Elastic scattering of 1 GeV protons from nuclei*

Girish K. Varma and Larry Zamick
Serin Physics Laboratory, Rutgers—The State University, Freylinghuysen Road, Piscataway, New Jersey 08854
(Received 28 March 1977)

We obtain expressions for proton-nucleus elastic scattering amplitudes in the Glauber approximation which include the effects of spin dependence and Coulomb interactions. These effects are shown to be important for determining the neutron and matter radii for all target nuclei. The results for neutron radii are compared with theoretical predictions.

[NUCLEAR REACTIONS ^{16}O, ^{40,48}Ca, ^{90}Zr, ^{208}Pb(p,p), $E=1$ GeV; calculated $\sigma(\theta)$, determined radii of neutron density distributions.]

Recently extensive high energy proton-nucleus elastic scattering measurements have been made at Gatchina (1 GeV)[1] and at Saclay (1.04 GeV).[2] It has been hoped that these measurements would provide a reliable means of studying the neutron distributions in nuclei since the scattering mechanisms at these energies are reasonably well understood. For example, the Glauber theory[3-6] provides a framework which is asymptotically correct for high energy scattering at small momentum transfers.[7] At 1 GeV, for the angular region where measurements exist, it provides a fairly accurate description of the collision processes.[8] However, if one wishes to use the theory to extract reliable information on neutron distributions, it is essential that the additional simplifications (required to make the calculations tractable) be made with considerable care. An earlier analysis[1] of Ca isotopes, for instance, neglected the effects due to spin dependence, Coulomb interaction, and center-of-mass correlations. The spin effects are important[5,9,10] and the Coulomb interaction must be treated carefully if neutron distributions are to be distinguished from proton distributions.[11,12] Since the neutron radii in Ca isotopes are of considerable interest (for example, in relation to the Nolen-Schiffer anomaly[13]), it is worthwhile to analyze the Ca data more carefully. In this paper we obtain theoretical expressions, in the Glauber approximation, for p-nucleus elastic scattering which include the leading order spin effects and contributions due to Coulomb interactions and c.m. correlations. The results are applied to the existing 1 GeV data[1] to determine the radii of neutron (and matter) distributions in target nuclei.

The amplitude for elastic scattering of protons from nuclei consisting of Z protons and N neutrons can be written as[3,11,12]

$$F(q)=\frac{ik}{2\pi}K(q)\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\left\{1-\prod_{p=1}^{Z}\left[1-\langle\phi_p|\Gamma_p(\vec{b}-\vec{s}_p)|\phi_p\rangle\right]\prod_{n=1}^{N}\left[1-\langle\phi_n|\Gamma_n(\vec{b}-\vec{s}_n)|\phi_n\rangle\right]\right\}, \tag{1}$$

where the ground state wave function of the target is assumed to be described by a product of single particle wave functions ϕ_j. Here $\hbar k$ is the incident momentum in the p-nucleus c.m. system, $\hbar\vec{q}$ is the momentum transfer, $\vec{b}$ is the impact parameter vector, $\vec{s}_j$ are projections of the nucleon coordinates $\vec{r}_j$ on the impact parameter plane, and $K(q)$ is a c.m. correlation function.[14] Γ_N are the NN profile functions which for the pp interaction can be written as[11,12]

$$\Gamma_p(\vec{b})=[1-e^{i\chi_C(\vec{b})}]+e^{i\chi_C(\vec{b})}\Gamma_p^s(\vec{b}), \tag{2}$$

where $\chi_C(b)$ denotes the phase shift function for the Coulomb interaction between the projectile and a bound proton. Γ_p^s is the profile function for pp strong interactions and is related to the strong interaction NN amplitudes f_N by

$$\Gamma_N^s(\vec{b})=(2\pi i k_N)^{-1}\int d^2q\, e^{-i\vec{q}\cdot\vec{b}}f_N(k_N,\vec{q}),\quad N=p,n, \tag{3}$$

where $\hbar k_N$ is the momentum in the NN c.m. system. It is a good approximation[11,12] to treat the protons as point charges in which case we have[15]

$$\chi_C(b)=2n\ln(kb), \tag{4}$$

where $n=Ze^2/\hbar v$ is the usual Coulomb parameter. The NN amplitude can be written as

$$f_N(k_N,\vec{q})=k_NA_N(\vec{q})+k_Nq\,\vec{\sigma}\cdot\hat{m}\,c_N(\vec{q}), \tag{5}$$

