Second-order pion-nucleus optical potential*

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The second-order pion-nucleus optical potential is expressed in momentum space in terms of the πN off-shell t matrix and the NN correlation function. The dependence of the πN t matrix on the total πN momentum is determined within the framework of relativistic particle quantum mechanics. Numerical results are presented for π-^{4}He scattering. It is found that the NN correlations tend to decrease the forward elastic cross sections by a factor of about 1.5 at low energy, $E_\pi \simeq 60$ MeV. At higher energies the main effect of the NN correlations is to increase the differential cross section at large angles. The effect of NN correlations on the total cross section and the pion wave function is also found to be important. A study of the dependence of the calculated cross sections on the range parameters of the πN t matrix and of the NN correlations suggests that the *low*-energy pion-nucleus scattering data cannot be adequately described by an optical model constructed from the first- and second-order terms in an expansion in terms of the πN t matrix.

[NUCLEAR REACTIONS Second-order π-nucleus optical potential studied in the multiple-scattering theory.]

I. INTRODUCTION

Recent studies of intermediate-energy pion-nucleus scattering have indicated that an important part of the pion-nucleus optical potential can be constructed within the impulse approximation from the πN t matrix. Within the framework of multiple-scattering theory,[1] this "scattering" part of the optical potential can be expanded in powers of the πN t matrix. The leading term $U^{(1)}$ in this expansion, the first-order optical potential, has been found[2-4] to yield a satisfactory description of forward-angle pion-nucleus elastic scattering at pion energies above 120 MeV. Difficulties have recently been encountered at lower pion energies[5,6] ($E_\pi \lesssim 100$ MeV).

To improve agreement with experiment at lower energies and to obtain better pion wave functions for other purposes, it is necessary to develop more accurate methods[6,7] for the calculation of $U^{(1)}$, and to investigate the influence of higher-order contributions to the pion-nucleus optical potential. If the multiple-scattering expansion of the pion-nucleus optical potential is valid, one would expect higher-order terms to improve the fit to experimental data at lower energies, without spoiling the "good fit" already achieved by $U^{(1)}$ alone at higher energies ($E_\pi \gtrsim 120$ MeV). In this paper we study the influence of the second-order optical potential $U^{(2)}$ on pion-nucleus elastic scattering.

Another purpose of this study stems from concern about future applications of pion-nucleus optical potentials. The second-order optical potential $U^{(2)}$ describes the effects of two-nucleon (NN) correlations on the motion of the pion inside the nucleus. Effects of NN correlations on the pion wave function are expected to be important in determining pion-nucleus reactions such as $A(\pi^+,\pi^-)B$ and $A(\pi^-,p)B$ whose predominant mechanism involves two or more nucleons.

We will develop a method to calculate the pion-nucleus second-order optical potential directly in momentum space. Our approach is thus distinctly different from that of Miller and Spencer,[8] who study the effect of $U^{(2)}$ in coordinate space using the equivalent local potential approach of Kujawski.[9] Since it is quite feasible to study pion-nucleus reactions directly in momentum space,[10] our approach will not create any major obstacle in practice to using our model for $U^{(1)}+U^{(2)}$. For our quantitative discussions of the effect of NN correlations, numerical results for π-^{4}He scattering will be presented.

As is usual in calculations of the first-order pion-nucleus optical potential, the impulse approximation will be assumed. The degree of validity of the impulse approximation is not precisely known and is not discussed in this paper. Nevertheless, in view of the general success achieved at $E_\pi \gtrsim 120$ MeV by the first-order pion-nucleus optical potentials, the impulse approximation must at least be a reasonable starting point. Also, within the impulse approximation, the second-order optical potential has been derived by Feshbach, Gal, and Hüfner[11] from the multiple-scattering formalism of Kerman, McManus, and Thaler.[1] In this paper, we follow Ref. 11 and construct the second-order pion-nucleus optical potential from the πN off-shell t matrix and the NN correlation functions.

A basic ingredient in the calculation of $U^{(1)}$ and $U^{(2)}$ is the πN off-shell t matrix for nonzero total momentum $\vec{Q}$. Phenomenological constructions of the optical potential relate it to the πN phase shifts which determine the πN t matrix only for $\vec{Q}=0$, i.e., in the πN c.m. frame. A procedure must therefore be devised to relate these two dynamically different objects. To do this we follow Heller, Bohannon, and Tabakin[12] and Lee and Coester.[13] This method has been used by Lee[6] to study the first-order π-^{4}He optical potential and will be reviewed briefly. We construct the desired off-shell t matrix from a simple πN model described in Ref. 6.

Another basic ingredient of $U^{(2)}$ is a model for the NN correlations. We follow the procedure of Feshbach, Gal, and Hüfner[11] to construct the NN correlations of ^{4}He from a simple shell-model wave function. The NN correlation function and the density for ^{4}He are both taken to be independent of spin and isospin.

In Sec. II, the first- and second-order pion-nucleus optical potentials are explicitly expressed in terms of the πN t matrix and nuclear wave functions. The method of constructing the πN t matrix for nonzero total momentum is presented in Sec. III. Section IV is devoted to a discussion of the NN correlation function for ^{4}He. In Sec. V, we present in detail the numerical procedures in momentum space for the calculation of the pion-nucleus T matrix from the optical potential $U^{(1)} + U^{(2)}$. Results for π-^{4}He elastic scattering are discussed in Sec. VI. In Sec. VII, we summarize our results.

II. FIRST- AND SECOND-ORDER PION-NUCLEUS OPTICAL POTENTIALS

In this section, we construct the first- and second-order pion-nucleus optical potentials in momentum space, using the formalism of Feshbach, Gal, and Hüfner (FGH).[11] The further approximations needed to permit numerical calculations will also be discussed.

The pion-nucleus optical potential can be expanded in powers of a many-body operator $\tau(E)$ for the scattering of a pion from a "bound" nucleon. From the formal derivation of FGH, we have, up to second order in τ,

$$U^{\mathrm{opt}}(E)=U_{00}(E)+\sum_{\alpha\neq 0}^{\infty} U_{0\alpha}(E)\frac{1}{E-\epsilon_\alpha-K_\pi-U_{\alpha\alpha}(E)+i\delta}U_{\alpha 0}(E), \tag{1}$$

where

$$U_{\alpha\beta}(E)=(A-1)\langle\Phi_\alpha|\tau(E)|\Phi_\beta\rangle \tag{2}$$

and

$$\tau(E)=v+v\frac{\mathcal{A}}{E-K_\pi-H_A+i\delta}\tau(E). \tag{3}$$

Here, v denotes the πN potential, $|\Phi_\alpha\rangle$ is an eigenstate of the nuclear Hamiltonian H_A with eigenvalue ϵ_α, K_π is the pion kinetic energy operator, $\mathcal{A}$ is the nuclear antisymmetrization operator, and E is the pion-nucleus collision energy. The goal is to introduce suitable approximations so that Eq. (1) can be evaluated from the free pion-nucleon scattering operator defined by

$$t(\omega)=v+v\frac{1}{\omega-K_\pi-K_N+i\delta}t(\omega), \tag{4}$$

where K_N is the nucleon kinetic energy operator and ω is a properly chosen πN collision energy.

