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Scattering of pions by deuterons at low and medium energies*

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We apply the multiple scattering method to calculate the differential cross sections for elastic πd scattering in the low and medium energy regions, showing the importance of the proper treatment of the kinematics in the two body collision. Our results are compared with all existing experimental data in these energy regions. We show that the large angle elastic scattering at medium energies is highly sensitive to details of the deuteron structure and of the calculation procedure, and provides an excellent ground to study the properties of the meson-deuteron and of the off-shell meson-nucleon interactions.

[NUCLEAR REACTIONS Pion deuteron elastic scattering. Multiple scattering calculation; Fermi motion, kinematical ambiguities, and off-energy-shell effects. Evaluation of $d\sigma/d\Omega$, low and intermediate energies.]

I. INTRODUCTION

The aims in the study of pion nuclear systems are simultaneously those of understanding the nature of the few-body dynamics, of investigating the nuclear structure, and of obtaining more information on the relevant two-particle interaction than what can be derived from direct two-body experiments. The difficulties one has to face when dealing with all these aspects simultaneously are sometimes beyond control, and it has not been always possible to develop a critical feeling for the value and limitations of the methods of analysis and calculation. Because of its comparative simplicity in all these aspects, the study of the pion-deuteron system is of fundamental importance in pion nuclear physics. Without a previous good description of the π-d system, little hope may exist that the behavior of more complicated systems can be understood. Thus, every effect towards a better understanding of pion-deuteron processes is justified.

In spite of this comparative simplicity, the description of the π-d interaction in the low and intermediate energy regions is still far from satisfactory. From a theoretical point of view, the π-d system is in a privileged position, if compared with other π-nuclei systems, as the Faddeev equations provide the basis for an exact formulation of three-body problems. Several attempts have been made[1-6] to solve the Faddeev equations for the π-d scattering. In particular, recent data[7] on pion absorption and elastic and breakup scattering at 47.5 MeV have been well fitted by these calculations. However, as soon as the energy goes above a limit which is still rather low, these calculations based on direct solution of Faddeev equations face limitations of practical nature, due to the large number of coupled angular momentum states involved. Fortunately at these higher energies the rather simple and model independent multiple scattering calculations are able to give a fairly good description of π-d scattering.

Several authors[8] have applied the multiple scattering method to evaluate pion deuteron cross sections. In general these calculations include terms representing single and double scattering of the incident pion. It is assumed that binding corrections, complicated three-body mechanisms and other effects, all difficult to evaluate quantitatively without use of particular and arbitrary models, give comparatively small contributions. Then no detailed dynamical knowledge of the system is required, and the calculation is based almost entirely on directly observable properties of the intervening two-body systems. However, important technical details, such as Fermi motion dependence of the amplitudes, structure of the deuteron, nucleon recoil, and so on, are not treated uniformly by the several authors. Also, each author considers only one, or a limited range of values of the energy, and comparing the results we observe that the performance of the calculations varies strongly with the energy. Besides that, the existing data are scarce, and of low accuracy, and must be used all as a whole if a meaningful analysis is to be made.

The purpose of the present work is to investigate the applicability of the multiple scattering method to elastic pion deuteron scattering at low and medium energies, confronting the results of calculations with all available experimental data in this energy range.

Our calculations include double scattering terms, allowing for nucleon recoil, and including both δ

function and the principal value parts originating from the pole in the propagator. We have observed that the corrections to the differential cross section arising from the double scattering terms are never large, so that it is unnecessary to include Fermi motion dependence in these terms. Fermi-motion effects are taken into account in the evaluation of the single scattering terms, and shown to be important, particularly in large angle scattering.

The inclusion of Fermi motion effects enhances the influence of the off-the-energy shell behavior of the two-body amplitudes and of the kinematical ambiguities characteristic of the impulse approximation and multiple scattering calculations. We have concentrated effort in the discussion of these points, and in particular we compare the results obtained using different prescriptions for the value of the energy parameter to be used in the definition of the pion-nucleon amplitude. We show that the proper treatment of π-d scattering in the impulse approximation as a three-particle system eliminates the ambiguity in the definition of the collision energy for the pion-nucleon system. This treatment is made in the framework of the multiple scattering series derived from the Faddeev equations, and is shown to lead to values of the differential elastic π-d cross sections which give better fitting to the experimental data in the medium energy region than what is obtained following other usual kinematical prescriptions.

In Sec. II we discuss the structure of the multiple scattering series, and mention some of the problems related to its possible relativistic extensions. In Sec. III we compare different prescriptions for the treatment of the kinematical and dynamical arbitrariness occurring in the explicit evaluation of terms of the multiple scattering series. Section IV presents the essential ingredients of the practical calculation, with care given to relativistic effects resulting from the comparatively low value of the pion mass. In Sec. V our calculations are confronted with the whole experimental data on π-d elastic scattering, with the purpose of investigating the conditions of applicability of the multiple scattering method, and of obtaining information on the influence of the technical aspects and details of the calculation. The overall output of this analysis is summarized and discussed in Sec. VI, where we mention some possible causes of observed discrepancies, and indicate lines for further study and development.

II. FADDEEV EQUATIONS AND THE MULTIPLE SCATTERING SERIES

The exact three-body amplitude for π-d scattering given by Faddeev equations can be expanded in terms of the two-particle collision operators, in the form of a multiple scattering series. In the explicit evaluation of the terms of the expansion, care must be taken when expressing the matrix elements of operators defined in the three-particle Hilbert space in terms of the usual two-body matrix elements.

Let the three particles be labeled by the indices 1, 2, and 3 with momenta $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ in the lab system of reference. Let us select a pair (2,3), and treat the particle 1 separately. We define the new momentum variables

$$\vec{K} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 ,$$
$$\vec{k}_1 = (m_3\vec{p}_2 - m_2\vec{p}_3)/(m_2+m_3) ,$$
and
$$\vec{q}_1 = \frac{(m_2+m_3)\vec{p}_1 - m_1(\vec{p}_2+\vec{p}_3)}{m_1+m_2+m_3} , \tag{1}$$

where $\vec{K}$ is the total momentum of the system, $\vec{k}_1$ is the internal momentum in the (2,3) pair relative to its center of mass, and $\vec{q}_1$ is the momentum of particle 1 with respect to the center of mass of the whole system. Defining the reduced masses

$$\mu_1 = m_2 m_3/(m_2+m_3) \tag{2}$$

and

$$M_1 = m_1(m_2+m_3)/(m_1+m_2+m_3)$$

the kinetic energy of the three particles in the nonrelativistic case can be written in the form

$$H_0 = \frac{K^2}{2(m_1+m_2+m_3)} + \frac{k_1^2}{2\mu_1} + \frac{q_1^2}{2M_1} . \tag{3}$$

The pair of particles arbitrarily selected can be any of the three possible choices, and new (not independent) sets of variables can be defined for each case. Each of these choices is usually called a channel.

