Threshold π^0 photoproduction from complex nuclear targets*

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The photoproduction of neutral pions near threshold from complex nuclear targets is investigated. Momentum dependent terms in the photoproduction operator and a two-nucleon charge exchange production mechanism are included. Various selection rules related to the quantum numbers of the nuclear states involved are discussed. Differential cross sections for pion energies <7.5 MeV from ^{4}He and ^{6}Li are calculated using realistic shell model wave functions. The effects of distortion of the pion wave function are found to be small, but the two-nucleon charge exchange mechanism is found to dominate the coherent production from $J\neq 0$ targets near threshold.

[NUCLEAR REACTIONS (γ,π^0) near threshold; realistic photoproduction operator; shell model wave functions; calculated differential cross section on ^{4}He and ^{6}Li.]

I. INTRODUCTION

In recent years the use of charged pions as probes for obtaining nuclear structure information has been recognized. For example, the usefulness of radiative absorption[1-3] of stopped negative pions has been established for the study of low-lying bound nuclear states as well as isovector resonances. Substantial progress has also been made employing charged pion photoproduction near threshold.[4-7]

The extraction of nuclear structure information from the above reactions relies on three basic assumptions: (a) The amplitude for the basic process $\gamma+N\rightarrow\pi+N$ is well known. (b) The reaction mechanism is well understood and the impulse approximation can be employed. (c) The pion wave function in the field of the nucleus is reliably known. For charged pions it seems that the above conditions are fairly well satisfied. In fact, the nuclear structure information obtained from the above reactions is of comparable quality to that obtained with other nuclear probes.[2,8,9]

Reactions involving neutral pions have as yet not been extensively employed although they offer certain advantages over charged pions. Unlike charged pions which cause only isovector excitations, neutral pions can excite both isoscalar and isovector modes. Furthermore, charged pion photoproduction occurs mainly in the region of the nuclear surface while neutral pion photoproduction can probe the nuclear matter distribution and pion wave function throughout the entire nuclear volume.[7]

At present experiments are underway to measure (γ,π^0) cross sections near threshold from d and ^{3}He.[10] The motivation for the present paper is to aid and encourage experimentalists to investigate (γ,π^0) reactions near threshold on other light nuclei. In particular, detailed results are presented for ^{4}He and ^{6}Li. We will attempt, however, to emphasize those conclusions which may be pertinent to other nuclear systems as well.

We must, of course, admit at the outset that, unlike (π^-,γ) and $(\gamma,\pi^\pm)$ reactions, the assumptions (a) and (b) mentioned above are not satisfied in the case of (γ,π^0) reactions. The amplitude for the basic reaction on a single nucleon

$$\gamma+N\rightarrow\pi^0+N$$

is given near threshold by small nonstatic terms which are absent in the Kroll-Ruderman interaction and is not very well known. The reaction mechanism in the case of a nuclear target is not well established. As discussed in Ref. 12, there could be a substantial contribution from a two-nucleon production process characterized by the production of a charged pion on one nucleon accompanied by charge exchange scattering on another. On the other hand, the neutral pion wave function in the presence of the nuclear medium may be better understood than that of a negative pion.

The above-mentioned theoretical problems coupled with the experimental difficulties, arising from the fact that the detection of neutral pions is difficult, the cross sections are exceedingly small and the available γ-ray beams are not monochromatic, may indicate that (γ,π^0) reactions are doomed. We suggest, however, that by a judicious choice of target even crude experiments should help to resolve some of the theoretical problems. The shape of the differential cross section in the presence of the rescattering mechanism is, in some cases, so vastly different than that associated with the direct mechanism alone that a rough measurement of this quantity should establish the reaction mechanism. Also, the powerful selection rules provided by the quantum numbers of the nucleus enable one to isolate the various pieces of the amplitude and facilitate its understanding in a way which is not possible in the case of production from a single nucleon.

The present paper does not intend to provide all the details of the formalism required in the study of the (γ, π^0) reaction on complex nuclei. This will be presented elsewhere. In Sec. II we discuss the π^0 photoproduction operator. Since we are interested here in complex nuclei we omit some of the refinements concerning the momentum and energy dependence of the basic amplitude[12] in order to keep the calculation tractable. In Sec. III we discuss the selection rules provided by the nucleus and how useful information can be extracted from typical nuclear targets. Some basic formulas needed for the calculation of the differential cross section are supplied in Sec. IV. We present and discuss the results of our calculations for ^{4}He$(\gamma, \pi^0)^4$He and ^{6}Li$(\gamma, \pi^0)^6$Li in Secs. V and VI, respectively. In Sec. VII the effects on the calculation of the distortion of the π^0 wave function due to the strong interaction are discussed. Finally, in Sec. VIII we summarize our results and conclusions.

II. PHOTOPRODUCTION OPERATORS

A. Direct process

An essential input to the calculation of pion photoproduction from a nucleus is a model for the pion photoproduction amplitude on a single nucleon. For charged pion production the empirical electric and magnetic multipole amplitudes[14] are quite well known and various analyses are compared in Ref. 15. On the other hand the amplitude for the reaction $\gamma + N \rightarrow \pi^0 + N$ is not well understood near threshold. In this work we therefore adopt the model for the basic amplitude described in Ref. 13. This model gives a good description of the data for $\gamma + p \rightarrow \pi^0 + p$ at the lowest available energy.

To keep the calculation for complex nuclei tractable, no attempt has been made to include all of the refinements given in Ref. 12 for the construction of the nuclear photoproduction operator. The transformation from the two-body to the nuclear c.m. frame has been neglected. Such a transformation would complicate matters since the effective operator in coordinate space will then contain derivatives with respect to the nucleon coordinates. Since the operator will ultimately be evaluated between harmonic oscillator wave functions we would expect a much smaller contribution from such derivative terms than was found for the case of the deuteron[12] with a Hulthén wave function since the harmonic oscillator has few high momentum components.

The amplitudes for pion photoproduction are given in terms of the quantities c_i, $i = 1, \ldots, 5$ (see Refs. 12 and 16). The quantity c_i is an operator in isospin space of the form

$$c_i = c_i^+ \delta_{\beta 3} + c_i^- \tfrac{1}{2}[\tau_\beta, \tau_3] + c_i^0 \tau_\beta ,$$

where β labels the isospin of the produced pion. Matrix elements of single nucleon states of the operator are[15]

$$\langle n\pi^+ | c_i | \gamma p \rangle = \sqrt{2}\,(c_i^0 + c_i^-) , \tag{1a}$$

$$\langle p\pi^- | c_i | \gamma n \rangle = \sqrt{2}\,(c_i^0 - c_i^-) , \tag{1b}$$

$$\langle p\pi^0 | c_i | \gamma p \rangle = c_i^+ + c_i^0 , \tag{1c}$$

$$\langle n\pi^0 | c_i | \gamma n \rangle = c_i^+ - c_i^0 . \tag{1d}$$

It is now convenient to introduce the notation of Ref. 15 for the combination of isospin amplitudes appearing in Eq. (1). The right-hand side of Eq. (1a) [Eq. (1b)] will be denoted by $A^\pm$, $B^\pm$, $C^\pm$, $D^\pm$, and $E^\pm$ for $i = 1$, 2, 3, 5, and 4. We also define an isoscalar amplitude $A_s^0 = c_1^+$, $B_s^0 = C_2^+$, etc., and an isovector amplitude $A_v^0 = -2c_1^0$, $B_v^0 = -2c_2^0$, etc. These coefficients were calculated using the model discussed in detail in Refs. 12 and 13 and the numerical values near threshold are tabulated in Table I (in units of $\hbar = c = m_\pi = 1$). Two sets of amplitudes are given. One set (to be denoted "with Δ") includes the contribution of an elementary Δ pole term as well as nucleon and pion pole graphs. In the other set the contribution of the Δ pole has been removed. While the Δ almost certainly has some effect on near threshold photoproduction, it is not completely clear that its approximation by a zero width pole is a good one. We therefore wish to investigate the sensitivity of neutron pion photoproduction from nuclear targets to the Δ contributions.

The coefficients $c_i^{\pm,0}$ used to calculate the coefficients appearing in Table I were evaluated at the single-nucleon threshold and their energy dependence was neglected. We estimate the corrections to their approximation to be less than 25%.

The pion photoproduction operator associated with the direct production of a pion with charge c can now be written as follows[15]:

$$\begin{aligned} H_{\text{eff}}^{(1)} &= (1 + m_\pi/m_n)(1/\sqrt{k'})\, H'_{\text{eff}} , \\ H'_{\text{eff}} &= \sum_{j=1}^{A} e^{-i\vec{k}\cdot\vec{r}_j} H_\lambda^c(j)\, \varphi_{lm}(\vec{r})\, \delta(\vec{r} - \vec{r}_j) , \\ H_\lambda^c(j) &= 2\pi i \sum_{\tau = s,v} 0_\tau^c [A_\tau^c \vec{\sigma}_j \cdot \hat{\epsilon}_\lambda + B_\tau^c (\vec{\sigma}_j \cdot \hat{\epsilon}_\lambda)(\vec{q} \cdot \vec{k}) \\ &\quad + C_\tau^c (\vec{\sigma}_j \cdot \vec{k})(\hat{\epsilon}_\lambda \cdot \vec{q}) + iD_\tau^c (\vec{k} \times \hat{\epsilon}_\lambda) \cdot \vec{q} \\ &\quad + E_\tau^c (\vec{\sigma}_j \cdot \vec{q})(\hat{\epsilon}_\lambda \cdot \vec{q})] , \end{aligned} \tag{2}$$

TABLE I. Threshold values for the coefficients A_s^0, B_s^0, C_s^0, D_s^0, E_s^0, $A_v^{\pm,0}$, $B_v^{\pm,0}$, $C_v^{\pm,0}$, $D_v^{\pm,0}$, $E_v^{\pm,0}$, $A_{0,0}^0$, $B_{0,0}^0$, $C_{0,0}^0$, $D_{0,0}^0$, $E_{0,0}^0$, $A_{0,1}^1$, $B_{0,1}^1$, $C_{0,1}^1$, $D_{0,1}^1$, $E_{0,1}^1$, and $b_\tau(T_1,T_2)$ defined in the text both with and without Δ (in natural units).

