PHYSICAL REVIEW C VOLUME 16, NUMBER 1 JULY 1977

Distorted-wave impulse-approximation (p,π) calculations: Effects of realistic wave functions and factors determining resonance position*

Harold W. Fearing
TRIUMF, Vancouver, British Columbia, Canada V6T 1W5
(Received 17 January 1977)

In a recent paper a general impulse approximation formalism for (p,π) reactions in nuclei was developed which incorporated distortion, spin, and antisymmetrization effects. Here we discuss two further considerations in connection with this model with particular reference to its application to the reactions $pd \to t\pi^+$ and $pd \to {}^3\text{He}\pi^0$. First, realistic wave functions for t and ^{3}He which produce the dip in the electromagnetic form factor are incorporated. We find results qualitatively similar to those found previously when simple analytic wave functions were used. This is in contrast to the work of Locher and Weber who found that the dip in the electromagnetic form factor produced a similar dip in the (p,π) cross section. In the present model correct antisymmetrization of the wave functions and inclusion of spin components of the wave functions, in particular the D state of the deuteron, fill the dip. Second, we examine the factors which affect the energy at which a bump due to the $\Delta(1232)$ resonance would appear in the (p,π) cross section. This is motivated by a recent experiment which found no evidence of a bump at 450 MeV where it would be expected from simple kinematic arguments. We find that distortion and form factor effects both tend to push the bump to lower energies and that the net result is that the model predicts a smooth falloff with energy of the fixed angle cross section above 350 MeV, in qualitative agreement with the experiment, and a bump below 350 MeV, where there are no data. The physical mechanisms responsible for the shift, and some possible approaches which might improve quantitative agreement, are then briefly discussed.

NUCLEAR REACTIONS (p,π) reactions in distorted-wave impulse approximation. Effects of realistic wave functions on cross section and of form factors and distortion on resonance position. Differential cross section and energy dependence of fixed angle cross section for $^2\text{H}(p,\pi^+)^3\text{H}$ and $^2\text{H}(p,\pi^0)^3\text{He}$ at 250–800 MeV.

I. INTRODUCTION

In a recent calculation[1] (hereafter referred to as I) a general formalism was developed for (p,π) reactions based on a distorted-wave impulse-approximation (DWIA) model. Unlike most previous discussions[2] which have emphasized the threshold region, this model was tailored for the resonance region, i.e., the region, roughly 300–600 MeV incident laboratory proton energy, where the effects of the $\Delta(1232)$ are expected to be most pronounced. The formalism was then applied rather successfully to the reaction $pd \to t\pi^+$ in this energy region.[1]

In this paper two further considerations in connection with this model are discussed, namely, we consider (a) the consequences of improved wave functions which reproduce the electromagnetic form factors, and (b) the energy dependence of the cross section at fixed angle, and in particular the role which distortion effects and form factors play in shifting the effective position of the resonance bump. The discussion will be limited to the reaction $pd \to t\pi^+$ and its isospin partner $pd \to {}^3\text{He}\pi^0$, though the substance of most of the remarks applies also to the general case.

In I simple analytic forms were used for the three-body wave functions, whereas here wave functions are used which fit the electromagnetic form factors. The end results show quantitative, but little qualitative, difference. This is in contrast to the results of Locher and Weber,[3] who found in a related calculation that the dip in the electromagnetic form factor led to a dip in the inelastic form factors and hence gave structure in the (p,π) cross section. This difference will be discussed and apparently can be understood in terms of spin and antisymmetrization effects which we have included and which seem to be necessary to obtain qualitatively correct results.

As a result of recent experiments[4,5] there are now enough data to look at the cross section as a function of energy for a few fixed angles. The results were at first sight puzzling,[4] as there was no evidence of the resonance bump in the 450 MeV region where it was expected. These results are discussed and we show that, within the context of this model, distortion and form factor effects shift the bump to below the region of the data so that the theoretical predictions for the energy dependence are in qualitative, though not particularly good quantitative, agreement with the data.

In the following sections we briefly review the theory and then discuss these two points in more detail.

II. REVIEW OF THE THEORY

To begin we recall the basic ingredients of the DWIA model developed in I. The main assumption (cf. Ruderman[6] and also the earlier work of Henley[7]) is that the interaction responsible for nuclear (p,π) reactions is the same as that appropriate for $pp\rightarrow\pi d$. This leads in impulse approximation to a nuclear cross section which is just a form factor involving wave function overlaps times the $pp\rightarrow\pi d$ cross section. Since the $\Delta(1232)$ resonance seemingly dominates $pp\rightarrow\pi d$ at medium energies, it presumably will dominate the nuclear process as well.