In the FGH approach, we first introduce a scattering operator t_i' for the ith nucleon,

$$t_i'(E)=v_i+v_i\frac{1}{E-K_\pi-H_A+i\delta}t_i'(E), \tag{5}$$

which, since it does not contain the antisymmetrizer $\mathcal{A}$, is a much simpler operator than τ. Next, we assume that the energy denominator $E-\epsilon_\alpha-K_\pi-U_{\alpha\alpha}$ is independent of the nuclear state α and take an average value $E-K_\pi-\bar{\epsilon}-\bar{U}_0$. The closure relation then gives U^{opt} in terms of the operator $t_i'(E)$ and the nuclear ground state wave function. We have, for the first- and second-order optical potentials,

$$U^{\mathrm{opt}}=U^{(1)}(E)+U^{(2)}(E),$$

where

$$U^{(1)}(E)=(A-1)\langle\Phi_0|\frac{1}{A}\sum_{i=1}^{A}t_i'(E)|\Phi_0\rangle \tag{6}$$

and

$$U^{(2)}(E)=(A-1)^2\Big[\frac{1}{A(A-1)}\sum_{i\neq j}\langle\Phi_0|t_i'(E)\frac{1}{E-K_\pi-\bar{\epsilon}-\bar{U}_0+i\epsilon}t_j'(E)|\Phi_0\rangle - \langle\Phi_0|\frac{1}{A}\sum_{i=1}^{A}t_i'(E)|\Phi_0\rangle\frac{1}{E-K_\pi-\bar{\epsilon}-\bar{U}_0+i\epsilon}\langle\Phi_0|\frac{1}{A}\sum_{i=1}^{A}t_i'(E)|\Phi_0\rangle\Big]. \tag{7}$$

We now discuss the approximations involved in the computation of Eqs. (6) and (7). First, it is assumed that binding effects can be neglected, so that

$$t'(E) \cong t(\omega_0) \tag{8}$$

with a properly chosen πN collision energy ω_0. Here, $t(\omega_0)$ is the free πN scattering operator defined by Eq. (4). The choice of ω_0 depends on the details of the model used and has been discussed by many authors. We will indicate our choice of ω_0 in the next section when we discuss the calculation of the matrix elements of $t(\omega_0)$ in momentum space. Next, we observe (following Landau and Thomas[7]) that with the harmonic-oscillator 1s single-particle wave function $\phi(p)$, which is appropriate for the ^{4}He ground state, one can express the first-order optical potential in the form

$$\begin{aligned}\langle \vec{k}' | U^{(1)}(E) | \vec{k}\rangle &= (A-1)\langle \vec{k}'\Phi_0 | \frac{1}{A}\sum_{i=1}^{A} t_i(\omega_0) | \vec{k}\Phi_0\rangle \\ &\equiv (A-1)F_{00}(\vec{q}) \int d\vec{p}\, |\phi(\vec{p})|^2 \\ &\qquad\times \langle \vec{k}', \vec{p}+\vec{p}_0-\vec{q} | t(\omega_0) | \vec{k}, \vec{p}+\vec{p}_0\rangle,\end{aligned} \tag{9}$$

where $\vec{k}$ and $\vec{k}'$ are pion momenta in the pion-nucleus c.m. frame,

$$\vec{p}_0 = -\vec{k}/A + \vec{q}(A-1)/2A, \tag{10}$$

with $\vec{q} = \vec{k}' - \vec{k}$, and $F_{00}(\vec{q})$ is the nuclear ground state form factor. In deriving Eq. (9), care has been taken to express the nucleon momentum of the intrinisic nucleon wave functions (defined in the nuclear c.m. frame) in terms of momenta defined in the pion-nucleus c.m. frame. This is necessary for ^{4}He since the c.m. recoil of light nuclei is important. *The above folded first-order potential is used in our calculations of* $U^{(1)}$.[19]

To evaluate $U^{(2)}$, we now use Eq. (9) to introduce the "factorization" approximation which consists of removing the t matrix from the integral and evaluating it at $\vec{p} = 0$:

$$\langle \vec{k}'\Phi_0 | \frac{1}{A}\sum_{i=1}^{A} t_i(\omega_0) | \vec{k}\Phi_0\rangle \simeq F_{00}(\vec{q})\langle \vec{k}', \vec{p}_0-\vec{q} | t(\omega_0) | \vec{k}, \vec{p}_0\rangle. \tag{11}$$

We generalize Eq. (11) to any nuclear state $|\Phi_\beta\rangle$:

$$\langle \vec{k}'\Phi_\beta | \frac{1}{A}\sum_{i=1}^{A} t_i(\omega_0) | \vec{k}\Phi_0\rangle = F_{\beta 0}(\vec{q})\langle \vec{k}', \vec{p}_0-\vec{q} | t(\omega_0) | \vec{k}, \vec{p}_0\rangle, \tag{12}$$

where $F_{\beta 0}(\vec{q})$ is the appropriate transition form factor. Using the above generalized factorization approximation, a straightforward derivation yields

$$\begin{aligned}\langle \vec{k}' | U^{(2)}(E) | \vec{k}\rangle = (A-1)^2 \int d\vec{k}'' d\vec{k}''' \langle \vec{k}', \vec{p}_0'-\vec{q}' | t(\omega_0) | \vec{k}'', \vec{p}_0'\rangle \\ \times C(\vec{k}'-\vec{k}'', \vec{k}'''-\vec{k})\langle \vec{k}'' | (E-\bar{\epsilon}-K_\pi-\overline{U}_0+i\epsilon)^{-1} | \vec{k}'''\rangle \langle \vec{k}''', \vec{p}_0-\vec{q} | t(\omega_0) | \vec{k}, \vec{p}_0\rangle,\end{aligned} \tag{13}$$

where $\vec{q} = \vec{k}''' - \vec{k}$, $\vec{p}_0 = -\vec{k}/A + \vec{q}(A-1)/2A$, $\vec{q}' = \vec{k}' - \vec{k}''$, $\vec{p}_0' = -\vec{k}''/A + \vec{q}'(A-1)/2A$, and

$$C(\vec{q}', \vec{q}) = \langle \Phi_0 | e^{i(\vec{q}'\cdot\vec{r}_1' + \vec{q}\cdot\vec{r}_2')} | \Phi_0\rangle - F_{00}(\vec{q}')F_{00}(\vec{q}) \tag{14}$$

is the correlation function in momentum space.[11] It should be noted here that our form of $U^{(2)}(E)$, Eq. (13), retains the explicit dependence of the t matrix on nucleon momentum and is therefore different from that of FGH.

