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"Nuclear matter" approach to the energy dependence of the real part of the proton-nucleus optical potential at intermediate incident energies*

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We present and discuss here the results of a calculation of the real part of the potential energy of a proton within nuclear matter, making use of the realistic Reid soft-core nucleon-nucleon interaction, which we justify in the intermediate-energy range. The problem of evaluating the volume of the real part of the proton-nucleus optical-model potential as a function of incident energy is solved with the help of a number of techniques previously developed in the literature of nuclear-matter theory. We find good agreement with results of various phenomenological studies over an energy range of 100 to 1050 MeV incident proton energy for targets ^{40}Ca and ^{208}Pb, which we adopt as test cases. Furthermore, we consider a number of higher-order corrections to our initial result; all of these have minimal effect in the energy regime we have considered except for the third-order correction proposed by Rajaraman, as we demonstrate and discuss. Finally, we show that the conventional approach to the nucleon-nucleus potential at medium energy, the so-called Rayleigh-Lax potential, which makes use of a straightforward impulse approximation and an empirical nucleon-nucleon scattering amplitude, is in gross disagreement both with phenomenological findings concerning the energy dependence of the optical potential and with results of the present calculations. We make an effort to shed some light on the physical reasons for the failure of the Rayleigh-Lax approach, as opposed to the success of other approaches relying upon realistic nucleon-nucleon interactions or phase shifts rather than an empirical medium-energy nucleon-nucleon T matrix.

NUCLEAR REACTIONS ^{40}Ca(p,p), ^{208}Pb(p,p), $E=100$–1050 MeV; nuclear matter approach used to estimate volume of real part proton-nucleus optical model potential, multiple-scattering formalism, numerical estimates of various higher-order terms.

I. INTRODUCTION

The nucleon-nucleus interaction potential, however defined, is energy-dependent. Over the past 25 years, a vast lore has built up concerning the empirical energy dependence of the local phenomenological nucleon-nucleus optical-model potential. This phenomenology now extends from a few MeV, with some gaps, up to and beyond a GeV. Over the years, there have been a number of efforts to understand and explain the energy dependence and magnitude, particularly of the real part of the phenomenological optical potential.

Out of necessity, most early studies covered low incident nucleon energy ($\lesssim$100 MeV) where the data seem to show a linear dependence on energy. The literature abounds in theoretical analyses of one kind or another which successfully reproduce this empirical linear kinetic-energy dependence. Typical is the work of Sinha[1] who achieves good agreement using a simplified nucleon-nucleon potential and a single-folding approach, with the exchange term playing an important role in producing the agreement. Such energy dependence is most convincingly discussed in terms of the volume-integral per nucleon of the potential, because of the well known geometry ambiguities which plague low-energy phenomenological analyses. Various authors have calculated such a volume integral and have obtained an adequate reproduction of the low-energy phenomenology.[2-4]

Interestingly, studies of the empirical optical potential which have extended to 1 GeV and beyond[5-8] suggest a rough overall *logarithmic* energy dependence of the volume of the real part of the optical potential. Naive extrapolation of the low-energy linear dependence indicates that the real potential should go to zero at about 250 MeV. However, recent analyses of data for incident energies between 180 and 560 MeV[7] and from 350 to 1150 MeV,[8] unpublished work by the present authors, and earlier incomplete studies by Van Oers *et al.*[5,6] are all consistent in indicating a zero which occurs near 500–600 MeV, the potential becoming increasingly repulsive thereafter.

Theoretical efforts to understand the logarithmic energy dependence, which has long been expected on the basis of simple dispersion-theoretic arguments like those of Passatore[9] and of Gall and Weigel,[10] have tended to give too *much* repulsion at medium energies, although adequately reproducing the low-energy transition from linear to logarithmic regimes.

A formal theory of the nucleon-nucleus optical potential exists which gives what might be called a "multiple-excitation series" for the nucleon-nucleus interaction.[11,12] Only the first term in this series is generally used for calculations; in momentum-transfer space this first term has the approximate form $\tau(q^2)\times F(q^2)$, where $\tau(q^2)$ is the nucleon-nucleon T matrix in nuclear matter, and $F(q^2)$ is the nuclear-matter distribution. In all numerical applications known to us to date, the further approximation is made of replacing $\tau(q^2)$ by $t(q^2)$, the empirical free nucleon-nucleon scattering T matrix. For short, following Boridy and Feshbach,[12] we will call the resulting potential the Rayleigh-Lax potential, or sometimes the T-matrix potential.

Calculations using the Rayleigh-Lax potential again suffer from too much repulsion at medium energies, and in addition give an energy dependence in obvious and serious disagreement with phenomenology. One is thus left with the puzzle of whether the discrepancy arises from the approximation $\tau\approx t$, as one would naively expect, or if instead it is due to higher-order terms in the multiple-excitation series. In the present work, we try to answer both of these questions, by avoiding the use of an empirical nucleon-nucleon T matrix, and also by investigating several types of higher-order correction terms.

We base our calculations on the Reid soft-core nucleon-nucleon potential[13] which we use at energies up to 1 GeV. Such use of this potential requires careful justification, which is included in Sec. III. Following the techniques of nuclear-matter theory,[14-16] we are able to obtain an improved estimate for the diagonal part of the nucleon-nucleon scattering amplitude inside the nucleus by taking into account certain many-body corrections to the diagonal part of the so-called G operator generally used in nuclear-matter calculations.

In Sec. II we briefly review the connection between the nucleon-nucleus optical potential and the nucleon-nucleon interaction, as obtained in our theoretical framework. In Sec. III, we present the details of the evaluation of the matrix elements of the Reid potential and summarize the results of our calculations, in comparison with data and with Rayleigh-Lax predictions, over the energy range from 100 to 1050 MeV. In Sec. IV, we estimate the size of certain higher-order corrections, and examine the range of validity of certain approximations used in this and prior work. Our conclusions and general suggestions for further work are given in the concluding Sec. V.

II. THEORY

Our aim in this work is the study of corrections to the conventional impulse approximation, $\tau(q^2)\approx t(q^2)$, with the hope of obtaining an improved estimate for $\tau(q^2)$. A number of calculations have already made it clear that the second- and higher-order terms in the multiple-excitation series, with the impulse approximation retained, do not provide the necessary corrections to the first-order calculation to explain the observed energy dependence.[17,18] Hence, our first effort should be to improve the first-order term and the impulse approximation. In doing so, however, we will retain a number of other approximations which are quite familiar to workers in the field, and easy to justify on the basis of numerical calculation and comparison with available data. For the sake of completeness and clarity, we will mention some of these further approximations, although in the literature it is customary to adopt them without any comment whatsoever.

In the Kerman-McManus-Thaler (KMT) multiple-excitation approach, the first-order optical potential is[11,12]

$$U_{\rm opt}=[\eta(A-1)/A]\langle\Phi_0|\tau|\Phi_0\rangle\,, \tag{1}$$

where Φ_0 is the fully antisymmetric ground state function of the target nucleus. The kinematic factor η and the "counting factor" $(A-1)/A$ are fully discussed in the literature[12,17] and to save space will usually be suppressed in subsequent equations. The potential $U_{\rm opt}$ is usually realized in momentum-transfer space by assuming that the dependence of τ on the lab momentum of the struck nucleon in the target is negligible.[11,12] The result is

$$\tilde U_{\rm opt}(q^2)=[\eta(A-1)/A]\tilde\rho(q^2)\tau(q^2)\,. \tag{2}$$

The meaning of the various quantities in Eq. (2) is obvious; $\tilde\rho(q^2)$ is the Fourier transform of the matter density of the target-nucleus ground state with respect to $\vec q=\vec k-\vec k'$, the center-of-momentum system momentum transfer, and $\tilde U_{\rm opt}(q^2)$ is similarly the Fourier transform of the optical potential. In general, of course, the microscopic optical potential is considered nonlocal. But in the medium-energy regime it has been repeatedly shown[1,2,19,20] that the effects of nonlocality are insignificant. We follow Feshbach[12] and others[21-23] in using a local potential throughout.