where $\hat{m}$ is a unit vector perpendicular to the NN scattering plane, $\vec{\sigma}$ is the projectile spin operator, and we have neglected terms linear in target nucleon spins $\vec{\sigma}_j$ which, for spin zero nuclei, do not contribute to the p-nucleus scattering amplitude in the first order. Utilizing Eqs. (2)–(5), the p-nucleus scattering amplitude can be reduced to the form

$$F(\vec{q}) = G(\vec{q}) + H(\vec{q})\vec{\sigma}\cdot\hat{n}, \quad (6)$$

where $\hat{n}$ is a unit vector perpendicular to the p-nucleus scattering plane, and

$$G(q) = ikK(q)\int_0^\infty J_0(qb)\{1 - g_p g_n - h_p h_n\} b\, db ,$$
$$H(q) = kK(q)\int_0^\infty J_1(qb)\{h_p g_n + g_p h_n\} b\, db , \quad (7)$$

where

$$g_p(b) = \sum_{j=0}^{Z}\binom{Z}{j}(\gamma_p)^{Z-j}(\delta_p)^j , \quad j\ \textit{even}\ \text{only} ,$$
$$h_p(b) = \sum_{j=1}^{Z}\binom{Z}{j}(\gamma_p)^{Z-j}(\delta_p)^j , \quad j\ \textit{odd}\ \text{only} . \quad (8)$$

Expressions for g_n and h_n can be obtained from Eq. (8) by the replacements $p \leftrightarrow n$, $Z \leftrightarrow N$. We have also

$$\gamma_n(b) = -i\int_0^\infty J_0(qb)A_n(q)S_n(q)q\, dq ,$$
$$\delta_n(b) = \int_0^\infty J_1(qb)C_n(q)S_n(q)q^2 dq , \quad (9)$$

where the form factors $S_N(q)$ are related to the single particle densities $\rho_N(\vec{r})$ by

$$S_N(q) = \int d^3r\, e^{i\vec{q}\cdot\vec{r}}\rho_N(\vec{r}) , \quad N = p, n . \quad (10)$$

In order to obtain simple expressions for γ_p and δ_p, it is necessary to choose specific forms for A_p and C_p. At high energies, one can use the Gaussian parametrization

$$A_N(q) = \overline{A}_N e^{-(1/2)a_N q^2} ,$$
$$C_N(q) = \overline{C}_N e^{-(1/2)c_N q^2}, \quad N = p, n . \quad (11)$$

γ_p and δ_p can then be reduced to the form

$$\gamma_p(b) = k^{2in}\int_0^\infty u^{2in+1} du \int_0^\infty J_0(qb)J_0(qu)S_p(q)q\, dq + i\overline{A}_p\Gamma(1+in)(2a_p k^2)^{in} \times \int_0^\infty J_0(qb)S_p(q)\,{}_1F_1(1+in;1;-\tfrac{1}{2}a_p q^2)q\, dq , \quad (12)$$

$$\delta_p(b) = \overline{C}_p\Gamma(2+in)(2c_p k^2)^{in}\int_0^\infty J_1(qb)S_p(q)\,{}_1F_1(2+in;2;-\tfrac{1}{2}c_p q^2)q^2 dq ,$$

where ${}_1F_1(a;b;z)$ is the confluent hypergeometric function.

For harmonic oscillator wave functions, γ_N and δ_N may be evaluated analytically. For general forms of $S(q)$, they must be evaluated by numerical integrations. For general forms of NN amplitudes (for example, from phase-shift analyses), γ_p and δ_p are quite difficult to evaluate. We therefore also consider an average phase approximation[6,11,12] which is usually made in optical model potential calculations and neglects the Coulomb-nuclear interference in individual collisions. In this approximation we obtain

$$G(q) = ik K(q) \times\int_0^\infty J_0(qb)\{1 - e^{i\chi_{CZ}(b)}[g_p^s g_n + h_p^s h_n]\} b\, db ,$$
$$H(q) = k K(q) \times\int_0^\infty J_1(qb)e^{i\chi_{CZ}(b)}\{h_p^s g_n + g_p^s h_n\} b\, db , \quad (13)$$

where $\chi_{CZ}(b)$ is the p-nucleus Coulomb phase shift function[6,12] and g_p^s, h_p^s no longer include the Coulomb interaction and hence can be obtained from g_n and h_n by simply letting $n \to p$. The elastic scattering intensities and polarization are given by

$$d\sigma/d\Omega = |G|^2 + |H|^2 ;$$
$$P = 2\,\mathrm{Re}(G^*H)/(d\sigma/d\Omega) . \quad (14)$$

It is worth mentioning at the outset that we have found that the more approximate (but much simpler) Eq. (13) for the inclusion of Coulomb effects yields quite accurate results for determining the densities. This is because Eq. (13) leads typically to few percent errors near the minima in cross sections. The important quantities, however, are the positions of the minima (which are sensitive mainly to the half-density radius) and the size of the subsidiary maxima (which are very sensitive to surface diffuseness). Therefore a moderate error in cross sections near the minima has essentially no effect on extraction of density parameters.

