathfrak{M}_{\mathrm{GT}}(q^2) \equiv \langle\psi_{^{12}\mathrm{C}}|\sum_{a=1}^{A} j_0(|\vec{q}|r^{(a)})\tau_+^{(a)}\sigma_z^{(a)}|\psi_{^{12}\mathrm{B},\xi(0)}\rangle, \tag{6a}$$

$$\mathfrak{M}_{\mathrm{L}}(q^2) \equiv \langle\psi_{^{12}\mathrm{C}}|\sum_{a=1}^{A} \frac{3j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})}\tau_+^{(a)}(\vec{r}^{(a)}\times\vec{p}^{(a)})_z|\psi_{^{12}\mathrm{B},\xi(0)}\rangle, \tag{6b}$$

$$\mathfrak{M}_{\sigma p z}(q^2) \equiv i\langle\psi_{^{12}\mathrm{C}}|\sum_{a=1}^{A} \frac{3j_1(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})}\tau_+^{(a)} z^{(a)}(\vec{\sigma}^{(a)}\cdot\vec{p}^{(a)})|\psi_{^{12}\mathrm{B},\xi(0)}\rangle, \tag{6c}$$

$$\mathfrak{M}_{\sigma Q}(q^2) \equiv q^2\langle\psi_{^{12}\mathrm{C}}|\sum_{a=1}^{A} \frac{15j_2(|\vec{q}|r^{(a)})}{(|\vec{q}|r^{(a)})^2}\tau_+^{(a)}\sigma_z^{(a)}[(r^{(a)})^2 - 3(z^{(a)})^2]|\psi_{^{12}\mathrm{B},\xi(0)}\rangle, \tag{6d}$$

where

$$\psi_{^{12}\mathrm{B},\xi(0)} \equiv \psi_{^{12}\mathrm{B},\xi(0)}(\ldots,\vec{r}^{(a)},\sigma_z^{(a)},\tau_z^{(a)},\ldots),$$

$$\psi_{^{12}\mathrm{C}} \equiv \psi_{^{12}\mathrm{C}}(\ldots,\vec{r}^{(a)},\sigma_z^{(a)},\tau_z^{(a)},\ldots),$$

$$\tau_\pm^{(a)} \equiv \tfrac{1}{2}(\tau_1^{(a)} \pm i\tau_2^{(a)}),$$

and $\xi(0)$ is the polarization four-vector describing the $^{12}\mathrm{B}$ state with $J_z=0$; as a rough estimate, we have $|\mathfrak{M}_{\mathrm{GT}}(q^2)| \approx |\mathfrak{M}_{\sigma p z}(q^2)| \gtrsim |\mathfrak{M}_{\mathrm{L}}(q^2)| \gg |\mathfrak{M}_{\sigma Q}(q^2)|$, the last inequality being a consequence of the approximate spatial sphericity of $|\psi_{^{12}\mathrm{B},\xi(0)}\rangle$. Further, the spin of the $^{12}\mathrm{B}$ nucleus arises largely from the appropriately coupled spins of its constituent nucleons so that $|\mathfrak{M}_{\mathrm{L}}(q^2)|$ is expected to be considerably smaller than $|\mathfrak{M}_{\mathrm{GT}}(q^2)|$. Finally, a nuclear-physics calculation[14] yields $\mathfrak{M}_{\mathrm{GT}}(q^2) = -|\mathfrak{M}_{\mathrm{GT}}(q^2)|$ so that [see Eqs. (7a) and 7(b) just below] $F_{M,A}(q^2) = |F_{M,A}(q^2)|$.

Using a method similar to that described by Delorme[10] and keeping terms up to the first order in $\Delta^-/2m_p$, we then obtain[14]

$$\sqrt{2}F_M^-(q^2) = \sqrt{2}\,[F_M^{(\mathrm{I})-}(q^2) - F_M^{(\mathrm{II})-}(q^2)]$$
$$\cong -[f_V^{(\mathrm{I})}(q^2) + f_M^{(\mathrm{I})}(q^2)][\mathfrak{M}_{\mathrm{GT}}(q^2) - \tfrac{1}{12}\mathfrak{M}_{\sigma Q}(q^2)] - f_V^{(\mathrm{I})}(q^2)\mathfrak{M}_{\mathrm{L}}(q^2)$$
$$\cong -[f_V^{(\mathrm{I})}(q^2) + f_M^{(\mathrm{I})}(q^2)]\mathfrak{M}_{\mathrm{GT}}(q^2) - f_V^{(\mathrm{I})}(q^2)\mathfrak{M}_{\mathrm{L}}(q^2)\,, \quad (7a)$$

$$\sqrt{2}F_A^-(q^2) = \sqrt{2}\,[F_A^{(\mathrm{I})-}(q^2) - F_A^{(\mathrm{II})-}(q^2)]$$
$$\cong -\left[f_A^{(\mathrm{I})}(q^2) + \frac{\Delta^-}{2m_p}f_E^{(\mathrm{II})}(q^2)\right][\mathfrak{M}_{\mathrm{GT}}(q^2) - \tfrac{1}{12}\mathfrak{M}_{\sigma Q}(q^2)]$$
$$\cong -\left[f_A^{(\mathrm{I})}(q^2) + \frac{\Delta^-}{2m_p}f_E^{(\mathrm{II})}(q^2)\right]\mathfrak{M}_{\mathrm{GT}}(q^2)\,, \quad (7b)$$

$$\sqrt{2}F_P^-(q^2) = \sqrt{2}\,[F_P^{(\mathrm{I})-}(q^2) - F_P^{(\mathrm{II})-}(q^2)]$$
$$\cong -\left[f_P^{(\mathrm{I})}(q^2) - \left(\frac{m_\pi}{2m_p}\right)^2 f_E^{(\mathrm{II})}(q^2)\right][\mathfrak{M}_{\mathrm{GT}}(q^2) + \tfrac{1}{6}\mathfrak{M}_{\sigma Q}(q^2)] - \frac{1}{4}\frac{m_\pi{}^2}{q^2}\left[f_A^{(\mathrm{I})}(q^2) + \frac{\Delta^-}{2m_p}f_E^{(\mathrm{II})}(q^2)\right]\mathfrak{M}_{\sigma Q}(q^2)$$
$$\cong -\left[f_P^{(\mathrm{I})}(q^2) - \left(\frac{m_\pi}{2m_p}\right)^2 f_E^{(\mathrm{II})}(q^2)\right]\mathfrak{M}_{\mathrm{GT}}(q^2)\,, \quad (7c)$$