The scattering amplitudes are directly related to the operators τ and t given[12] by

$$\tau=v+v(\mathcal{Q}/e^N)\tau, \tag{3}$$

and

$$t=v+v(1/e^F)t\ . \tag{4}$$

Here, as usual, $e^N=E^{(+)}-K-H_N$ and $e^F=E^{(+)}-K$

$-K_i$, where $E^{(+)}$ is the energy of the incident nucleon in the projectile-nucleus center-of-momentum system plus $i\epsilon$; K is the projectile kinetic-energy operator, H_N is the Hamiltonian of the target nucleon, and K_i is the kinetic-energy operator of the struck free nucleon. The nucleon-nucleon interaction is v and $\mathcal{Q}$ is an operator which selects out the subspace of antisymmetric target-nuclear states, thus allowing only physical target-nucleus states as intermediate states in the "multiple-excitation" series.[11,12]

The operators t and τ are related by the familiar iterative series[11]

$$\tau = t + t(\mathcal{Q}/e^N - 1/e^F)\tau \ . \tag{5}$$

The impulse approximation that is the basis of most applications of Eq. (2) amounts to keeping only the first term in the series Eq. (5).

To avoid the impulse approximation, we turn to the realm of nuclear matter, where it is conventional, on the basis of the Goldstone many-body formalism,[14] to write the effective two-body interaction within nuclear matter as

$$G^N = v + v(Q/e^N)G^N \ , \tag{6}$$

where Q is the Pauli operator, a full-antisymmetrization operator for *all* nucleons in the system, projecting all intermediate nucleon-nucleon states onto a basis of unoccupied two-nucleon states above the Fermi surface. In direct analogy with Eq. (4), we define the effective two-body interaction in free space as

$$G^F = v + v(1/e^F)G^F \ , \tag{7}$$

so that G^N and G^F are related by[15]

$$G^N = G^F + G^F(Q/e^N - 1/e^F)G^N \ . \tag{8}$$

What the accumulated lore of nuclear-matter theory tells us is how G^N in fact is related to G^F and what the numerically important differences are. If we can therefore relate G^F and G^N to t and τ, respectively, we can apply nuclear-matter results directly in order to answer the questions concerning the impulse approximation that we have raised.

In this program, we must be guided by physical and numerical considerations rather than empty formal relations. If we are to use the nuclear-matter approach, we are imagining the interaction of the incident nucleon to take place with infinite nuclear matter. At medium energies, say 1 GeV, the wavelength of the incident nucleon is quite small in comparison with the nuclear radius, particularly for the heavy nuclei we consider in this work and it is relatively straightforward to apply nuclear-matter results to the problem.

A direct comparison of Eqs. (4) and (7) gives for the general off-shell matrix elements of the operators G^F and t that

$$\langle \vec{k} | t | \vec{k}' \rangle = \langle \vec{k} | G^F | \vec{k}' \rangle \ . \tag{9}$$

The connection between G^N and τ is more subtle; one needs to recall that at high incident momentum there is a large separation in momentum space between incident nucleon and target nucleons, and thus the need to provide full antisymmetrization of incident *and* target nucleons diminishes with increasing incident energy. Since our local optical potential involves only on-shell matrix elements, for medium energies and heavy nuclei the differing subspaces projected out by $\mathcal{Q}$ and Q will not invalidate the approximation

$$\langle \vec{k} | \tau | \vec{k}' \rangle \equiv \tau(q^2) \approx \langle \vec{k} | G^N | \vec{k}' \rangle \ . \tag{10}$$

Equations (8), (9), and (10) are the basis of what follows. A direct transposition of the techniques of nuclear-matter theory, developed to handle the corrections to the matrix elements of G^F which will give us a good approximation to the (unknown) matrix elements of G^N, can now be made to the present problem if we restrict ourselves to diagonal matrix elements—this restriction is not necessary, but is made only for convenience since here we are interested only in the volume integral of the real potential, and this is given in terms of $\tilde{U}_{\rm opt}(q^2=0)$ directly by inspection.

We therefore obtain, in lowest order in G^F, an expression equivalent to the usual Rayleigh-Lax prescription for the nucleon-nucleus optical potential[11,12] volume integral

$$\overline{U}A = [\eta(A-1)/A](Z\langle \vec{k}_0 | G^F_{pp} | \vec{k}_0\rangle + N\langle \vec{k}_0 | G^F_{pn} | \vec{k}_0\rangle) \ , \tag{11}$$

and our aim herein is to study the energy dependence of and certain higher order corrections to this expression. We can begin to apply the ideas of nuclear-matter theory at once by noting

$$\langle \vec{k}_0 | G^F | \vec{k}_0\rangle = \langle \phi_0 | G^F | \phi_0 \rangle \ , \tag{12}$$

where ϕ_0 is the plane-wave state of the incident nucleon, and by using the on-shell identity $G\phi = v\psi$ for $\psi = \phi + e^{-1}G\phi$. We explicitly use the Reid soft-core potential for v, and construct its eigenstates ψ_0 such that $G^F\phi_0 = v\psi_0$. In this, we understand that $\vec{k}_0$ is the physical relative momentum of the incident and target nucleons, and appears as such in G^F. We suppress the isospin dependence in Eq. (11) for notational convenience in subsequent equations, and introduce an overall factor c_i which absorbs all the multiplicative factors in Eq. (11), divided again by A. We now make the usual partial wave expansion for the plane wave ϕ_0 and the distorted wave ψ_0, and obtain for the potential volume integral per nucleon:

$$\overline{U}=c_i\sum_{L=0}^{\infty} 4\pi(2L+1)\,\frac{1}{k_0}\int_0^{\infty} j_L(k_0r)\nu_L(r)u_L(k_0r)r\,dr\,, \tag{13}$$

where the radial part of ψ_0 is $u_L(k_0r)/k_0r$ and we allow the interaction to be different in each relative angular momentum state. This expression is valid for singlet states only. It may be generalized for the triplet states if we average over the spin magnetic quantum number M_s, thus giving

$$\overline{U}=c_i\left(\frac{1}{2S+1}\right)\times\sum_{L,M_s} 4\pi(2L+1)\,\frac{1}{k_0}\int_0^{\infty} j^*_{L,M_s}(k_0r)\nu_{L,M_s}(r)\times u_{L,M_s}(k_0r)r\,dr\,, \tag{14}$$

where u_{L,M_s} is given in terms of the usual u_{LJ} wave functions for triplet states as follows. If we compare the partial wave expansions of the total wave function ψ_0 with no spin to that case where the total spin of the two nucleons is 1 then the wave function for the spin 1 system may be expanded as

$$\psi_0(\vec{r},S=1)=\sum_{LM_sM_L}\frac{1}{k_0r}\,u_{LM_s}i^LY_{LM_L}\,, \tag{15a}$$

where

$$u_{L,M_s}(k_0r)=\left(\frac{1}{2L+1}\right)\sum_{JM'_L}u_{LJ}(k_0r)(LM'_L1M_s\,|\,JM_J)\chi_{1M_s}\,. \tag{15b}$$