Let us call v_1 the potential acting between particles 2 and 3, v_2 the potential acting between 1 and 3, and so on. The Hamiltonian of the system in the center-of-mass system ($\vec{K}=0$) is

$$H = \frac{k_\alpha^2}{2\mu_\alpha} + \frac{q_\alpha^2}{2M_\alpha} + \sum_{\beta=1}^{3} v_\beta = H_0 + \sum_{\beta=1}^{3} v_\beta , \tag{4}$$

where the index α ($\alpha = 1,2,3$) indicates the channel which has been selected. An important concept is that of the channel Hamiltonian

$$h_\alpha = (k_\alpha^2/2\mu_\alpha) + (q_\alpha^2/2M_\alpha) + v_\alpha , \tag{5}$$

where there appears interaction only between the two particles forming the pair in channel α. The so-called channel resolvent is

$$g_\alpha(z) = (z - h_\alpha)^{-1} . \tag{6}$$

We are dealing with a three-particle system, and these operators are defined in the Hilbert space of three particles. Now, if our operators are channel operators, that is, if they depend on the relative coordinates of only two particles, their matrix element between free particle states can be expressed in terms of operators defined in the two-body Hilbert space. We call

$$\hat{h}_\alpha = \frac{k_\alpha^2}{2\mu_\alpha} + v_\alpha \tag{7}$$

the two-body Hamiltonian in channel α, and

$$\hat{g}_\alpha(z) = (z - \hat{h}_\alpha)^{-1} \tag{8}$$

the corresponding two-body resolvent. We can then reduce a three-body channel matrix element writing

$$\begin{aligned}\langle \vec{k}_\alpha \vec{q}_\alpha | g_\alpha(z) | \vec{k}'_\alpha \vec{q}'_\alpha \rangle &= \langle \vec{q}_\alpha | \vec{q}'_\alpha \rangle \cdot \langle \vec{k}_\alpha | \left[z - \left(\frac{k_\alpha^2}{2\mu_\alpha} + \frac{q_\alpha^2}{2M_\alpha} + v_\alpha \right) \right]^{-1} | \vec{k}'_\alpha \rangle \\ &= \delta(\vec{q}_\alpha - \vec{q}'_\alpha) \langle \vec{k}_\alpha | \left[\left(z - \frac{q_\alpha^2}{2M_\alpha} \right) - \hat{h}_\alpha \right]^{-1} | \vec{k}'_\alpha \rangle \\ &= \delta(\vec{q}_\alpha - \vec{q}'_\alpha) \langle \vec{k}_\alpha | \hat{g}_\alpha \left(z - \frac{q_\alpha^2}{2M_\alpha} \right) | \vec{k}'_\alpha \rangle . \end{aligned} \tag{9}$$

The shift in the value of the argument of the resolvent is very important for us here.

Now let us write the Faddeev equations. The full three-body transition matrix $T(z)$ is written as a sum

$$T = T_1 + T_2 + T_3, \tag{10}$$

where T_1, T_2, and T_3 satisfy the coupled equations

$$\begin{aligned} T_1 &= t_1 + t_1 g_0 (T_2 + T_3), \\ T_2 &= t_2 + t_2 g_0 (T_1 + T_3), \\ T_3 &= t_3 + t_3 g_0 (T_1 + T_2). \end{aligned} \tag{11}$$

Here

$$g_0(z) = (z - H_0)^{-1} \tag{12}$$

is the resolvent for three free particles, and

$$t_\alpha(z) = v_\alpha + v_\alpha g_\alpha(z) v_\alpha \tag{13}$$

are channel α transition operators acting in the three-particle Hilbert space, and satisfying the reduction relation

$$\langle \vec{k}_\alpha \vec{q}_\alpha | t_\alpha(z) | \vec{k}'_\alpha \vec{q}'_\alpha \rangle = \delta(\vec{q}_\alpha - \vec{q}'_\alpha) \langle \vec{k}_\alpha | \hat{t}_\alpha \left(z - \frac{q_\alpha^2}{2M_\alpha} \right) | \vec{k}'_\alpha \rangle, \tag{14}$$

where now $\hat{t}_\alpha$ is the transition operator in the two-body Hilbert space for the two particles forming a pair in channel α.

The Faddeev version of the multiple scattering series is obtained in an obvious way by iterating the coupled integral equations written above. For the elastic scattering of particle 1 by the (2,3) bound pair the transition operator $T(z)$ can be expanded in the form of a multiple scattering series

$$\begin{aligned} T(z) &= t_2(z) + t_3(z) + t_2(z) g_0(z) t_3(z) \\ &\quad + t_3(z) g_0(z) t_2(z) + \cdots , \end{aligned} \tag{15}$$

where the interpretation of the terms is the usual one, and all operators are defined in the three-particle Hilbert space.

Care must be exercised when evaluating explicitly the matrix elements of the terms above taken between states of three free particles, so that the reduction to matrix elements of two-body operators can be made with the appropriate shift corresponding to the energy of the particle which, in each term, does not participate in the process.

Let E be the value of the total kinetic energy of the particle-deuteron system in the center-of-mass system, $\vec{P}$ the nucleon (particle 1) lab momentum, and $\vec{p}$ ($\vec{p}'$) the initial (final) meson (particle 3) momentum in the lab system. For the term with particle 2 as spectator,

$$\langle \vec{P}', -\vec{P}', \vec{p}' | t_2(E) | \vec{P}, -\vec{P}, \vec{p} \rangle = \delta(\vec{q}'_2 - \vec{q}_2) \delta(\vec{K}' - \vec{K}) \langle \vec{k}'_1 | \hat{t}_2 \left(E - \frac{q_2^2}{2M_2} \right) | \vec{k}_1 \rangle, \tag{16}$$

where $\vec{K}$ ($\vec{K}'$) is the total initial (final) momentum of the three particles, $\vec{q}_2$ ($\vec{q}'_2$) is the initial (final) momentum of the spectator with respect to the center of mass, $\vec{k}_1$ ($\vec{k}'_1$) is the initial (final) momentum of the meson relative to the center of mass of the interacting meson-nucleon system, M_2 is given by

$$M_2 = m_N (m_N + m_\pi)/(2m_N + m_\pi), \tag{17}$$

and $\hat{t}_2$ is the usual two-body collision operator.

The argument of the two-body transition operator $\hat{t}_2$ then reads

$$E - \frac{q_2^2}{2M_2} = E - \frac{[(2m_N + m_\pi)\vec{P} + m_N \vec{p}]^2}{2m_N(m_N + m_\pi)(2m_N + m_\pi)} . \tag{18}$$

In the evaluation of the double scattering terms, one introduces complete sets of three free particle states between the operators, and the reduction to the two-body operators takes place in a manner analogous to that described above.

We must remark that pions are relativistic even at rather low energies, while the formalism developed above is completely nonrelativistic. However, the only result of consequence in our computation is Eq. (16), and the approximation involved in its use is expected to be very reasonable, as the spectator particle, whose energy is subtracted from the total energy available, is always a nonrelativistic nucleon.

There are generalizations of Faddeev equations to the relativistic case,[9] which give rise to a multiple scattering series which is of the same structure as Eq. (15), but where the Hamiltonian H_0 appearing in the three-particle resolvent $g_0(z)$ is not of the same simple form as given in Eq. (3). The essential problem comes from the fact that in the relativistic generalization of Faddeev equations obtained from Bethe-Salpeter equation, the denominator in the three free particle resolvent is not linear in the energy of the particles.[9] Then the spectator particle cannot be removed from the matrix element of the collision operator in three-body Hilbert space. Consequently we are not able to write a simple expression to reduce the three-body to a two-body matrix element, but for the purpose of performing practical calculations we may adopt the nonrelativistic Faddeev prescription such as given by Eq. (16) to fix the value of the energy parameter to be used in the two-body transition operator.

Starting from a relativistic Schrödinger equation we can write a Watson multiple scattering series such that the denominator occurring in the propagator is linear in the energy of the three particles.[10] An analogous series can be obtained from Feynman diagram rules, with prescriptions to relate the vertex functions to the elementary amplitudes and to the nuclear wave function.[11] However, these approaches do not solve relativistically the problem of defining the energy for the two-body collision operators in the terms of the series, and a recipe such as the one mentioned above must be adopted.

III. KINEMATICAL AND DYNAMICAL AMBIGUITIES IN THE EVALUATION OF TWO-BODY AMPLITUDES: SOME SELECTED PRESCRIPTIONS

In the previous section we have discussed in some detail the kinematical structure of the terms of the multiple scattering series as derived from the Faddeev equations, considering pion deuteron scattering as a three-body problem. We have shown how, in the nonrelativistic case, a prescription is obtained for the value of the energy parameter to be used in the two-body matrix elements. There appears a shift relative to the total energy of the system, which is due to the amount of energy carried by the particles behaving as a spectator in each two-body collision. For our future reference we call that *prescription* A.