$$\sqrt{2}F_E^-(q^2) = \sqrt{2}\,[F_E^{(\mathrm{I})-}(q^2) - F_E^{(\mathrm{II})-}(q^2)]$$
$$\cong -[f_A^{(\mathrm{I})}(q^2) - f_E^{(\mathrm{II})}(q^2)][\mathfrak{M}_{\mathrm{GT}}(q^2) + \tfrac{1}{6}\mathfrak{M}_{\sigma Q}(q^2)] - f_A^{(\mathrm{I})}(q^2)\left[2\mathfrak{M}_{\sigma p z}(q^2) + \frac{m_p\Delta^-}{2q^2}\mathfrak{M}_{\sigma Q}(q^2)\right]$$
$$\cong -[f_A^{(\mathrm{I})}(q^2) - f_E^{(\mathrm{II})}(q^2)]\mathfrak{M}_{\mathrm{GT}}(q^2) - 2f_A^{(\mathrm{I})}(q^2)\mathfrak{M}_{\sigma p z}(q^2) \quad (7d)$$

so that

$$\frac{F_M^{(\mathrm{II})-}}{F_M^{(\mathrm{I})-}} \cong 0\,, \quad (8a)$$

$$\left|\frac{F_A^{(\mathrm{II})-}}{F_A^{(\mathrm{I})-}}\right| \cong \left| -\frac{\Delta^-}{2m_p}\frac{f_E^{(\mathrm{II})}}{f_A^{(\mathrm{I})}}\right| \ll 1\,, \quad (8b)$$

$$\left|\frac{F_P^{(\mathrm{II})-}}{F_P^{(\mathrm{I})-}}\right| \cong \left|\left(\frac{m_\pi}{2m_p}\right)^2\frac{f_E^{(\mathrm{II})}}{f_P^{(\mathrm{I})}}\right| \ll 1\,, \quad (8c)$$

$$\frac{F_E^{(\mathrm{II})-}}{F_E^{(\mathrm{I})-}} \cong \frac{f_E^{(\mathrm{II})}}{f_A^{(\mathrm{I})}}\left(1 + 2\,\frac{\mathfrak{M}_{\sigma p z}}{\mathfrak{M}_{\mathrm{GT}}}\right)^{-1}, \quad (8d)$$

$$\frac{F_M^{(\mathrm{I})-}}{F_A^{(\mathrm{I})-}} \cong \frac{f_V^{(\mathrm{I})} + f_M^{(\mathrm{I})}}{f_A^{(\mathrm{I})}} + \frac{f_V^{(\mathrm{I})}}{f_A^{(\mathrm{I})}}\frac{\mathfrak{M}_{\mathrm{L}}}{\mathfrak{M}_{\mathrm{GT}}}\,, \quad (8e)$$

$$\frac{F_P^{(\mathrm{I})-}}{F_A^{(\mathrm{I})-}} \cong \frac{f_P^{(\mathrm{I})}}{f_A^{(\mathrm{I})}}\,, \quad (8f)$$

$$\frac{F_E^{(\mathrm{I})-}}{F_A^{(\mathrm{I})-}} \cong 1 + 2\,\frac{\mathfrak{M}_{\sigma p z}}{\mathfrak{M}_{\mathrm{GT}}} = -\left(1 + 2\,\frac{\mathfrak{M}'_{\sigma p z}}{\mathfrak{M}'_{\mathrm{GT}}}\right);$$

$$\mathfrak{M}'_{\sigma p z}, \mathfrak{M}'_{\mathrm{GT}} \equiv \mathfrak{M}_{\sigma p z}, \mathfrak{M}_{\mathrm{GT}} \quad \text{with } \tau_+^{(a)} \to \tau_-^{(a)} \text{ and } |{}^{12}\mathrm{B}\rangle \to |{}^{12}\mathrm{C}\rangle. \quad (8g)$$

We recall that the contributions of $V_\lambda^{(\mathrm{II})}$ and $A_\lambda^{(\mathrm{II})}$ may be particularly sensitive to deviations from IA (nucleon-off-mass-shell and meson-exchange effects) so that $F_{M,A,P,E}^{(\mathrm{II})-}$ could be quite different from the values indicated in Eqs. (7a)–(8d).[8] Nonetheless, the conclusion from these equations that, given $f_E^{(\mathrm{II})} \approx f_A^{(\mathrm{I})}$ only $F_E^{(\mathrm{II})-}$ is important, is likely to be correct.

We are now ready to set down the reduced $T$ matrix elements for ${}^{12}\mathrm{B} \to {}^{12}\mathrm{C}e^-\bar{\nu}_e$, ${}^{12}\mathrm{N} \to {}^{12}\mathrm{C}e^+\nu_e$, and $\mu^-{}^{12}\mathrm{C} \to \nu_\mu{}^{12}\mathrm{B}$. These are

$$T({}^{12}\mathrm{B} \to {}^{12}\mathrm{C}e^-\bar{\nu}_e) = \frac{G}{\sqrt{2}}\langle {}^{12}\mathrm{C}(p_1)|[V_\lambda(0) + A_\lambda(0)]|{}^{12}\mathrm{B}(p_2,\xi)\rangle \frac{i}{(2\pi)^3}\bar{u}_e(p_e)\gamma_\lambda(1+\gamma_5)v_{\nu_e}(p_{\nu_e})\,, \quad (9a)$$

$$T(^{12}\mathrm{N}\rightarrow{}^{12}\mathrm{C}e^{+}\nu_e)=\frac{G}{\sqrt{2}}\langle{}^{12}\mathrm{C}(p_1)|[V_\lambda^\dagger(0)+A_\lambda^\dagger(0)]|{}^{12}\mathrm{N}(p_2,\xi)\rangle\frac{i}{(2\pi)^3}\bar{u}_{\nu_e}(p_{\nu_e})\gamma_\lambda(1+\gamma_5)v_e(p_e)\,, \tag{9b}$$

$$T(\mu^{-\,12}\mathrm{C}\rightarrow\nu_\mu{}^{12}\mathrm{B})=\frac{G}{\sqrt{2}}\langle{}^{12}\mathrm{B}(p_2,\xi)|[V_\lambda^\dagger(0)+A_\lambda^\dagger(0)]|{}^{12}\mathrm{C}(p_1)\rangle\frac{i}{(2\pi)^3}\bar{u}_{\nu_\mu}(p_{\nu_\mu})\gamma_\lambda(1+\gamma_5)u_\mu(p_\mu) \tag{9c}$$

with

$$\langle{}^{12}\mathrm{B}(p_2,\xi)|[V_\lambda^\dagger(0)+A_\lambda^\dagger(0)]|{}^{12}\mathrm{C}(p_1)\rangle=\langle{}^{12}\mathrm{C}(p_1)|[V_\lambda(0)+A_\lambda(0)]|{}^{12}\mathrm{B}(p_2,\xi)\rangle^*(1-2\delta_{4\lambda})\,.$$