		With Δ					Without Δ				
	Quantity	A $10^{-2}m_\pi$	B $10^{-2}m_\pi^3$	C $10^{-2}m_\pi^3$	D $10^{-2}m_\pi^3$	E $10^{-2}m_\pi^2$	A $10^{-2}m_\pi$	B $10^{-2}m_\pi^2$	C $10^{-2}m_\pi^2$	D $10^{-2}m_\pi^2$	E $10^{-2}m_\pi^2$
Direct	π^+	−3.1010	0.3199	−3.8838	−1.3994	3.4052	−2.9830	−0.0635	−3.4258	−1.1231	3.4205
	π^-	3.5486	−0.7742	4.3004	1.4333	−3.9282	3.4301	−0.3907	3.8444	1.1569	−3.9435
	π^0(isocs.)	0.0653	−1.1404	1.3318	1.2362	−0.1380	0.0565	−0.9140	0.9250	0.2954	−0.1175
	π^0(isov.)	−0.3162	0.3212	−0.2957	−0.0239	0.3698	−0.3162	0.3212	−0.2457	−0.0239	0.3698
Rescattering (π^0 production)	M_{00}^0	0.4646	−0.0764	0.5725	0.1980	−0.5126	0.4480	−0.0228	0.5082	0.1594	−0.5147
	M_{01}^1	−0.0312	0.0317	−0.0292	−0.0024	0.0366	−0.0312	0.0317	−0.0292	−0.0024	0.0366
	$b_0(0,0)$	0.4646	−0.0764	0.5725	0.1980	−0.5126	0.4480	−0.0228	0.5082	0.1594	−0.5147
	$b_0(1,1)$	0.2682	−0.0441	0.3305	0.1143	−0.2959	0.2587	−0.0132	0.2934	0.0920	−0.2972
	$b_1(0,1)$	0.0540	−0.0549	0.0506	0.0042	−0.0634	0.0540	−0.0549	0.0506	0.0042	−0.0634
	$b_2(1,1)$	0.8482	−0.1395	1.0452	0.3615	−0.9359	0.8180	−0.0417	0.9278	0.2910	−0.9398

where

$$0_\tau^c = \begin{cases} 0 & \text{if } \tau = s \text{ and } c = + \text{ or } -, \\ 1 & \text{if } \tau = s \text{ and } c = 0, \\ t_c & \text{if } \tau = v \text{ and } c = +, -, \text{ or } 0. \end{cases}$$

The operators t_+, t_-, and t_0 obey ordinary angular momentum commutation rules. The rest of the symbols are essentially the same as those defined in Ref. 15. $\vec{k}$ and $\hat{\epsilon}_\lambda$ refer to the momentum and polarization of the photons, $\vec{q}$ is the pion momentum, and $\vec{\sigma}_j$ and $\vec{r}_j$ are the spin and coordinate of the jth nucleon. φ_{lm} is the wave function for the outgoing pion which will be assumed to be a plane wave. Distorted pion wave functions will be discussed in Sec. VII.

The direct photoproduction process can be described by the isoscalar A_s^0, B_s^0, C_s^0, D_s^0, and E_s^0 amplitudes (neutral pions) and the isovector amplitudes A_v^0, B_v^0, C_v^0, D_v^0, and E_v^0 (neutral pions) and $A_v^\pm$, $B_v^\pm$, $C_v^\pm$, $D_v^\pm$, and $E_v^\pm$ associated with positive and negative pions.

B. Rescattering process

A glance at the amplitudes appearing in Table I, in particular a comparison between the A coefficients (momentum independent terms) which are expected to dominate very close to threshold, suggests that the direct process described above may not be adequate in describing (γ, π^0) production. Rescattering contributions such as those described above are going to be very important near threshold.[12] We found it convenient to transform the amplitudes from the invariant isospin amplitudes

$$M_{\gamma\pi} = c_i^+ \delta_{\beta 3} + c_i^- \tfrac{1}{2}[\tau_\beta, \tau_3] + c_i^0 \tau_\beta ,$$
$$M_{\pi N} = M_{\pi N}^+ \delta_{\beta 3} + M_{\pi N}^- \tfrac{1}{2}[\tau_\beta, \tau_3] ,$$

to amplitudes with explicit reference to the isospin quantum numbers of the initial and final two-nucleon states. This can be done starting from all possible charge combinations of the pions and nucleons involved. Thus for each operator indicated above by A, B, C, D, and E we obtain five possible isospin combinations, i.e., the amplitude is described, in general, by a set of 25 coefficients. The effective two-nucleon amplitude which contributes to the neutral pion photoproduction in coordinate space is given as follows:

$$H_{\text{eff}}^{(2)} = \sum_{i<j} (\chi_{ij} M_{ij} + \chi_{ji} M_{ji}) , \qquad (3)$$

where

$$X_{ij} = e^{i\vec{k}\cdot\vec{r}_i} e^{-i\vec{q}\cdot\vec{r}_j} .$$

The operator M_{ij} is

$$M_{ij}=2\pi i\Big\{a_{T_z}^{T,T'}\vec{\sigma}_i\cdot\hat{\epsilon}_\lambda z_0(q,x)+i[b_{T_z}^{T,T'}(\vec{\sigma}_i\cdot\hat{\epsilon}_\lambda)(\vec{k}\cdot\hat{x})+c_{T_z}^{T,T'}(\vec{\sigma}_i\cdot\vec{k})(\hat{\epsilon}_\lambda\cdot\hat{x})+id_{T_z}^{T,T'}(\vec{k}\times\hat{\epsilon}_\lambda)\cdot\hat{x}]z_1(q,x)$$
$$+e_{T_z}^{T,T'}\left[\vec{\sigma}_i\cdot\hat{\epsilon}_\lambda\frac{z_1(q,x)}{x}+(\vec{\sigma}_i\cdot\hat{x})(\hat{\epsilon}_\lambda\cdot\hat{x})z_2(q,x)\right]\Big\},$$

where

$$\hat{x}=\frac{\vec{x}}{|\vec{x}|},\quad \vec{x}=\vec{r}_1-\vec{r}_2;$$

$$z_n(q,x)=(-iq)^{n+1}y_n(-iqx),$$

$$y_0(x)=e^{-x}/x,\quad y_1(x)=(1+1/x)e^{-x}/x,$$

$$y_2(x)=(1+3/x+3/x^2)e^{-x}/x,$$

and all quantities are expressed in units of the pion mass ($\hbar=c=1$, $m_\pi=1$). The rescattering operator was derived assuming pion propagation with frozen nucleons and no "off shell" form factors at the production or rescattering vertices. The term proportional to $a_{T_z}^{TT'}$, which involves s-wave pion production and rescattering, should not be greatly affected by these approximations. The momentum dependent terms are more singular and neglecting form factors tends to overestimate these contributions. Although the term proportional to $e_{T_z}^{T,T'}$ (the most singular term) is carried along in our formalism, it is not included in the numerical results in Sec. VI.

The quantities $a_{T_z}^{T,T'}$, etc., depend on the isospin quantum numbers of the two nucleon states between which the above operator is going to be inserted ($T_z=T_z'$ since the rescattering amplitude for a neutral pion in the final state conserves the third component of isospin T_z). The above 25 amplitudes, denoted collectively by $\omega_{T_z}^{T,T'}$, can be expressed in terms of the $c_i^{\pm,0}$ amplitudes for direct pion photoproduction, denoted collectively by $M_{\gamma\pi}^{\pm,0}$, and the invariant πN scattering amplitudes $M_{\pi N}^{\pm}$ as follows:

$$\omega_0^{0,0}=\frac{1}{4\pi}(M_{\gamma\pi}^+M_{\pi N}^+-2M_{\gamma\pi}^-M_{\pi N}^-),$$

$$\omega_0^{1,1}=\frac{1}{4\pi}(M_{\gamma\pi}^+M_{\pi N}^++2M_{\gamma\pi}^-M_{\pi N}^-),$$

$$\omega_0^{0,1}=\frac{1}{4\pi}M_{\gamma\pi}^0(M_{\pi N}^+-2M_{\pi N}^-)=\omega_0^{1,0},$$

$$\omega_1^{1,1}=\frac{1}{4\pi}(M_{\gamma\pi}^+-M_{\gamma\pi}^0)M_{\pi N}^+,$$

$$\omega_{-1}^{1,1}=\frac{1}{4\pi}(M_{\gamma\pi}^++M_{\gamma\pi}^0)M_{\pi N}^+. \tag{4}$$

For soft pions we approximate $M_{\pi N}$ by the scattering length and write $M_{\pi N}^{\pm}=4\pi(1+m_\pi/M_N)a^{\pm}$ with the quantities $a^{\pm}$ given from experiment.