In I a number of refinements were made in this simple approach, particularly in the way in which one calculates the form factor. Provision was made for general wave functions, spin and antisymmetrization effects were included, as was the D-state component of the deuteron wave function which appears in the initial and intermediate states. The major improvement of I, however, was the inclusion of the distortion effects which result from the secondary scatterings and interactions of the incoming proton and outgoing pion. These effects, which were included in Glauber approximation using experimental NN and πN cross sections as input, are primarily absorptive and for the most part just reduce the differential cross section by an overall factor which is nearly independent of angle for any given energy.

The basic physical ingredients of the improved theory of I are summarized in Fig. 1, namely, impulse $pp\rightarrow\pi d$ cross section taken from experiment, pion and proton distortion, and a form factor coming from a Fourier transform of wave function overlaps.

An important qualitative feature of the theory is that both shape and normalization of the differential cross section are well determined and the shape is relatively insensitive to the input quantities. No truly free parameters are involved in the theory. However, changes in wave function or distorting potential parameters do change the normalization, and equally reasonable choices can lead to changes in normalization of factors of 2 or 3. Similarly, as extensively discussed in I, different ways of incorporating distortion effects lead to upper and lower estimates for the normalization of the results [cf. curves (b) and (c) of Figs. 5–7 of I], which differ by amounts of the same order. Thus realistically speaking, at the present time one must assess a theoretical uncertainty in the normalizations of the results of the order of a factor of 2 or 3, which is comparable with or less than the uncertainties in most other models.[2]

As shown in I (cf. also Fig. 3 below) the agreement between theory and the data then available was rather good in the resonance region, even with simple wave functions. At 470 MeV incident proton energy both normalization and shape fit the data except for a backward peak which is not reproduced; at 590 MeV the shape is also good and the normalization, though a bit low, is probably within the uncertainties of the theory. Both distortion effects and to a lesser extent the D-state part of the deuteron are important and work to improve the agreement between theory and experiment. Similar results subsequently have been obtained[8] for $p\,{}^3\mathrm{He}\rightarrow{}^4\mathrm{He}\pi^+$ with good agreement of both normalization and shape near resonance and good shape but somewhat too low normalization above, in this case far above, resonance. Thus in general the theory gives satisfactory agreement near resonance, and we can proceed to look at further refinements and interesting consequences.

III. WAVE FUNCTION IMPROVEMENTS

In I rather simple analytic wave functions were used for the three-body system. These wave functions were easy to handle but did not reproduce the electromagnetic form factors at large momentum transfer. Thus, for example, the exponential wave function used produced a charge form factor which simply averaged over the dip seen experimentally, while the Gaussian wave function gave a form factor which also produced no dip and was far too low in the region of the second maximum.

Here we consider the effects of using wave functions which do correctly reproduce the charge form factor, including the dip and second maximum. The motivation for this is twofold. In the first place, the structures of the inelastic form

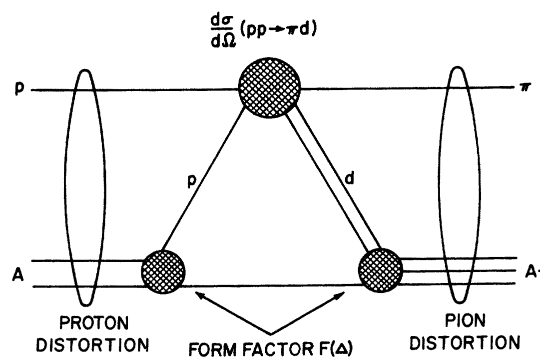

FIG. 1. Physical input to the DWIA model.

factors and the electromagnetic form factors are similar, and thus at sufficiently high momentum transfer in (p,π) reactions one might expect to see some effect of the dip which appears in the charge form factor. It is also obvious that one should use the best wave function practicable. Secondly, Locher and Weber[3] in a very interesting calculation actually tried such wave functions. They found that indeed the inelastic form factors did have dips and, furthermore, one could generate interesting structure in the differential cross section from the interference of the dips in the inelastic form factor corresponding to the diagram we consider and that of another diagram considered by Barry[9] and Bhasin and Duck.[10] Their theory, however, did not have a number of the refinements of the present model; in particular spin and antisymmetrization effects were not built in, nor was the D state of the deuteron. Thus it is interesting to see if this phenomenon persists in a more complete model.