The $e^\mp$ decay energy and angular distributions and the corresponding asymmetry coefficients $\mathfrak{A}^\mp$ are then[14,15]

$$d^3\Gamma(e^\mp)\cong\frac{G^2}{8\pi^4}[\sqrt{2}F_A^\mp(0)]^2F_\mp(Z,E_e)p_eE_e(\Delta^\mp-E_e)^2(1+\eta_\mp+a_\mp E_e)\,dE_e\,d\Omega_e$$

$$\times[1\mp(h_1-h_{-1})(1+\alpha_\mp E_e)\cos\theta_e+(1-3h_0)\alpha_\mp E_e(\tfrac{3}{2}\cos^2\theta_e-\tfrac{1}{2})]\,,$$

$$\mathfrak{A}^\mp\equiv\frac{d^3\Gamma(e^\mp)_{\theta_e=0}-d^3\Gamma(e^\mp)_{\theta_e=\pi}}{d^3\Gamma(e^\mp)_{\theta_e=0}+d^3\Gamma(e^\mp)_{\theta_e=\pi}}$$

$$\cong\mp(h_1-h_{-1})\{1+\alpha_\mp[1-(1-3h_0)]E_e\}\,, \tag{10}$$

where

$F_\mp(Z,E_e)\equiv$Fermi function for the $e^\mp$ decays ,

$h_1,h_{-1},h_0\equiv$populations of the $J_z=1,-1,0$ states of $^{12}$B or $^{12}$N normalized so that $h_1+h_{-1}+h_0=1$,

$\cos\theta_e\equiv\hat{p}_e\cdot\hat{z}$ ,

$$a_\mp\equiv\pm\frac{4}{3m_p}\frac{F_M^\mp(0)}{F_A^\mp(0)}\,,\quad\alpha_\mp\equiv\frac{1}{3m_p}\left(\pm\frac{F_M^\mp(0)}{F_A^\mp(0)}-\frac{F_E^\mp(0)}{F_A^\mp(0)}\right), \tag{11}$$

$$\eta_\mp\equiv\frac{\Delta^\mp}{3m_p}\left(\mp2\frac{F_M^\mp(0)}{F_A^\mp(0)}+\frac{F_E^\mp(0)}{F_A^\mp(0)}\right),$$

whence, with the replacement[16]

$$F_\mp(Z,E_e)p_eE_e(\Delta^\mp-E_e)^2dE_e\rightarrow\zeta_\mp p_eE_e(\Delta^\mp-E_e)^2dE_e\,, \tag{12a}$$

where

$$\zeta_\mp\equiv\frac{\int_{b_\mp}^1dx\,x(x^2-b_\mp^2)^{1/2}(1-x)^2F_\mp(Z,x)}{\int_{b_\mp}^1dx\,x(x^2-b_\mp^2)^{1/2}(1-x)^2}$$

$$\cong\frac{\int_{b_\mp}^1dx\,x(x^2-b_\mp^2)^{1/2}(1-x)^2F_\mp(Z,x)}{(1/30-b_\mp^2/6)}$$

$$\equiv(f)_{e^\mp}b_\mp^5(1/30-b_\mp^2/6)^{-1}\,,\quad b_\mp\equiv\frac{m_e}{\Delta^\mp}<\tfrac{1}{25} \tag{12b}$$

we get

$$\Gamma(e^\mp)=\frac{G^2(\Delta^\mp)^5}{2\pi^3}\zeta_\mp[\tfrac{1}{30}-\tfrac{1}{6}(m_e/\Delta^\mp)^2][\sqrt{2}F_A^\mp(0)]^2$$

$$\times\left(1+\frac{F_E^\mp(0)}{F_A^\mp(0)}\frac{\Delta^\mp}{3m_p}\right). \tag{13}$$

Using the appropriately calculated $f$ values[17]:

$$(f)_{e^-}=(5.6113\pm0.0026)\times10^5\,,$$
$$(f)_{e^+}=(1.1327\pm0.0017)\times10^6 \tag{14}$$

we obtain

$$\zeta_-=1.146\,,\quad\zeta_+=0.880\,, \tag{15}$$

so that, with $G=(1.140\pm0.006)\times10^{-11}$ MeV$^{-2}$,

$$\Gamma(e^-)=124.51\ \mathrm{sec}^{-1}[F_A^-(0)]^2\left(1+0.005\frac{F_E^-(0)}{F_A^-(0)}\right), \tag{16a}$$

$$\Gamma(e^+)=251.28\ \mathrm{sec}^{-1}[F_A^+(0)]^2\left(1+0.006\frac{F_E^+(0)}{F_A^+(0)}\right), \tag{16b}$$

while from the experimental values of the half-lives[18]

$$(t_{1/2})_{e^-}=21.02\pm0.06\ \mathrm{msec}\,,$$
$$(t_{1/2})_{e^+}=11.63\pm0.04\ \mathrm{msec} \tag{17}$$

we have

$$\Gamma(e^-)=32.98\pm0.10\ \mathrm{sec}^{-1}\,,$$
$$\Gamma(e^+)=59.60\pm0.20\ \mathrm{sec}^{-1}. \tag{18}$$

Equations (16a), (16b), and (18) specify the values of $F_A^\mp(0)$ rather accurately since the terms in $F_E^\mp(0)/F_A^\mp(0)$ are multiplied by very small coefficients.