In application at 1 GeV we have considered two forms of NN amplitudes. The first form (I) is

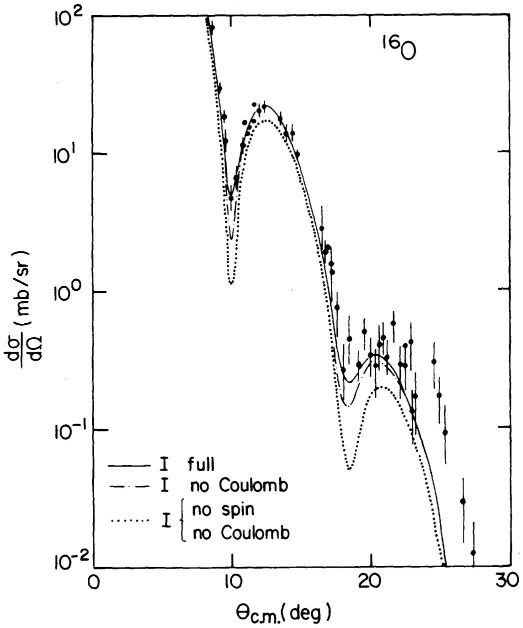

FIG. 1. Results for p-^{16}O elastic scattering at 1 GeV obtained from NN amplitude set I as input, together with the data. Also shown are curves obtained by neglecting Coulomb and spin effects.

given by Eq. (11) with

$$\bar{A}_p = \frac{4.75}{4\pi}(i - 0.07)\ \text{fm}\ ,\quad \bar{A}_n = \frac{3.85}{4\pi}(i - 0.4)\ \text{fm}\ ,$$

$$a_p = a_n = 0.22\ \text{fm}^2\ , \qquad (15)$$

$$\bar{C}_p = \bar{C}_n = 10.12(0.8i - 1)/(8m_N\pi)\,\text{fm}^3\ ,$$

$$c_p = c_n = 0.6\ \text{fm}^2\ ,$$

where m_n is the nucleon mass. The parameters for $\bar{A}_N$ are taken from NN measurements[16] and $\bar{C}_N$ from Ref. 10. The second form (II) is taken from the phase-shift analysis (970 MeV) of Hoshizaki.[17] The form factors $S_p(q)$ can be obtained from the measured charge form factors $\bar{S}_p(q)$ by the relation

$$S_p(q) = \bar{S}_p(q)/[K(q)F_p(q)]\ ; \qquad (16)$$

$$F_p(q) = [1 + q^2/0.71\ (\text{GeV}/c)^2]^{-2}\ ,$$

$F_p(q)$ being the proton form factor. For c.m. correction $K(q)$ we take the form[6,14] $\exp(q^2\langle r_m^2\rangle/6A)$ where $\langle r_m^2\rangle$ is the mean square radius of the nuclear matter distribution. We also define, for convenience, a form factor $\bar{S}_n$ by Eq. (16).

Before analyzing the calcium data, we have applied our results to the other available light and heavy nuclei in order to have more confidence in the NN amplitudes. The NN amplitude set I was shown in Ref. 10 to give a reasonable description of 1.04 GeV p-α data. We have applied it also to 1 GeV p-^{16}O data[18] and the recent Gatchina data[1] on ^{90}Zr and ^{208}Pb, assuming the neutron and proton distributions to be equal. For ^{16}O we take the harmonic oscillator form fitted to electron scattering.[19] Since error bars in this data are quite large we did not try to obtain a better fit. For heavier nuclei the densities corresponding to $\bar{S}_N$ can be parametrized by

$$\bar{\rho}_N(\vec{r}) \propto (1 + wr^2/R_N^2)\{1 + \exp[(r - R_N)/z_N]\}^{-1}\ , \qquad (17)$$