We thus extended the calculation of I to include two new wave functions for the three-body system:

(a) Correlated Gaussian:

$$\phi_3(x_0, x_1, x_2) = N_3 e^{-\alpha^2 u} \prod_{j>i=0}^{2} [1 - C\exp(-\beta^2|\vec{x}_i - \vec{x}_j|^2)] ,$$

$$u = |\vec{x}_0 - \vec{x}_1|^2 + |\vec{x}_0 - \vec{x}_2|^2 + |\vec{x}_1 - \vec{x}_2|^2 ,$$

$$N_3 = 1.83\times 10^6 \text{ MeV}^3, \quad \alpha = 62.8 \text{ MeV}, \quad \beta = 232 \text{ MeV}, \quad C = 0.925 .$$

(b) Cluster wave function[3]:

$$\phi_3(x_0, x_1, x_2) = N_3\left(\frac{\exp(-\beta|\vec{x}_0 - \vec{x}_1|) - \exp(-\gamma|\vec{x}_0 - \vec{x}_1|)}{|x_0 - x_1|}\right)\frac{e^{-\alpha x}}{x}(1 - e^{-\eta x})^4 ,$$

$$x = |\vec{x}_2 - \tfrac{1}{2}(\vec{x}_0 + \vec{x}_1)| ,$$

$$N_3 = 125.6 \text{ MeV}, \quad \beta = 45 \text{ MeV}, \quad \gamma = 270 \text{ MeV}, \quad \alpha = 88.8 \text{ MeV}, \quad \eta = 346 \text{ MeV}.$$

The correlated Gaussian is a wave function of the form suggested by Khanna.[11] The parameters differ, however, because Khanna apparently neglected to fold the proton and neutron form factors into the overall form factor before making the fit. The parameters above do take this into account and are a (probably nonunique) set which give a reasonable fit (though not good in the χ^2 sense) to the triton charge form factor. The cluster wave function is one of those used in Ref. 3. It also gives a good fit to the charge form factor. Note, however, that it is not symmetric in the three nucleon coordinates as is required for the three-body S state in the formalism of I, in which symmetrization effects have been carefully treated.

The results are shown in Fig. 2 for an incident proton energy of 800 MeV. This energy was chosen to be sufficiently high so that the momentum transfer is high enough that effects of the form factor dip show up in the angular distribution, although based on previous experience at that energy the normalization may be too low.

The results are rather interesting. The dotted lines show comparison between exponential, correlated Gaussian (CG), and cluster wave functions for the case when only the S-state part of the deuteron wave function has been kept. Observe that the cluster and CG wave functions give rather similar results except that the CG wave function, which does give a dip in the charge form factor, does not give a real dip in the inelastic process. For the cluster wave function, however, the dip shows up clearly. Both CG and cluster wave functions reproduce the dip in the charge form factor. The primary difference between them is that the CG wave function is properly symmetrized while the cluster wave function is not. Apparently the extra terms coming from symmetrization change the overlap enough to remove the dip. Thus one concludes that proper symmetrization of the wave function is important to get the right qualitative behavior.

In the solid curves, the D-state component of the deuteron has been included in both the initial and intermediate deuteron states, as described in I. Now all three curves are qualitatively the same. The addition of the D state has also been sufficient to eliminate the dip which originally appeared with the cluster wave function.

The implications of these results can be summarized as follows:

(1) It is apparently important to properly include correct symmetry properties for the wave functions, and perhaps to a lesser degree the correct spin components. If this is not done, one can get qualitatively incorrect results even with what seem to be reasonable radial wave functions.