According to the intuitive ideas supporting the impulse approximation calculations, the deuteron is viewed as a wave packet of two nucleons, with a momentum distribution determined by the deuteron wave function. The incident particle collides with one of the nucleons at a time, while the other nucleon remains as a spectator. These ideas have led to the most usually adopted prescription for the definition of the kinematics governing the two-body collision, which we call here *prescription* B. It assumes that the incident particle collides with an on-shell physical nucleon. If Fermi-motion effects are taken into account, for each value and each direction of the nucleon momentum inside the deuteron, a different value is used for the relative energy between the incident particle and the nucleon.

Another interesting way to solve the ambiguity has been suggested by the experiments in which there is a breakup of the deuteron, and where an identification has been made between the spectator and the struck nucleons. These experiments show that the spectator nucleon recoils with a momentum distribution which is, in good approximation, the same as expected from the deuteron wave function. We are thus led to the assumption, here called *prescription* C, that the spectator nucleon behaves from beginning to end as an on-shell particle. The nucleon which participates in the collision must then be treated as an unphysical particle in the initial and final states. To fulfill energy conservation, the energy of the participant nucleon is equal to the deuteron mass m_d minus the energy $m_N + P^2/2m_N$ carried by the spectator nucleon, where P is the Fermi-motion momentum. Thus the participant nucleon behaves as having an effective mass m_{eff} such that

$$m_{\text{eff}} + P^2/2m_{\text{eff}} = m_d - m_N - P^2/2m_N . \tag{19}$$

The value of m_{eff} depends on the momentum P. The relative energy in the center-of-mass frame is evaluated applying Lorentz transformation to the laboratory system motion of an incident physical meson, and a particle of mass m_{eff} and momentum $\vec{P}$.

In a certain sense, prescriptions B and C exchange the roles of the spectator and of the struck nucleons. At zero Fermi momentum the two prescriptions nearly coincide, as then $m_{\text{eff}} = m_d - m_N \approx m_N$.

We have thus described three ways of defining the value of the energy to be used in the evaluation of the off-shell matrix element of the two-body amplitude. Prescription B has been often used in multiple scattering calculations of πd processes,[8] while prescription C has only been used in the analysis of pion deuteron breakup scattering.[12]

While prescription B seems to be intuitively appealing, according to the ideas giving support to impulse approximation calculations, and prescription C finds support in the experimental observation of spectator spectra in breakup processes, prescription A has a safer theoretical basis. As

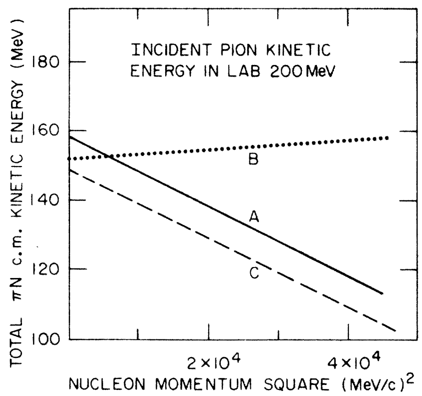

FIG. 1. Values of the total kinetic energy (rest masses excluded) in the πN c.m. system, according to prescriptions A, B, and C described in the text, against Fermi-motion momentum squared. The energy values are averaged over all directions for a given magnitude of Fermi momentum. The diagram is drawn for incident pions of 200 MeV kinetic energy.

we are dealing with the evaluation of off-energy-shell matrix elements, which are not intuitive quantities, we should rather rely on the more formal approach. The nucleons are not free physical particles inside the deuteron, and prescription A tells us how to take partially into account the effect in our calculation of the presence of two particles in the target nucleus. As shown in Sec. V, the kinematical prescription adopted can have a strong influence in the results of the calculation.

In Fig. 1 are shown the values of the kinetic energy (excluded rest masses) in the center-of-mass system of the two colliding particles, as a function of the Fermi-motion momentum. The relative energy depends not only on the magnitude, but also on the direction of the Fermi-motion momentum, and the lines drawn represent the average value over all directions for a fixed magnitude P of the momentum. In prescription B, the value plotted for the energy does not depend much on the value of the Fermi momentum, and remains almost constant, while in cases A and C the variation is strong. We can thus expect that Fermi-motion effects may be stronger in cases A and C than in case B. These predictions have been confirmed by our calculations, covering the interval of energies from zero up to about 400 MeV. A main observation is that Fermi-motion effects are extremely important for the correct evaluation of large angle scattering, because the strong cancellations which occur in the evaluation of the cross sections are sensitive to the proper account of the variation of the values of the integrand as a consequence of these effects. A factor of up to 4 in the differential cross section can appear in the backward angles as the Fermi-motion effect is switched on and off. On the other hand, we may expect that in the cases of prescriptions A and C the calculations are more sensitive to changes in the large momentum tail of the deuteron wave function than they are in case B.

In the usual multiple scattering calculation the binding forces in the deuteron are ignored, and the values of the relative momentum used in the evaluation of the two-body matrix elements are not the same in the initial and final states, and are not related to the energy of the whole πd system. Thus we deal essentially with off-the-energy shell matrix elements of two-body transition operators. In general, these matrix elements are not known, and the values to be used have to be guessed, following some chosen prescription, from the on-the-energy-shell values which are obtained from direct two-body experiments.

For each partial wave we must evaluate an off-shell amplitude $\langle k'|f_l(y)|k\rangle$ where k, k' are the initial and final relative momenta of the colliding pair, and y is the energy parameter defined according to each of the prescriptions adopted. These three quantities are not related among themselves through the usual on-shell relation. Integration is made over all initial and final values of the nucleon momentum and the values of k, k', and y vary rather disconnectedly. We must define the matrix element as a function of these variables.

A simple and direct way, which we have used in our computations, consists in writing the separation

$$\langle k'|f_l(y)|k\rangle = \frac{1}{(kk')^{1/2}} b_l(y), \qquad (20)$$

where $b_l(y)$ is defined as the value of

$$(kk')^{1/2}\langle k'|f_l(y)|k\rangle$$

calculated for k and k' on the energy shell defined by the value of y. As y is fixed in a unique way for each of the prescriptions defined, the computational procedure becomes completely specified. This form of off-the-energy-shell extrapolation is suggested by writing the on-shell scattering amplitude for a given partial wave in the form

$$\langle k|f_l(y)|k\rangle = (1/k)\sin\delta_l(y)\exp i\delta_l(y) \qquad (21)$$

and letting $k \rightarrow (kk')^{1/2}$ when the initial and final values of k do not coincide. We have verified in our actual computations of the πd cross section that, due to the integrations performed, which smooth the effect of the separate dependence, it makes almost no difference to write $(kk')^{1/2}$ or k in Eq. (20).