Equation (13) is still a complicated object. Further simplification is possible on the assumption that the average nuclear fluctuation from the ground state is small during the propagation of the pion; hence the average optical potential operator $\overline{U}_0$ is approximated by a constant value

$$\overline{U}_0 \cong U_0(E) = \langle \vec{k}_0 | U^{(1)}(E) | \vec{k}_0\rangle, \tag{15}$$

where $\vec{k}_0$ is the incident momentum. Then the propagator of Eq. (13) becomes diagonal in $\vec{k}$ space[20]:

$$\langle \vec{k}'' | (E-\bar{\epsilon}-K_\pi-\overline{U}_0+i\epsilon)^{-1} | \vec{k}'''\rangle \equiv \delta(\vec{k}''-\vec{k}''')[E-\bar{\epsilon}_0-U_0(E)-(\mu^2+k''^2)^{1/2}-(M_A^2+k''^2)^{1/2}+i\epsilon]^{-1}, \tag{16}$$

where μ and M_A are, respectively, the masses of the pion and the nucleus. Substituting Eq. (16) into Eq. (13), we obtain

$$\begin{aligned}\langle \vec{k}' | U^{(2)}(E) | \vec{k}\rangle = (A-1)^2 \int d\vec{k}'' \langle \vec{k}', \vec{p}_0'-\vec{q}' | t(\omega_0) | \vec{k}'', \vec{p}_0'\rangle C(\vec{k}'-\vec{k}'', \vec{k}''-\vec{k})\langle \vec{k}'', \vec{p}_0-\vec{q} | t(\omega_0) | \vec{k}, \vec{p}_0\rangle \\ \times \frac{1}{E-\bar{\epsilon}_0-U_0(E)-(\mu^2+k''^2)^{1/2}-(M_A^2+k''^2)^{1/2}+i\epsilon}.\end{aligned} \tag{17}$$

Equation (17) is used in our numerical studies of NN correlation effects on pion-nucleus elastic scattering. In the next two sections, we will discuss the construction of the πN off-shell t matrix and the NN correlation function.

III. πN OFF-SHELL t MATRIX

To calculate the optical potentials of Eqs. (9) and (17), we need to construct the πN off-shell t matrix for nonzero total momentum. We follow Refs. 12 and 6 to construct this object using relativistic particle quantum mechanics.[21] In this approach, it is assumed that the πN off-shell t matrix $\tilde{t}(\tilde{\omega}_0)$ in the πN c.m. frame has been constructed from a model of the πN mass operator (i.e., the Hamiltonian in the πN c.m. frame). Neglecting the spin-dependent term, the matrix element of $\tilde{t}(\tilde{\omega}_0)$ can be written as

$$\tilde{t}(\vec{\kappa}', \vec{\kappa}, \tilde{\omega}_0) = \sum_{I\,l}\left\{\sum_{j=l\pm 1/2}(j+\tfrac{1}{2})P^{(I)}\tilde{t}^{I}_{lj}(\kappa', \kappa, \tilde{\omega}_0)\right\}P_l(\hat{\kappa}'\cdot\hat{\kappa}), \quad (18)$$

where $P^{(I)}$ is the isospin projection operator, and $\vec{\kappa}$ and $\vec{\kappa}'$ are relative momenta. In each eigenchannel $\alpha(Ilj)$, a simple separable form $\tilde{t}_\alpha(\kappa', \kappa, \tilde{\omega}_0)=\tilde{t}_\alpha(\tilde{\omega}_0)g_\alpha(\kappa')g_\alpha(\kappa)/g_\alpha{}^2(\kappa_0)$, with $g_\alpha(\kappa) = \kappa^l e^{-\beta_\alpha \kappa^2}$, is used. Here $\tilde{t}_\alpha(\tilde{\omega}_0)$ is the on-energy-shell t matrix determined from the CERN phase shifts.[14] Our task is to express the desired matrix element $\langle\vec{k}', \vec{p}+\vec{p}_0-\vec{q}\,|\,t(\omega_0)\,|\,\vec{k}, \vec{p}+\vec{p}_0\rangle$ in terms of $\tilde{t}(\vec{\kappa}', \vec{\kappa}, \tilde{\omega}_0)$. Following Ref. 12, it can be shown that

$$\begin{aligned}\langle\vec{k}', \vec{p}+\vec{p}_0-\vec{q}\,|\,t(\omega_0)\,|\,\vec{k}, \vec{p}+\vec{p}_0\rangle &= [J(\vec{k}', \vec{p}+\vec{p}_0-\vec{q}, \vec{\kappa}')J(\vec{k}, \vec{p}+\vec{p}_0, \vec{\kappa})]^{1/2} \\ &\quad\times\{\tilde{t}(\vec{\kappa}', \vec{\kappa}, (\omega_0^2-Q^2)^{1/2}) + O([\vec{Q}/(M+\mu)]^2) + \cdots\},\end{aligned} \quad (19)$$

where $\vec{Q}=\vec{k}+\vec{p}+\vec{p}_0$ and

$$J(\vec{k}, \vec{p}, \vec{\kappa}) = \frac{[E_\pi(\vec{k})+E_N(\vec{p})][E_\pi(\vec{\kappa})E_N(\vec{\kappa})]}{[E_\pi(\vec{\kappa})+E_N(\vec{\kappa})]\,[E_\pi(\vec{k})E_N(\vec{p})]}.$$

In the energy region under consideration, terms of the order of $[Q/(M+\mu)]^2$ or higher should not be important and are therefore neglected. Carrying out appropriate boost transformations, the relative momenta $\vec{\kappa}$ and $\vec{\kappa}'$ can be related to the momentum variables $\vec{k}$, $\vec{k}'$, and $\vec{p}$. Keeping only the lowest-order term in the nucleon momentum,[22] we get

$$\begin{aligned}\vec{\kappa} &= \frac{-W_1(k)}{2W_3(k)}\left\{a_1(k)\left[\vec{p}+\frac{A-1}{2A}(\vec{k}+\vec{k}')\right]-a_2(k)\vec{k}\right\}, \\ \vec{\kappa}' &= \frac{-W_1(k')}{2W_3(k')}\left\{a_1(k')\left[\vec{p}+\frac{A-1}{2A}(\vec{k}+\vec{k}')\right]-a_2(k')\vec{k}'\right\},\end{aligned} \quad (20)$$

where

$$W_1(k)=\{[m+E_\pi(k)]^2-k^2\}^{1/2},$$
$$W_3(k)=m+E_\pi(k)+W_1(k),$$
$$a_1(k)=1+2E_\pi(k)/W_1(k)+(\mu^2-m^2)/W_1{}^2(k),$$

and

$$a_2(k)=2+2E_\pi(k)/W_1(k)+2m/W_1(k).$$

To complete the definition of the off-shell t matrix, the πN collision energy ω_0 must be defined. As our model,[23] we assume for a total πN momentum $\vec{Q}$ that

$$\omega_0{}^2=W_0{}^2+\vec{Q}^2,$$

where W_0 is an "invariant mass" determined by the incident pion momentum $\vec{k}_0$, i.e.,

$$W_0{}^2=[E_\pi(k_0)+E_N(k_0/A)]^2-[(A-1/A)k_0]^2.$$

To calculate the matrix elements of the optical potential in each pion-nucleus partial wave, we need the partial-wave decomposition of Eq. (19). For simplicity, only the off-shell t matrix Eq. (19) for $\vec{p}=0$ will be presented. From Eq. (20), we note that, as $\vec{p}=0$, the magnitudes κ', κ and angle $\hat{\kappa}'\cdot\hat{\kappa}$ can be explicitly expressed in terms of k', k, and angle $\hat{k}'\cdot\hat{k}$. Keeping only the s- and p-wave πN interaction, we can then write

$$\tilde{t}(\vec{\kappa}', \vec{\kappa}, \tilde{\omega}_0)=\sum_l \tilde{t}_l(k', k, \tilde{\omega}_0)P_l(\hat{k}'\cdot\hat{k}), \quad (21)$$

where

$$\begin{aligned}\tilde{t}_l(k', k, \tilde{\omega}_0) &= G^0_l(k', k, \tilde{\omega}_0) - [A(k)B(k')k^2+B(k)A(k')k'^2]\frac{G^1_l(k', k, \tilde{\omega}_0)}{k_0{}^2} \\ &\quad+\sum_{l'}(2l'+1)\begin{pmatrix} l & 1 & l' \\ 0 & 0 & 0\end{pmatrix}^2[A(k)A(k')+B(k)B(k')]G^1_l(k', k, \tilde{\omega}_0).\end{aligned} \quad (22)$$