where the densities are normalized to unity. For Zr and Pb, in order to keep the number of parameters to a minimum, we take $w = 0$ and vary R and z to obtain a best fit (this procedure yields the matter distribution). The fits are shown in Figs. 1 and 2. Also shown in Fig. 2 are the results obtained from NN amplitude set II. We note that the two sets differ only in the predictions near the minima (this is mainly because Set II, which is obtained from 970 MeV data, yields a positive Re$\bar{A}_p$. Set I yields a negative Re$\bar{A}_p$ which is consistent with 1 GeV pp Coulomb-nuclear interference measurements and dispersion relation calculations[16]). Again this discrepancy has essentially no effect on determination of the nuclear radii. (The two sets, however, lead to significantly different p-nucleus polarizations.)

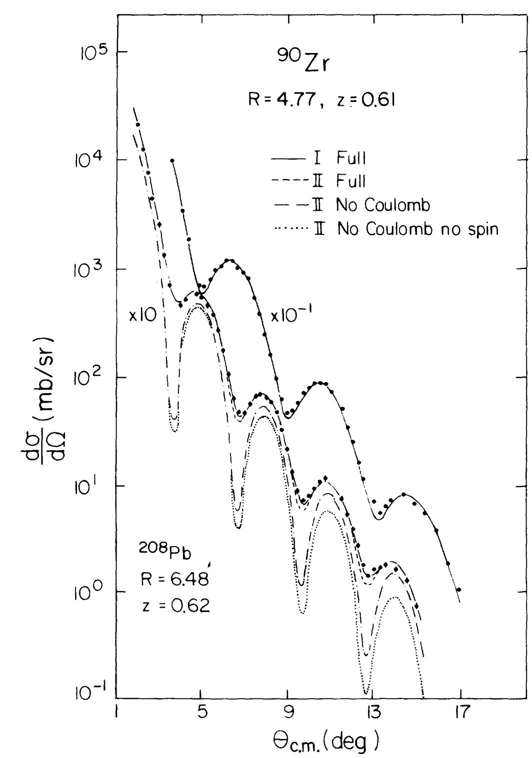

FIG. 2. Results for p-^{90}Zr and p-^{208}Pb elastic scattering at 1 GeV together with the data. For ^{208}Pb results are shown with both the NN amplitudes I and II. The density parameters shown are in fm.

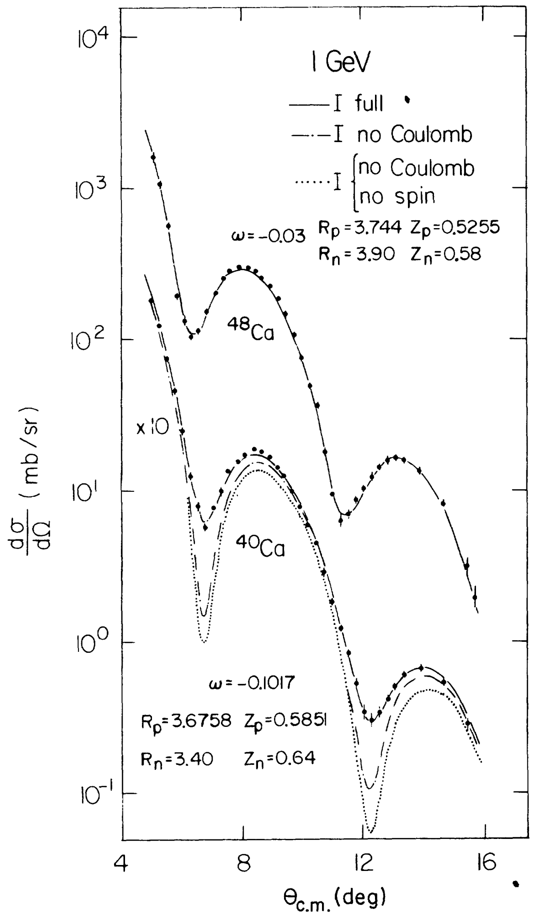

FIG. 3. Same as Fig. 1 for ^{40,48}Ca.

In case of ^{40,48}Ca we have obtained proton distributions from electron scattering measurements[20] and determined the parameters for neutron distributions (R_n and z_n being the only free parameters). The fits together with the parameters are shown in Fig. 3. Also shown in Figs. 1–3 are the results of neglecting Coulomb and spin effects. In all cases both these effects give quite significant contributions. Near the third maximum in ^{16}O, spin effects increase the cross sections by ~37%. Near the third maximum in ^{40}Ca, they increase it by ~22% and Coulomb effects increase it further by ~13%. It is evident that these effects must be treated in any realistic analysis.