Another possible method of specifying the off-the-energy shell extrapolation of the scattering amplitudes consists in using a separable potential

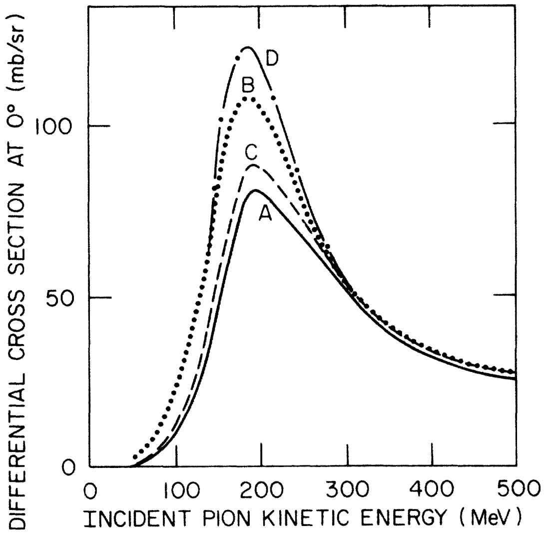

FIG. 2. Forward differential cross section for πd elastic scattering, with the Coulomb interaction switched off, comparing results obtained with kinematical prescriptions described in the text. The solid curve represents results obtained with a shift in the value of the energy parameter as determined by the reduction from three-particle to two-particle matrix elements (prescription A). The dotted curve is obtained with the struck nucleon on shell (prescription B), and the dashed curve shows the results obtained with on-shell spectator nucleon (prescription C). The dashed-dotted curve (D) is obtained by eliminating Fermi motion. The peak in the solid curve is displaced about 6 MeV towards higher energies as compared with the other cases.

model for each partial wave amplitude.[3,5,6,12]

As a practical example of comparing the results of calculations made using the three above mentioned prescriptions for the value of the energy in the two-body collision, we show in Fig. 2 the curves for the purely nuclear (Coulomb interaction switched off) forward differential cross section for elastic πd scattering as a function of the meson incident energy. We also include in the figure the results of a calculation without accounting for Fermi-motion effects. In forward scattering, as in the value of the total cross section, Fermi-motion effects are comparatively much less important than in backward scattering. Near the P_{33} resonance the influence of Fermi motion in the forward cross section can be about 35% in the case of prescription C and 15% in prescription B.

We see in Fig. 2 that the position of the peak due to the P_{33} resonance is nearly the same in all cases, with a shift of about 6 MeV towards higher values of the energy observed in the case of prescription A. This is an important, although rather obvious, result, as we expect a displacement to occur in the position of the peak as a consequence of the shift in the value of the energy caused by the reduction from three-body to two-body operators. This result is also true of the total cross section, as the elastic πd scattering is almost completely forward. It is interesting to remark that larger shifts are expected to occur in the scattering by heavier nuclei.

We must call attention to the result, shown in the figure, that the values of the total and forward cross sections, evaluated with prescription A in the resonance region, are remarkably lower than the values obtained in the other two cases.

We wish here to remember a remark made by Brayshaw,[2] that the peak observed in the cross section for πd scattering does not correspond to a resonance in the usual sense (a zero in the real part of the πd amplitude).

We must remark that the influence of Fermi motion and of the treatment of the kinematical ambiguities on the forward and total cross sections tends to disappear at higher energies, as all curves then become superposed. As we shall see, this stability is in contrast to what happens in large angle scattering.

We must also remark that the values obtained for the forward differential cross section do not depend much on details of deuteron structure (e.g., the amount of D wave) and of the interaction mechanism, and it is completely dominated by the single scattering term of the multiple scattering series.

IV. INGREDIENTS OF THE CALCULATION

The relativistic kinematical variables we use to describe the πd scattering in the impulse approximation have been defined previously.[12,13] We have now extended the calculation to include the deuteron D-wave component, and the contribution of the double scattering terms, including in these terms both the pole and the principal value contributions arising from the propagator.

Let us write the elastic πd amplitude in the lab system separating the single $T_{fi}^{(S)}$ and double scattering $T_{fi}^{(D)}$ contributions in the form

$$T_{fi}(\vec{p},\vec{p}\,')=T_{fi}^{(S)}(\vec{p},\vec{p}\,')+T_{fi}^{(D)}(\vec{p},\vec{p}\,'), \tag{22}$$

where i and f represent, respectively, the initial and final states of the system, and $\vec{p}(\vec{p}\,')$ is the initial (final) pion momentum. The differential cross section in the lab frame is given by

$$\frac{d\sigma}{d\Omega}=\frac{p'^3E_d'}{p(W_Lp'-E_{p'}\vec{p}\cdot\vec{p}\,')}\times\left(\frac{1}{3}\right)\sum_{i,f}|T_{fi}|^2, \tag{23}$$

where the sum extends over deuteron polarization states, E_d' is the deuteron final energy, $E_{p'}$ is the total energy for a meson of momentum $\vec{p}\,'$, and $W_L=m_d+E_p$ is the total energy of the system in the lab frame of reference.

Let us write the operator representing the deuteron wave function in the two-nucleon spin space in the form

$$\psi(\vec{P})=\phi_0(\vec{p}) - \frac{1}{\sqrt{8}}\phi_2(\vec{p})\left(\frac{3}{P^2}\vec{\sigma}_p\cdot\vec{P}\vec{\sigma}_n\cdot\vec{P}-\vec{\sigma}_p\cdot\vec{\sigma}_n\right), \tag{24}$$

where the indices p and n refer to proton and neutron, $\vec{P}$ is the Fermi momentum, and ϕ_0 and ϕ_2 represent, respectively, the S- and D-wave parts of the deuteron wave function.

We can then write for the matrix element representing the single scattering contribution

$$T_{fi}^{(S)}(\vec{p},\vec{p}')=\int d_3\vec{P}\sum_{j,m}\psi^\dagger_{fm}(\vec{p}+\tfrac{1}{2}\vec{\Delta}) \times \mathcal{T}_{mj}(\vec{P},\vec{p},\vec{p}+\vec{\Delta})\psi_{ji}(\vec{P}). \tag{25}$$

In this expression

$$\mathcal{T}_{mj}(\vec{P},\vec{p},\vec{p}+\vec{\Delta})=\sum_{\mu\nu\mu'\nu'}C^{(1)*}_{m\mu'\nu'}C^{(1)}_{j\mu\nu}\langle\mu',\vec{p}+\vec{\Delta},\vec{P}-\vec{\Delta}\,|t_{\pi p}|\,\mu,\vec{p},\vec{P}\rangle\delta_{\nu'\nu}+(\text{proton}\to\text{neutron}), \tag{26}$$

where $\vec{\Delta}=\vec{p}'-\vec{p}$, μ (or μ'), and ν (or ν') label, respectively, the proton and neutron spin states, and $C^{(1)}_{j\mu\nu}$ is the Clebsch-Gordan coefficient coupling two spin $\frac{1}{2}$ particles of z spin components μ and ν to form a spin 1 state with a z component given by the value of j. The quantities $t_{\pi p}$ and $t_{\pi n}$ stand, respectively, for the pion proton and pion neutron collision operators, and $\psi_{ji}(\vec{P})$ represents the matrix element of $\psi(\vec{P})$ evaluated between deuteron polarization states of z spin components j and i.

In the whole region of interest for our calculations, the double scattering terms give small contributions as compared with the single scattering terms. We thus find that neglecting Fermi-motion effects in the amplitudes and treating the deuteron as a pure S state in the double scattering contributions are very reasonable approximations. This assumption simplifies substantially the calculation, and we can write

$$T_{fi}^{(D)}(\vec{p},\vec{p}') =\int d_3\vec{p}''\frac{\rho_0(\vec{p}''-\vec{p}+\Delta/2)D_{fi}(\vec{p},\vec{p}',\vec{p}'')}{E_p-E_{p''}-(\vec{p}-\vec{p}'')^2/2m_N+i\epsilon}, \tag{27}$$

where

$$\rho_0(\vec{\Delta})=\int d_3\vec{P}'\psi_0(\vec{P}'+\vec{\Delta})\psi_0(\vec{P}') \tag{28}$$

is the (pure S-wave) deuteron form factor and

$$D_{fi}(\vec{p},\vec{p}',\vec{p}'') =\sum_{\alpha\beta}\sum_{\mu\mu'\nu\nu'}C^{(1)*}_{f\mu'\nu'}C^{(1)}_{i\mu\nu} \times\langle\mu',\vec{p}',\vec{p}''-\vec{p}'\,|t_{\pi\alpha}|\,\mu,\vec{p}'',0\rangle \times\langle\nu',\vec{p}'',\vec{p}-\vec{p}''\,|t_{\pi\beta}|\,\nu,\vec{p},0\rangle. \tag{29}$$

The sums over the isospin indices α,β take into account all possible intermediate charge states, and include the charge exchange contribution. To deal properly with the isospin variables, we must include isospin dependence in the definition of the collision operators, and isospin quantum numbers in the definition of the states. As the deuteron is an isospin zero state the double charge exchange term comes out with opposite sign relative to the charge preserving double scattering contribution.