Note the averaging over M'_L in the definition of u_{L,M_s}. Using the plane wave for triplets we define $j_{L,M_s}=j_L\chi_{1M_s}$ since the radial part depends on the orbital angular momentum only. The potential ν_{L,M_s} is that which produces the eigenstate u_{L,M_s}. The u_{LJ} are found by solving the usual coupled equations, the origin of the coupling being of course the tensor force. Hence we solve[24]

$$\left[\frac{d^2}{dr^2}-\frac{L(L+1)}{r^2}+k_0^{\,2}-\frac{2\mu}{\hbar^2}\,\nu_L^C(r)\right]u_{LJ}(r)=\frac{2\mu}{\hbar^2}\,\nu_L^T(r)\sum_{L'}\langle L1JM\,|\,S_{12}\,|\,L'1JM\rangle u_{L'J}(r)\,, \tag{16}$$

where $\nu_L(r)=\nu_L^C(r)+S_{12}\nu_L^T(r)$, the central and tensor parts of the nuclear force, S_{12} is the standard tensor operator, and μ is the two-nucleon system reduced mass. Since the spin-orbit potential is rather small we have neglected it, thereby allowing the fortunate simplifications resulting from L-S coupling. Note that for energies up to 1 GeV we make a relativistic correction to the Schrödinger kinematics by using the exact relativistic two-nucleon center-of-momentum (c.m.) system wave number k_0 and interpreting μ to be the "reduced energy,"[25] [i.e., $\mu=\epsilon_1\epsilon_2/(\epsilon_1+\epsilon_2)$, ϵ_i being the relativistic c.m. energy of particle i in amu]. The method of solution of the coupled equations and the boundary conditions of the wave functions u_{LJ} are fully discussed in the literature and will not be mentioned further.[26]

The iterative solution of Eq. (8) is of course

$$G^N=G^F+G^F(Q/e^N-1/e^F)G^F+\cdots\,. \tag{17}$$

We postpone discussion of the corrections due to the second term in this expansion until Sec. IV. Also in Sec. IV we make estimates of corrections due to motion of target nucleons, as well as remarks concerning the importance of three-body correlations which have been shown to be significant in calculations of the binding energy of nuclear matter.[27,28]

Nuclear-matter studies have indicated that it is the *third-order* contribution in Eq. (17) which is most significant for high relative momenta, and it is to a discussion of this correction we now turn. Physically, the third-order terms account for the possible particle-hole interactions involved in the collision of the incident nucleon with a target nucleon. The initial interaction of incident nucleon and target nucleon produces an intermediate particle-hole state, with which the incident nucleon interacts to return the two nucleons to their original momentum-space configurations. There are two distinct particle-hole interactions, the one in which the incident nucleon in its intermediate state scatters from the hole state, and the other in which the particle and hole states annihilate to produce a second particle-hole state. The first of these interactions is "direct," the second "exchange." The inclusion of these processes, as studied by Rajaraman,[29] can be accomplished at high relative energies by a relatively simple device. See Fig. 1.

Rajaraman[29] used a central, spin-independent potential and computed the general off-shell matrix elements in order to estimate the contributions of the various possible particle-hole interactions. He found that the important third-order corrections can be properly included in a first-order calculation by the device of neglecting the nuclear interaction in the relative odd angular momentum states, and increasing the statistical weight of the relative even angular momentum state interactions by a common factor of $\frac{4}{3}$. Details of this correction are discussed in Sec. III.

However, as pointed out by Sprung, Bhargava, and Dahlblom,[30] the Rajaraman correction is poor for relative momenta of the order of k_F, the Fermi momentum; for the higher energies we consider, the Rajaraman correction is accurate and as we will see gives good agreement with such empirical

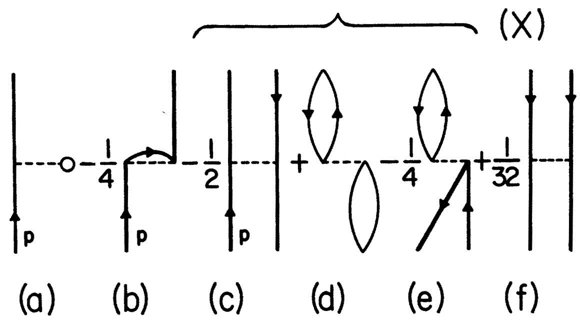

FIG. 1. Contributions to the single-particle potential energy in nuclear matter after Rajaraman (Ref. 29). Diagrams (a) and (b) represent the simplest direct and exchange processes. The hole-hole interaction (f), marked here by (×), was neglected by Rajaraman. The Rajaraman correction discussed in the text accounts for the processes represented by diagrams (c) through (e), as indicated by the upper curly bracket. For details, see Ref. 29.

results as are known; a low-energy breakdown will also be apparent, as expected here.

It is also noteworthy that the Rajaraman correction was tested only for central forces, and hence is only approximately valid in our case since we include tensor forces as well. However, we shall show in the next section that the influence of the tensor force also is diminished at the higher relative momenta, so that again our approximations remain good in the 400–1000 MeV range in which we are mainly interested. A considerably more ambitious and tedious approach along the present lines would be required to connect smoothly to the low-energy (5–50 MeV) phenomenology!

We need to emphasize here also the advantage of the present approach as opposed to the more familiar Rayleigh-Lax-type calculation in which a phenomenological nucleon-nucleon T matrix, $t(q^2)$, is used in Eq. (2) in place of $\tau(q^2)$. One simply has not the vaguest notion of how to go about modifying the empirical free matrix element $t(q^2)$ in the direction of the unknown $\tau(q^2)$ for nuclear matter. But by one further approximation, Eq. (10), numerically as well justified as the impulse approximation with which one is anyhow forced to begin, one can turn to the decades of insight provided by nuclear-matter theory and finally begin to approach $\tau(q^2)$ itself. The price one pays is the necessity to fall back upon an empirical nucleon-nucleon potential, or set of phase shifts; this is considerably trickier and more difficult than to plug in some simple parametrization of $t(q^2)$ mindlessly and grind away. But as we shall see in Sec. III, the simple parametrization does not work. To get out more physics, one has to put in some more physics.

Note finally that we have not attempted to replace the systematic KMT "multiple-excitation" theory of the optical potential by an *alternate* "G-matrix theory." Our aim throughout is to use the numerical and theoretical knowledge of G^F and G^N accumulated in nuclear-matter studies in order to guide us in improving the usual approximation to the first term in the KMT multiple-excitation series.[11,12,17,18]

III. NUMERICAL RESULTS

The potentials as calculated in Eqs. (13) and (14) are the interactions for target protons or neutrons with the incoming nucleon, for either singlet or triplet spin states and for either even or odd relative angular momentum. If we consider protons incident on a target nucleus with neutron (proton) fraction c_n (c_p), then the full expression for the volume of the real part of the proton-nucleus optical potential is given by[22,31]

$$\overline{U} = c_p(\tfrac{1}{4}\overline{V}_{se} + \tfrac{3}{4}\overline{V}_{to}) + c_n(\tfrac{1}{8}\overline{V}_{se} + \tfrac{3}{8}\overline{V}_{te} + \tfrac{1}{8}\overline{V}_{so} + \tfrac{3}{8}\overline{V}_{to}) \ , \qquad (18)$$

where $\overline{V} = \overline{U}/c_i$ [see Eqs. (11)–(14)] and the subscripts refer to singlet or triplet and even or odd states. If we calculate $\overline{U}$ using the Rajaraman correction[29] we obtain:

$$\overline{U} = c_p[\tfrac{1}{3}\overline{V}_{se}] + c_n[\tfrac{1}{6}\overline{V}_{se} + \tfrac{1}{2}\overline{V}_{te}] \ . \qquad (19)$$