The functions $G^{l_s}_l(k', k, \tilde{\omega}_0)$ are determined by the relation

$$G^{l_s}_l(k', k, \tilde{\omega}_0) = \frac{2}{2l+1}\int_{-1}^{+1}P_l(x)\left[\sum_{\substack{j=l_s\pm 1/2 \\ I}}(j+\tfrac{1}{2})P^{(I)}\left(\frac{\kappa_0{}^2}{\kappa\kappa'}\right)^{l_s}\tilde{t}^{I}_{l_s j}(\kappa', \kappa, \tilde{\omega}_0)\right]dx, \quad (23)$$

with $x=\hat{k}'\cdot\hat{k}$. Other quantities in Eq. (22) are given by

$$A(k)=\frac{W_1(k)}{2W_3(k)}\left[a_2(k)-\frac{A-1}{2A}a_1(k)\right],\quad B(k)=\frac{W_1(k)}{2W_3(k)}\left[\frac{A-1}{2A}a_1(k)\right]. \tag{24}$$

With the expansion

$$[J(\vec{k}',\vec{p}_0-\vec{q},\vec{\kappa}')J(\vec{k},\vec{p}_0,\vec{\kappa})]^{1/2}=\sum_l J_l(k',k)P_l(\hat{k}'\cdot\hat{k}), \tag{25}$$

we obtain from Eqs. (21) and (25) the relation

$$\langle\vec{k}',\vec{p}_0-\vec{q}\,|\,t(\omega_0)\,|\,\vec{k},\vec{p}_0\rangle=\sum_L t_L(k',k,W_0)P_L(\hat{k}'\cdot\hat{k}), \tag{26}$$

where

$$t_L(k',k,W_0)=\sum_{ll'}(2L+1)\begin{pmatrix} l' & l & L\\ 0 & 0 & 0\end{pmatrix}^2 J_l(k',k)\tilde{t}_{l'}(k',k,W_0). \tag{27}$$

Equation (26) is needed to calculate the second-order optical potential Eq. (17).

To calculate the folded first-order potential, Eq. (9), we have used a similar but more complicated procedure to expand the desired matrix element $\langle\vec{k}',\vec{p}+\vec{p}_0-\vec{q}\,|\,t(\omega_0)\,|\,\vec{k},\vec{p}+\vec{p}_0\rangle$ in terms of Legendre functions $P_l(\hat{k}'\cdot\hat{k})$, $P_l(\hat{k}\cdot\hat{p})$, and $P_l(\hat{k}'\cdot\hat{p})$. We can write, in general, for any off-shell model,

$$\langle\vec{k}',\vec{p}+\vec{p}_0-\vec{q}\,|\,t(\omega_0)\,|\,\vec{k},\vec{p}+\vec{p}_0\rangle=\sum_{l_1l_2l_3}t^{l_1}_{l_2l_3}(k',k,p,\omega_0)P_{l_1}(\hat{k}'\cdot\hat{k})P_{l_2}(\hat{k}\cdot\hat{p})P_{l_3}(\hat{k}'\cdot\hat{p}). \tag{28}$$

A three-dimensional integration is needed to obtain $t^{l_1}_{l_2l_3}(k',k,p,\omega_0)$. To save computation time, the quantity $t^{l_1}_{l_2l_3}(k',k,p,\omega_0)$ is evaluated using the expansion

$$E=(m_0^2+(\vec{a}+\vec{b})^2)^{1/2}\simeq(m_0^2+a^2+b^2)^{1/2}\left[1+\frac{\vec{a}\cdot\vec{b}}{(m_0^2+a^2+b^2)^{1/2}}\right] \tag{29}$$

for energy variables. With this approximation and our simple Gaussian form of $\tilde{t}^{I}_{lj}(\kappa',\kappa,\tilde{\omega}_0)$ we can use Racah algebra to obtain $t^{l_1}_{l_2l_3}(k',k,p,\omega_0)$. The three-dimensional integration over the angles between $\hat{k}$, $\hat{k}'$, and $\hat{p}$ are therefore avoided. The resulting complicated form of $t^{l_1}_{l_2l_3}(k',k,p,\omega_0)$ is not relevant to the rest of our discussion and is not presented here.

IV. TWO-NUCLEON CORRELATION FUNCTION OF ^{4}He

For simplicity, we follow Feshbach *et al.*[11] to construct the two-nucleon correlation function of ^{4}He from an assumed shell-model wave function. Three different correlations are obtained by using the shell-model wave function to calculate $C(\vec{q}',\vec{q})$, Eq. (14). First, since the nucleus is required to recoil as a whole in defining the optical potential, a correlation due to this kinematic restriction will naturally exist. This correlation, called the c.m. correlation, is expected to be small for heavy nuclei, but must be considered in studying π-^{4}He scattering. Secondly, because of the antisymmetrization of nuclear wave functions, Pauli correlations occur. Finally, the short-range dynamical correlation due to the NN interaction must be included.

Since detailed shell-model calculations of the ^{4}He ground state are not available, the three correlation functions are evaluated by the following approximate procedure. It is assumed that the c.m. and Pauli correlations can be evaluated from the determinantal wave function for four nucleons moving independently in 1s oscillator orbitals. This wave function is expected to be the largest component in the ^{4}He ground state. For this wave function the Pauli correlation vanishes. The derivation of the c.m. correlation from the 1s oscillator shell-model wave function is given in Ref. 11 and hence is omitted. To account for the short-range dynamical correlation, it is postulated that the short-range part of the two-nucleon relative wave function of the independent particle model must be modified. This is equivalent to a modification of the intrinsic two-body density matrix of the form

$$\rho(\vec{r}_1',\vec{r}_2')=W(4/b^2,\vec{X})W(1/2b^2,\vec{r})-CW(4/b^2,\vec{X})W(1/2b^2,\vec{r})[1+g(\vec{r})], \tag{30}$$

where $\vec{X}=\frac{1}{2}(\vec{r}_1'+\vec{r}_2')$, $\vec{r}=\vec{r}_1'-\vec{r}_2'$, and $W(\gamma,\vec{X})=(\gamma/\pi)^{3/2}e^{-\gamma X^2}$. The short-range correlation $g(\vec{r})$ is subject to the constraints that $g(\vec{r})\to 0$ as $r\to\infty$, $g(0)=-1$. A convenient parametrization is

$$g(\vec{r})=-e^{-(r^2/2l_c^2)}. \tag{31}$$

We define l_c as a correlation length that measures the strength of the dynamical correlations due to the NN interaction. The constant C in Eq. (30) is found to be $1/[1-(l_c/b)^3]$, which is determined by the requirement that the one-body density is normalized in the presence of $g(r)$.