(2) Once these effects are included there is much

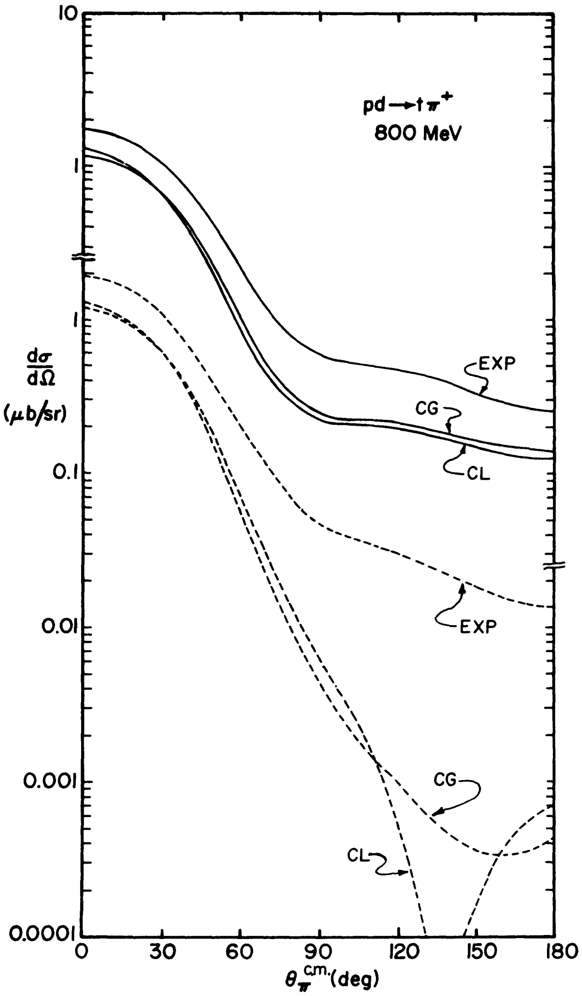

FIG. 2. Comparison of differential cross sections for $pd \rightarrow t\pi$ at 800 MeV using exponential (EXP), correlated Gaussian (CG), and cluster (CL) wave functions for the triton. The D-state component of the deuteron wave function has (has not) been included in the upper solid (lower dotted) curves. Note the change of scale.

less difference among the results using different wave functions than one might expect. Thus the simple exponential wave function used extensively in I gives qualitatively the same results as the more accurate CG wave functions. There are quantitative differences, however, both in normalization and shape of the angular distributions, at the level of factors of two so that clearly one should prefer the CG wave function which does reproduce the charge form factor.

(3) The structure in the differential cross sections observed in Ref. 3 and resulting from the interference of dips in the inelastic form factors is presumably not present in a more complete calculation which includes proper symmetrization of the wave functions and/or the deuteron D state.

Finally we show in Fig. 3 a comparison of the theoretical results using the improved (CG) wave function with the data now available. In particular, the new data of Ref. 4 on $pd \rightarrow {}^3\text{He}\pi^0$ has been included so that there are now two modern experiments with at least partial angular distributions at several energies. For convenience we have plotted all data as $pd \rightarrow {}^3\text{He}\pi^0$, using the isospin relation $\sigma(pd \rightarrow t\pi^+) = 2\times\sigma(pd \rightarrow {}^3\text{He}\pi^0)$. Other older points and ones at isolated angles[12] have also been included. In each case where necessary we have used the theory to scale data at different energies so as to bring them to a common energy. This amounts to only a 5% change at 462 MeV and 15% at 590 MeV for the two modern experiments of Refs. 4 and 5. However, the adjustment required to bring the 325-MeV Chapman data[12] and the 340-MeV Frank data[12] to 377 MeV is as much as 30% for some of the points, and so these points perhaps should be interpreted with caution.

The theoretical curves differ from those of I in that CG wave functions which fit the charge form factor have been used, the deuteron D state is included, and a slightly different choice of energy for $pp \rightarrow \pi d$ has been made (see the discussion below). The curve plotted is the average of the upper and lower estimates of distortion effects which were discussed in I [i.e., the average of curves analogous to curves (b) and (c) of Figs. 5–7 of I]. This curve has then been scaled by eye to fit the data, with emphasis on the two modern experiments.[4,5] The scale factors required are 0.88 at 377 MeV, 1.90 at 462 MeV, 6.5 at 590 MeV. For the lower two energies these factors are not large and are within the expected factor of 2 or 3 normalization freedom allowed by various reasonable changes in the parameters. For 590 MeV the factor is a bit larger than might be expected and seems to reflect the result to be discussed below that the theory tends to fall too low as the energy increases above resonance.

In general, the various improvements have each helped to increase the quantitative agreement of theory and experiment as compared with I, although the qualitative aspects of the curves remain the same. The most striking improvement is at 377 MeV and results from the fact that the new data[4] are significantly different from the older data[12] available in I and now agree well with the theory. At both 590 and 462 MeV the shape has been improved somewhat but the backward peak seen in the 470 MeV data of Ref. 5 is still not reproduced. Note also that the 450 MeV points of Crewe[12] are either high, scaled for energy improperly (reduced by about 6%), or there is a suggestion of a forward peak as well.

Thus the DWIA model of I, as modified and improved here, seems to give rather good fits to the available data, and we can proceed to discuss a further interesting aspect of the theory.

IV. EFFECTS OF DISTORTION AND FORM FACTORS ON RESONANCE POSITION

Let us look now at the energy dependence of the cross section taken at fixed angle, which was not considered in I. In the related reaction $pp\rightarrow\pi d$ the dominant feature of such cross section is a bump at roughly 600 MeV, which is presumably due to the $\Delta(1232)$ resonance.[13] In the context of the impulse approximation one would expect this bump to be reflected in a similar bump in the (p,π) cross section on nuclei. Simple kinematic arguments[9,14] indicate that the bump should appear at about 450 MeV for $pd\rightarrow t\pi$, and in fact such a bump was seen at about this energy in the plane wave impulse approximation calculation of Ingram[14] and seemed to be suggested by the two or three data points then available.