Although the above 25 quantities $\omega_{T_z}^{T_1,T_2}$ (i.e., $a_{T_z}^{T_1,T_2}$, etc.) completely describe the isospin nature of the operator, in nuclear physics it is convenient to express the above quantities in terms of isoscalar isovector and isotensor components in the usual way:

$$\Omega_0^\tau(T_1,T_2)=\sum_{T_z}\langle T_1T_zT_2-T_z|\tau 0\rangle(-)^{T_2-T_z}\omega_{T_z}^{T_1,T_2}.$$

All the essential information is contained in the reduced matrix elements $b_\tau(T_1,T_2)$ of the above operator which are defined as follows:

$$\langle T_1T_z|\Omega_0^\tau|T_2T_z\rangle=\frac{1}{(2T_1+1)^{1/2}}\langle T_2T_z\tau 0|T_1T_z\rangle b_\tau(T_1,T_2),$$

where $\tau=0,1,2$. The quantities $b_\tau(T_1,T_2)$ are given as follows:

A. Isoscalar ($\tau=0$)

$$b_0(T_1,T_2)=\frac{\delta_{T_1T_2}}{(2T_1+1)^{1/2}}M_{T_1,T_2}^0,$$

B. Isovector ($\tau=1$)

$$b_1(0,1)=-b_1(1,0)=-\sqrt{3}M_{0,1}^1;$$

$$b_1(1,1)=\sqrt{6}M_{1,1}^1,$$

C. Isotensor ($\tau=2$)

$$b_2(1,1)=\sqrt{\tfrac{10}{3}}M_{1,1}^2.$$

The new independent quantities are:

$$M_{0,0}^0=\frac{1}{4\pi}(M_{\gamma\pi}^+M_{\pi N}^+-2M_{\gamma\pi}^-M_{\pi N}^-),$$

$$M_{1,1}^0=\frac{1}{4\pi}[-3M_{\gamma\pi}^+M_{\pi N}^+-2M_{\gamma\pi}^-M_{\pi N}^-],$$

$$M_{0,1}^1=M_{1,0}^1=\frac{1}{4\pi}M_{\gamma\pi}^0(M_{\pi N}^+-2M_{\pi N}^-), \tag{5}$$

$$M_{1,1}^1=\frac{1}{4\pi}M_{\gamma\pi}^0M_{\pi N}^+,$$

$$M_{1,1}^2=-\frac{1}{4\pi}2M_{\gamma\pi}^-M_{\pi N}^-.$$

The scattering lengths are $a^-=(0.086\pm0.005)\ m_\pi^{-1}$, $a^+=(0.005\pm0.005)\ m_\pi^{-1}$. Since a^+ is consistent with zero, there are only two nonzero basic amplitudes,

$M^0_{0,0}$ and $M^0_{0,1}$. The rest are either zero or expressed in terms of these two. In particular $M^0_{1,1} = M^2_{1,1} = M^0_{0,0}$; $M^1_{1,0} = M^1_{0,1}$, and $M^1_{1,1} = 0$. The quantities $M^0_{0,0}$ and $M^1_{0,1}$ and the relevant reduced matrix elements $b_\tau(T_1, T_2)$ associated with each of the A, B, C, D, and E terms are given in Table I both with and without Δ.

III. SELECTION RULES

A mere glance at Eqs. (2) and (3) will easily convince everyone that the transition operators $H^{(1)}_{\text{eff}}$ and $H^{(2)}_{\text{eff}}$ are rather complicated. They will appear more complicated if one expands them in terms of spherical tensors, which is essential for nuclear calculations. Hence quantitative conclusions can be drawn only after rather involved computer calculations which will involve detailed nuclear structure with the associated uncertainties. A number of qualitative conclusions can, however, be derived from the form of the above operators. At the present state of experimental situation such conclusions are very useful. The wealth of selection rules provided by the nuclear quantum numbers may enable us to get information not only about the basic amplitude itself and the reaction mechanism, but about the structure of the nucleus as well. The numerical calculations performed in the case of ^{4}He and ^{6}Li as well as the previously reported calculations[12] on d, ^{3}He, and ^{3}H will provide the testing ground for these qualitative conclusions. There are two kinds of selection rules, those associated with isospin and those associated with spin and parity.

A. Isospin selection rules

Such selection rules exist both for the direct and the rescattering transition operator.

(i) Rescattering operator. This operator has been found to dominate near threshold[12] for the deuteron. It is essentially of isoscalar and isotensor character (the isovector component is negligible). Therefore if this operator indeed dominates the cross section for $J \neq 0$, $T = 0$ targets near threshold, one should see strong transitions to final states with $T = 0$ and extremely weak transitions to $T = 1$ final states. Although transition to the $T = 2$, $T_z = 0$ final states can in principle be strong, such states, if they exist at all, must be at very high energies.

(ii) Direct operator. This operator contains both isoscalar and isovector components. From Table I we can see that near threshold, when the momentum independent term dominates, the isovector contributions must be about 10 times stronger than the isoscalar contribution. Although this naive conclusion may be altered by the fact that for isovector transitions the spatial overlap is perhaps less favorable compared with that entering quasielastic scattering, it explains perhaps why the direct contribution[12] dominates in ^{3}H(γ, π^0)-^{3}H. Further from threshold, when the momentum dependent terms begin to dominate, one does not expect to see such simple behavior. We expect, however, that the isoscalar amplitude will be larger.

One advantage associated with $T = 0$ targets is that, if the direct mechanism dominates, one obtains separately and directly the coefficients $M^+_{\gamma\pi}$ from the isoscalar transitions and the $M^0_{\gamma\pi}$ invariant amplitudes from the isovector transitions. These amplitudes cannot be separated from (γ, π^0) reactions on the nucleon.

B. Angular momentum selection rules

Since the pion photoproduction amplitude contains both spin independent terms (D term) and spin dependent terms one can have a variety of selection rules depending on the spin of the targets. These rules can be organized as follows:

(i) *Transitions* $0^+ \to 0^+$. Such transitions isolate the spin independent amplitude for the direct and rescattering process. The differential cross section due to the direct term is going to behave like $\sin^2\theta$, where θ is the angle between the oncoming photon and the outgoing pion. In the case of the rescattering term the differential cross section will again be proportional to $\sin^2\theta$, but it will contain an additional but rather small dependence on angle. Thus, unfortunately, in this case one cannot extract the reaction mechanism from the angular distributions. Another interesting feature of the above transitions is that in the case of the direct term the contribution of the s-wave pion is identically zero. Hence the cross section will be dominated by p waves even just above threshold (at higher energies, of course, one expects p wave dominance).

From Table I it is seen that the inclusion of Δ in the basic amplitudes greatly affects the direct spin independent amplitude. Thus the cross section will be about 20 times larger depending upon whether the Δ contribution is there or not. It is hoped that such big factors may enable even rather crude experiments to settle such questions about the amplitude.

(ii) $J^\pi \to J^\pi$ *with* $J \neq 0$. Such transitions will enable one to determine whether or not the contribution of the rescattering terms is important. Unlike the $J = 0$ case discussed above, in which the angular dependence of the differential cross section of the direct and rescattering terms was not very different, in the $J \neq 0$ case the cross sections are vastly different. Near threshold the reduced cross section $(k/q)(d\sigma/d\Omega_{\rm res})$ will have a maximum at $\theta=0°$ and will drop as a function of the angle with a minimum at $\theta=180°$. Of course, the maximum and minimum values are functions of the energy of the pion. On the other hand, the cross section due to the direct term remains pretty flat near threshold with a minimum around $\theta=90°$. The present calculations on ^{6}Li (isoscalar transitions) using the amplitudes of Table I not only confirm these expectations but, in addition, show that the rescattering term dominates near threshold. The above conclusions about the shape of the differential cross section are not expected to depend strongly on the model used to compute the amplitudes. Hence we suggest that even the simplest experiments may be able to determine whether the rescattering terms are important or not.

(iii) *Heavy nuclei.* The above two cases [(i) and (ii)] may not be clearly distinguished in the case of coherent production (quasielastic scattering) $J,\ T=0\to J,\ T=0$. One may have spin-dependent contributions from the valence nucleons if $J\neq 0$ and spin-independent contributions both from the valence nucleons and the core. As the number of core nucleons increases the contribution of the core will become more and more important and the $\sin^2\vartheta$ shape will dominate around $\theta=90°$. Even for heavy nuclei, however, the direct and rescattering cross sections will be vastly different near $\theta=0°$ or $\theta=180°$. Therefore we expect that one will be able to draw conclusions about the importance of the rescattering term by studying the shape of the coherent differential cross section with $J\neq 0$ targets.

IV. DIFFERENTIAL CROSS SECTION

Before one is able to calculate the differential cross section in coordinate space, one has to expand the effective transition operators into spherical tensors. The resulting operators are not very different from those met in other nuclear processes[15] except that: (1) the radial part is more complicated, (2) many more multipoles may contribute, and (3) it is possible to have interference between the various multipoles. Nuclear recoil is neglected.