To obtain the $\overline{V}$ quantities appearing in Eqs. (18) and (19) we begin by solving the coupled equations (16) of Sec. II for the u_{LJ} eigenstates, or the uncoupled form of Eq. (16) (right-hand side equal to zero) for singlet eigenstates u_L. Using Eq. (15b) we then construct the triplet eigenstates u_{LM_s}. From the asymptotic boundary conditions we then obtain the usual C matrix [$e^{i\sigma_L}F_L + C_L(G_L + iF_L)$, where F_L and G_L are the usual regular and irregular Coulomb functions]. Because all our matrix elements [Eqs. (11) and (12)] are on shell and diagonal, we do not in fact need to compute such overlap integrals as Eqs. (13) and (14), since they are given in terms of the C matrix. We have the well-known result[32]

$$C_L = (-2\mu/\hbar^2)\int_0^\infty r\,dr\,j_L(k_0 r)v_L(r)u_L(k_0 r) \qquad (20)$$

and an analogous result for the triplet states. In these calculations we have in fact neglected the Coulomb interaction, since at 1 GeV the Sommerfeld parameter n is of the order 10^{-2} for one proton incident upon another. We considered angular momentum values up to $L = 6$ at 1 GeV in order to test the convergence of the series involved in Eqs. (13) and (14). The convergence is excellent even at

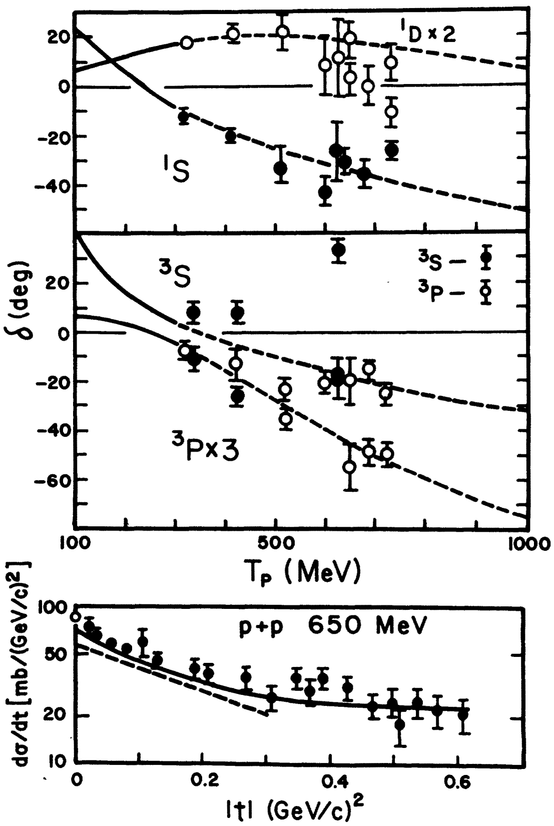

FIG. 2. (Upper) Experimental nucleon-nucleon phase shift data for ^1S, ^3S, ^3P, and ^1D relative states following Simmons (Ref. 33) and references therein. The solid curves are polynomial fits to the lower-energy phase shifts, which are indistinguishable from the predictions of the Reid soft-core potential shown by the dashed curves. (Lower) Proton-proton elastic scattering angular distribution at 0.65 GeV incident laboratory energy, with data from Refs. 33–37. The prediction of the Reid soft-core nucleon-nucleon interaction is shown as the solid curve. The dashed line is the absolute prediction of the crude but generally used parametrization [Eq. (23)] with parameters appropriate to 0.65 GeV from the tabulation of Bystricki and Lehar (Ref. 37). The open circle at t $(=q^2)=0$ represents the cross section resulting from adding the real part of the forward scattering amplitude predicted by the Reid soft-core potential to the imaginary part obtained from the experimental total cross section via the optical theorem.

$L=6$ at 1 GeV, with higher relative angular momenta contributing a percent or so to $\overline{U}$.

Since the Reid potential, which we use to construct the C matrix, was developed for much lower energies than 1 GeV, one may well question the advisability of using it at all in these calculations. It is obvious that the precise nature of the repulsive core is more important near 1 GeV, and that a full complex effective nucleon-nucleon interaction is required in order to account for the flux loss from the elastic channel resulting from meson production, nucleon-resonance channels, etc., which open in the upper part of the nucleon-nucleon incident energy range. The Reid potential certainly does rather well at lower ($\lesssim 350$ MeV) energies. We use it here, first of all, because of the blunt fact that the current state of medium-energy nucleon-nucleon phenomenology is insufficient both experimentally and theoretically[33-38] to provide an all-encompassing energy-independent interaction, either microscopic or phenomenological.

Phase shift data are available for the dominant $^1S_0$, $^3S_1$, and $^1D_2$ nucleon-nucleon states as well as the $^3P_0$ state for energies up to about 700 MeV.[33] Our calculations with the Reid potential reproduce these experimental phase shifts throughout the energy range where we have data, as seen in Fig. 2. In the lower portion of Fig. 2, we also show for 0.65 GeV a comparison of the proton-proton elastic scattering cross section, calculated using the Reid potential, with the available data[33-37] and again we see good agreement in shape and magnitude. Finally, we have checked that at least the real part of the forward scattering amplitude (which is all we need in the present work) as predicted by the Reid potential, even at 1 GeV, is not inconsistent with phenomenology.[34-38] Until something better comes along, we see no basis for ignoring the Reid and similar low-intermediate-energy potentials in intermediate-energy applications.

The reader will have noticed that, since continuum boundary conditions were as usual imposed on the eigenstates u_L and u_{LM_S}, the right-hand sides of Eqs. (13) and (14) are nontrivially complex, although we are interested here only in the real part of the nucleon-nucleus optical potential, and have used a purely real nucleon-nucleon interaction.[39]

Therefore, we write [compare Eq. (13)]

$$\overline{V}_{L,S=0}=(4\pi/k_0)\mathrm{Re}\left(\int_0^\infty r\,dr\,j_L(k_0r)v_L(r)u_L(k_0r)\right), \tag{21}$$

and this is the quantity exhibited in Figs. 3 and 4, for the singlet states, while the corresponding expression for the triplet states is

$$\overline{V}_{L,S=1}=(2S+1)^{-1}(4\pi/k_0)\times\mathrm{Re}\left(\sum_{M_S}\int_0^\infty r\,dr\,j^*_{L,M_S}(k_0r)v_L(r)u_{L,M_S}(k_0r)\right). \tag{22}$$

Figure 3 shows the ^1S, ^3S, ^1P, and ^3P contribu-

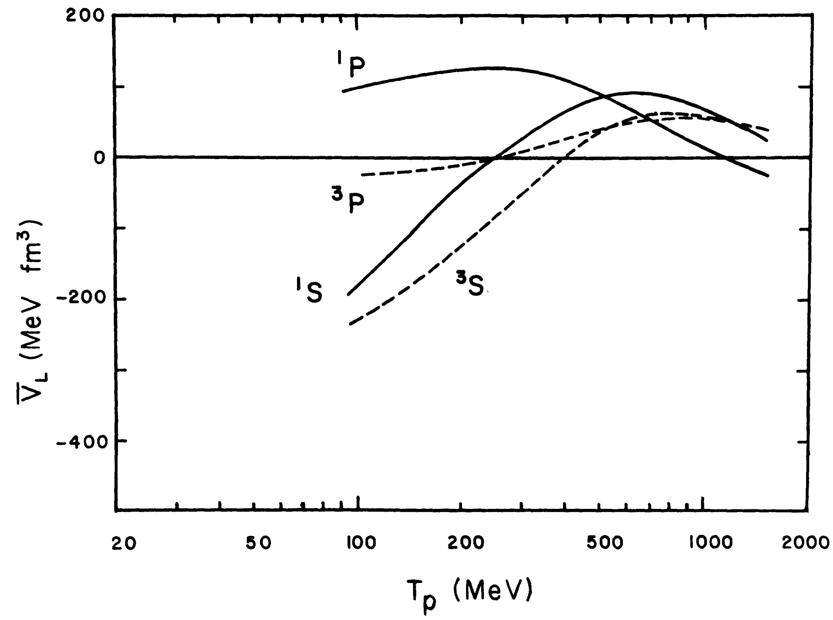

FIG. 3. Contributions to the volume of the nucleon-nucleus optical potential from the ^1S, ^3S, ^1P, and ^3P states of the nucleon-nucleon system, as a function of laboratory incident proton kinetic energy T_p. The expressions used are Eqs. (21) and (22) of the text. Note the important contribution of the ^1P term even at 100 MeV, the greater attraction for the ^3S term compared with the ^1S (as a result of the tensor force), and the gradual convergence of the ^1S-^3S and ^1P-^3P terms as T_p approaches 1 GeV.