Since then, however, data have become available from a single experiment[4] at 377, 462, and 576 MeV, energies which span the supposed resonance position. The experiment of Ref. 5 provided additional points at 470 and 590 MeV, and the two experiments seem to be in reasonable agreement. The results are puzzling in light of the above remarks in that there is no evidence for the expected bump in the 450 MeV region. Instead, although there is some scatter in the points, the general trend of the data is a rather gradual decrease with increasing energy at fixed angle (cf. Fig. 4) in the region above 350–375 MeV.

When the older data are included there is a suggestion of a bump at some angles, but at energies of roughly 375 MeV, significantly lower than the expected 450 MeV, and one exceptionally high point at 35° and 450 MeV. This high point is from the experiment of Crewe,[12] which seems from the angular distribution of Fig. 3 to be anomalously high. Similarly, the points at 340 MeV, which provide the only evidence for the downturn in the cross section at lower energies, come from the experiment of Frank[12] which seems from the angular distribution of Fig. 3 to be anomalously low.

In any case, however, there seems to be no strong evidence for a bump at 450 MeV as predicted by earlier theories.[9,14] Thus it is worth looking carefully at the DWIA model of I to see what it predicts regarding the resonance position, particularly since it seems to give the angular distributions in this energy region rather well.

We thus focus our attention on the model of I to understand the aspects which determine the resonance position and the ways such a determination differs from earlier calculations. Our main qualitative conclusion is that the resonance bump should be present but should be shifted down by distortion and form factor effects to the 325–375 MeV range. Hence the theory predicts a falloff with increasing energy in qualitative agreement with the recent data. Quantitative agreement, however, is not as good as for the angular distributions, as the theory gives results which fall much too rapidly with increasing energy.

The precise position of the resonance bump in $pd\rightarrow t\pi$ is determined in the context of this model by several things: the position of the resonance in $pp\rightarrow\pi d$, the prescription one uses for relating the energy in the nuclear process to the energy at which one evaluates the $pp\rightarrow\pi d$ cross section, the modulating effects of the form factor, and distortion effects.

The energy prescription simply gives the energy in the nuclear process for which the $pp\rightarrow d\pi$ cross section is evaluated at the resonance energy of about 600 MeV. The simple prescription used in earlier calculations[9,14] gives an angle-independent position for the resonance of roughly 450 MeV. As discussed in I, however, this prescription does not satisfy all of the kinematic constraints. The more detailed prescription of I leads to a resonance position which depends slightly on angle but is also about 450–460 MeV, so this refinement of the model does not contribute to a shift of the resonance.

The actual position of the bump can be further modulated by the form factor, which is basically just the Fourier transform of the wave function overlaps with respect to the momentum transfer Δ. The form factor falls very rapidly with increasing Δ. At fixed angle, Δ increases with increasing incident energy. Hence the form factor suppression increases with increasing energy at fixed angle and thus has the effect of shifting the resonance bump to lower energies as well as drastically suppressing the cross section at higher energies. The PWIA curves of Fig. 4 show this effect. It is larger for larger angles and has the net result of moving the bump from the naive 450 MeV position to the 350–425 MeV region. Note that this shift is somewhat larger than that obtained by Ingram,[14] which presumably is a result of the more rigorous formula for the form factors used in I. Finally, the form factor shift is slightly dependent on the choice of wave functions. The CG wave functions produce roughly 10–20 MeV more downward shift than the exponential wave functions.

The next effect to be considered is that of distortion. For the angular distribution the distortion effects served primarily to reduce the magnitude