Further, the muon capture rate is specified by[5,9,9(a)]

$$\Gamma(\mu^{-\,12}\mathrm{C}\rightarrow\nu_\mu{}^{12}\mathrm{B})=\beta({}^{12}\mathrm{C},{}^{12}\mathrm{B})\left(\frac{F_A^-(q_m^2)}{F_A^-(0)}\right)^2R(q_m^2)\,, \tag{19}$$

where

$$q_m^2 \equiv (p_2 - p_1)_m^2 = (p_\mu - p_\nu)^2 = -m_\mu^2 + 2E_\nu m_\mu = 0.740 m_\mu^2 = 0.212\ \text{fm}^{-2}, \quad (20a)$$

$$\beta(^{12}\text{C}, ^{12}\text{B}) \equiv \pi E_\nu^2 \left(1 - \frac{E_\nu}{m_\mu + M_2}\right) C(^{12}\text{C}) \times \left(\frac{Z(^{12}\text{C})}{137} \frac{m_\mu M_1}{m_\mu + M_1}\right)^3 \left(\frac{\Gamma(e^-)}{m_e^5 (f)_{e^-}}\right) = (3.55 \pm 0.01) \times 10^3\ \text{sec}^{-1} \quad (20b)$$

and

$$R(q_m^2) \equiv 2\left(1 + \frac{F_M^-(q_m^2)}{F_A^-(q_m^2)} \frac{E_\nu}{2m_p}\right)^2 + \left(1 + \frac{F_P^-(q_m^2)}{F_A^-(q_m^2)} \frac{m_\mu E_\nu}{m_\pi^2} - \frac{F_E^-(q_m^2)}{F_A^-(q_m^2)} \frac{E_\nu}{2m_p}\right)^2 \quad (20c)$$

and where we have used

$$E_\nu = m_\mu - \Delta^- - \frac{(m_\mu - \Delta^-)^2}{2M(^{12}\text{B})} = 91.41\ \text{MeV},$$

$$Z(^{12}\text{C}) = 6,$$

$C(^{12}\text{C}) \equiv$ correction factor arising from the nonpoint charge distribution of $^{12}$C, $= 0.841$ [Refs. 9 and 9(b)].

Finally, the recoil $^{12}$B polarization in $\mu^{-12}\text{C} \to \nu_\mu {}^{12}\text{B}$ is given by[5]

$$P_{\text{av}} = \frac{2}{3}\left(1 - \frac{P(q_m^2)}{R(q_m^2)}\right) \quad (21a)$$

with

$$P(q_m^2) \equiv \left[\frac{F_P^-(q_m^2)}{F_A^-(q_m^2)} \frac{m_\mu E_\nu}{m_\pi^2} - \left(\frac{F_M^-(q_m^2)}{F_A^-(q_m^2)} + \frac{F_E^-(q_m^2)}{F_A^-(q_m^2)}\right) \frac{E_\nu}{2m_p}\right]^2. \quad (21b)$$

If we now assume the validity of strong CVC, we can immediately determine the numerical value for $F_M^\mp(0)$. The transition rate for the $\gamma$ decay $^{12}\text{C}^* \to {}^{12}\text{C}\gamma$ is specified by

$$\Gamma(^{12}\text{C}^* \to {}^{12}\text{C}\gamma) = \frac{\alpha}{3} \frac{E_\gamma^3}{m_p^2} |\sqrt{2}\ \mu(0)|^2,$$

$$E_\gamma = 15.10\ \text{MeV} \quad (22a)$$

and, using the experimental value[19]

$$\Gamma(^{12}\text{C}^* \to {}^{12}\text{C}\gamma) = 37.0 \pm 1.1\ \text{eV} \quad (22b)$$

we obtain from Eqs. (22a), (22b), (2a)

$$F_M^\mp(0) = \sqrt{2}\ \mu(0) = 1.97 \pm 0.02. \quad (23)$$

As regards the relevance of PCAC to the evaluation of $F_P^\mp(q^2)/F_A^\mp(q^2)$, or, in view of Eqs. (3b) and (3c) to the evaluation of $\epsilon^\mp(q^2)$, we first assume

$$\frac{F_D^{(I)\mp}(q^2)}{F_D^{(I)\mp}(0)} \cong \frac{F_D^{(II)\mp}(q^2)}{F_D^{(II)\mp}(0)} \cong \frac{F_A^\mp(q^2)}{F_A^\mp(0)} \cong \frac{F_E^\mp(q^2)}{F_E^\mp(0)} \cong \frac{F_M^\mp(q^2)}{F_M^\mp(0)} \cong \frac{\mu(q^2)}{\mu(0)} \quad (24)$$

and, remembering that $m_{\pi'} \gg m_\pi$, obtain from Eqs. (3a)–(3c),

$$\epsilon^\mp(q^2) \cong \frac{\pm F_D^{(II)\mp}(0)}{F_D^{(I)\mp}(0) \mp F_D^{(II)\mp}(0)} = \frac{\pm[F_A^{(II)\mp}(0) + (\Delta^\mp/2m_p) F_E^{(II)\mp}(0)]}{[F_A^{(I)\mp}(0) + (\Delta^\mp/2m_p) F_E^{(I)\mp}(0)] \mp [F_A^{(II)\mp}(0) + (\Delta^\mp/2m_p) F_E^{(II)\mp}(0)]}. \quad (25)$$

Equation (24) corresponds to the assumption that the $q^2$ dependence of $F_D^{(I),(II)\mp}(q^2)$, $F_A^\mp(q^2)$, $F_E^\mp(q^2)$, $F_M^\mp(q^2)$, and $\mu(q^2)$ is essentially determined by the dimensions of the initial and final nuclei for the range of $q^2$ under consideration and so is very similar for all the various form factors. The impulse approximation, i.e., Eqs. (7a)–(7d), yields $F_A^{(II)-}(0) + (\Delta^-/2m_p) F_E^{(II)-}(0) = 0$ so that $\epsilon^-(q^2) \cong 0$. More generally, it is reasonable to assume $|F_A^{(II)\mp}(0)| \ll |F_A^{(I)\mp}(0)|$ and $|(\Delta^\mp/2m_p) F_E^{(I),(II)\mp}(0)| \ll |F_A^{(I)\mp}(0)|$ so that $\epsilon^\mp(q^2)$ is expected to be small even without invocation of the impulse approximation.