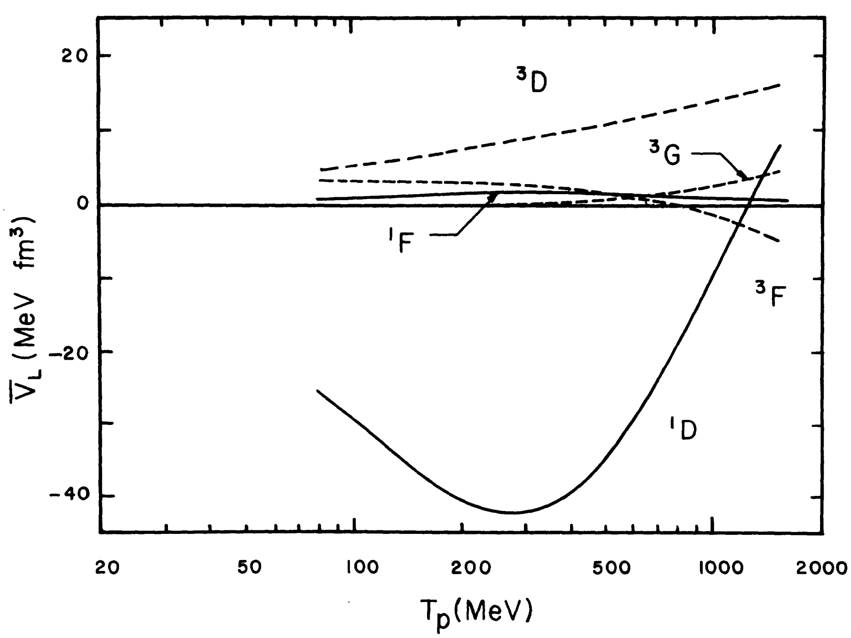

FIG. 4. Same as Fig. 3, except that the contributions of the ^1D, ^3D, ^1F, ^3F, and ^3G nucleon-nucleon states are shown. Note the convergence of ^1D and ^3D past 1 GeV. Note also that the partial waves $L=3$, 4 do not become significant in terms of their contribution to $\bar{U}$ until an energy of 1.0–1.5 GeV is reached. Finally, note that this figure uses a different scale than does Fig. 3.

TABLE I. Spin and isospin state contributions to the real optical potential.

E_{lab} (MeV)	100	200	300	550	720	1040	1500
$\overline{V}_{se}$ (MeV fm^3)	−324	−240	−188	−66.8	−22.5	14.0	61.2
$\overline{V}_{te}$ (MeV fm^3)	−199	−82.3	−3.09	114	142	152	164
$\overline{V}_{so}$ (MeV fm^3)	267[a]	384	325[a]	248	160	37.4	−53.9
$\overline{V}_{to}$ (MeV fm^3)	−36[a]	−3.53	54[a]	129	148	143	92.4

[a]Interpolated from other calculated values.

tions as labeled in the drawing. Figure 4 has the corresponding results for the ^1D, ^3D, ^1F, ^3F, and ^3G states. The remaining states we considered, ^1G, ^1H, ^3H, ^1I, and ^3I, are too small to distinguish from the zero line and are omitted from the drawings for clarity. Note the differing scale in the two figures. We see clearly that the dominant even partial wave at the lower energies is of course the S wave, with the D wave becoming significant by 1 GeV. The P wave dominates the odd-relative-state forces throughout the energy range considered, with the $L=3$ states not becoming significant until about 1.5 to 2 GeV. We also observe the influence of the tensor force in splitting the singlet and triplet contributions to be weakened as we approach 1 GeV, as mentioned in Sec. II.

In Table I we give the values of the real parts of the overlap integrals for the singlet-even, triplet-even, singlet-odd, and triplet-odd relative states for selected incident nucleon laboratory energies. These are the $\overline{V}$ quantities of Eqs. (21)–(22).

To give a general idea of the phenomenological trend of the energy dependence of the real optical potential volume integral, we have obtained the volumes of a number of tabulated optical model potentials from a variety of sources[5-8,25,40,41] mainly for the cases $p+{}^{40}$Ca and $p+{}^{208}$Pb over a range of energies from 30 to 1040 MeV. The results of van Oers[5,6] indicate that for the heavier nuclei, the real optical potential volume is roughly independent of target mass number A, which is also borne out by our calculations, and we have therefore included in the figures recent data obtained for $p+{}^{16}$O and $p+{}^{90}$Zr, which was communicated to us privately[7,41] after our calculations were completed, and serve to fill in some wide gaps in the $p+{}^{40}$Ca and $p+{}^{208}$Pb data.

In Fig. 5 we see the available data for the real part of the optical potential volume for $p+{}^{40}$Ca, with a few $p+{}^{16}$O points added[7] near 500 MeV, compared with our predictions. The solid curve represents the calculation using the Rajaraman correction, as in Eq. (19), whereas the dashed curve gives the result of using an impulse prescription, Eq. (18). The $p+{}^{40}$Ca points are mainly from the tabulation of Perey and Perey[40] for the lower energies. The 1 GeV point comes from previous phenomenological and microscopic analyses of the Saclay $p+{}^{40}$Ca data.[5,6,25]

The crosshatched region in Fig. 5 gives the prediction of the conventional Rayleigh-Lax potential determined from the free-space parametrized nucleon-nucleon T matrix which is almost invariably used in medium-energy calculations. This parametrization is

$$t(q^2)=(ik_0\sigma_T/4\pi)(1-i\alpha)\exp(-\beta q^2/2)\ , \qquad (23)$$

where q is the usual four-momentum transfer, σ_T is the total nucleon-nucleon cross section at the appropriate energy, and β is obtained by fitting the forward slope of the experimental nucleon-nucleon angular distribution at the appropriate energy.[34,37] Note that the *magnitude* of the experimental cross section is not reproduced by this parametrization, since the remaining parameter α is fixed independently, from rather delicate and uncertain Coulomb-nuclear interference analyses near $q^2=0$. An entirely typical result is shown by the dashed line in the lower part of Fig. 2.