A. Direct term

The spherical decomposition of $H_{\rm eff}^{(1)}$ proceeds in an analogous fashion as in the radiative pion absorption which has been discussed in detail previously (see Ref. 15). The pion wave function is now a plane wave or a distorted wave and is expanded in multipoles in the usual way:

$$\varphi_{\vec q}(\vec r)=\sum_{l m}(4\pi)\,i^l\varphi_l(q,r)\,Y_m^l(\hat r)\,[Y_m^l(\hat q)]^*\,, \tag{6}$$

where $\varphi_l(q,r)$ is a Bessel function $j_l(qr)$ for plane wave pions, or a function calculated numerically using appropriate pion-nucleus optical potential when distortions are necessary (see Sec. VII). The reduction of the operator proceeds now as in Ref. 15 except that instead of the transition probability one calculates the differential cross section, which in the notation of Ref. 15 is written as follows (in natural units):

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}(q^2+m_\pi{}^2)^{1/2}|ME|^2\,, \tag{7}$$

$$|ME|^2=\sum_{\substack{\alpha,\beta\\ a,b\\ \Lambda}} i^{l_b-l_a}g_\alpha^*(a)g_\beta(b)\,\frac{F_{\alpha,\beta}(a,b,\Lambda)}{2J+1}\,\delta_{J_aJ}\delta_{J_bJ}$$
$$\times\langle J_f\|h_\alpha^{(a)}(r)\,T^{S_\alpha,\mathcal{L}_a,J_a}\|J_i\rangle^*\,\frac{p_\Lambda(\hat k\cdot\hat q)}{4\pi}$$
$$\times\langle J_f\|h_\beta^{(b)}(r)\,T^{S_\beta,\mathcal{L}_b,J_b}\|J_i\rangle\,.$$

The summation labels α and β run from 1–5 correspond to the coefficients $A,\ B,\ C,\ D,$ and E. (a) and (b) stand for a collection of quantum numbers associated with each matrix element. In fact, $(a)\equiv\mu_a',\mu_a,l_a,l_a'',\mathcal{L}_a,J_a$ and $(b)\equiv\mu_b',\mu_b,l_b,l_b',l_b'',\mathcal{L}_b,J_b$. These quantum numbers are the same as those defined in Ref. 15 and will be described only briefly here: μ' is the multipolarity of the photon ($\mu'\geq 1$) obtained by expanding the photon wave function $e^{i\vec k\cdot\vec r}$. μ is obtained by coupling the photon polarization vector $\hat\epsilon_\lambda$ with the photon multipolarity μ' ($\mu\equiv|\mu'-1|,\ \mu',\ \mu'+1$). l is the multipolarity of the pion. l' is the multipolarity obtained when the gradient operates on the pion wave function in the case of $B,\ C,\ D,$ and E terms ($l'=l-1,\ l+1$). As in Ref. 15, we assume here that the gradient operates in the pion wave function only and not in the nuclear wave function. l'' is the multipolarity obtained when the gradient operates on the l' multipolarity ($l''=|l'-1|,\ l'+1$). This occurs only in the case of the E term. $\mathcal{L}$ and J indicate the spatial and total angular momentum ranks of the operator. S_α and S_β indicate the spin rank of the operator (zero if α and $\beta=D$ and unity otherwise). $J=|\mathcal{L}\pm J|$ or $J=\mathcal{L}$. The qualities $g_\alpha(a)$ and $g_\beta(b)$ can be read off directly from Table II of Ref. 15. The function $F_{\alpha,\beta}(a,b,\Lambda)$ is different in the case of photoproduction as compared to radiative pion capture and is given in Appendix A. The quantity Λ indicates the order of the Legendre polynomial entering in the differential cross section. Integrating over the angles, only $\Lambda=0$ survives and yields the total cross section. One has

$$|l_a-l_b|\leq\Lambda\leq l_a+l_b$$

(assuming that the amplitudes do not depend on the momentum transfer). The differential cross section can be written as

$$\frac{d\sigma}{d\Omega}=\frac{1}{4\pi}\sum_{\Lambda}\sigma_{\Lambda}(k,q)P_{\Lambda}(\hat{k}\cdot\hat{q}).$$

If only s and p pion waves contribute to the cross sections near threshold, $\Lambda=0,1,2$. One notes that it is possible to have interference between the s and p pion waves in the differential cross section. For $J_\pi=0^+$ targets, independently of the structure of the nuclear wave function, one has $\sigma_1=0$ and $\sigma_2=-\sigma_0$. Hence the cross section becomes

$$\frac{d\sigma}{d\Omega}(0^+\to 0^+)=\frac{\sigma_0}{4\pi}\frac{3}{2}\sin^2\theta.$$

We emphasize that the essential ingredients entering our calculation are the basic amplitudes for the π^0 photoproduction process, the pion wave function (distorted or plane wave), and the nuclear wave functions appearing in the reduced matrix elements. Thus the direct term was treated exactly except for the two approximations on the basic amplitudes already mentioned, namely the fact that we neglected their energy dependence and did not transform them to the pion nucleus c.m. system.

B. Rescattering term

In order to be able to perform calculations with nuclear wave functions, it is important to transform the wave function of the two interacting particles and the operator $H_{\text{eff}}^{(2)}$ into relative and center-of-mass coordinates. The first of these tasks can be achieved with harmonic oscillator wave functions by performing a Moshinski-Brody transformation.[17] For the 1s and 1p shells required in the present calculation, the required coefficients have been tabulated.[17] Furthermore, we note that the transition operator M_{ij} of Eq. (3) is already in this form. The product of the photon and pion wave functions can be simply written in terms of relative and c.m. coordinates only if the pion wave function is undistorted, in which case

$$e^{-i\vec{k}\cdot\vec{r}_i}e^{i\vec{q}\cdot\vec{r}_j}=e^{i\vec{p}\cdot\vec{r}}e^{i\vec{P}\cdot\vec{R}},$$

where

$$\vec{r}=\frac{1}{\sqrt{2}}(\vec{r}_i-\vec{r}_j),\quad \vec{R}=\frac{1}{\sqrt{2}}(\vec{r}_i+\vec{r}_j),$$

$$\vec{p}=-\frac{1}{\sqrt{2}}(\vec{k}+\vec{q}),\quad \vec{P}=-\frac{1}{\sqrt{2}}(\vec{k}-\vec{q}).$$

As we shall see later, the pion distortions are small and the above procedure can be easily applied.

A multipole expansion analogous to that of the direct term was carried out for the rescattering amplitude as follows:

(i) expand $e^{i\vec{p}\cdot\vec{r}}$ into multipoles (indicated by λ);
(ii) expand $e^{i\vec{P}\cdot\vec{R}}$ into multipoles (indicated by μ);
(iii) couple the λ multipoles with the operator M_{ij} which depends on the relative coordinate $\hat{x}$ to get a multipole λ_1 ($\lambda_1=|\lambda-1|$, $\lambda+1$ for B, C, and D terms, $\lambda_1=\lambda$ for A term, $\lambda_1=|\lambda-2|$, λ, $\lambda+2$ for the E term);
(iv) couple λ_1 and μ to get the total spatial rank of the operator $\mathcal{L}$;
(v) couple the operator $\mathcal{L}$ with $\vec{\Sigma}_{12}$ to obtain the total tensorial rank of the operator (not relevant in the case of the D term).

$\vec{\Sigma}_{12}$ is $\vec{\sigma}_1+\vec{\sigma}_2$ when $\lambda=$ even or $\vec{\sigma}_1-\vec{\sigma}_2$ when $\lambda=$ odd. We will not bother the reader with the details of the above reduction. The differential cross section is given by an expression similar to that of Eq. (5) except that $|ME|^2$ will be a summation over the above quantum numbers of products of geometric factors and two reduced matrix elements $\langle J_f\|0_{\lambda,\lambda_1,\mu,\mathcal{L},J}\|J_i\rangle^*$ and $\langle J_f\|0_{\lambda',\lambda_1',\mu',\mathcal{L}',J}\|J_i\rangle$. The operator $0_{\lambda,\lambda_1,\mu,\mathcal{L},J}$ has the following form:

$$0_{\lambda,\lambda_1,\mu,\mathcal{L},J}=j_\lambda(pr)z_i(q,\sqrt{2}r)j_\mu(PR)\times[[Y^{\lambda_1}(\hat{r})\otimes Y^{\mu}(\hat{R})]^{\mathcal{L}}\times\vec{\Sigma}_{12}]^J \tag{8}$$

where $j_\lambda(pr)$ and $j_\mu(PR)$ are spherical Bessel functions. The operator with primed subscripts is defined similarly, and the functions $z_i(q,\sqrt{2}r)$, $i=0,1,2$ were defined in Eq. (3). The calculations of the reduced matrix elements follow the usual Racah techniques used to reduce the many-particle to a two-particle matrix element. The two-particle matrix element is calculated using a Moshinski-Brody transformation.[17] The radial integrals in the relative and center-of-mass coordinates are performed numerically.

The geometric factors are rather complicated. For the special cases of interest here they are given in Appendix B. One distinguishes two matrix elements, one involving the rescattering amplitude only (RR) and the other involving the interference between the direct and the rescattering amplitudes (DR). The angular dependence arises in two ways. Explicitly from terms of the type $[Y^{\Lambda_1}(\hat{p})\times Y^{\Lambda_2}(\hat{P})]_0^{\Lambda}$ with $Y^{\Lambda_1}(\hat{p})$ and $Y^{\Lambda_2}(\hat{P})$ spherical harmonics in the momentum coordinates p and P resulting by the following couplings:

$$\vec{\lambda}+\vec{\lambda}'=\vec{\Lambda}_1,\quad \vec{\mu}+\vec{\mu}'=\vec{\Lambda}_2,$$

$$|\lambda-\lambda'|\leq\Lambda_1\leq\lambda+\lambda',\quad |\mu-\mu'|\leq\Lambda_2\leq\mu+\mu'.$$

The above terms can be expressed in terms of Legendre polynomials $P_L(\hat{k}\cdot\hat{q})$. (Such transformations for the cases considered in the present paper are trivial. For more realistic cases the formulas will be given elsewhere.) There is, however, an additional angular dependence which arises implicitly from the radial integrals involving $j_\lambda(pr)$ and $j_\mu(PR)$ which are functions of the angle between k and q given as follows:

$$p=\tfrac{1}{2}[k^2+q^2+2kq\cos\theta]^{1/2},$$
$$P=\tfrac{1}{2}[k^2+q^2-2kq\cos\theta]^{1/2}.$$

This latter angular dependence can only be computed numerically.