Our results for the radii of neutron density distributions are summarized in the Table I together with typical Hartree-Fock calculations. The largest discrepancy is in case of ^{208}Pb. However, our results are consistent with those obtained by Glauber and Mathiae from p-Pb data at 19.3 GeV/c.[6] For other nuclei our results for the radii of neutron excesses are in good agreement with the Hartree-Fock calculations. We should point out that the results of our analysis are appreciably different from those obtained in Ref. 1 from the analysis of the same data (for example, for ^{48}Ca we obtain $\langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2} = 0.21$ fm in comparison with 0.13 fm obtained in Ref. 1). Also of interest in Ca isotopes are the differences $\langle r_n^2\rangle_{48}^{1/2} - \langle r_n^2\rangle_{40}^{1/2} = 0.27$ fm and $\langle r_m^2\rangle_{48}^{1/2} - \langle r_m^2\rangle_{40}^{1/2} = 0.15$ fm. They agree well with the Skyrme II (SKII) [density dependent Hartree-Fock (DDHF)] predictions of 0.28 (0.31) fm and 0.17 (0.19) fm, respectively.

We now make some comments about the significance of the results. The fact that for ^{40}Ca the proton radius is bigger than the neutron radius is reasonable. The Coulomb repulsion between the protons pushes them away from each other. The large difference (0.27 fm) in neutron radii of ^{48}Ca and ^{40}Ca is consistent with the naive model in which eight neutrons are put in the $f_{7/2}$ shell and there is negligible polarization of the neutron core. If harmonic oscillator radial wave functions are used (with length parameter $b = 1.96$ fm) the following expression results:

TABLE I. The results for rms radii obtained from the 1 GeV p-nucleus data. $\langle \bar{r}_{p,n,m}^2\rangle^{1/2}$ denote the radii (including the finite size of the nucleon) of proton, neutron, and matter distributions, respectively. For Zr and Pb, $\langle \bar{r}_n^2\rangle^{1/2}$ is obtained by the relation $A\langle \bar{r}_m^2\rangle = Z\langle \bar{r}_p^2\rangle + N\langle \bar{r}_n^2\rangle$. The last two columns represent the theoretical predictions of the density dependent Hartree-Fock (DDHF) theory [J. W. Negele, Phys. Rev. C 1, 1260 (1970); C 9, 1054 (1974)] and of Hartree-Fock calculations using Skyrme II (SKII) interactions [D. Vautherin and D. M. Brink, Phys. Rev. C 5, 626 (1972)]. The results of the density matrix expansion [J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972)] are very close to that of SKII and hence are not shown.

Nucleus	$\langle \bar{r}_p^2\rangle^{1/2}$	$\langle \bar{r}_n^2\rangle^{1/2}$	$\langle \bar{r}_m^2\rangle^{1/2}$	$\langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ (fm) This paper	DDHF	SKII
^{40}Ca	3.487	3.42	3.45	−0.07	−0.04	−0.05
^{48}Ca	3.476	3.69	3.60	0.21	0.23	0.18
^{90}Zr	4.263	4.39	4.34	0.07	0.12	0.08
^{208}Pb	5.497	5.54	5.52	0.04	0.20	0.20

$$\langle r_n^2\rangle_{48} - \langle r_n^2\rangle_{40} = \frac{8}{28}[\langle r^2\rangle_{f_{7/2}} - \langle r^2\rangle_{\text{core}}]$$

$$= \frac{8}{28}[4.5b^2 - 3b^2]\,,$$

which leads to

$$\langle r_n^2\rangle_{48}^{1/2} - \langle r_n^2\rangle_{40}^{1/2} = 0.24 \text{ fm}\,.$$

If there had been no neutron skin ^{48}Ca, this could have been interpreted as being due to the valence $f_{7/2}$ neutrons polarizing the neutrons in the core and shrinking the core. Such a core polarization would have been helpful in explaining the Nolen-Schiffer anomaly,[13] i.e., the large mass difference of the mirror pairs such as ^{41}Sc-^{41}Ca, but could be troublesome in other respects.[21] The contribution to the effective interaction between two $f_{7/2}$ neutrons due to the exchange of a monopole phonon would then be very large and negative. This would make it all the more difficult to explain the effective repulsion between like particles as noted by Talmi.[21] The proton scattering data at 1 GeV, however, indicates that this type of core polarization is quite small.[22]

We wish to thank Dr. S. J. Wallace for providing the *NN* amplitudes from phase-shift analyses.