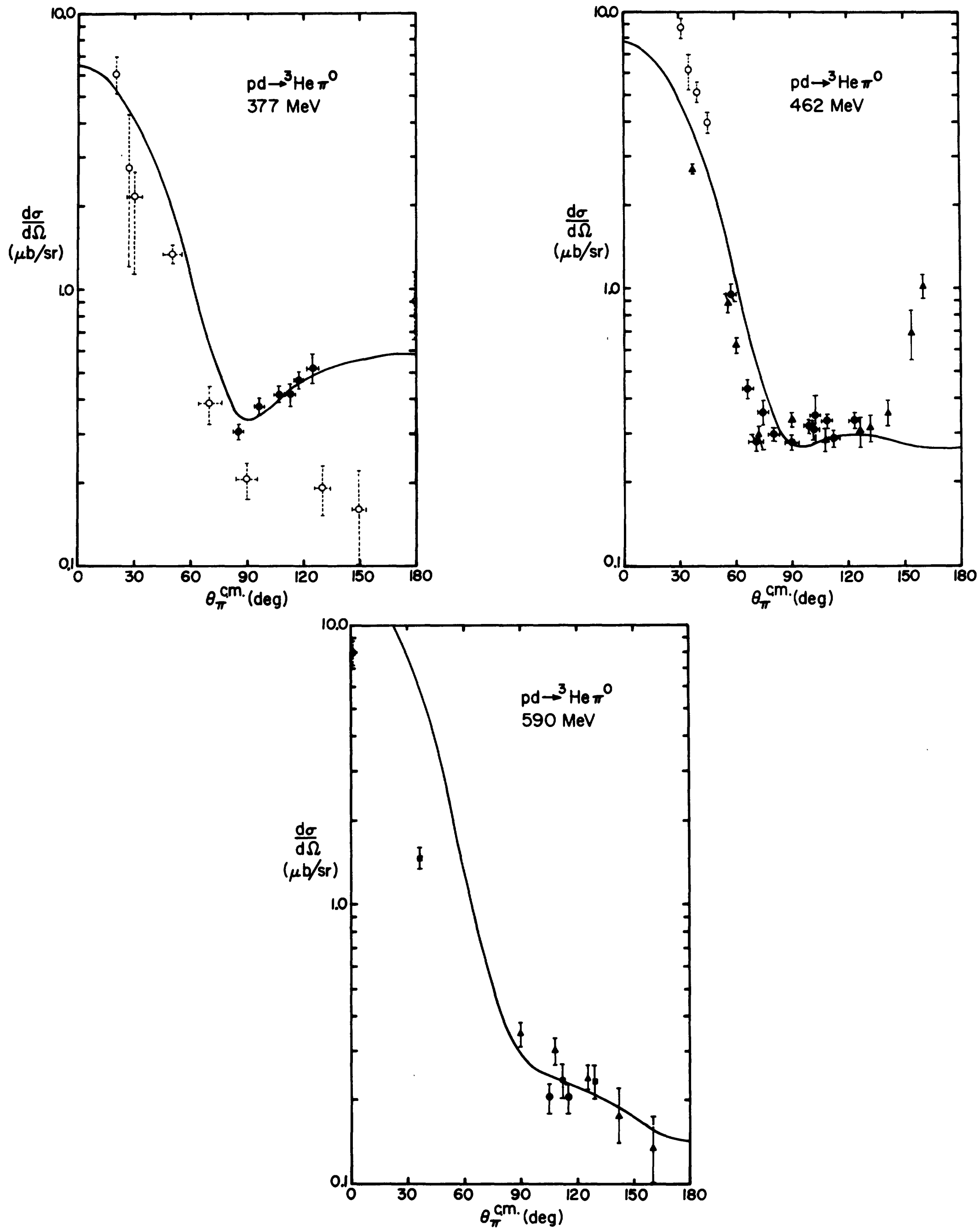

FIG. 3. Comparison of the DWIA model using CG wave functions as described in the text with the differential cross-section data for $pd \to {}^3\text{He}\pi^0$ and $pd \to t\pi^+$. The triangles, heavy dots, and squares are data from Dollhopf (Ref. 5), Fredrickson (Ref. 4), and Harting (Ref. 12). The lighter points are data from Ref. 12 which were either old, available at only isolated angles, or required sizable scaling in the energy.

of the cross section by an amount more or less independent of angle. The effects of distortion on the resonance position can be seen in Fig. 4. For definiteness we have plotted the average of the upper and lower estimates of the distortion effects, the details of which were described in I. One can see that at all energies these effects are primarily absorptive and reduce the cross section, but by an amount which is strongly energy dependent and asymmetrical with respect to the position of the bump. The net effect is to push the bump to even lower energies, 310–360 MeV, which is clearly