As in the case of the direct term, the multipole expansion does not converge very rapidly. However, the multipoles are restricted by the angular momentum couplings involved in the expression of Eq. (8) and the conditions imposed by the angular momentum and parity selection rules, i.e., $(-)^{\mathcal{L}}=\pi_i\pi_f$ and $|J_i-J_f|\leqslant J\leqslant J_i+J_f$. Further restrictions are imposed from the restrictions imposed by the Moshinski-Brody transformations, namely,

$$|N_i-N_f|\leqslant\lambda\leqslant N_i+N_f\,,\quad |N_i-N_f|\leqslant\mu\leqslant N_i+N_f\,,$$

where N_i and N_f are the harmonic oscillator quanta of the initial and final two-body nuclear states.

V. RESULTS FOR ^{4}He $(\gamma,\pi^0)^4$He

The reaction ^{4}He$(\gamma,\pi^0)^4$He is very interesting both for experimental and theoretical reasons. From an experimental point of view ^{4}He is a good target. It is very bound and it does not have any other bound states or low-lying resonances. Thus the coherent cross section can unambiguously be extracted from the data. From the theoretical point of view it has the desired features exhibited by isoscalar $0^+\to0^+$ selection rules discussed previously. Furthermore, since all other nuclei are in some form or another supposed to have a ^{4}He-like core, knowledge of the cross sections for this nucleus is essential in understanding coherent production from heavier nuclei. It has of course the disadvantage that only the spin-independent amplitude contributes and the reduced cross sections $(k/q)(d\sigma/d\Omega)$ will be proportional to k^2q^2, i.e., very small near threshold.

In calculating the differential cross section we will use the formulas given in the appendixes. In the calculation of the reduced matrix elements we intend to use the harmonic oscillator shell model.

For the direct contribution two calculations were performed. The first involved a naive $(1s)^4$ configuration. The second involved a shell model space including 0p-0h (i.e., $1s^4$), 1p-1h, 2p-2h, and 4p-4h, i.e., excitations up to $4\hbar\omega$. The shell model Hamiltonian matrix was constructed using the Brown-Kuo-Lee[18] realistic two-nucleon effective interaction and a one-body Hamiltonian determined from the experimental single particle energies [those of case (b) of Ref. 19]. The resulting wave function contains 23 components. The dominant ones are:

$$|0^+(\text{g.s.})\rangle=0.89978\,|1s_{1/2}{}^4\rangle+0.37105\,|1s_{1/2}{}^3 2s_{1/2}\rangle-0.15652\,|1s_{1/2}{}^2 J=1,\ T=0;\ (1p_{3/2}1p_{1/2})J=1,\ T=0\rangle$$
$$(97\%\text{ of the total wave function}).\qquad(9)$$

The other essential parameter is the harmonic oscillator parameter $\hbar\omega$. Although the extraction of this parameter from electron scattering experiments is somewhat model dependent, the currently accepted value is $\hbar\omega=24$ MeV (i.e., $b=1.3$ fm). Partly to illustrate how the results depend on $\hbar\omega$ and partly because there will be contributions from a ^{4}He-like core in the coherent production from all p-shell nuclei for which $\hbar\omega=14$ MeV, we repeated the calculation for $\hbar\omega=14$ MeV.

From the formulas given in the appendixes it can be shown that the reduced differential cross section takes the form

$$\frac{k}{q}\frac{d\sigma}{d\Omega}=\frac{\sigma_0^R}{4\pi}\frac{3}{2}\sin^2\theta\,.$$

In the case in which there is no contribution from the rescattering term, σ_0^R depends only on q and represents the total reduced cross section. It is given as follows:

$$\sigma_0^R(q)=\tfrac{1}{3}(D_0)^2(kq)^2(R_0-R_2)^2,$$

where R_0 and R_2 are just radial integrals involving the $A=4$ system. We have seen that only p-wave ($l=1$) pions contribute, i.e., $l'=0$ or 2. For plane wave pions we have

$$R_\lambda=\langle 0^+\|\sum_{i=1}^{4}j_\lambda^0(qr_i)j_\lambda^0(kr_i)\|0^+\rangle\,.$$

The quantity σ_0^R is given as a function of the pion energy in Table II for the various values of the parameters entering the problem.

Since the differential cross section is proportional to q^3 it is very small near threshold. In fact,

TABLE II. The total reduced cross section (in μb) associated with the direct production mechanism in the ^{4}He$(\gamma,\pi^0)^4$He reaction for the various cases discussed in the text. For notation see text.

E_π	No Δ				With Δ			
	Correlated		Uncorrelated		Correlated		Uncorrelated	
(MeV)	$\hbar\omega=14$	$\hbar\omega=24$	$\hbar\omega=14$	$\hbar\omega=24$	$\hbar\omega=14$	$\hbar\omega=24$	$\hbar\omega=14$	$\hbar\omega=24$
1.8	0.1207	0.1545	0.1114	0.1571	2.1148	2.7063	1.9524	2.7527
3.6	0.2406	0.3113	0.2204	0.3155	4.2144	3.4526	3.8615	5.5268
5.8	0.3640	0.4762	0.3310	0.4809	6.3757	8.3407	5.7976	8.4243
7.2	0.4907	0.6491	0.4428	0.6533	8.5958	11.3704	7.7562	11.4438

for energies a few MeV above threshold, the total cross section due to the direct term is only a few hundredths of a microbarn. It depends rather crucially on the model used to calculate the basic amplitudes (with or without Δ), as can be seen from Table II. Its dependence on the harmonic oscillator parameter $\hbar\omega$ and the nuclear wave function used is not as dramatic.

The rescattering operator is somewhat more complicated. However, the nuclear wave function is dominated by $L=0$, and the formulas given in Appendix B can be used. Since in this case the spin does not appear in the operator of Eq. (8) and the wave function is dominated by s nucleons, we get $\lambda'=\lambda=1$, $\lambda_1'=\lambda_1=\mu=\mu'=0$. Hence the differential cross section is simplified. We get:

(i) *RR contribution.*

$$|ME|^2=\tfrac{1}{3}q^2k^2(2D^0_{00})^2 \times\frac{|\langle 0^+\| J_1(pr)J_0(PR)z_1(q,\sqrt{2}r)\| 0^+\rangle^2}{2p^2} \times[1-P_2(\hat{q}\cdot\hat{k})].$$

(ii) *DR contribution.* Again as in the case of the direct amplitude, $l=1$, $l'=0$ and 2, $\lambda=1$, $\mu=\lambda_1=0$, using the formulas given in Appendix B we get

$$|ME|^2=\tfrac{4}{3}D_0D^0_{00}(kq)^2\,[1-P_2(\hat{k}\cdot\hat{q})](R_0-R_2) \times\frac{\langle 0^+\| J_1(pr)J_0(PR)\,\mathrm{Re}\,z_1(q,\sqrt{2}r)\| 0^+\rangle}{\sqrt{2}p},$$

where $\mathrm{Re}\,z_1(q,\sqrt{2}r)$ = Real part of $z_1(q,\sqrt{2}r)$, and where the quantities R_0 and R_2 have been discussed in the case of the direct term.

The reduced matrix elements entering in all the above equations depend on the nuclear wave functions. Since 97% of the wave function is contained in the three components given in Eq. (9) it is a good approximation to neglect the small amplitudes comprising the 3% of the wave function in calculating the reduced matrix element of the rescattering operator. Furthermore, near $q=0$ we found that the reduced matrix element involving the wave function of Eq. (9) is only 12% larger compared to that of a pure $(1s)^4$ wave function. We expect that at higher pion momenta the results will be scaled by about the same factor. Hence in Figs. 1(a) and 1(b) we present the results obtained with the simple $(1s)^4$ wave function. In Fig. 1(a) we plot the reduced differential cross section as a function of θ for $\hbar\omega$ = 24 MeV for various energies of the outgoing pion. On the same plot we present the contribution of the direct term only. One notices that the differential cross section increases by an order of magnitude when the amplitudes, which include Δ, are used. The contribution of the direct terms represents 75% of the total cross section. A typical cross section evaluated with $\hbar\omega=24$ MeV at say $E_\pi=3.5$ MeV is 0.1 μb without Δ and 1.6 μb with Δ. The calculation was repeated with $\hbar\omega=14$ MeV. The results are now somewhat smaller. The total cross sections at $E_\pi=3.5$ MeV are 0.08 μb without Δ and 1.1 μb with Δ. The rescattering contribution is again important but smaller than that of the direct term [see Table II and Fig. 1(b)]. One cannot deduce much information from the shape of the cross section since the 1-$p_2(\hat{k}\cdot\hat{q})$ angular dependence dominates even the case of the rescattering term.