To accelerate the convergence to zero of the integrand in Eq. (27) as $p''\to\infty$ we have adopted the nonrelativistic form for the pion energy $E_{p''}$ in the propagator, so as to have a p''^2 behavior in the denominator, instead of a linear p''. This procedure may be considered as a prescription for the off-energy-shell behavior of the two-body amplitudes, and does not have a fundamental influence in our calculation, as the double scattering contributions to the differential cross sections are very small.

At this point we may remark that at very high energies, where the eikonal limit is reasonable, it is known that the principal value part of the double scattering integral is cancelled by higher order terms.[14]

V. RESULTS OF CALCULATIONS AND CONFRONTATION WITH DATA

In what follows, we present our results of multiple scattering calculations, comparing the different prescriptions for the kinematical variables used in the evaluation of the two-body amplitudes. The calculations include double scattering terms, allowing for nucleon recoil and including both the δ function and the principal value parts originated from the pole in the propagator. Corrections to the differential cross section arising from the double scattering terms never amount to more than 10% in the whole range of energies where the multiple scattering calculation makes sense (let us say above 85 MeV). It is thus unnecessary to include Fermi-motion dependence in the double scattering terms, which brings an important simplification in the numerical computations. The comparatively small contribution obtained for the double scattering terms makes us confident that higher order terms of the series can be neglected. The calculations account for Fermi-motion effects in the single scattering terms, and are made with a Moravcsik wave function, with a 7% d-wave component.[15] For the pion-nucleon phase shifts we have used the parametrization of Roper, Wright, and Feld,[16] except at 47.5 MeV, where CERN phase shifts were used.

As explained in the Introduction, the main purpose of the present work is to test calculations with the multiple scattering method against the available experimental information in the low and medium energy region. Unfortunately the data on πd elastic scattering in the low and medium energy regions are scarce, many rather old, with low statistics and large error bars. The only new data obtained in the recent years come from the experiment at 47.5 MeV,[7] and the expected results of the measurements at 347 MeV/c (234.4 MeV kinetic energy) and 443 MeV/c (324.9 MeV kinetic energy) performed by a collaboration of the groups at the University of Virginia and at Los Alamos Scientific Laboratory.[17] There are reported experimental results on the elastic πd differential cross section for incident pions at 61,[18] 85,[19] 140,[20,21] 182,[22] 224,[23] 256,[24] 300,[25] and 330 MeV.[26] For large angle scattering, between 140 and 180° in the laboratory system, there are results obtained by Schroeder *et al.*[27] at 375.7, 412.4, and 469.6 MeV and higher energies. The work of Gabathuler *et al.*[24] also includes measurements of the backward cross section at 160° lab scattering angle for incident pions of 141, 163, 185, and 208 MeV.

Angular distributions

In Figs. 3 to 6 we present the results of our calculations of angular distributions for πd elastic scattering for pion lab kinetic energies ranging from 47.5 to 324.9 MeV. At all energies except the lowest ones, there is a large variation in magnitude in the differential cross section between the forward and the backward directions, and in order to show more clearly the behavior of the curves and of the data we have used separate scales for the forward and the backward angles. We have used a highly expanded scale for the differential cross sections at large angles so as to exhibit rather than to hide discrepancies, and thus avoid the inconveniences of a logarithmic scale. This is important, as it is in the large angle elastic scattering that the effects of the deuteron structure

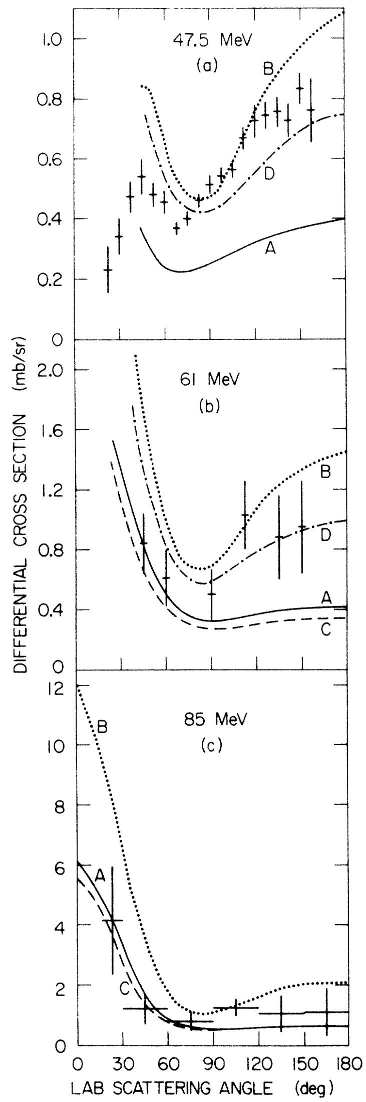

FIG. 3. (a)–(c) Data on πd elastic scattering differential cross sections and theoretical curves representing results of multiple scattering calculations. The labels A (solid), B (dotted), and C (dashed) refer to the kins of kinematical prescription described in the text. Curves D (dot-dashed) at 47.5 and 61 MeV are obtained without accounting for Fermi motion. The experimental results are from Refs. 7 (47.5 MeV), 18 (61 MeV), and 19 (85 MeV). We think that the two lowest energies (47.5 and 61 MeV) are too small for the application of the multiple scattering method and that these calculations seem to make sense only for incident pion kinetic energies above 85 MeV.

and of details in the treatment of the system can be seen more clearly.

As shown in Fig. 3, the experimental results[10] obtained at 47.5 MeV are reasonably well fitted by a multiple scattering calculation with the most usual treatment of the two-body kinematics, name-

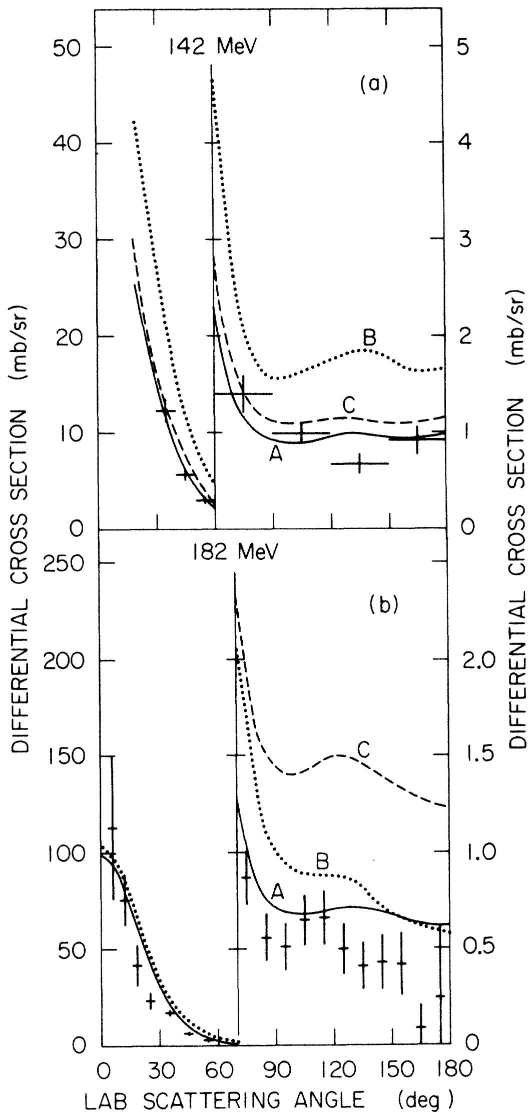

FIG. 4. (a), (b) Curves for the differential cross section for πd elastic scattering at 142 and 182 MeV, obtained in a multiple scattering calculation involving single and double scattering terms, and accounting for Fermi-motion and nucleon recoil effects. The solid curves are calculated using the value of the energy parameter obtained from a proper treatment of the three-body kinematics, as described in the text (prescription A). The dotted curve (B) is obtained with the prescription which puts the struck nucleon on the mass shell, while for the dashed curve (C) the spectator nucleon is on shell. The experimental results are from Refs. 21 (142 MeV) and 22 (182 MeV). We have used an expanded scale for the large angles, so as to exhibit more clearly the observed discrepancies.