Use of such a T matrix within the Rayleigh-Lax approach[12,18,23] amounts to use of a Gaussian effective nucleon-nucleon interaction; the parametrization, Eq. (23), is clearly limited by the accuracy with which the various parameters, particularly α and β, are known. The width of the shaded region in Figs. 5 and 6 is a reflection of the experimental uncertainty in the parameters, particularly α.[37] In looking at Figs. 5 and 6 it is worth recalling that the Rayleigh-Lax potential has an energy-dependent *geometry* through the parameter β in Eq. (23), which has an *extremely* strong energy dependence in the range 300–1000 MeV.[37] This strong energy dependence, and associated experimental uncertainties, might account in some measure for the drastic difference between the Rayleigh-Lax or T-matrix prediction and the obvious empirical trend. However, for whatever reason, it remains clear that the Rayleigh-Lax potential, which has almost invariably been used in medium-energy proton-nucleus scattering and reaction calculations, is in hopeless disagreement with phenomenology, the zero in the real potential occurring

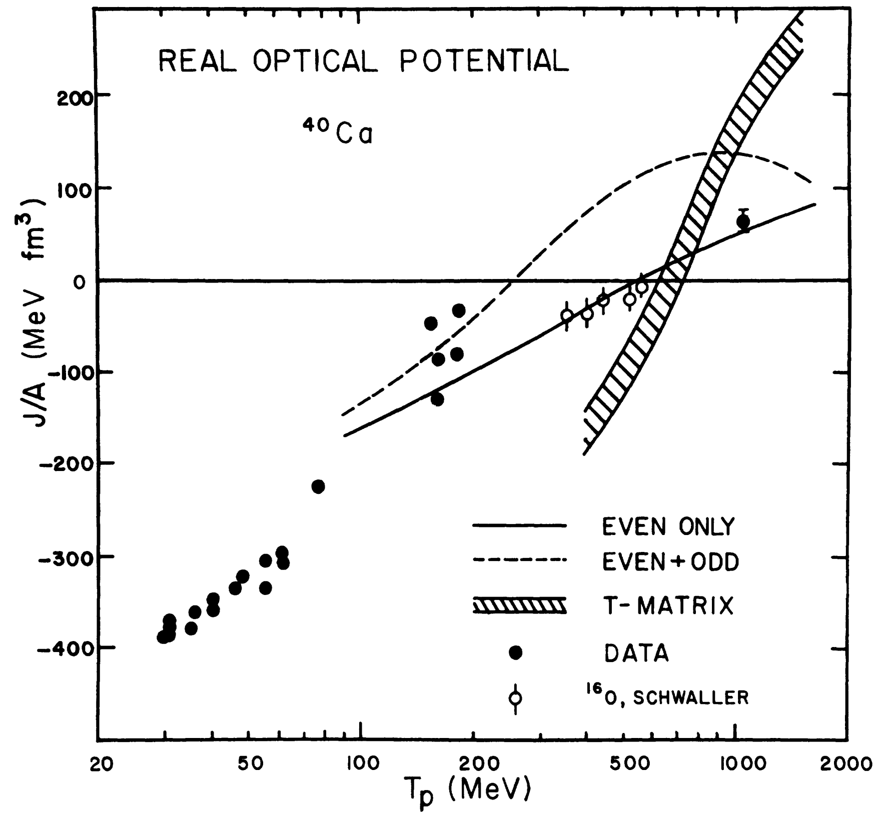

FIG. 5. The quantity $\overline{U}$ [Eqs. (18) and (19)], usually called J/A, as a function of incident laboratory proton kinetic energy for the target nucleus ^{40}Ca. The solid curve is the calculation using only even states [Eq. (19)] with statistical weight unity; the similar dashed curve is the full calculation using both odd and even states [Eq. (18)]. The trend of the predictions of the Rayleigh-Lax or T matrix potential is given by the shaded region, while the data points were taken from the literature of the phenomenological optical potential for $p+{}^{40}$Ca elastic scattering. It is seen that, as expected, the solid curve (which includes the Rajaraman correction) gives a good account of the data at medium energies. The failure of the usual potential constructed from the parametrization of $t(q^2)$ ("T matrix") is also apparent. Data obtained by Schwaller *et al.* (Ref. 7) for $p+{}^{16}$O are included near 500 MeV, to define the zero of J/A more clearly.

nearer 700 than 500 MeV, and there being too much repulsion predicted at higher energies.

We observe that the even-state Rajaraman-corrected calculation gives good agreement with phenomenology in the medium-energy range, although it gives increasingly too weak a potential at the low-energy end of the scale. Note, in both Figs. 5 and 6, that we have denoted $\overline{U}$, the volume integral of the real part of the proton-nucleus optical potential by its more familiar if clumsier symbol of J/A, widely used in the low-energy literature.

Figure 6 shows the results for ^{208}Pb. As in Fig. 5, the solid curve represents the even-state-only calculation with $\frac{4}{3}$ enhancement, as per Rajaraman, while the dashed curve included both even and odd contributions. The crosshatched region is again the T-matrix prediction, and the phenomenological results were taken from the literature.[6,8,25,40,41] As in Fig. 5, we have supplemented the ^{208}Pb data with more recent data for ^{90}Zr, in the energy region 100–200 MeV.[41]

Our calculations were not extended below about 100 MeV, since the approximations we have made are justified only at the higher energies. For example, we have assumed that the incident proton is well localized within the finite nucleus, so that it may be treated in much the same way as a nucleon within infinite nuclear matter. This assumption clearly becomes increasingly realistic as the incident proton energy increases, and the wavelength of the proton decreases in proportion to the nuclear radius. Also the value of k_0, the relative nucleon-nucleon system momentum, which is needed in constructing v_L and u_L [Eq. (16)] is taken to be the same for all target nucleons, and again this approximation breaks down as the incident nucleon energy approaches the nuclear Fermi kinetic energy per nucleon of about 30 MeV. Finally,

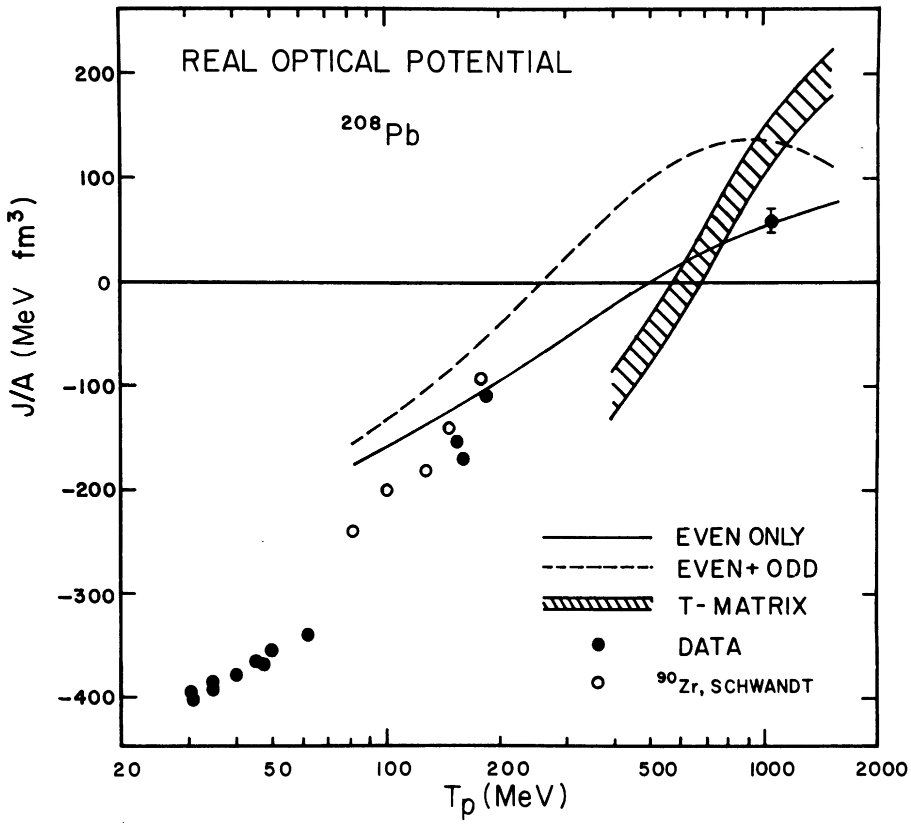

FIG. 6. The meaning of all symbols is the same as in Fig. 5, except that the solid data points are for $p+{}^{208}$Pb phenomenology. We have also included some data for $p+{}^{90}$Zr between 80 and 180 MeV, as provided by Schwandt *et al.* (Ref. 41), which show well the approximate A independence of J/A previously noted by van Oers *et al.* (Ref. 5) in comparison with the ^{208}Pb data and our calculations. Again, the solid curve (which includes the Rajaraman correction) is in best agreement with the data, and the failure of the "T-matrix" approach is equally obvious.

the second-order term in Eq. (17) becomes more important at lower energies, as shown in Sec. IV.