VI. RESULTS FOR ^{6}Li$(\gamma,\pi^0)^6$Li

The reaction ^{6}Li$(\gamma,\pi^0)^6$Li from the point of view of the quantum numbers is similar to the corresponding one on the deuteron. Even though ^{6}Li is an extensively studied nucleus, it is not, of course, as well understood as the deuteron. ^{6}Li has definite experimental advantages, however, since it is much more tightly bound than the deuteron and therefore the coherent cross section can be measured even though the incoming γ rays are not monochromatic. Unfortunately, there are some excited states at low energies, e.g., 3^+ $T=0$ at 2.185 MeV and 0^+ $T=1$ at 3.562 MeV which, with

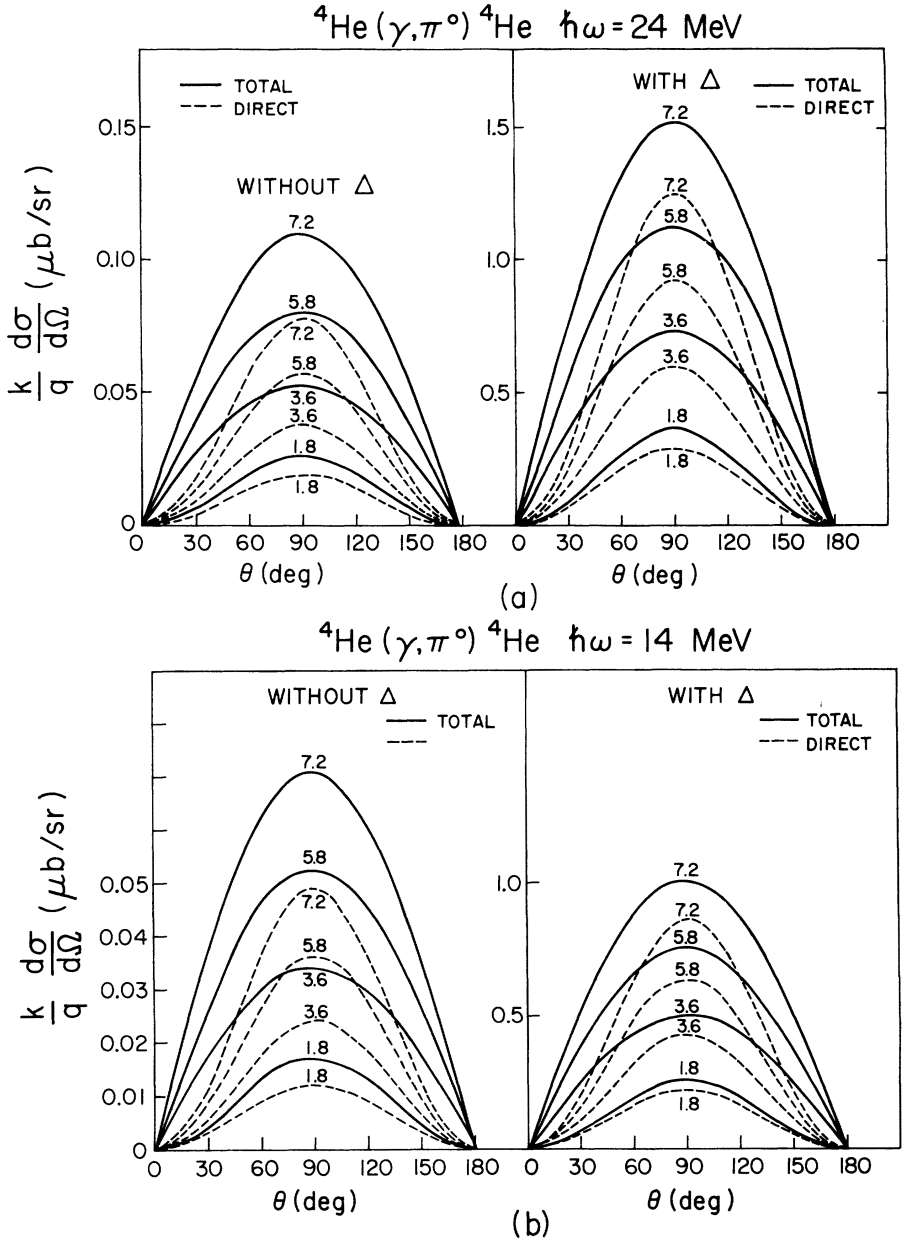

FIG. 1. (a) The reduced differential cross-section $(k/q)(k\,\sigma/d\Omega)$ for the ^{4}He$(\gamma,\pi^0)^4$He reaction, with harmonic oscillator parameter $\hbar\omega = 24$ MeV plotted as a function of the angle for various π^0 energies (notation explained in the text). (b) The same as (a) with $\hbar\omega = 14$ MeV.

present day resolutions, may not be resolved. We shall show, however, that the cross section for exciting these levels is going to be very small. Hence this nucleus is the simplest deuteronlike nucleus accessible to present day experiments.

The coherent cross section ^{6}Li$(\gamma,\pi^0)^6$Li is characterized by exactly the same isospin selection rules as ^{4}He. It probes, however, all five isoscalar amplitudes, not just the spin independent part.

There are many nuclear wave functions describing this particular nucleus.[8,9] In the present calculation the ^{6}Li wave functions were obtained by considering two $1p$-shell nucleons outside an inert ^{4}He core. The calculation of these wave functions has been described previously (Set 1b of Ref. 19). The g.s. wave function expressed in the L-S coupling scheme is

$$(1^+\ T=0)=[0.9716\,|\,L=0\rangle+0.2240\,|\,L=1\rangle$$
$$+0.0771\,|\,L=2\rangle\,](1s)\,.^4$$

The harmonic oscillator parameter used was $\hbar\omega$ $=11$ MeV, both for the valence nucleons and the inert core. It was obtained from electron scattering data.[8] In the calculation of the direct term the entire wave function was used. In the case of the rescattering term only the $L=0$ p-shell component was retained.

In calculating the contribution of the $(1s)^4$ inert core the formulas given in the previous section may be used. One must, of course, scale the results by the correct statistical factor $1/(2J_i+1)$ $=\frac{1}{3}$ and use the harmonic oscillator parameter appropriate for ^{6}Li. The results of the direct term for the various values of the parameters used in the calculation for various values of the pion energy are given in Table IV. In the same table we present the results including distortions of the pion wave function due to strong interactions.

For the rescattering term for the $L=0$ nuclear wave function one finds that the transition operator is characterized by $J=1$ and $\mathfrak{L}=\mathfrak{L}'=0$. This implies $\lambda=\mu$ and $\lambda'=\mu'$. On the other hand, the fact that the two nucleons are confined in the $1p$ shell implies that $\lambda=\lambda'=0$; with these simplifications the formulas given in Appendix B become

(i) *RR contribution (due to the A term).*

$$|ME|^2$$
$$=\tfrac{8}{3}(A^0_{00})^2\langle(1p^2)L=0\,|j_0(pr)z_0(q,\sqrt{2}\,r)\,j_0(PR)\,|(1p)^2L=0\rangle$$
$$=\tfrac{8}{3}(A^0_{00})^2R_0{}^2\ ,$$

where

$$R_0=\tfrac{1}{2}[\langle 1s\,|\,j_0(pr)\,z_0(q,\sqrt{2}r)\ 1s\rangle\,\langle 2s\,|\,j_0(PR)|\ 2s\rangle$$
$$-2\langle 1s|\,j_0(pr)\,z_0(q,\sqrt{2}r)|\ 2s\rangle\,\langle 2s|\,j_0(PR)|\ 1s\rangle$$
$$+\langle 2s|\,j_0(pr)\,z_0(q,\sqrt{2}r)|\ 2s\rangle\,\langle 1s|\,j_0(PR)|\ 1s\rangle]\,.$$

(ii) *DR contribution (due to the A term).*

$$|ME|^2=\tfrac{16}{3}A_0A^0_{00}R_0\sum_l(2l+1)^{1/2}R_lP_l(\hat{k}\cdot\hat{q})\ ,$$

where P_l are Legendre polynomials and

$$R_l=\langle 1p|\,j_l(kr)\,j_l(qr)|\ 1p\rangle,\quad l=0,1\,.$$

The interference between the A_{resc} and all the other direct terms (B_{dir}, C_{dir}, D_{dir}, and E_{dir}) was also included in the calculation. The interference between the A_{resc} and all the other rescattering terms was also calculated and was found small. All the other rescattering contributions are expected to be negligible and were not considered.

The results of the calculations including all the above contributions are plotted in Fig. 2 both with and without Δ. The contribution of the direct term is presented separately. From this figure it is apparent that the rescattering term manifests itself dramatically both in the magnitude and angular dependence of the differential cross section.

The direct term due to the valence nucleons is more or less constant near threshold and exhibits small oscillations at higher energies when higher Legendre polynomials contribute significantly (see Table III and Fig. 2). The cross section due to the rescattering term has an entirely different shape with a maximum at $\theta=0°$ and a minimum at $\theta=180°$. The maximum increases with energy until the oscillations of $z_0(q,r)$ begin to set in. These conclusions are modified as soon as core contributions begin to be important. Then both the direct and rescattering cross sections are affected by an additional 1-$p_2(\hat{k}\cdot\hat{q})$ dependence which becomes pronounced near $\theta=90°$. If the core contribution is important, as e.g., in heavy nuclei, the direct and rescattering will still have different shapes away from $\theta=90°$. In this important sense the coherent differential cross section for nuclei with $J\neq 0$, $T=0$ is expected to be different from that of the deuteron,[12] especially away from threshold. In our calculation we see the effects of the core, even in a light nucleus like ^{6}Li, using the amplitudes with Δ, which make the core contribution 20 times larger. Note that at threshold the total reduced cross sections are close, i.e., 1.16 and 1.24 without and with Δ, respectively, since, in this case, there is no core contribution.

As expected in view of the selection rules given earlier, the rescattering contribution to the 0^+ $T=1$ state is 450 times smaller compared to the one associated with the coherent production. This is due to the fact that the isovector amplitude is very small. The contribution of the rescattering process to the 3^+ $T=0$ state is also very small (20 times smaller than that of coherent production). In this case the hindrance is not due to isospin but due to the poor overlap of the radial wave functions.