ly, prescription B. Surprisingly Fermi-motion effects do not contribute to improve the quality of this theoretical curve. The other two prescriptions perform badly at this energy, giving results which

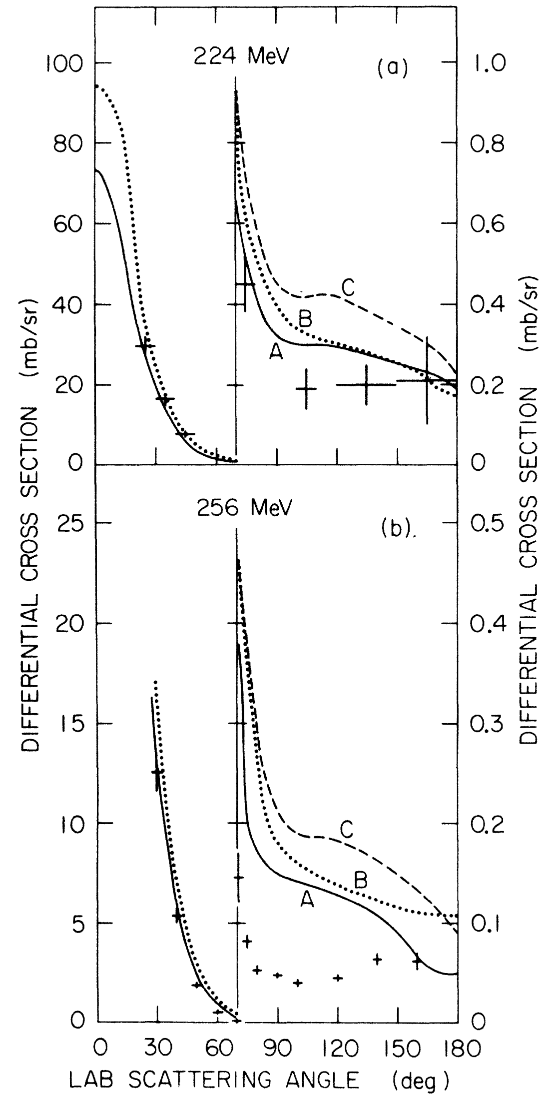

FIG. 5. Differential cross section at 224 and 256 MeV obtained in multiple scattering calculations using different prescriptions for the treatment of the kinematical variables. The solid curve (prescription A) gives the best results, but the large angle scattering data are not very well reproduced by the theoretical calculations. The experimental points are from Refs. 23 (224 MeV) and 24 (256 MeV).

are much lower than the data. The curve for case C has not been drawn because it is even worse than the curve obtained with prescription A. The results obtained at 61 MeV, shown in Fig. 3(b), present nearly these same characteristics.

At this point we wish to call attention to the danger of drawing conclusions from observations made in a restricted energy interval, in calculations of this kind. We can see that the situation already becomes very different at slightly higher energies.

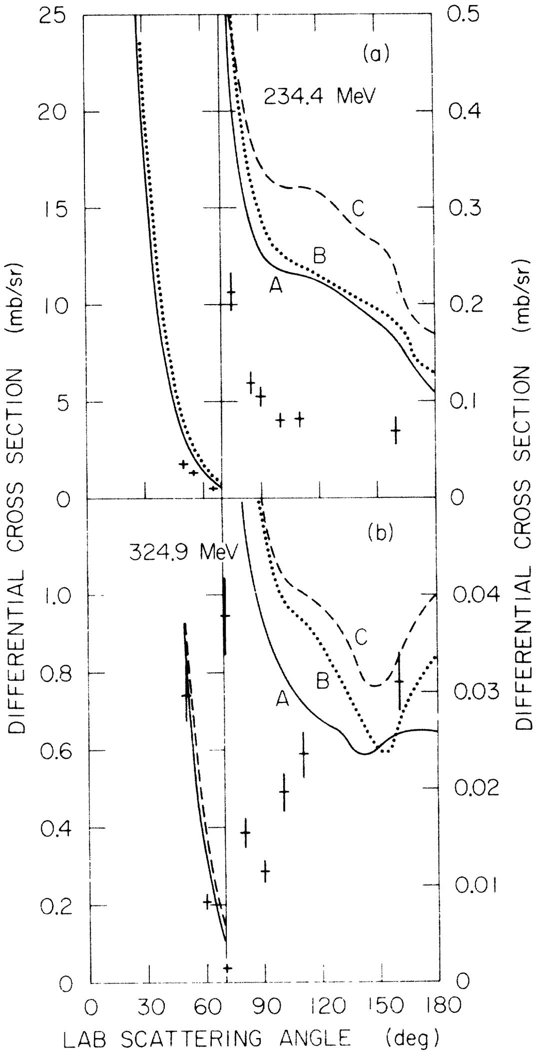

FIG. 6. (a), (b) Preliminary data at 234.4 and 324.9 MeV from Ref. 17, and theoretical curves for the different cross section obtained in multiple scattering calculations using differential prescriptions for the treatment of the kinematical variables in the two-body collision. Again prescription A performs better than the other two cases, but the calculations are clearly nonsatisfactory at large angles.

In fact, in Fig. 3(c), for 85 MeV, the case which performed best at 47.5 MeV (namely prescription B) is now the worst. The solid line, which in all figures represent prescription A, has already taken a reasonable standing. The experimental errors are rather large, which makes things apparently easier, but looking at Fig. 3(c) we feel that the theoretical calculation makes some sense at this energy.

Figures 4(a) and 4(b) present experimental and theoretical results at 142 and 182 MeV; the solid curves, representing prescription A, give a good fitting to the data, and make us confident that this is a proper way to perform these calculations. The dotted and dashed curves (cases B and C, respectively) are not so close to the experimental points, and invert their relative positions from 142 to 182 MeV. The poor results obtained with prescription A at 47.5 and 61 MeV should thus be taken as demonstrating that this energy is too low for a multiple scattering calculation involving only single and double scattering terms. Binding corrections, off-the-energy-shell extrapolations, or complicated three-body mechanisms might play important roles at such low energies.

In Figs. 5(a) and 5(b) and 6(a) and 6(b) for 224, 256, 234.4, and 324.9 MeV, again prescription A gives the best results, but here there appears regularly a rather strong discrepancy in a wide angular region above 70°, with the experimental data lying much below the calculated points. At small angles up to about 70° in the lab system the calculated values are reasonable, but at large angles the calculations are wrong by a factor of about 2. Here the theoretical results must be considered as nonsatisfactory, and there is an obvious need for improvement and interpretation.

At the energies of the measurements of angular distribution in the backward directions by Schroeder *et al.*,[27] namely at and above 375.7 MeV, our calculations confirm the above mentioned trend, giving results which are below the experimental points by a factor lying between 2 and 3. It seems that some change must be introduced, either in the description of the deuteron structure, or in the treatment of the collision mechanism. To our knowledge, no reasonable quantitative or qualitative interpretation has been given of these experimental results, which are already five years old.