Combining the c.m. and dynamical correlations and carrying out the Fourier transform, we obtain the input function $C(\vec{q}',\vec{q})$ of the second-order optical potential:

$$C(\vec{q}',\vec{q})=\left[e^{-(1/16\nu)(\vec{q}+\vec{q}')^2}e^{-(1/8\nu)(\vec{q}-\vec{q}')^2}-e^{-(3/16\nu)(\vec{q}^2+\vec{q}'^2)}\right]-\left[\left(\frac{\nu}{\nu+\gamma}\right)^{3/2}e^{-(1/16\nu)(\vec{q}+\vec{q}')^2}e^{-(1/8(\nu+\gamma))(\vec{q}-\vec{q}')^2}\right]$$
$$+\left(\frac{l_c}{b}\right)^3\left[e^{-(1/16\nu)(\vec{q}+\vec{q}')^2}e^{-(1/8\nu)(\vec{q}-\vec{q}')^2}-\left(\frac{\nu}{\nu+\gamma}\right)^{3/2}e^{-(1/16\nu)(\vec{q}+\vec{q}')^2}e^{-1/8(\nu+\gamma)(\vec{q}-\vec{q}')^2}\right.$$
$$\left.+e^{-(3/16\nu)\vec{q}^2}e^{-(1/16\nu)\vec{q}'^2}+e^{-(1/16\nu)\vec{q}^2}e^{-(3/16\nu)\vec{q}'^2}-2e^{-(3/16\nu)(\vec{q}^2+\vec{q}'^2)}\right]. \quad (32)$$

Here we have defined $\nu=1/b^2$ and $\gamma=1/l_c^2$ [since $l_c \ll b$, the terms of order higher than $(l_c/b)^3$ are neglected].

V. CALCULATION METHOD IN MOMENTUM SPACE

From Eqs. (9) and (17), it can be seen that the dependence of the pion-nucleus optical potential $U=U^{(1)}+U^{(2)}$ on the momentum and energy variables is not simple. To calculate the pion-nucleus T matrix from this nonlocal potential, a relativistic Lippmann-Schwinger equation is solved directly in momentum space. It is interesting to note here that this method provides a significantly different treatment of the nonlocalities of $U^{(1)}$ and $U^{(2)}$ from what has been given in the r-space study.[8,9] In this section we briefly outline our calculation procedure.

The first step is to partial-wave decompose the matrix elements of the optical potential. Using the relation

$$\int d\hat{p}P_{l_2}(\hat{k}'\cdot\hat{p})P_{l_3}(\hat{p}\cdot\hat{k})=\delta_{l_2l_3}\frac{4\pi}{2l_2+1}P_{l_2}(\hat{k}'\cdot\hat{k}) \quad (33)$$

and expanding the ^{4}He form factor determined from electron scattering,[15]

$$F(\vec{q})=\sum_{l_1}F_{l_1}(k',k)P_{l_1}(\hat{k}'\cdot\hat{k}), \quad (34)$$

we find from Eqs. (9) and (28) that

$$\langle\vec{k}'|U^{(1)}|\vec{k}\rangle=\sum_l U_l^{(1)}(k',k)\frac{2l+1}{4\pi}P_l(\hat{k}'\cdot\hat{k}), \quad (35)$$

where

$$U_l^{(1)}(k',k)=(4\pi)^2(A-1)\sum_{\substack{l_1l_2\\l_3l_4}}\frac{(2l_4+1)}{(2l_3+1)}\begin{pmatrix}l & l_1 & l_4\\0&0&0\end{pmatrix}^2\begin{pmatrix}l_2 & l_3 & l_4\\0&0&0\end{pmatrix}^2F_{l_1}(k',k)\int_0^\infty p^2dp\,|\phi(p)|^2t_{l_3l_3}^{l_2}(k',k,p,\omega_0). \quad (36)$$

The partial-wave decomposition of the matrix element of $U^{(2)}$ is obtained by projecting out the angular dependences of the correlation function $C(\vec{q}',\vec{q})$,

$$C(\vec{q}',\vec{q})=\sum_{\substack{l_1l_2\\l_3}}C_{l_2l_3l_1}(k',k,k'')P_{l_1}(\hat{k}'\cdot\hat{k})P_{l_2}(\hat{k}'\cdot\hat{k}'')P_{l_3}(\hat{k}''\cdot\hat{k}). \quad (37)$$

For example, the leading term of $C(\vec{q}',\vec{q})$ can be given in the form

$$C^{(1)}=e^{-(1/16\nu)(\vec{k}+\vec{k}')^2}e^{-(1/8\nu)(\vec{k}'+\vec{k}-2\vec{k}'')^2}$$
$$=\sum_{l_1l_2l_3}[(2l_1+1)(2l_2+1)(2l_3+1)(-1)^{l_1}]I_{l_1}\left(\frac{3kk'}{8\nu}\right)e^{-(1/16\nu)[k^2+k'^2-10kk']}e^{-(1/8\nu)(k'+k-2k'')}$$
$$\times I_{l_2}\left(\frac{k'k''}{2\nu}\right)I_{l_3}\left(\frac{kk''}{2\nu}\right)P_{l_1}(\hat{k}'\cdot\hat{k})P_{l_2}(k'\circ k'')P_{l_3}(\hat{k}''\cdot\hat{k}), \quad (38)$$

where

$$I_l(z)=e^{-z}i_l(z).$$

Substituting Eq. (37) and the off-shell t matrix for $\vec{p}=0$, Eq. (26), into Eq. (17) and using the relation Eq. (33), we get

$$\langle\vec{k}'|U^{(2)}|\vec{k}\rangle=\sum_l U_l^{(2)}(k',k)\frac{2l+1}{4\pi}P_l(\hat{k}'\cdot\hat{k}), \quad (39)$$

where

$$U_l^{(2)}(k',k)=\sum_{\substack{l_1 l_2 l_3\\ \bar{l}_1\bar{l}_2 l'}}(4\pi)^2(2l'+1)\begin{pmatrix}l_1 & \bar{l}_1 & l'\\ 0 & 0 & 0\end{pmatrix}^2\begin{pmatrix}l_2 & \bar{l}_2 & l'\\ 0 & 0 & 0\end{pmatrix}^2\begin{pmatrix}l_3 & l' & l\\ 0 & 0 & 0\end{pmatrix}^2$$

$$\times\int k''^2dk''\frac{t_{\bar{l}_1}(k',k'',\omega_0)C_{l_1l_2l_3}(k',k,k'')t_{\bar{l}_2}(k'',k,\omega_0)}{E-\bar{\epsilon}_0-U_0(E)-(\mu^2+k''^2)^{1/2}-(M_A{}^2+k''^2)^{1/2}+i\epsilon}\,. \quad (40)$$

The resulting matrix element of the optical potential $U_l(k',k)=U_l^{(1)}(k',k)+U_l^{(2)}(k',k)$ is then substituted into a relativistic Lippmann-Schwinger equation:

$$T_l(k',k,E)=U_l(k';k)+\int\frac{U_l(k',k'')T_l(k'',k,E)}{E-(\mu^2+k''^2)^{1/2}-(M_A{}^2+k''^2)^{1/2}+i\delta}k''^2dk''\,, \quad (41)$$

which is then solved by matrix inversion.[3,10] The pion-nucleus differential cross sections are then calculated from the T matrix.