With $\epsilon^\mp(q^2)$ and $(\Delta^\mp/2m_p) F_E^\mp(q^2)$ neglected, Eq. (3b) becomes

$$\frac{F_P^\mp(q^2)}{F_A^\mp(q^2)} \cong -\frac{1}{1 + q^2/m_\pi^2}, \quad (26a)$$

a result reminiscent of that obtained in the nucleon case through use of PCAC.[13]

We also note that, in agreement with Eq. (26a), the impulse approximation of Eqs. (7a)–(7d) yields directly

$$\frac{F_P^\mp(q^2)}{F_A^\mp(q^2)} \cong \frac{f_P^{(I)}(q^2)}{f_A^{(I)}(q^2)} \cong -\frac{1}{1 + q^2/m_\pi^2}, \quad (26b)$$

where the second equality follows with rather high precision from PCAC applied to the nucleon case.[13] Hence a gross deviation of the $\epsilon^\mp(q^2)$ from zero indicates the failure not only of the assumption of similar $q^2$ dependence of the various form factors [Eq. (24)] but also of the impulse approximation and, in fact, detailed theoretical estimates of $\epsilon^-(q_m^2)$ in the $^{12}C \rightarrow {}^{12}B$ case correspond to a rather small deviation, i.e., to a value of about −0.15 ("suitably corrected" PCAC).[20]

To throw further light on the relative magnitudes of $F_A^-(q_m^2)/F_A^-(0)$, $F_M^-(q_m^2)/F_M^-(0)$, and $\mu(q_m^2)/\mu(0)$, we define

$$\mathfrak{M}_{GT}(q^2;{}^{12}C^* \rightarrow {}^{12}C) \equiv \langle \psi_{^{12}C} | \sum_{a=1}^{A} j_0(|\vec{q}|\, r^{(a)}) \tfrac{1}{2}\tau_3^{(a)}\sigma_z^{(a)} | \psi_{^{12}C^*, \zeta(0)} \rangle \,, \tag{27a}$$

$$\mathfrak{M}_{L}(q^2;{}^{12}C^* \rightarrow {}^{12}C) \equiv \langle \psi_{^{12}C} | \sum_{a=1}^{A} \frac{3 j_1(|\vec{q}|\, r^{(a)})}{(|\vec{q}|\, r^{(a)})} \frac{\tau_3^{(a)}}{2} (\vec{r}^{(a)} \times \vec{p}^{(a)})_z | \psi_{^{12}C^*, \zeta(0)} \rangle \tag{27b}$$

and denote $\mathfrak{M}_{GT}(q^2)$ and $\mathfrak{M}_L(q^2)$ defined in Eqs. (6a) and (6b) by $\mathfrak{M}_{GT}(q^2;{}^{12}B \rightarrow {}^{12}C)$ and $\mathfrak{M}_L(q^2;{}^{12}B \rightarrow {}^{12}C)$. We then have

$$\frac{\mathfrak{M}_L(q^2;{}^{12}C^* \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(q^2;{}^{12}C^* \rightarrow {}^{12}C)} \cong \frac{\mathfrak{M}_L(q^2;{}^{12}B \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(q^2;{}^{12}B \rightarrow {}^{12}C)} \tag{28}$$

with the small difference between the two ratios determined essentially by the amount of isospin-symmetry breaking between $\psi_{^{12}B, \zeta(0)}$ and $\psi_{^{12}C^*, \zeta(0)}$. In addition, the connection between EPT and IA yields

$$\sqrt{2}\; \mu(q^2) \cong -\{[e_p(q^2) - e_n(q^2)] + [\mu_p(q^2) - \mu_n(q^2)]\}\, \mathfrak{M}_{GT}(q^2;{}^{12}C^* \rightarrow {}^{12}C) - [e_p(q^2) - e_n(q^2)]\, \mathfrak{M}_L(q^2;{}^{12}C^* \rightarrow {}^{12}C)\,, \tag{29}$$

where $e_p(q^2)$ $[e_n(q^2)]$ and $\mu_p(q^2)$ $[\mu_n(q^2)]$ are the electric charge and anomalous magnetic moment form factors of the proton (neutron). Note that $e_p(0)=1$, $e_n(0)=0$, and $\mu_p(0) - \mu_n(0) = 3.706$. Hence, using the EPT-IA connection of Eqs. (7a) and (7b)

$$\begin{aligned}\frac{F_M^-(q_m^2)}{F_M^-(0)} &\cong \frac{F_A^-(q_m^2)}{F_A^-(0)} \left( \frac{f_V^{(I)}(q_m^2) + f_M^{(I)}(q_m^2)}{f_V^{(I)}(0) + f_M^{(I)}(0)} \Big/ \frac{f_A^{(I)}(q_m^2)}{f_A^{(I)}(0)} \right) \\ &\quad \times \left[ \left( 1 + \frac{f_V^{(I)}(q_m^2)}{f_V^{(I)}(q_m^2) + f_M^{(I)}(q_m^2)} \frac{\mathfrak{M}_L(q_m^2;{}^{12}B \rightarrow C^{12})}{\mathfrak{M}_{GT}(q_m^2;{}^{12}B \rightarrow {}^{12}C)} \right) \Big/ \left( 1 + \frac{f_V^{(I)}(0)}{f_V^{(I)}(0) + f_M^{(I)}(0)} \frac{\mathfrak{M}_L(0;{}^{12}B \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(0;{}^{12}B \rightarrow {}^{12}C)} \right) \right] \\ &\equiv \frac{F_A^-(q_m^2)}{F_A^-(0)} [1 + \epsilon'^-(q_m^2)]\;. \end{aligned} \tag{30}$$

Here $\epsilon'^-(q_m^2)$ can be estimated as $\lesssim 2\%$ for any reasonable nuclear model[10] so that $F_M^-(q^2)/F_M^-(0) = F_A^-(q^2)/F_A^-(0)$ can be considered as valid to a few percent within the context of the EPT-IA connection. Further, Eqs. (28), (29), and (7b) yield