In the present work, we have emphasized the energy dependence of the volume of the real part of the optical potential, because most authorities are agreed that this quantity is determined, relatively unambiguously, by phenomenological analyses. We have tried to show that, while the approach sketched here gives a good account of phenomenology in the medium-energy range insofar as it is known, by contrast the usual parametrization of $t(q^2)$ [Eq. (23)] is in *gross* disagreement both with phenomenology and with our present results.

The reader will appreciate, from Sec. II, that the present approach can be used as in the usual application of the Rayleigh-Lax potential[12,23] to give the full magnitudes and geometries of the real and imaginary parts of the nucleon-nucleus optical model potential. In a subsequent paper, we will present a full analysis of the available medium-energy nucleon-nucleus elastic scattering data, using the approach we have discussed here.[25]

IV. ESTIMATES OF HIGHER-ORDER CORRECTIONS

In this section we will briefly give estimates of several higher-order corrections, these being: the second-order term in Eq. (17); the effect of three-body correlations; and effects due to motion of target nucleons. Furthermore, we will discuss the breakdown of the Rajaraman approximation and mention other improvements which could be made in the calculations of Sec. III.

The second-order correction to the potential $\overline{U}^{(2)}$ is given by

$$\overline{U}^{(2)}=\langle\phi_0|G^{F\dagger}(Q/e^N-1/e^F)G^F|\phi_0\rangle\ , \qquad (24a)$$

which may be written out more fully as

$$\overline{U}^{(2)}=\frac{2\mu}{\hbar^2}\frac{1}{(2\pi)^3}\int d^3k'\,|\langle\phi'|G^F|\phi_0\rangle|^2 \times\left[\frac{Q(k')}{k_0{}^2-k'^2+k_H{}^2}-\frac{1}{k_0{}^2-k'^2-k_i{}^2}\right]\ , \qquad (24b)$$

where $k_i^2 = 2\mu\langle K_i\rangle/\hbar^2$ and $k_H^2 = 2\mu\langle -H_n\rangle/\hbar^2$; K_i and H_n have been introduced in Sec. II and k' is the relative momentum of the two nucleons in the intermediate state $|\phi'\rangle$. In order to obtain a rough estimate of the size of this correction factor we let $\psi \cong \phi_0$ and let each $\nu_L = \nu_{av}$, some average L-independent effective potential. Hence

$$\langle\phi'|G^F|\phi_0\rangle = \langle\phi'|\nu|\psi\rangle \cong \langle\phi'|\nu|\phi_0\rangle$$

$$= 2\pi\int_0^\infty \nu_{av}(r)\{j_0[(k_0-k')r] + j_0[(k_0+k')r]\}r^2dr\ , \tag{25}$$

where only even angular momentum states are considered and we have used the fact that

$$\sum_{\substack{\text{even}\\ L=0}}^{\infty} (2L+1)j_L(k'r)j_L(k_0r) = \tfrac{1}{2}\{j_0[(k_0-k')r]+j_0[(k_0+k')r]\}\ . \tag{26}$$

We adjust the strength of $\nu_{av}(r)$ so that when $k'=k_0$ we have

$$\langle\phi_0|G^F|\phi_0\rangle \cong \langle\phi_0|\nu_{av}|\phi_0\rangle = \overline{U}\ . \tag{27}$$

If we take an exponential form for $\nu_{av}(r)$, then the integral in Eq. (24b) can be carried out analytically. Further we assume $Q(k')=0$ if $k'<k_F$ and $Q(k')=1$ if $k'\geq k_F$ where $k_F = 1.36\ \text{fm}^{-1}$, the Fermi momentum. Using $\langle K_i\rangle = 28$ MeV and $\langle -H_n\rangle = 22$ MeV we find $\overline{U}^{(2)}$ to be fairly small, varying from about +10 MeV fm^3 at 100 MeV to about +8 MeV fm^3 at 1.04 GeV. Thus the second-order correction is seen to be quite small, as is the case in similar nuclear-matter calculations.[14-16]

The literature of nuclear-matter calculations has been concerned for the last 10 years with estimating three-body correlation effects upon the predicted binding energy and saturation density of nuclear matter.[27,28] These "three-body graphs" constitute additional intermediate interactions between the incident nucleon and target nucleons, and certainly make additional contributions to the nucleon-nucleus optical potential. However, Bethe[27] has shown that the contributions of all such three-body graphs to the single-nucleon potential, identified as the real part of the nucleon-nucleus optical potential, when summed to all orders of the interaction, may be expressed in terms of a matrix element involving the two-nucleon effective interaction, the propagator Q/e, and a three-body state function. If we assume that the two target nucleons involved have small momenta as compared with the incoming nucleon, and take the on-shell values for the two-nucleon operators τ describing the effective two-nucleon interaction, we may follow Ref. 27 in writing

$$\overline{U}^{(3)}(\text{3-body}) \approx \sum_i c_i\rho_i \int \tilde{\tau}_{pi}(r)F_1(r)d^3r\ . \tag{28}$$

Here the sum is over $i=p,n$, $\tilde{\tau}_{pi}(r)$ is the local two-body proton-nucleon effective interaction given as the Fourier transform of $\tau(q^2)$ as in Eq. (10), and the function $F_1(r)$ is the "suppression factor" of Bethe,[27] which has been calculated for the Reid hard-core potential by Dahlblom[28] and is found to be roughly equal to 1.3 fm^3 for all r, when averaged over singlet and triplet states. We take it here as a constant. The quantities ρ_i are estimates of the proton and neutron matter densities within the nucleus. We have taken $\rho_p=\rho_n=0.088\ \text{fm}^{-3}$ for ^{40}Ca, and have used $\rho_p = 0.064\ \text{fm}^{-3}$ and $\rho_n = 0.093\ \text{fm}^{-3}$ for ^{208}Pb. In both cases, the proton density is obtained by unfolding the proton charge form factor from the nuclear charge distribution as determined from electron scattering measurements.[42] For the neutron density, we have used the predictions of the density-matrix-expansion variant of Hartree-Fock theory, as presented by Negele and Vautherin,[43] and as embodied in the program written by Negele.[43] Further details are given in another paper by Ray and Coker[25] which deals with full optical-model analyses of elastic scattering of nucleons from nuclei at 1 GeV, using a microscopic potential constructed along lines developed from the present work.

With $F_1 = 1.3$ fm^3 we see that the three-body correction is directly proportional to the volume of the two-body effective interaction, which is nothing but $\tau(q^2=0)$, and under the approximations stated in Sec. II may be approximately expressed in terms of the volume of the first-order potential $\overline{U}$, calculated in Sec. III. The result is

$$\overline{U}^{(3)}(\text{3-body}) \approx [\rho_n+\rho_p]F_1\overline{U}/2 \tag{29}$$

and amounts to about +10% of the first-order real potential. This gives a value of about +5 MeV fm^3 at 1 GeV incident proton energy, and about −20 MeV fm^3 at 100 MeV. This is, of course, a rough estimate but it serves to show that this correction is also rather small.

Next, using the Fermi gas model for the target nucleus, we estimate the correction due to the motion of target nucleons. If we assume the nucleons inside the nucleus have randomly oriented velocities, and speeds which are uniformly distributed between 0 and the Fermi velocity $V_f = \hbar k_F/m$, then for a fixed incident proton velocity the relative momentum between the incoming proton and each target nucleon covers a range of values as determined by V_f. Hence, the optical potential for a given incident lab kinetic energy T_p should be a weighted average of the integral $\langle\phi|\nu|\psi\rangle$ as calculated in Secs. II and III over this range of relative velocities.