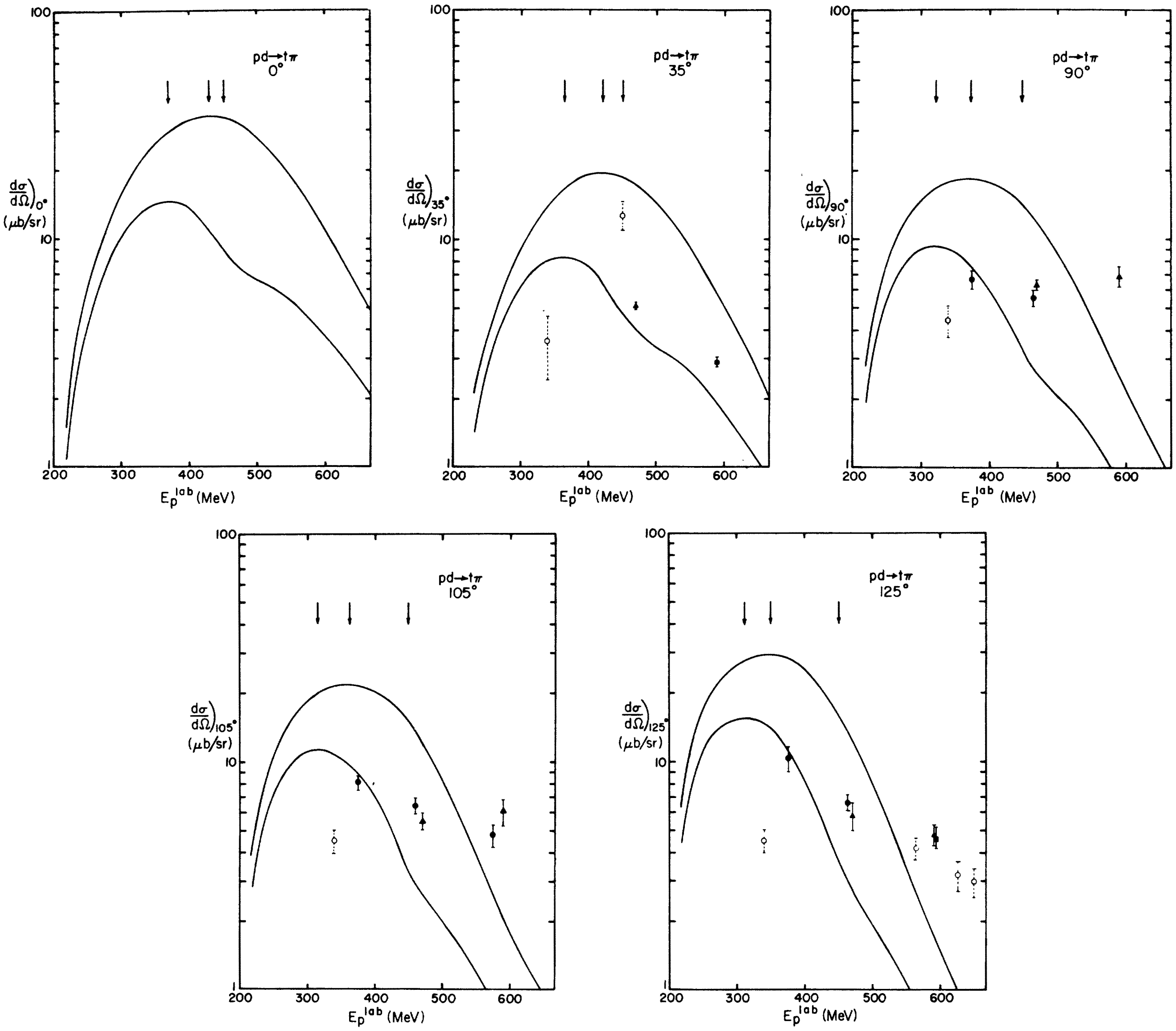

FIG. 4. Differential cross section for $pd \to t\pi$ at fixed angle as a function of energy using CG wave functions and the DWIA model described in the text. The upper curves are without distortion (PWIA); the lower curves include distortion (DWIA) which is taken as the average of the two options described in Ref. 1. The arrows show, from left to right, the position of the peak in the DWIA curve, in the PWIA curve, and the position expected from kinematics (Refs. 9 and 14). Data points are as in Fig. 3.

below the lowest energy measured in either of the two recent experiments.

There is a fairly simple and general physical explanation for this shift of the effective resonance position due to distortion effects. Recall that the distortion effects are predominantly absorptive. In the Glauber approach which has been used the absorptive or imaginary part of the potential is simply proportional to the total cross section for pN and πN scattering. For about 450-MeV incident protons the pion is produced with an energy appropriate to form a $\Delta(1232)$. Hence in this energy region the $\Delta(1232)$ dominates the πN total cross section and makes the imaginary potential large. Thus the final result is suppressed more in this energy region than either above or below. At 0° this suppression produces a noticeable dip in the cross section, and at all angles the position of the bump in the cross section is shifted toward lower energies. Similar effects have been previously noticed in elastic π-nucleus scattering.[15] Thus one could say that it is the importance of the $\Delta(1232)$ in πN interactions which helps to suppress evidence of its importance in $pd \to t\pi$ in the expected 450-MeV energy range.