$$\begin{aligned}\frac{\mu(q_m^2)}{\mu(0)} &\cong \frac{F_A^-(q_m^2)}{F_A^-(0)} \left( \frac{e_p(q_m^2) - e_n(q_m^2) + \mu_p(q_m^2) - \mu_n(q_m^2)}{e_p(0) - e_n(0) + \mu_p(0) - \mu_n(0)} \Big/ \frac{f_A^{(I)}(q_m^2)}{f_A^{(I)}(0)} \right) \left( \frac{\mathfrak{M}_{GT}(q_m^2;{}^{12}C^* \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(0;{}^{12}C^* \rightarrow {}^{12}C)} \Big/ \frac{\mathfrak{M}_{GT}(q_m^2;{}^{12}B \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(0;{}^{12}B \rightarrow {}^{12}C)} \right) \\ &\quad \times \left[ \left( 1 + \frac{e_p(q_m^2) - e_n(q_m^2)}{e_p(q_m^2) - e_n(q_m^2) + \mu_p(q_m^2) - \mu_n(q_m^2)} \frac{\mathfrak{M}_L(q_m^2;{}^{12}C^* \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(q_m^2;{}^{12}C^* \rightarrow {}^{12}C)} \right) \Big/ \right. \\ &\quad \left. \left( 1 + \frac{e_p(0) - e_n(0)}{e_p(0) - e_n(0) + \mu_p(0) - \mu_n(0)} \frac{\mathfrak{M}_L(0;{}^{12}C^* \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(0;{}^{12}C^* \rightarrow {}^{12}C)} \right) \right] \\ &\cong \frac{F_A^-(q_m^2)}{F_A^-(0)} \left[ \left( 1 + \frac{1}{4.7} \frac{\mathfrak{M}_L(q_m^2;{}^{12}B \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(q_m^2;{}^{12}B \rightarrow {}^{12}C)} \right) \Big/ \left( 1 + \frac{1}{4.7} \frac{\mathfrak{M}_L(0;{}^{12}B \rightarrow {}^{12}C)}{\mathfrak{M}_{GT}(0;{}^{12}B \rightarrow {}^{12}C)} \right) \right] \\ &\equiv \frac{F_A^-(q_m^2)}{F_A^-(0)} [1 + \epsilon''^-(q_m^2)]\,, \end{aligned} \tag{31}$$

where $\epsilon''^-(q_m^2) = \epsilon'^-(q_m^2)$ if strong CVC holds and $\epsilon''^-(q_m^2) \cong \epsilon'^-(q_m^2)$ even without strong CVC. Examination of Eq. (31) shows that it is hardly possible to reconcile with the EPT-IA connection a value for $|1 - [F_A^-(q_m^2)/F_A^-(0)]/[\mu(q_m^2)/\mu(0)]|$ of more than 5%. Such a conclusion is independent of the validity of strong CVC.

We now proceed to analyze the situation on the basis oi the following experimental data as *reliable* inputs:

(1) $[\Gamma(e^-)]_{exp} = 32.98 \pm 0.10\ \text{sec}^{-1}$, $[\Gamma(e^+)]_{exp} = 59.60 \pm 0.20\ \text{sec}^{-1}$ (Ref. 18);

(2) $[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp} = (6.2 \pm 0.3) \times 10^3\ \text{sec}^{-1}$ (Ref. 21); (32)

(3) $[\Gamma({}^{12}\text{C}^* \to {}^{12}\text{C}\gamma)]_{exp} = 37.0 \pm 1.1$ eV (Ref. 19) and $(\mu(q_m^2)/\mu(0))_{exp} = 0.750 \pm 0.013$ (Ref. 22).

### Hypothesis A

*Assumptions*:

$$F^{(\text{II})\mp}_{M,A,P,E,D}(q^2) = 0;$$

$$\frac{F_P^\mp(q^2)}{F_A^\mp(q^2)} = -\frac{1+\epsilon^\mp(q^2)}{1+q^2/m_\pi^2} \quad \text{with } \epsilon^-(q_m^2) = -0.15 \text{ (Ref. 20)};$$

$$\frac{F_M^\mp(q^2)}{F_M^\mp(0)} = \frac{F_A^\mp(q^2)}{F_A^\mp(0)} = \frac{F_E^\mp(q^2)}{F_E^\mp(0)}; \tag{33}$$

$$\frac{F_M^-(0)}{F_M^+(0)} = \frac{F_A^-(0)}{F_A^+(0)} = \frac{F_E^-(0)}{F_E^+(0)} \text{ (Ref. 23)};$$

$$\alpha_- = (\alpha_-)_{exp} = (3.1 \pm 0.6)/\text{GeV},$$

$$\alpha_+ = (\alpha_+)_{exp} = -(2.1 \pm 0.7)/\text{GeV (Ref. 1)}.$$

*Predictions*:

Eqs. (11) and (33) yield

$$\alpha_\mp = \frac{1}{3m_p}\left(\pm\frac{F_M^\mp(0)}{F_A^\mp(0)} - \frac{F_E^\mp(0)}{F_A^\mp(0)}\right) = \frac{1}{3m_p}\left(\pm\frac{F_M^-(0)}{F_A^-(0)} - \frac{F_E^-(0)}{F_A^-(0)}\right) = (\alpha_\mp)_{exp} \tag{34}$$

so that

$$\frac{F_M^\mp(0)}{F_A^\mp(0)} = (7.32 \pm 1.30), \quad \frac{F_E^\mp(0)}{F_A^\mp(0)} = -(1.41 \pm 1.30). \tag{35}$$

Equation (35), together with Eqs. (16a), (16b), and (18), yields

$$F_A^-(0) = 1.055 F_A^+(0) = 0.516$$

which in turn implies

$$F_M^-(0) = 1.055\, F_M^+(0) = 3.78 \pm 0.67 = (1.92 \pm 0.34)[F_M^-(0)]_{\text{CVC}}.$$

Equations (19)–(21b), (35), and (33) and the value of $[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp}$ then give

$$\frac{F_A^-(q_m^2)}{F_A^-(0)} = 0.639 \pm 0.030$$

and

$$P_{av} = 0.61 \pm 0.02$$

This value of $F_A^-(q_m^2)/F_A^-(0)$ yields

$$1 - \left(\frac{F_A^-(q_m^2)}{F_A^-(0)}\right) \Big/ \left(\frac{\mu(q_m^2)}{\mu(0)}\right)_{exp} = (15 \pm 4)\% \tag{36}$$

which is in serious disagreement with what is expected from the hypothesis of similar $q^2$ dependence of the various form factors [see Eq. (24) *et seq.*] or from the impulse approximation [see Eq. (31) *et seq.*].

Alternatively, if we use $F_A^-(q_m^2)/F_A^-(0) = [\mu(q_m^2)/\mu(0)]_{exp}$ instead of $[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp}$ in our input and the same assumptions as before and calculate $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})$ from Eqs. (19)–(20c), we obtain

$$\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}) = (8.6 \pm 0.8) \times 10^3\ \text{sec}^{-1} \tag{37}$$

which is much too large to reconcile with

$$[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp} = (6.2 \pm 0.3) \times 10^3\ \text{sec}^{-1}.$$

Further, since

$$\left(1 + \frac{F_P^-(q_m^2)}{F_A^-(q_m^2)}\frac{m_\mu E_\nu}{m_\pi^2} - \frac{F_E^-(q_m^2)}{F_A^-(q_m^2)}\frac{E_\nu}{2m_p}\right)^2 \geq 0$$

in the expression of $R(q_m^2)$ [see Eq. (20c)], we obtain

$$1 - \left(\frac{F_A^-(q_m^2)}{F_A^-(0)}\right) \Big/ \left(\frac{\mu(q_m^2)}{\mu(0)}\right)_{exp} \geq (9 \pm 5)\% \tag{38}$$

or, alternatively, leaving $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})$ to be calculated,

$$\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}) \geq (7.3 \pm 0.7) \times 10^3\ \text{sec.}^{-1} \tag{39}$$

Both values are again too large to reconcile with Eq. (24) or $[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp}$.