VI. RESULTS AND DISCUSSION

In this section, we first study the effect of the NN correlations on π-^{4}He scattering by comparing the elastic cross sections calculated from $U^{(1)}$ and $U^{(1)}+U^{(2)}$. We then discuss possible ways to resolve the difficulties[5,6] encountered in fitting the low-energy π-nucleus elastic data.

In the calculation of $U^{(1)}$, the nuclear form factor $F_{00}(\vec{q})$ is assumed to be the charge form factor determined by electron scattering.[18] The oscillator parameter needed to calculate the folding integration of Eq. (9) and the NN correlation function Eq. (32) is taken to be $b=1.32$ fm. The π-^{4}He second-order optical potential then depends on two range parameters—the short-range NN correlation length l_c, defined by Eq. (31), and the range parameter β_α of the πN off-shell t matrix. Consistent with the present knowledge of NN correlations, we assume that l_c must be of the order of the radius of the repulsive core of NN interactions. Hence, the range of values considered for l_c will be 0.2 fm $\lesssim l_c \lesssim$ 0.8 fm. The parameter β_α of the πN off-shell t matrix is related to the range of πN interaction. In a simple isobar model,[13] the πN interaction in the P_{33} channel can be fitted with the choice $\beta_\alpha=0.2$ fm^2. Hence, acceptable values of β_α probably lie in the region 0.1 fm$^2\leq\beta_\alpha\leq$ 0.4 fm^2.

The effects of NN correlation on the π-^{4}He elastic differential cross sections, $d\sigma/d\Omega$, are displayed in Figs. 1(a) and 1(b). The solid and dashed curves, respectively, are calculated from $U^{(1)}+U^{(2)}$ and $U^{(1)}$, with parameters $\beta_\alpha=0.2$ fm^2 and $l_c=0.4$ fm. In Fig. 1(a), both $U^{(1)}$ and $U^{(2)}$ are calculated using the factorization approximation [Eq. (11) and Eq. (12)]. This consistent comparison reveals, at least qualitatively, the importance of the effect of $U^{(2)}$ relative to $U^{(1)}$. In Fig. 1(b), we also attempt to compare our theoretical predictions with the experimental data.[16,17] In this regard, the folded $U^{(1)}$, Eq. (9), is used in the calculation in order to partially take into account the important effect of Fermi motion. The results shown in Fig. 1(b) will motivate our later discussion about improving the fit to the data at low energies by phenomenologically adjusting the parameters l_c and β_α of $U^{(2)}$.

We see from Fig. 1 that the effects of NN correlations on $d\sigma/d\Omega$ at low and high energy are different. At $E_\pi=60$ MeV, both the forward and backward differential cross sections are strongly influenced by the NN correlation. Except at angles near 100°, the NN correlations tend to decrease the cross section at $E=60$ MeV by a factor of about 1.5. As the pion energy increases, the qualitative influence of the NN correlations rapidly changes. At $E_\pi\geq 110$ MeV, the NN correlations tend to *significantly increase* the backward cross section by factors of 3 to 8, but increase the forward cross section only slightly. Considering the available low-energy data, the significant decrease of the forward cross section strongly indicates that the NN correlations should be included in all optical potentials for low-energy pion-nucleus scattering. Since the effects of NN correlations become more asymmetric in angles for high energies, measurements of large angle cross sections at higher energies will yield quantitative information on the NN correlation length l_c.

The total and total elastic cross sections as functions of pion energy are also relevant quantities in assessing π-nucleus optical potentials. We therefore show in Fig. 2 the effects of NN correlations on σ_t and σ_{el}. The main effect of NN correlations is to decrease both cross sections at low energies, and to increase them at higher energies. The resonant shape of σ_t accordingly becomes more pronounced. The peak position of σ_{el} is also shifted from ~70 MeV to ~110 MeV. Thus the effects of the NN correlation should be taken into account in any serious attempt to study total cross sections.

The pion elastic scattering wave functions generated from the optical potential are essential in the study of pion-nucleus reactions. It is therefore important to see the effects of NN correla-

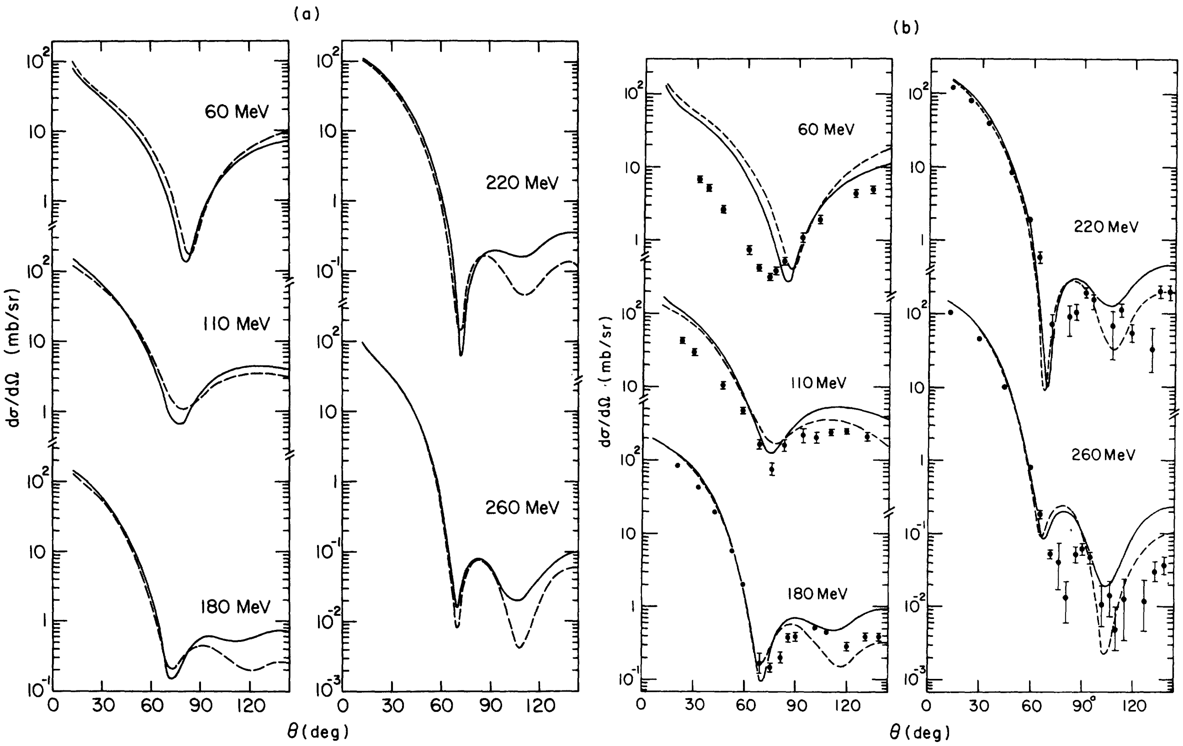

FIG. 1. The π^--^{4}He elastic differential cross sections calculated from $U^{(1)} + U^{(2)}$ (solid curves) and $U^{(1)}$ (dashed curves) are compared. The data are from Refs. (16) and (17). The chosen range parameters of the optical potential are $\beta_\alpha = 0.2$ fm^2 and $l_c = 0.4$ fm. The results of (a) and (b) are calculated based on the factorized and the folded form of $U^{(1)}$, respectively. In both cases, $U^{(2)}$ is calculated using the factorization approximation.