Energy dependence of backward cross section

The πd elastic differential cross section falls rapidly as a function of the scattering angle. The backward cross section, in the energies here considered, can be as much as a thousand times smaller than the forward differential cross section.

For large values of the momentum transfer, $\vec{\Delta}$, due to the large structure of the deuteron in configuration space, and consequently to its short range distribution in momentum space, the product $\psi(\vec{P})\psi(\vec{P}+\vec{\Delta}/2)$ in Eq. (25) varies rapidly with $|\vec{P}|$. Thus the result of the integral leading to the amplitude for scattering by a deuteron is very sensitive to the $\vec{P}$ dependence of the other terms occurring in the integrand of the expression leading to the differential cross section, Eq. (23). This is seen in Fig. 7, where we plot all the experimental

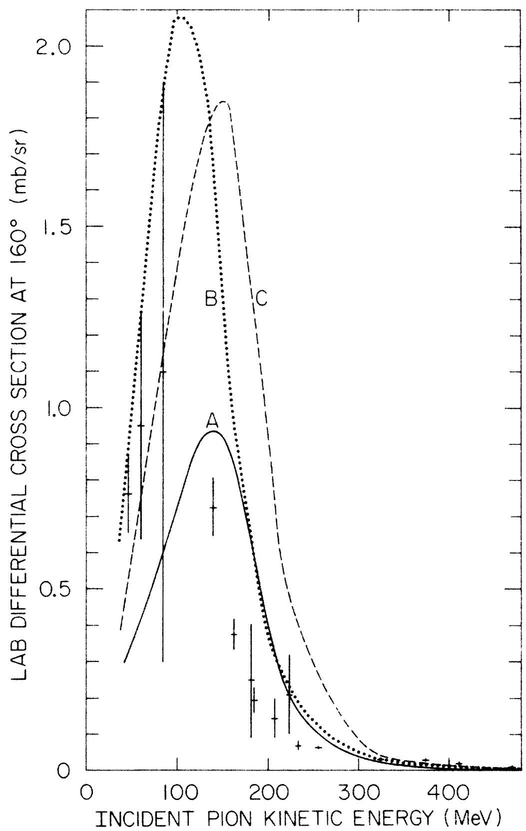

FIG. 7. Energy dependence of the backward (160° lab scattering angle) differential cross section for πd elastic scattering. The experimental data are from Refs. 7 and 17–27. The curves represent the calculations made with three different prescriptions for the treatment of the kinematics of the two-body collision, as described in the text. The values obtained for the backward differential cross section are very sensitive to details of the multiple scattering calculations, such as Fermi-motion effects, deuteron structure, and off-shell behavior of amplitudes. The figure shows that prescription A (solid curve) gives reasonable results in the energy range from 140 to 250 MeV.

data for $(d\sigma/d\Omega)$ at 160°,[7,17-27] together with the results of our calculations. We show curves for the three cases of kinematical prescription, all calculated taking account of Fermi motion in the single scattering term, and for nucleon recoil in the double scattering contribution.

We see from the figure that prescription A, based on Faddeev's equation, gives a good account for the data in the region from 140 to 300 MeV. The other two prescriptions fail in this region, and are more reasonable at the lower energies (see the experimental points at 47.5, 61, and 85 MeV). At the energies of the Schroeder[27] experiment (375.7, 412.4, and 468.6 MeV) all calculations made give too low values when compared with the data.

We tried to improve the fitting of the experimental results in the low energy extreme by using other sets of low energy pion-nucleon phase shifts, such as the CERN phase shifts, and allowed for some reasonable fluctuations in these numbers but the improvement obtained was much smaller than desired. We think that the good performance of prescriptions B and C in the lowest energies is purely accidental, and we again call attention to the danger of extracting conclusions from analysis of results obtained at only one value or in a narrow region of values of the incident energy. Of course it would not be reasonable to use arbitrarily chosen prescriptions for different sets of data. We should rather consider that it is not reasonable to expect that a simple multiple scattering calculation, without reliable corrections for binding effects and other complications, can appropriately describe elastic πd experiments at the very low energies. The approach based on the direct solution of the Faddeev equations (1), (2), and (3) is a more reliable and adequate method for energies below 100 MeV.

At the energies of the experiment by Schroeder *et al.* (375.7 MeV and above) all calculated values are below the data. Again in this extreme region prescription A seems to be worse than the other two. It is not difficult to find possible reasons for this disagreement. As mentioned above, the differential cross sections at high momentum transfers is highly sensitive to the form of the deuteron wave function. The very low values of the backward differential cross section at these energies, which are of the order of 0.03, to 0.01 mb, are from one to three thousand times smaller than the forward cross section. This enormous cancellation is due to the factor $\psi(\vec{P})\psi(\vec{P}+\vec{\Delta}/2)$ in the integrand, and can be modified by a change in the high momentum tail of the deuteron wave function. For example, it has already been pointed out[28] that the difficulty found in the multiple scattering calculations to explain Schroeder's results might be an indication of the presence of a Δ isobar component in the deuteron wave function. We find that this is an interesting line for further investigation.

VI. COMMENTS AND CONCLUSIONS

We have applied the multiple series method to the πd elastic scattering in the low and medium energy regions, concentrating some effort in the analysis of an effect which is of large practical importance in these calculations, namely that of the arbitrariness in the determination of the values of the kinematical variables fixing the two-body amplitudes. We have reviewed prescriptions previously used, and compared them with the prescription derived from the proper consideration of the fact that the two-body operators appearing in the multiple scattering series for πd scattering are initially defined in a three-particle Hilbert space. We can then account properly for the energy carried by the particle which acts as spectator in each term evaluated, and thus the fact that we are dealing with a three-body problem is not overlooked in each term of the calculation.

We have performed calculations covering the whole interval of low and medium energies, from zero to about 400 MeV, and in comparing the results with the existing experimental data on πd elastic scattering. This is important, as the results reported in the literature are sometimes contradictory, and we have shown that the observation of the performance of these multiple scattering calculations in a narrow interval of energies may easily lead us to wrong conclusions.

Our calculations have shown, or rather confirmed, that Fermi-motion effects are extremely important in large angle pion-deuteron scattering at low and medium energies. If prescription A, based on the proper consideration of the three-body kinematics, is used to fix the value of the energy which enters as the argument of the two-body collision operator, it is of course essential that the nucleon momentum be treated as a variable, according to Eq. (16). As mentioned in Sec. III, inspection of Fig. 1 leads us to expect that in the case of prescription B the Fermi-motion effects are less important than in the two other cases. Still, results show that in this case accounting for Fermi-motion effects can reduce the calculated backward (at 160° of scattering angle in the lab system) differential cross section by a factor of about 2 in the energy region from 180° to 260 MeV. Below and up to 100 MeV the Fermi-motion effect is such as to increase the calculated value of the backward differential cross section by about 50%. At 90° lab scattering angle, the effect is not so strong, amounting up to a reduction of about 30% in the differential cross section, for an incident energy of 180 MeV.

The differential elastic cross sections are well reproduced by a calculation using prescription A, in the interval of incident pion energies from 140 to about 230 MeV. Above this energy strong discrepancies occur for scattering angles larger than 70°.

We may speculate on what may be the cause of the observed discrepancies. We notice that the strong reduction in the large angle experimental cross section, as compared with the calculated values, occurs suddenly as the energy goes above 230 MeV. At this energy some new dynamical phenomenon may have started to play a role. We may think for example that pion production and consequent reabsorption by the other nucleon may have started to contribute significantly. At these energies, which are above the threshold for pion production, this essentially three-body mechanism which is not included in terms of the multiple scattering series, could eventually be responsible for a change in the dynamics of the process. The diagram in Fig. 8 gives a simplified representation for this kind of mechanism, which accounts for part of the binding corrections to the single scattering term (other corrections would involve production and absorption of two or more pions). If the intermediate pion is charged, there is charge exchange between the struck and the spectator nucleons.