### Hypothesis B

*Assumptions*:

$$F_M^\mp(q^2) = \sqrt{2}\,\mu(q^2) \text{ as in Eqs. (2a), (23), and (32)};$$

$$\frac{F_P^\mp(q^2)}{F_A^\mp(q^2)} = -\frac{1+\epsilon^\mp(q^2)}{1+q^2/m_\pi^2} \quad \text{with } \epsilon^-(q_m^2) = -0.15 \text{ (Ref. 20)};$$

$$\frac{F_A^\mp(q^2)}{F_A^\mp(0)} = \frac{F_E^\mp(q^2)}{F_E^\mp(0)}; \quad \frac{F_E^{(\text{I})-}(0)}{F_E^{(\text{I})+}(0)} = \frac{F_E^{(\text{II})-}(0)}{F_E^{(\text{II})+}(0)} \cong 1 \text{ (Ref. 23)};$$

$$\alpha_- = (\alpha_-)_{exp} = (3.1 \pm 0.6)\text{GeV}, \tag{40}$$

$$\alpha_+ = (\alpha_+)_{exp} = -(2.1 \pm 0.7)/\text{GeV (Ref. 1)}.$$

*Predictions*:

Equations (16a), (16b), (18), and (40) together with

$$\alpha_\mp = \frac{1}{3m_p}\left(\pm\frac{F_M^\mp(0)}{F_A^\mp(0)} - \frac{F_E^\mp(0)}{F_A^\mp(0)}\right) = (\alpha_\mp)_{exp}$$

yield

$$\begin{aligned} F_A^-(0) &= F_A^{(I)-}(0) - F_A^{(II)-}(0) = 0.521\,, \\ F_E^-(0) &= F_E^{(I)-}(0) - F_E^{(II)-}(0) = -(2.58\pm 0.86); \\ F_A^+(0) &= F_A^{(I)+}(0) + F_A^{(II)+}(0) = 0.484\,, \\ F_E^+(0) &= F_E^{(I)+}(0) + F_E^{(II)+}(0) = +(0.89\pm 0.95) \end{aligned} \tag{41a}$$

so that

$$\begin{aligned} F_E^{(I)\mp}(0) &= -(0.85\pm 0.64)\,, \\ F_E^{(II)\mp}(0) &= +(1.74\pm 0.64). \end{aligned} \tag{41b}$$

Equations (19)–(21b), (40), and (41a) and the value of $[\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)]_{exp}$ then give

$$\frac{F_A^-(q_m{}^2)}{F_A^-(0)} = 0.677\pm 0.021$$

and

$$P_{av} = 0.65\pm 0.01\,.$$

This value of $F_A^-(q_m{}^2)/F_A^-(0)$ yields

$$1-\left(\frac{F_A^-(q_m{}^2)}{F_A^-(0)}\right)\Big/\left(\frac{\mu(q_m{}^2)}{\mu(0)}\right)_{exp} = (10\pm 3)\% \tag{42}$$

which [as in the case of Eq. (36)] is in disagreement with the hypothesis of similar $q^2$ dependence of the various form factors or with the impulse approximation. Alternatively, if we use $F_A^-(q_m{}^2)/F_A^-(0) = [\mu(q_m{}^2)/\mu(0)]_{exp}$ instead of $[\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)]_{exp}$ in our input and the same assumptions as before, and calculate $\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)$ from Eqs. (19)–(20c) we obtain

$$\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B) = (7.4\pm 0.4)\times 10^3\ \text{sec}^{-1} \tag{43}$$

which [as in the case of Eq. (37)] is too large to reconcile with

$$[\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)]_{exp} = (6.2\pm 0.3)\times 10^3\ \text{sec}^{-1}\,.$$

We note that the error in the above prediction of $\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)$ is considerably smaller than in the corresponding prediction for hypothesis A because here the error in $F_M^-(q^2)$ [Eqs. (23) and (32)] is very small.

Finally, we emphasize that for both hypothesis A and hypothesis B, the predicted $P_{av}$: 0.61 $\pm 0.02$ and $0.65\pm 0.01$ are in poor agreement with $(P_{av})_{exp} = 0.53\pm 0.04$ as obtained by Possoz *et al.*[6]

### Hypothesis C

*Assumptions*:

$$F_{M,A,P,E,D}^{(II)\mp}(q^2) = 0;$$

$F_M^\mp(q^2) = \sqrt{2}\,\mu(q^2)$ as in Eqs. (2a), (23), and (32);

$$\frac{F_P^\mp(q^2)}{F_A^\mp(q^2)} = -\frac{1+\epsilon^\mp(q^2)}{1+q^2/m_\pi{}^2}\ \text{with}\ \epsilon^-(q_m{}^2) = -0.15\ \text{(Ref. 20)};$$

$$\frac{F_M^\mp(q^2)}{F_M^\mp(0)} = \frac{F_A^\mp(q^2)}{F_A^\mp(0)} = \frac{F_E^\mp(q^2)}{F_E^\mp(0)}\,;\quad \frac{F_A^-(0)}{F_A^+(0)} = \frac{F_E^-(0)}{F_E^+(0)}\ \text{(Ref. 23)}; \tag{44}$$

$\dfrac{F_E^\mp(0)}{F_A^\mp(0)} = 3.64\pm 0.08$ from Eq. (8g) and a nuclear-physics calculation (Ref. 14).

*Predictions*:

Equations (16a), (16b), (18), (44) yield $F_A^-(0) = 1.058F_A^+(0) = 0.510$, and, using also $F_A^-(q_m{}^2)/F_A^-(0) = [\mu(q_m{}^2)/\mu(0)]_{exp}$, Eqs. (19)–(21b) and (44) give

$$\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B) = (6.2\pm 0.2)\times 10^3\ \text{sec}^{-1} \tag{45}$$

which is clearly consistent with

$$[\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)]_{exp} = (6.2\pm 0.3)\times 10^3\ \text{sec}^{-1}$$

and

$$P_{av} = 0.57\pm 0.01 \tag{46}$$

which agrees with $(P_{av})_{exp} = 0.53\pm 0.04$ within the quoted uncertainties.