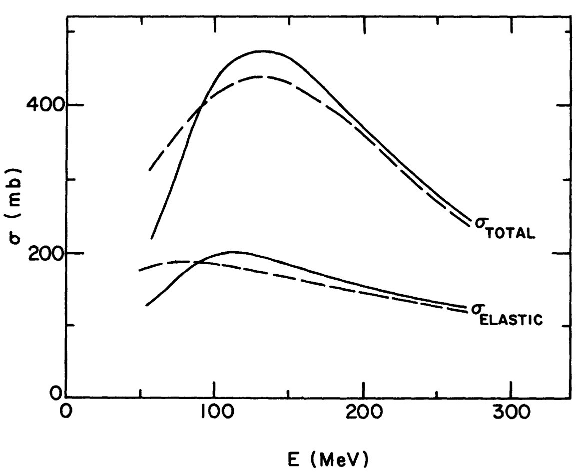

FIG. 2. The π^--^{4}He total and total elastic cross sections calculated from $U^{(1)} + U^{(2)}$ (solid curves) and $U^{(1)}$ (dashed curves) are compared. The chosen range parameters of the optical potential are $\beta_\alpha = 0.2$ fm^2 and $l_c = 0.4$ fm.

tions on the pion wave functions generated from our optical model. In Fig. 3, we compare the p-wave pion-nucleus wave function calculated at $E = 60$ and 220 MeV from $U^{(1)}$ and $U^{(1)} + U^{(2)}$. It is clear that low-energy pion wave functions are strongly modified by the NN correlations. Therefore, the cross sections for reactions involving low-energy pions are likely to be sensitive to the NN correlations. At higher energies, the effect of NN correlations on pion wave functions becomes much smaller but is apparently not negligible.

The cross sections shown in Fig. 1 do not fit the data very well. In particular, the calculated forward cross sections at low energies, $E_\pi = 60$, 110 MeV, are much too high. These results were obtained using somewhat arbitrary values of β_α and l_c. It is interesting to see the extent to which the fit to the experimental data can be improved if these two range parameters are varied.

The dependence of cross sections on l_c is studied by comparing cross sections calculated using various values of l_c ranging from 0.2 fm to 0.8 fm,

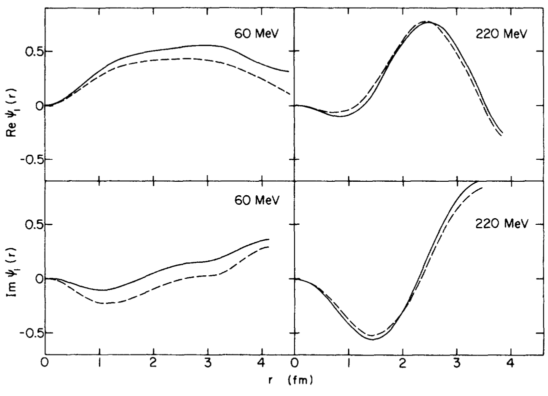

FIG. 3. The p-wave π^- ^{4}He radial wave functions $\Psi_{l=1}(r)$ calculated from $U^{(1)}+U^{(2)}$ (solid curves) and $U^{(1)}$ (dashed curves) are compared. The chosen range parameters of the optical potentials are $\beta_\alpha = 0.2$ fm^2 and $l_c = 0.4$ fm.

while β_α is fixed at 0.2 fm^2. The differential cross sections for $l_c = 0.2$ fm and 0.6 fm are compared in Fig. 4. We see that a large value of l_c gives a better fit to the small-angle cross sections at 60 MeV. However, even in a purely phenomenological calculation, the values of l_c chosen should be subject to the condition that a reasonable fit to the data at higher energies is maintained. At $E_\pi \geq 180$ MeV, we see that a larger value of l_c yields a shallower second minimum in the cross sections at $\theta \simeq 110°$. As l_c is increased beyond 0.7 fm, we find that the second minimum at $E_\pi = 180$ MeV is completely washed out. That is inconsistent with the data (see Fig. 1). The second minimum of $d\sigma/d\Omega$ at $E_\pi = 220$ MeV (260 MeV) disappears as l_c exceeds 0.8 fm (0.95 fm). Hence, the higher-energy data clearly indicate that l_c should not exceed 0.7 fm. This result is also consistent with our remark that the value of l_c must be of the order of the repulsive-core radius of the NN interaction. Thus, although the inclusion of NN correlations tends to yield a better fit to the low-energy data, the present model treatment does not succeed in resolving the discrepancy completely.

We now investigate the possibility of improving the fit to the data by varying the range parameter β_α of the πN interaction. In Fig. 5, we compare the cross sections calculated from $U^{(1)}+U^{(2)}$ using $\beta_\alpha = 0.2$ fm^2 (dashed curve) and $\beta_\alpha = 0.35$ fm^2 (solid curve). The value of l_c is fixed at 0.4 fm. We see that a larger β_α (corresponding to a longer πN interaction range) yields a smaller forward cross section at low energy, $E_\pi = 60$ MeV. At higher energies, the cross sections are less sensitive to the off-shell behavior of the πN t matrix. This result indicates that a larger β_α can drastically improve the fit to the low-energy data, but still maintain the quality of the fit to the data at higher

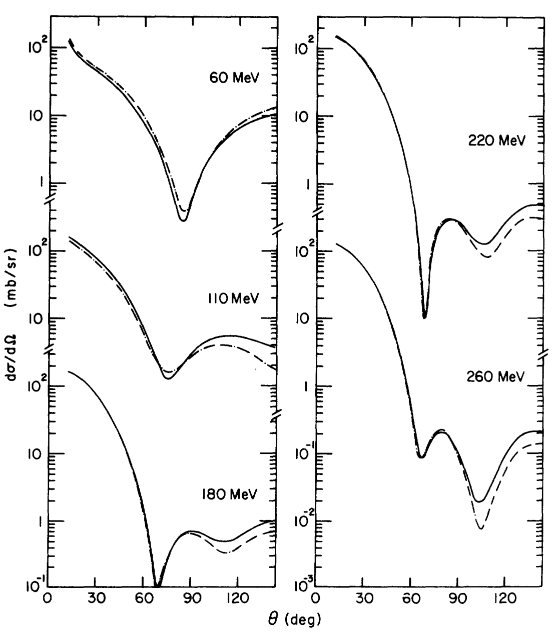

FIG. 4. The π^- ^{4}He elastic differential cross sections calculated from $U^{(1)}+U^{(2)}$ by using $l_c = 0.6$ fm (solid curves) and $l_c = 0.2$ fm (dash-dotted curves) are compared. The other range parameter β_α is fixed to be 0.2 fm^2 for all calculations in this figure.