*Supported in part by the National Science Foundation.

[1] G. D. Alkhazov *et al.*, Phys. Lett. 57B, 47 (1975); Report No. LINP-244, Leningrad, 1976 (unpublished).

[2] R. Bertini *et al.*, Phys. Lett. 45B, 119 (1973); G. D. Alkhazov *et al.*, Nucl. Phys. A274, 443 (1976).

[3] R. J. Glauber, in *Lectures in Theoretical Physics*, edited by W. E. Brittin and L. G. Dunham (Interscience, New York, 1959), Vol. I, p. 315.

[4] V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966); R. J. Glauber and V. Franco, *ibid.* 156, 1685 (1967).

[5] V. Franco, Phys. Rev. Lett. 21, 1360 (1968).

[6] R. J. Glauber and G. Mathiae, Nucl. Phys. B21, 135 (1970).

[7] D. R. Harrington, Phys. Rev. 184, 1745 (1969).

[8] See for example, S. J. Wallace, Phys. Rev. C 12, 179 (1975); C. W. Wong and S. K. Young, *ibid.* 12, 1301 (1975); C. W. Wong and S. K. Young (unpublished).

[9] E. Lambert and H. Feshbach, Ann. Phys. (N.Y.) 76, 80 (1973); E. Kujawaki and J. P. Vary, Phys. Rev. C 12, 1271 (1975).

[10] J. P. Auger, J. Gillespie, and R. J. Lombard, Nucl. Phys. A262, 372 (1976).

[11] G. K. Varma, Ph.D. thesis, 1976 (unpublished).

[12] V. Franco and G. K. Varma, Phys. Rev. C 12, 225 (1975).

[13] J. A. Nolen and J. P. Schiffer, Phys. Lett. 29B, 396 (1969); L. Zamick (unpublished).

[14] This is valid if c.m. and intrinsic wave functions factorize (as in the case of harmonic oscillator wave functions). We assume that this is true for other wave functions as well.

[15] The Coulomb phase-shift function in general, also involves a screening constant. However, this only leads to an overall phase in $F(q)$ and does not contribute to the cross sections (see Ref. 12).

[16] D. V. Bugg *et al.*, Phys. Rev. 146, 980 (1966); T. J. Devlin *et al.*, Phys. Rev. D 8, 136 (1973); O. Benary *et al.*, UCRL Report No. 2000-NN, 1970 (unpublished); A. A. Vorobyov *et al.*, Phys. Lett. 41B, 639 (1975).

[17] N. Hoshizaki, Fiz. El. Chast. Atom. Yad. 4, 79 (1973) [Sov. J. Part. Nucl. 4, 34 (1973)]. This analysis does not determine all the *NN* parameters. The value of C_{np} has been fixed by fitting the p-α data at 1.03 GeV [S. J. Wallace (private communication)].

[18] H. Palevsky *et al.*, Phys. Rev. Lett. 18, 1200 (1967).

[19] For ^{16}O, P. Goldhammer, Rev. Mod. Phys. 35, 40 (1963). The rms charge radii for Zr and Pb (listed in the table) are the average values of the slightly different radii listed in C. W. De Jager *et al.*, At. Data Nucl. Data Tables 14, 479 (1974).

[20] R. F. Frosch *et al.*, Phys. Rev. 174, 1380 (1968). Electron scattering data at different energies yield slightly different distributions. We have picked the distributions which yield rms radii closest to the model independent analysis of Sick. See, for example, I. Sick, Phys. Lett. 53B, 15 (1974).

[21] L. Zamick, Phys. Lett. 39B, 471 (1972); I. Talmi, Rev. Mod. Phys. 34, 704 (1962).

[22] Our results for $\langle r_n^2\rangle_{48}^{1/2} - \langle r_p^2\rangle_{48}^{1/2} = 0.21$ fm is significantly larger than 0.03 ($\pm$0.08) fm from 79 MeV α scattering by A. Bernstein *et al.*, Phys. Rev. C 12, 778 (1975). Our result $\langle r_n^2\rangle_{48}^{1/2} - \langle r_n^2\rangle_{40}^{1/2} = 0.27$ fm is also larger than 0.14 fm obtained from pion scattering by M. J. Jakobson *et al.*, Phys. Rev. Lett 38, 1201 (1977).