The contribution of the direct term for transitions to the 3^+ $T=0$ state is also small. This is, however, hardly surprising due to the high multipolarity of the transition. The contribution of the direct term to the 0^+ $T=1$ states is larger compared to that of the quasielastic scattering near threshold. In fact, $(k/q)\,\sigma(1^+\to 0^+)_{\text{dir}}=3.6(k/q)\,\sigma(1^+\to 1^+)_{\text{dir}}$. Isospin considerations alone would imply a ratio of about 5. The difference is attributed to the $(1s)^4$ core which contributes in the coherent process and the fact that the matching of the spin-space wave functions is not as good for transitions to 0^+ $T=1$ as it is in the coherent case. In any event the two cross sections ought to be comparable in the absence of the rescattering contribution, and, if such

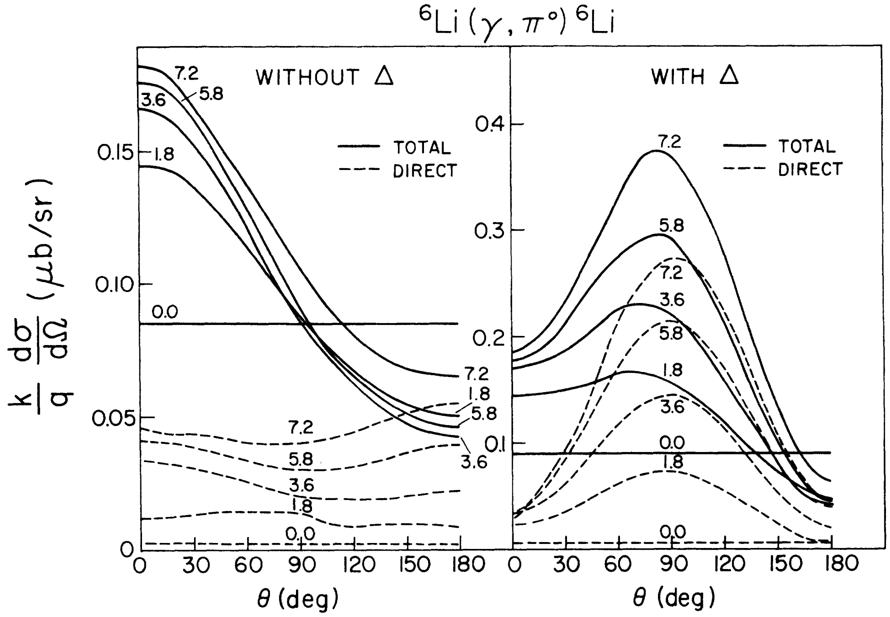

FIG. 2. The reduced differential cross section $(k/q)(d\sigma/d\Omega)$ for ^{6}Li$(\gamma,\pi^0)^6$Li (g.s.) with $\hbar\omega = 11$ MeV plotted as a function of the angle for various π^0 energies (notation explained in the text).

transitions are ever going to be resolved, will give additional information about the nature of the rescattering amplitudes.

It is interesting to note that in the case of transitions to the 0^+ $T=1$ state the reduced cross section decreases as a function of the pion momentum. The quantities σ_0^R, σ_1^R, and σ_2^R due to the direct term for the 0^+ $T=0$ states are presented in detail in Table IV.

VII. DISTORTION OF THE PION WAVE FUNCTION

Both in the direct operator $H_{\text{eff}}^{(1)}$ and in the rescattering operator $H_{\text{eff}}^{(2)}$ the wave function of the outgoing pion appears. Since the neutral pions carry no charge, the distortion of their wave functions is due to the strong interactions. Such distortions are expected to be small, but may be essential for quantitative calculations. In the present calculation we have calculated such distortions for various energies of the outgoing pions near threshold. This was achieved by including in the equation for the pion wave function an optical potential of the type[20]

$$V_\pi = -\frac{4\pi}{2m_\pi}[b_0\rho(\vec{r}) + b_1(\rho_n(\vec{r}) - \rho_p(\vec{r})) + B_0\rho^2(\vec{r}) - \vec{\nabla}q(\vec{r})\vec{\nabla}] ,$$

TABLE III. The coefficients σ_L^R of the Legendre polynomials $p_L(\hat{k}\cdot\hat{q}')$ entering the expression

$$\frac{k}{q}\frac{d\sigma}{d\Omega} = \frac{1}{4\pi}\sum_{L=0}^{2}\sigma_L^R P_L(\hat{k}\cdot\hat{q})$$

associated with the direct ^{6}Li$(\gamma,\pi^0)^6$Li (g.s.) reaction for various energies of the outgoing pion and $\hbar\omega = 11$ MeV.

	Without Δ						With Δ		
	No distortion			Distortion			No distortion		
E_π	σ_0^R	σ_1^R	σ_2^R	σ_0^R	σ_1^R	σ_2^R	σ_0^R	σ_1^R	σ_2^R
0.0	0.0301	−0.0000	0.0000	0.0230	0.0001	0.0000	0.0805	0.0501	−0.0246
1.8	0.1478	0.1108	0.0726	0.1557	0.1433	0.0643	1.0710	0.1734	−0.6416
3.6	0.2598	0.0946	0.1355	0.2624	0.1417	0.1191	2.0469	0.1309	−1.2657
5.8	0.3706	0.0430	0.1914	0.3693	0.1019	0.1666	3.0063	0.0272	−1.8947
7.2	0.4803	−0.0418	0.2402	0.4765	0.0365	0.2063	4.1318	−0.1163	−2.7113

TABLE IV. The same as in Table III, but for transition to excited states in the final nucleus.

	$1^+T=0\to 0^+T=1$			$1^+T=0\to 3^+T=0$		
E_π	σ_0^R	σ_1^R	σ_2^R	σ_0^R	σ_1^R	σ_2^R
			With Δ			
0.0	0.1039	0.0062	0.0000	0.0155	−0.0006	0.0041
1.8	0.0941	0.0290	0.0009	0.1269	−0.0020	−0.1088
3.6	0.0849	0.0375	0.0017	0.2428	−0.0012	−0.2180
5.8	0.0763	0.0421	0.0026	0.3627	0.0005	−0.3313
7.2	0.0682	0.0445	0.0034	0.4864	0.0028	−0.4484
			Without Δ			
0.0	0.1178	−0.0000	0.0000	0.0062	−0.0004	0.0000
1.8	0.1060	0.0328	0.0012	0.0451	−0.0125	−0.0334
3.6	0.0961	0.0427	0.0024	0.0835	−0.0159	−0.0675
5.8	0.0868	0.0483	0.0035	0.1234	−0.0172	−0.1034
7.2	0.0780	0.0514	0.0047	0.1646	−0.0170	−0.1410

where $q(\vec{r}) = c_0\rho(\vec{r}) + c_1(\rho_n(\vec{r}) - \rho_p(\vec{r})) + i\,\mathrm{Im}C_0\rho^2(\vec{r})$. For ^{6}Li the s-wave parameters are[20] (in units of m_π^{-4})

$$b_0 = -0.0134, \quad b_1 = 0.0873,$$

$$\mathrm{Re}B_0 = -0.059, \quad \mathrm{Im}B_0 = 0.0610.$$

The p-wave parameters[21] are (in units of m_π^{-6})

$$c_0 = 0.17, \quad c_1 = 0.22, \quad \mathrm{Im}C_0 = 0.036, \quad \mathrm{Re}C_0 = 0.$$

The parameters b and c are related to the elementary πN scattering lengths, while the parameters B and C are connected to the π-deuteron interaction. The solution was obtained numerically by matching it to plane waves at the distance of 9.0 fm.

The results of our calculations using distorted pion waves are presented in Table III. We see that the difference between the distorted and undistorted solutions is small. The distorted cross section at threshold is 16% less than the distorted one (s-pion dominance; A term only) while it is higher by 20%, 15%, 13%, and 12% at $E_\pi = 1.8$, 3.6, 5.8, and 7.2 MeV, respectively (p-wave dominance, nonstatic terms). Thus the uncertainties resulting from the pion wave function itself, at the energies considered here for the direct term, are much smaller compared to the other theoretical uncertainties entering the problem. For the rescattering term the inclusion of pion distortions is not trivial, since the convenient separation into relative and center of mass coordinates enjoyed by the plane waves is lost. Judging from the results of the direct term calculations, it is safe to assume that modification of $H_{\mathrm{eff}}^{(2)}$ due to pion distortions is small.

Even though the effect of distortions on the higher Legendre polynomials is greater and the contribution of the individual amplitudes fluctuates, the effects of distortions can be neglected in the present state of such calculations.

VIII. SUMMARY AND CONCLUSIONS

In this paper we study (γ, π^0) reactions from nuclear targets near threshold and provide the experimentalists with predictions about the magnitude and the shape of the expected differential cross sections. The emphasis has been on the differences in cross sections that arise due to uncertainties in the elementary amplitude and in the reaction mechanism. By a judicious choice of nuclear target it should be possible to shed some light on the reaction mechanism. Our calculations show, for example, that the shape of the differential cross section of $^6\mathrm{Li}(\gamma,\pi^0)^6\mathrm{Li}$ (g.s.) is highly modified by the inclusion of charge exchange rescattering mechanisms. Also, using the selection rules of Sec. III, one can isolate various pieces of the basic amplitude. In particular with 0^+ targets only the spin independent part of the amplitude contributes. This is also the amplitude which is most affected by the inclusion of Δ. Experiments on such targets as ^{4}He can therefore provide useful information on the single nucleon photoproduction model. Once the reaction mechanism and the basic amplitudes are known, one can feel confident that neutral pion photoproduction will develop into a useful tool for obtaining nuclear structure information probing the entire nuclear volume.

Of course, the experimental problems to be surmounted are tremendous. Present experiments are designed to provide only yields, that is, the differential cross section folded with the incident

γ-ray spectrum and a function $A(q,\theta)$ which characterizes the acceptance of the experimental apparatus. Furthermore, even absolute yields are not at present measured. All this makes it very difficult for the theorist to compare his calculations with experiment. Perhaps the best procedure would be to do experiments on a wide variety of nuclear targets and to try to determine the reaction mechanism and basic amplitude which will give a reasonable description of all the data. As we have seen, even crude data, like those which seem feasible at present, will be able to settle such questions. As the amount of data increases one would begin to focus on particular targets with the aim of extracting nuclear structure information.