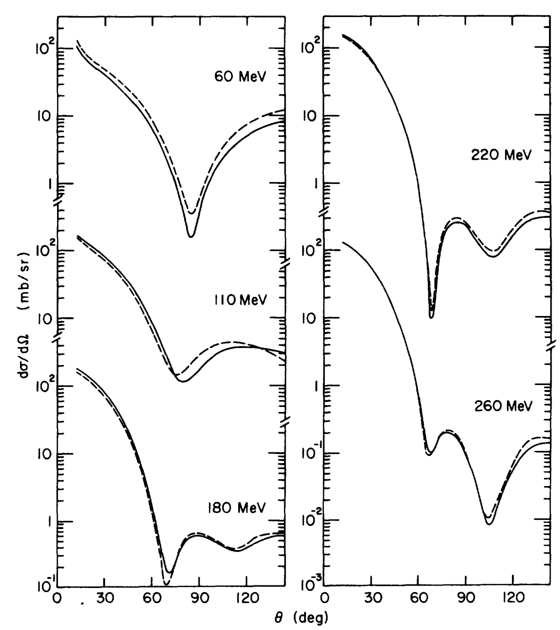

FIG. 5. The π^- ^{4}He elastic differential cross sections calculated from $U^{(1)}+U^{(2)}$ by using $\beta_\alpha = 0.35$ fm^2 (solid curves) and $\beta_\alpha = 0.2$ fm^2 (dashed curves) are compared. The other range parameter l_c is fixed to be 0.4 fm for all calculations in this figure.

energies $E_\pi \geq 180$ MeV. However, unless β_α is changed to a value (~1.2 fm^2) much larger than the value 0.2 fm^2 obtained from the simple isobar model,[13] the forward cross section at $E_\pi = 60$ MeV cannot be reduced to a value close to data. Furthermore, we also find that, with such a large value of β_α, the large angle cross section is roughly a factor of 10 smaller than the data. We conclude that the difficulties encountered in fitting the low-energy data cannot be resolved by simple adjustments in β_α. From the above study of the dependence of $d\sigma/d\Omega$ on the range parameters l_c and β_α, it is clear that other improvements of the pion-nucleus optical potential are necessary to resolve the difficulties at low pion energies.

VII. CONCLUSIONS

We have given a method in momentum space for the calculation of the second-order pion-nucleus optical potential. The influence of *NN* correlations on π-^{4}He elastic scattering are investigated in detail. It is found that *NN* correlations significantly *decrease* the forward cross section at low energy $E_\pi \simeq 60$ MeV, and increase the cross section at higher energy in the region near and beyond the second minimum. The corresponding effects on total cross sections and pion wave functions are also found to be significant, especially at low energies.

In a semiphenomenological approach in which the range parameters l_c and β_α of $U^{(2)}$ are allowed to vary, no fully satisfactory fit to low-energy data can be obtained within our model of $U^{(1)}+U^{(2)}$. On the other hand, the fits to the data at higher energies $E \gtrsim 120$ MeV are satisfactory. This suggests that the expansion of the pion-nucleus optical potential in terms of the πN t matrix converges at energies above 120 MeV. Finally, it is not difficult to generalize our method to calculate the second-order optical potential for heavier nuclei in a form appropriate for the momentum-space study[10] of pion-nucleus reactions involving pions with $E \gtrsim 120$ MeV.

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[1]K. M. Watson, Phys. Rev. 118, 886 (1960); A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).
[2]L. S. Kisslinger, Phys. Rev. 98, 761 (1955); E. H. Auerbach, D. M. Fleming, and M. M. Sternheim, Phys. Rev. 162, 1683 (1967); 171, 1781 (1968); H. K. Lee and H. McManus, Nucl. Phys. A167, 257 (1971).
[3]R. H. Landau, S. C. Phatak, and F. Tabakin, Ann. Phys. (N.Y.) 78, 299 (1973); Phys. Rev. C 7, 1804 (1973); M. G. Piepho and G. E. Walker, *ibid.* 9, 1352 (1974); R. H. Landau, Ann. Phys. (N.Y.) 92, 205 (1975); E. R. Siciliano and G. E. Walker, Phys. Rev. C 13, 257 (1975).
[4]L. S. Celenza, L. C. Liu, and C. M. Shakin, Phys. Rev. C 13, 2451 (1976); 12, 721 (1975); C. Schmit, J. P. Dedonder, and J. P. Maillet, Nucl. Phys. A239, 445 (1975); F. Lenz, Ann. Phys. (N.Y.) 95, 348 (1975).
[5]J. F. Amann, P. D. Barnes, M. Doss, S. A. Dytman, R. A. Eisenstein, and A. C. Thompson, Phys. Rev. Lett. 35, 426 (1975).
[6]T.-S. H. Lee, Phys. Lett. 67B, 282 (1977).
[7]R. H. Landau and A. W. Thomas, Phys. Lett. 61B, 361 (1976).
[8]G. A. Miller and J. E. Spencer, Phys. Lett. 53B, 328 (1974); and Los Alamos report, 1975 (unpublished).
[9]E. Kujawski, Phys. Rev. C 1, 1651 (1970).
[10]T.-S. H. Lee and F. Tabakin, Nucl. Phys. A226, 253 (1974); T.-S. H. Lee and S. Pittel, *ibid.* A256, 509 (1976).
[11]H. Feshbach, A. Gal, and J. Hüfner, Ann. Phys. (N.Y.) 66, 20 (1971).
[12]L. Heller, G. E. Bohannon, and F. Tabakin, Phys. Rev. C 13, 742 (1976).
[13]T.-S. H. Lee and F. Coester (unpublished).
[14]D. H. Herndon, A. Barbaro-Galtieri, and A. H. Rosenfeld, LRL Report No. UCRL-20030 πN, 1970 (unpublished).
[15]R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearin, Phys. Rev. 160, 874 (1967).
[16]K. Crowe, A. Fainberg, J. Miller, and A. Parsons, Phys. Rev. 180, 1349 (1969).
[17]F. Binon, P. Duteil, M. Gouanére, L. Hugon, J. Jansen, J.-P. Langnaux, H. Palevsky, J.-P. Pigneux, M. Spighel, and J.-P. Stroot, Phys. Rev. Lett. 35, 145 (1975).
[18]R. F. Frosch, J. S. McCarthy, R. E. Rand, and M. R. Yearian, Phys. Rev. 160, 874 (1967).
[19]The necessity of using the folded form of $U^{(1)}$, Eq. (9), to study the low-energy π-^{4}He scattering has been discussed by Lee in Ref. (6).
[20]Note that $\bar{\epsilon}_0$ in Eq. (16) is the average nuclear excitation energy in the nuclear c.m. frame. Since we are working in the π-nucleus c.m. frame, the nucleus c.m. energy has been explicitly written as $(M_A{}^2+k^2)^{1/2}$ for a π-nucleus relative momentum $\vec{k}$.
[21]B. Bakamjian and L. H. Thomas, Phys. Rev. 92, 1300 (1953); F. Coester, Helv. Phys. Acta 38, 7 (1965); L. L. Foldy and K. A. Krajcik, Phys. Rev. Lett. 18, 1025 (1974).
[22]Equation (20) for $\vec{\kappa}$ is obtained from Eq. (1) of Ref. (12) by using approximations $H_0 \simeq E_\pi(k)+m$ and $h_0 \simeq \{[m+E_\pi(k)]^2-k^2\}$ $[1+\cdots]$.
[23]Our choice of ω_0 is related to a naive three-body model involving π, N, and a "frozen" nuclear core of the $(A-1)$

nucleons. The recoil energy of the nuclear core is ignored. A precise definition of ω_0 should be given by a chosen model space in which the impulse approximation can be deduced from the bound operator τ. In this way, one can then define precisely the corrections to the impulse approximation for a given choice of ω_0. This problem is under investigation and will be published elsewhere.