We first consider a nonrelativistic expression where the incident velocity is given as $V_p = (2T_p/m_p)^{1/2}$, m_p being the proton rest mass. The real optical potential when averaged over the allowed range of relative velocities is

$$\overline{U}_{av}(V_p) = \int_{-V_p - V_f}^{-V_p + V_f} P(V_{rel})\overline{U}(|V_{rel}|)dV_{rel} , \quad (30)$$

where $P(V_{rel})$ is the probability of the incident proton and target nucleon having the relative velocity V_{rel} and is given by[14]

$$P(V_{rel})dV_{rel} = \frac{3}{2V_f^3}\left(\frac{V_f^2 - V_p^2 - V_{rel}^2}{2|V_p||V_{rel}|} + 1\right) \times V_{rel}^2 dV_{rel} . \quad (31)$$

The potential $\overline{U}(|V_{rel}|)$ in Eq. (30) is just the first-order potential calculated in Secs. II and III.

For the higher energies, we must use a relativistic treatment, which can be simplified if we neglect transverse relative velocities as compared with the relative longitudinal velocity of the two nucleons. The incident proton velocity is given by $V_p = c(1 - 1/\gamma^2)^{1/2}$ where $\gamma = (T_p + m_p c^2)/m_p c^2$. Thus the optical potential corresponding to an incident proton velocity of V_p, when averaged over the appropriate range of relative velocities, is

$$\overline{U}_{av}(V_p) = \int_{-V'_L}^{V'_L} P(V'_{rel})\overline{U}(|V'_{rel}|)dV'_{rel} , \quad (32)$$

where $\pm V'_L = (\pm V_f - V_p)/(1 \mp V_f V_p/c^2)$ and the probability $P(V'_{rel})$ is

$$P(V'_{rel})dV'_{rel} = \frac{3}{4V_f^3}\left[V_f^2 - \frac{(V'_{rel} + V_p)^2}{(1 + V'_{rel}V_p/c^2)^2}\right] \times \left[\frac{1 - V_p^2/c^2}{(1 + V'_{rel}V_p/c^2)^2}\right] dV'_{rel} . \quad (33)$$

In this expression V'_{rel} denotes the relative longitudinal velocity of the two nucleons in the direction of the incident proton and in its rest frame.

Using the $\overline{U}(|V_{rel}|)$ as calculated in Eq. (19) and with $k_F = 1.36\ \text{fm}^{-1}$ we obtain corrections to the optical potential of -20 MeV fm^3 at 100 MeV incident proton energy, -5.0 MeV fm^3 at 300 MeV, and finally about -1.0 MeV fm^3 at $T_p = 1$ GeV. Thus we see that the approximation $K_i \approx 0$ in Eq. (7) is adequate for incident proton energies greater than about 300 MeV, but that at lower energies, as one would naively expect, this approximation leads to significant errors.

The correction of Rajaraman[29] for the third-order terms in Eq. (17), as we mentioned in Sec. II, has been shown to fail at low incident relative momentum, of the order of the Fermi momentum. But Sprung *et al.*,[30] conclude that the Rajaraman approximation is valid for a momentum greater than about twice k_F, the Fermi momentum. Thus the solid even-state-only curves in Figs. 5 and 6 should apply for incident proton energies of 150 MeV or more. Below this limit, the various medium-energy approximations made in this and other comparable studies render our results unreliable.

An improvement which could be made at the lower energies is the inclusion of an exchange potential, although according to Sinha[1,31] and Scheerbaum[44] this correction amounts to only a few percent of the real optical potential at 100 MeV, and is quite negligible at 1 GeV. Also, one could if necessary adopt a nuclear phenomenology given entirely in terms of nucleon-nucleon phase shifts, as determined phenomenologically, once more accurate nucleon-nucleon data become available in the energy range 600 MeV to 1 GeV.

To summarize, then, we have found that the sum of the second-order, three-body-correlation, and Fermi-motion correction terms discussed in this section amount approximately to -30 MeV fm^3 at 100 MeV and +12 MeV fm^3 at 1 GeV incident proton energy. As can be seen in Figs. 5 and 6, these small corrections would tend to improve the general agreement between our predicted trend for $\overline{U}$ and phenomenological expectations. As is also seen, however, these corrections are too small to have a significant effect on our original predictions.

V. CONCLUSIONS

In this paper, we have made use of the basic physical input of the Reid soft-core nucleon-nucleon interaction as well as certain techniques borrowed from the theory of infinite nuclear matter, in order to estimate the energy dependence of the volume per nucleon of the real part of the nucleon-nucleus optical-model potential $\overline{U}$ (or J/A). We have shown that $\overline{U}$ calculated as described herein agrees well with the trend of the empirical optical-model phenomenology from 100 to 1050 MeV incident proton energy for target nuclei as diverse as ^{40}Ca and ^{208}Pb.

The more usual Rayleigh-Lax or T-matrix approach to the medium-energy nucleon-nucleus potential yields results which disagree markedly with the phenomenological, roughly logarithmic, energy dependence[5-9] of the nucleon-nucleus interaction. Since the G-matrix approach, as described here, permits the estimation of a number of corrections to the usual first-order term in the potential, we have made a study of various higher-order corrections. Of these corrections, the most important by far at medium energies turns out to be the Rajaraman[29] version of inclusion of the effects of the third-order terms in the "impulse" expansion, Eq. (17), for G^N (or τ) in terms of G^F (or t). By contrast, as in nuclear-matter calculations,[14-16] the second-order terms are relatively unimportant at high relative momentum, as are exchange and certain correlation effects (three body).

This puts us in a position to say something about why the T-matrix approach, which is overwhelmingly the most often used in various medium-energy nuclear reaction and scattering studies, fails as it does. First, the parametrization of the nucleon-nucleon scattering amplitude represented by Eq. (23) is extraordinarily crude, and a glance at the available nucleon-nucleon angular distribution data shows that it is in general valid only for very small momentum transfer, $q^2=0.4$ (GeV/c)^2 or less, while for certain specific energies in the medium-energy range it seems completely invalid.[34-37] While these facts tend to make one suspicious of tabulated values for the parameter β, it is equally apparent by inspection that the isospin-averaged parameters $\frac{1}{2}(\alpha_{pp}+\alpha_{pn})$ behave as a function of energy in a way quite inconsistent with nucleon-*nucleus* phenomenology, having a zero at about 650 MeV whereas nucleon-nucleus phenomenology requires it to be at about 500 MeV,[5-7] and having an energy dependence which (like that of β) is much too extreme to be adopted uncritically.[37]

The T-matrix approach has the great advantage of being completely straightforward, which probably explains its overwhelming popularity and its continual uncritical use. Because of its very straightforwardness, one sees almost by inspection[37] that it predicts an incorrect energy dependence for the nucleon-nucleus potential—an energy dependence which cannot be avoided within the approach *because* of this very straightforwardness.

We thus venture to make two conclusions on the basis of the present work. First, the parametrization [Eq. (23)] of $t(q^2)$ is not to be trusted in general in the medium-energy range, and in careful work one should have recourse to other parametrizations, to phase shifts, or to effective nucleon-nucleon interactions of one kind or another, in order to obtain $t(q^2)$. Second, as we have seen, the Rajaraman-type corrections to the impulse approximation cause a sizable difference between $t(q^2)$ and $\tau(q^2)$ throughout the medium-energy range.

The advantage of the type of approach sketched out here is therefore twofold; first, it allows one to make corrections of the Rajaraman type in a transparent way, and second, it shows one way to avoid the overused parametrizations of $t(q^2)$—an alternate way is shown, for instance in the work of Lambert and Feshbach.[17] Our aims in this work have been relatively modest. We hope, however, that our success indicates the value of further attempts to marry the knowledge gained in nuclear-matter studies to the pressing problems of medium-energy nuclear physics.

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