The prediction for the variation with energy of the asymmetry coefficients is given by Eqs. (11) and (44)

$$\alpha_- = (0.08\pm 0.03)/\text{GeV},\ \alpha_+ = -(2.75\pm 0.03)/\text{GeV} \tag{47}$$

which is, of course, in contradiction with $(\alpha_-)_{exp} = (3.1\pm 0.6)/\text{GeV}$ and $(\alpha_+)_{exp} = -(2.1\pm 0.7)/\text{GeV}$ obtained by Sugimoto *et al.*[1]

Further, instead of assuming a particular value for $F_E^\mp(0)/F_A^\mp(0)$ from Eq. (8g) and a nuclear-physics calculation, we can consider how $\Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B)$, $P_{av}$, and $\alpha_\mp$ vary with $F_E^\mp(0)/F_A^\mp(0)$. We obtain from Eqs. (19)–(21b), (11), (44)

$$\begin{aligned} \Gamma(\mu^{-12}C\to\nu_\mu{}^{12}B) &= (6.16\pm 0.21)\times 10^3\ \text{sec}^{-1}:\ F_E^\mp(0)/F_A^\mp(0) = +4\,, \\ &= (6.60\pm 0.23)\times 10^3\ \text{sec}^{-1}:\ F_E^\mp(0)/F_A^\mp(0) = 0\,, \\ &= (7.21\pm 0.25)\times 10^3\ \text{sec}^{-1}:\ F_E^\mp(0)/F_A^\mp(0) = -4\,, \end{aligned} \tag{48}$$

$$\begin{aligned} P_{av} &= 0.565: F_E^\mp(0)/F_A^\mp(0) = +4\,, \\ &= 0.619: F_E^\mp(0)/F_A^\mp(0) = 0\,, \\ &= 0.658: F_E^\mp(0)/F_A^\mp(0) = -4\,, \end{aligned} \tag{49}$$

$$\alpha_- = -0.05/\text{GeV}, \quad \alpha_+ = -2.88/\text{GeV}: \; F_E^{\mp}(0)/F_A^{\mp}(0) = +4,$$
$$\alpha_- = +1.36/\text{GeV}, \quad \alpha_+ = -1.44/\text{GeV}: \; F_E^{\mp}(0)/F_A^{\mp}(0) = 0, \tag{50}$$
$$\alpha_- = +2.77/\text{GeV}, \quad \alpha_+ = +1\times 10^{-3}/\text{GeV}: \; F_E^{\mp}(0)/F_A^{\mp}(0) = -4$$

so that $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})$, $P_{av}$ and $\alpha_\mp$ are all sensitive to the value of $F_E^{\mp}(0)/F_A^{\mp}(0)$, while

$$\alpha_- - \alpha_+ = \frac{\sqrt{2}\mu(0)}{3m_p}\left(\frac{1}{F_A^-(0)} + \frac{1}{F_A^+(0)}\right) \cong 2.80/\text{GeV}$$

is essentially independent of this value. It is to be noted on the basis of Eqs. (48) and (49) that, in contradistinction to the *negative* value for $F_E^{\mp}(0)/F_A^{\mp}(0)$ required by $(\alpha_-)_{exp}$ and $(\alpha_+)_{exp}$ [see Eq. (35)], a *positive* value for $F_E^{\pm}(0)/F_A^{\mp}(0)$, as predicted by Eq. (8g) and the various nuclear-physics calculations,[14,3] is in addition favored by both $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})_{exp}$ and $(P_{av})_{exp}$.

We also note that $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})$ [Eqs. (19)–(20c)] and $P_{av}$ [Eqs. (21a)–(21b)] contain the same combination of $F_P^-(q_m^2)/F_A^-(q_m^2)$ and $F_E^-(q_m^2)/F_A^-(q_m^2)$ so that $F_P^-(q_m^2)/F_A^-(q_m^2)$ and $F_E^-(q_m^2)/F_A^-(q_m^2)$ cannot be determined individually even if $(\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})_{exp}$ and $(P_{av})_{exp}$ are known precisely.

Finally, it is rather interesting to compare our results with those obtained for the nucleon case. There we have[13]

$$\begin{aligned}\Gamma(\mu^- p \to \nu_\mu n; S_{\mu^- p} = 0) &= 753 \pm 16\ \text{sec}^{-1} \quad \text{for } f_M^{(I)} = 1.9(f_M^{(I)})_{CVC} = 7.0,\ f_E^{(II)} = 0: \\ &\qquad \text{similar to hypothesis A}, \\ &= 762 \pm 16\ \text{sec}^{-1} \quad \text{for } f_M^{(I)} = (f_M^{(I)})_{CVC} = 3.7,\ f_E^{(II)} = 1.7(f_M^{(I)})_{CVC} = 6.3: \\ &\qquad \text{similar to hypothesis B}, \\ &= 659 \pm 15\ \text{sec}^{-1} \quad \text{for } f_M^{(I)} = (f_M^{(I)})_{CVC} = 3.7,\ f_E^{(II)} = 0: \\ &\qquad \text{similar to hypothesis C}\end{aligned} \tag{51}$$

which are to be compared with $[\Gamma(\mu^- p \to \nu_\mu n; S_{\mu^- p} = 0)]_{exp} = 651 \pm 57\ \text{sec}^{-1}$.[24] Note that, in the nucleon case, there is no appreciable ambiguity arising from the $q^2$ dependence of the various form factors. The striking analogy between Eqs. (37), (43), and (45) for $\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})$ vs $[\Gamma(\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B})]_{exp}$ and Eq. (51) for $\Gamma(\mu^- p \to \nu_\mu n; S_{\mu^- p} = 0)$ vs $[\Gamma(\mu^- p \to \nu_\mu n; S_{\mu^- p} = 0)]_{exp}$ constitutes another argument in favor of hypothesis C.[25]

## CONCLUSIONS

We summarize our conclusions by emphasizing that the gross violation of ("strong") CVC *or* the introduction of relatively large second-class axial currents, with either as required by the experiment of Sugimoto *et al.*,[1] results in predicted muon capture rates by $^{12}$C (and predicted polarizations of the recoil $^{12}$B) in disagreement with the measured values. A remeasurement of the variation with energy of the asymmetry coefficients in $^{12}$B and $^{12}$N is thus urgently needed.

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