The nuclear wave functions used in our calculations are not the most realistic possible with present day nuclear structure theory. Phenomenological wave functions like those of Bergstrom, Auer, and Hicks[8] in the case of ^{6}Li may be more appropriate for quantitative predictions. However, in light of the theoretical and experimental uncertainties mentioned above, our wave functions are adequately realistic and our conclusions will remain unchanged. The total cross section for the coherent production from ^{6}Li at $E_\pi = 2$ MeV is 0.3 μb with Δ and 0.2 μb without Δ. The same quantity for ^{4}He at a typical energy, say $E_\pi = 3.5$ MeV, can be anywhere between 0.1 and 1.5 μb depending on the model and the reaction mechanism. These predictions will be basically unaffected by reasonable changes in the wave functions.

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APPENDIX A: DIRECT TERM

The calculation of the function $F_{\alpha\beta}(a,b,\Lambda)$ which appears in the expression of the differential cross section in (γ,π^0) reactions proceeds in a fashion similar to that employed in the calculation of the corresponding quantity in the radiative π^- absorption.[15] Assuming that the basic amplitudes do not depend on the momentum transfer, one obtains after straightforward algebra:

$$F_{\alpha,\beta}(a,b,\Lambda) = \sum_{x_a,x_b} \tilde{q}_\alpha(a,x_a)\,\tilde{q}_\beta(b,x_b)\,\psi(a,b,x_a,x_b,\Lambda)\,,$$

where, in the notation of Ref. 15,

$$\psi(a,b,x_a,x_b,\Lambda) = \frac{(2l_a+1)(2l_b+1)(2x_a+1)}{2J+1} \times \left[1+(-1)^{\mu_a'+\mu_b'+\Lambda+\eta}\right](-)^{l_a+l_b+\mu_a'+J}\,U(\mu_a' l_a \mu_b' l_b; J\Lambda)\langle l_a 0 l_b 0 | \Lambda 0\rangle \langle \mu_a' 1 \Lambda 0 | \mu_b' 1\rangle\,,$$

and

$$\tilde{q}_\alpha(a,x_a) = \begin{cases} \delta_{x_a \mu_a'}\,, & \alpha = A, E\,, \\ (-)^{x_a+\mu_a'+1}\,U(\mathfrak{L}_a l_a \mu_a' J; l_a' x_a)\langle 10 x_a 1 | \mu_a' 1\rangle\,, & \alpha = B\,, \\ (-)^{J+1+\mathfrak{L}_a}\,U(1 J \mu_a' l_a; \mathfrak{L}_a x_a)\langle 10 x_a 1 | \mu_a' 1\rangle\,, & \alpha = C\,, \\ U(11\mu_a \mu_a'; 1 x_a)\,U(l_a 1 \mathfrak{L}_a \mu_a; l_a' x_a)\dfrac{\langle 10 x_a 1 | \mu_a' 1\rangle}{(2x_a+1)^{1/2}}\,, & \alpha = D\,. \end{cases}$$

Similarly for $\tilde{q}_\beta(b,x_b)$:

$$\eta = \begin{cases} \eta_1 & \text{no interference}\,, \\ \eta_1 + 1 & \text{interference;} \end{cases}$$

$$\eta_1 = \begin{cases} 1, & AB,\ AC,\ \text{and}\ AD\ \text{terms}\,, \\ 0, & \text{otherwise}\,. \end{cases}$$

"no interference" means[15] $\mu_a' = \mu_a$ and $\mu_b' = \mu_b$ or $\mu_a' \neq \mu_a$ and $\mu_b' \neq \mu_b$.

APPENDIX B: RESCATTERING TERM

We will not present here general formulas involving the rescattering term, but we will specialize to the case in which the nuclear wave functions entering the problem are characterized by $L = 0$. There are contributions which involve rescattering terms only (RR) and those which involve interference between the direct and the rescattering terms (DR).

Spin dependent terms

Near threshold the most important spin dependent terms involve the momentum independent amplitudes (A terms).

(1) *RR contribution.* Since $\mathcal{L}=0$, λ_1 is redundant, we have $\lambda=\mu$, $\lambda'=\mu'$. Then

$$|ME|^2=\frac{1}{2(2J_i+1)}\frac{2}{3}(A^0_{00})^2(4\pi)^3$$

$$\times\sum_{\lambda,\lambda'}[(2\lambda+1)(2\mu+1)]^{1/2}(-)^{\lambda-\lambda'}\langle J_+\|0^A_\lambda\|J_i\rangle\langle J_+\|0^A_{\lambda'}\|J_i\rangle\sum_{\Lambda=|\lambda-\lambda'|}^{\lambda+\lambda'}[Y^\Lambda(\hat{P})\otimes Y^\Lambda(\hat{P})]^0_0,$$

where

$$0^A_\lambda=j_\lambda(pr)z_0(q,\sqrt{2}r)j_\lambda(PR)[Y^\lambda(\hat{r})\otimes Y^\lambda(\hat{R})]^0\vec{\Sigma}_{12}.$$

Similarly for $0^A_{\lambda'}$ (summations over all particles understood).

(2) *DR contribution.*

$$|ME|^2=\frac{4}{(2J_i+1)}A_0A^0_{00}(4\pi)^2\sum_\lambda(-)^\lambda\langle J_f\|\Omega_l\|J_i\rangle(\mathrm{Re}\langle J_f\|0_\lambda\|J_i\rangle)[Y^1_0(\hat{q})(Y^\lambda(\hat{p})\otimes Y^\lambda(\hat{P})]^0_0,$$

where

$$\Omega_l=\sum_i j_l(kr_i)\,\varphi_l(qr_i)\vec{\sigma}_i\,,$$

and $\varphi_l(qr)$ is the radial pion wave function.

Spin independent terms

(1) *RR contribution.* In this case the operator M_{ij} is proportional to x. Hence $\lambda_1=|\lambda-1|,\lambda+1$. Since $\mathcal{L}=0$ (s-wave nuclear wave function) we must have $k=0$ and $\lambda_1=\mu$. Hence the expansion in this case depends on two quantum numbers λ and μ.

Hence we have

$$|ME|^2=\frac{k^2}{2J_i+1}(D^0_{00})^2(4\pi)^3\sum_{\substack{\lambda,\mu\\ \lambda',\mu'}}(-)^{(\lambda+\mu+\lambda'+\mu')/2}(2\lambda+1)(2\lambda'+1)$$

$$\times\langle\lambda 0\;10|\mu_0\rangle\langle\lambda'0\;10|\mu'0\rangle\langle J_f\|0^D_{\lambda,\mu}|j_i\rangle\langle j_f\|0^D_{\lambda',\mu'}\|J_i\rangle F^D_{RR}(\lambda,\mu,\lambda',\mu'),$$

where

$$0^D_{\lambda,\mu}=\begin{cases}2j_\lambda(pr)z_1(q,\sqrt{2}r)j_\mu(PR)[Y^\mu(\hat{r})\times Y^\mu(\hat{R})]^0,\lambda\text{ odd}\\ 0,\lambda\text{ even}\end{cases}$$

and similarly for $0^D_{\lambda',\mu'}$. $F^D_{RR}(\lambda,\mu,\lambda',\mu')$ is given by

$$F^D_{RR}(\lambda,\mu,\lambda',\mu')=\sum_{\substack{\Lambda_1,\Lambda_2\\ \Lambda\text{ even}}}(-)^{\Lambda_1+\Lambda_2}\begin{Bmatrix}\lambda' & \mu' & 1\\ \lambda & \mu & 1\\ \Lambda_1 & \Lambda_2 & \Lambda\end{Bmatrix}\langle\lambda 0\lambda'0|\Lambda_1 0\rangle\langle\mu 0\mu'0|\Lambda_2 0\rangle\langle 111-1|\Lambda 0\rangle[Y^{\Lambda_1}(\hat{p})\times Y^{\Lambda_2}(\hat{P})]^\Lambda_0\,.$$

(2) *DR contribution.* The selection rules involving the direct term are the same as those given in Appendix A. The allowed combination of the quantum numbers due to the rescattering term are given in the RR case. Hence

$$|ME|^2=\frac{k^2}{2J_i+1}D_0D^0_{00}(4\pi)^2\sum_{\substack{\lambda,\mu\\ l,l'}}(-)^{(\lambda+\mu+l-l')/2+\lambda+\mu+1}(2l+1)\left(\frac{2\lambda+1}{2\mu+1}\right)^{1/2}\langle\lambda 0\;10|\mu 0\rangle$$

$$\times\mathrm{Re}[F^D_{DR}(l,l',\lambda,\mu)\langle J_f\|0^D_{\lambda,\mu}|J_i\rangle\langle J_f\|\Omega^D_{l,l'}|J_i\rangle],$$

where

$$\Omega^D_{l',l}=j_{l'}(kr)\,D_{l',l}\,\varphi_l(q,r);\qquad D_{l',l}=\begin{cases}\left(\dfrac{l+1}{2l+3}\right)^{1/2}\left(\dfrac{d}{dr}-\dfrac{l}{r}\right) & l'=l+1\\[2ex] -\left(\dfrac{l}{2l-1}\right)^{1/2}\left(\dfrac{d}{dr}+\dfrac{l+1}{r}\right) & l'=l-1\,,\end{cases}$$

and

$$F^D_{\mathrm{DR}}(l,l',\lambda,\mu)=\frac{2}{3\sqrt{3}}\sum_{\Lambda\ \mathrm{even}}\frac{1}{2\Lambda+1}\left[\delta_{l'\Lambda}+\tfrac{5}{2}\langle l'020\,|\,\Lambda 0\rangle U(ll'12;\,1\Lambda)\right]\{Y^l(\hat{q})\otimes[Y^\lambda(\hat{p})\otimes Y^\mu(\hat{P})]^1\}^\Lambda_0\,.$$

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