Thus the net effect of distortion and form factor effects is to push the resonance bump down to energies of about 310–360 MeV, in any case to energies lower than those measured in Ref. 4. Thus within the context of DWIA theory it is no surprise that no bump was seen at 450 MeV. Instead, the theory predicts a gradual falloff with energy over the region greater than 360 MeV. This is in qualitative accord with the data, particularly that of Ref. 4.

Quantitatively, however, the rate of falloff predicted by the theory is too great so that at high energies the magnitude of the cross section is low, even though the shape of the angular distribution is good. A similar effect was seen for $p\,{}^3\mathrm{He} \to \pi\,{}^4\mathrm{He}$.[8]

This difficulty may be related to how one chooses the energy at which to evaluate the $pp \to \pi d$ cross section. The prescription used in I was the standard one for DWIA calculations, namely, one extracts the supposedly slowly varying $pp \to \pi d$ amplitude from an integral and evaluates it an average value of the energy or momentum corresponding to the maximum of the integrand. This leads to an expression [Eq. (24) of I] for $\bar{q}$, which is essentially the average momentum of the target nucleon in the p-nucleus center of mass. It turns out that this result is actually a large A approximation. More precisely, one should minimize $\tilde{q} \equiv q - (1/A)p_A$, essentially the relative momentum of the target nucleon, where p_A is the momentum of the target nucleus. This leads to a much more complicated formula for $\bar{q}$ which reduces to the original one as $A \to \infty$. Now $\tilde{q}$ rather than $\bar{q}$ is approximately in the direction $\vec{p}_\pi - \vec{p}_p$. This more accurate prescription was used in obtaining the numerical results above, but has little practical effect, as it changes the results by only about 20% at 90° and has no effect at 0° and 180°.

With either prescription, for incident energies in the 350–550 MeV range, depending somewhat on angle, the energy choice which maximizes the integrand also evaluates the $pp \to \pi d$ amplitude near resonance. It is this region where the theory agrees best with the data. As the incident energy increases, the energy given by the prescription becomes much higher than the resonance in $pp \to \pi d$. Hence the $pp \to \pi d$ cross section used is much reduced, leading to a reduced cross section. The decrease in the cross section for this reason alone is of the order of a factor of 7 from 500 to 800 MeV.

Basically the problem is the following. In the original distorted-wave integrand there are two peaked functions—the $pp \to \pi d$ amplitude and the other factors in the integrand. As long as both peak at roughly the same energy results are good, but when the peaks are widely separated the simple prescription together with on-shell $pp \to \pi d$ amplitude does not reproduce the overall magnitude as well as it does on resonance. It may be that to resolve this problem one must include off-shell effects in some way and carry out the original integral in more detail.

A possible alternative approach would be to evaluate the $pp \to \pi d$ amplitude on resonance regardless of incident energy. This was done for $p\,{}^3\mathrm{He} \to \pi\,{}^4\mathrm{He}$ in Ref. 16 and in that case did improve agreement with the data. However, this procedure is somewhat *ad hoc* and one cannot always satisfy the various kinematic constraints, while remaining within the physical region for the on-shell $pp \to \pi d$ amplitude. It changes the curves of Fig. 4 mainly above 600 MeV and below 300 MeV and so here does not improve agreement with data.

The problem may also result from the extremely rapid falloff of the form factor with momentum transfer. If this is the case one should look at mechanisms which reduce the effective momentum transfer in the form factor, perhaps by sharing among more than two nucleons. Such a reduction in effective momentum transfer would have the effect of reducing the rate at which the fixed angle cross section falls with energy and at the higher energies would tend to raise the backward angle differential cross sections relative to the forward angle ones. Both effects could improve agreement with data.

Alternatively, it is quite possible that completely different mechanisms may become important as the energy increases above resonance. Any such mechanism must give angular distributions similar to the present one, however, as the angular distribution is good even far above resonance.

To summarize this section, we have seen that the combination of form factor and distortion effects pushes the effective position of the resonance bump to an energy much lower than the 450 MeV position originally expected from simple kinematic arguments. Thus the predictions of the theory are in at least qualitative agreement with the data,[4] which showed no evidence of a bump at this energy.

ACKNOWLEDGMENTS

The author has benefited from discussions with many colleagues. He would like to especially thank Dr. A. Stetz for calling the experiment of Ref. 4 to his attention. He is also grateful for the hospitality extended to him at Aspen Center for Physics where portions of this work were completed.

*Work supported by the Atomic Energy Control Board and the National Research Council of Canada.

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