Another possible explanation for the observed discrepancy is that we may have entered a range of momentum transfer where the effects of our insufficient knowledge of the deuteron structure may have started to affect the calculations. A change in the large momentum tail in the deuteron wave function, such as that caused by the presence of a hard core in the neutron-proton interaction, may substantially change the value of the integral over internal Fermi momentum in the expression of the differential cross section, Eqs. (23) and (25).

These effects due to changes in the deuteron structure or in the meson-nucleon interaction might be expected to be small at first sight. However, we must notice that the value calculated for the πd differential cross section at large angles is several orders of magnitude smaller than the forward cross section, due to strong cancellations occurring in the integration procedure. The results obtained after such cancellations have a delicate and strong dependence on the quantities in the integrand. As an example, we mention that the introduction of the d-wave component in the deuteron wave function causes an increase by a factor 2 in the calculated cross section at large angles in the energies of Schroeder experiment (375.7 MeV and over).

We must also remark that we have given an arbitrary treatment to the off-energy-shell behavior

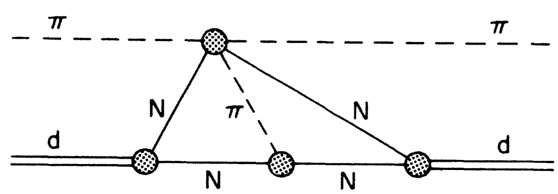

FIG. 8. A possible dynamical mechanism, not included in the terms of the usual multiple scattering series, which may be responsible for discrepancies observed at large scattering angles for energies above the threshold for pion production.

of the two-body amplitudes, as this behavior can only be fixed if some specific dynamical model is adopted for the pion-nucleon interaction. Of course changes in the prescribed off-shell behavior of the amplitude can alter the theoretical results obtained.

The double scattering terms do not give a strong contribution to the evaluated cross section at these energies and angles, so that we do not expect that a substantial change could come from the third and higher order terms of the multiple scattering expansion. Also binding corrections have been shown[29] to give contributions to the forward scattering amplitudes which are only of the order of the double scattering terms.

Whatever may be the cause for the failure of the present multiple scattering calculation at large angles above 230 MeV, we note that the extreme sensitivity of the backward πd elastic cross section at large angles provides an excellent ground to study the deuteron structure and properties of the meson-deuteron and meson-nucleon interaction.

We find that more accurate experiments on differential cross sections for πd scattering should be performed as soon as possible. The region of energies around and above 200 MeV should be carefully studied, as important changes in the process seem to take place in this region.

On the other hand, it is obvious that the theoretical effort must also be increased, both in the calculations with a multiple scattering method and in direct solutions of Faddeev integral equations. A combination of the two methods, joining the nice features of each, may be an interesting and rewarding program of investigation.

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[1]I. R. Afnan and A. W. Thomas, Phys. Rev. C 10, 109 (1974).

[2]D. D. Brayshaw, Phys. Rev. C 11, 1196 (1975).

[3]A. W. Thomas, Nucl. Phys. A258, 415 (1976); R. M. Woloshyn, E. J. Moniz, and R. Aaron, Phys. Rev. C 13, 286 (1976).

[4]V. B. Mandelzweig, H. Garcilazo, and J. M. Eisenberg, Nucl. Phys. A256, 461 (1976).

[5]A. W. Thomas, in *Proceedings of the Topical Conference on Meson-Nuclear Physics, Pittsburgh, 1976*, edited by P. D. Barnes, R. A. Eisenstein, and L. S. Kisslinger (AIP, New York, 1976), p. 375.

[6]D. Brayshaw, in *Proceedings of the Topical Conference on Meson-Nuclear Physics, Pittsburgh, 1976*, (see Ref. 5), p. 443.

[7]D. Axen, G. Duesdieker, L. Felawka, Q. Ingram, R. Johnson, G. Jones, D. Lepatourel, M. Salomon, W. Westlund, and L. Robertson, Nucl. Phys. A256, 387 (1976).

[8]Multiple scattering calculations of πd elastic scattering: R. M. Rockmore, Phys. Rev. 105, 256 (1957); A. Ramakrishnan, V. Devanathan, and K. Venkatesan, Nucl. Phys. 29, 680 (1962); H. N. Pendleton, Phys. Rev. 131, 1833 (1963); C. Carlson, Phys. Rev. C 2, 1224 (1970); D. S. Beder, Nucl. Phys. B34, 189 (1971); W. R. Gibbs, Phys. Rev. C 3, 1127 (1971); R. L. Landau, Nucl. Phys. B35, 390 (1971); J. M. Wallace, Phys. Rev. D 5, 1840 (1972); K. Gabathuler and C. Wilkin, Nucl. Phys. B70, 215 (1974); E. Ferreira, L. P. Rosa, and Z. D. Thomé, Nuovo Cimento A20, 277 (1974); M. A. Braun and V. B. Senyushkin, Yad. Fiz. 21, 279 (1975) [Sov. J. Nucl. Phys. 21, 147 (1975)].

[9]V. A. Allessandrini and R. L. Omnès, Phys. Rev. 139, B167 (1965); R. Blankenbecler and R. Sugar, *ibid.* 142, 1051 (1966).

[10]M. L. Goldberger and K. M. Watson, *Collison Theory* (Wiley, New York, 1964).

[11]L. Bertocchi and A. Capella, Nuovo Cimento A51, 369 (1967); G. Alberti, Instituto Nazionale di Fisica Nucleare, Trieste, Report No. INFN-AE 73/1, 1973 (unpublished).

[12]E. Ferreira, L. P. Rosa, and Z. D. Thomé, Nuovo Cimento A21, 187 (1974); Lett. Nuovo Cimento 9, 707 (1974); Nuovo Cimento A33, 216 (1976).

[13]E. Ferreira, L. P. Rosa, and Z. D. Thomé, Nuovo Cimento A20, 277 (1974).

[14]J. M. Eisenberg, Ann. Phys. (N.Y.) 71, 542 (1972).

[15]M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).

[16]L. D. Roper, R. W. Wright, and B. T. Feld, Phys. Rev. 138, B190 (1965).

[17]University of Virginia and Los Alamos Scientific Laboratory Collaboration-Preliminary Data, Los Alamos Report No. LA-6156-R (unpublished).

[18]A. M. Sachs, H. Winick, and B. A. Wooten, Phys. Rev. 109, 1733 (1958).

[19]K. C. Rogers and L. N. Lederman, Phys. Rev. 105, 247 (1957).

[20]E. Arase, G. Goldhaber, and S. Goldhaber, Phys. Rev. 90, 160 (1953).

[21]E. G. Pewitt, T. H. Fields, G. B. Yodh, J. G. Fetkovich, and M. Derrick, Phys. Rev. 131, 1826 (1963).

[22]J. H. Norem, Nucl. Phys. B33, 512 (1971).

[23]J. L. Acioli, Ph.D. thesis, University of Chicago, 1968 (unpublished).

[24]K. Gabathuler, C. R. Cox, J. J. Domingo, J. Rohlin,

N. W. Tanner, and C. Wilkin, Nucl. Phys. B55, 397 (1973).
[25]L. S. Dul'kova, I. B. Sokolova, and M. G. Shafranova, Zh. Eksp. Teor. Fiz. 35, 313 (1958) [Sov. Phys.-JETP 35, 217 (1959)].
[26]G. Brunhart, G. S. Faughn, and V. P. Kenney, Nuovo Cimento 24, 1162 (1963).
[27]L. S. Schroeder, D. G. Crabb, R. Keller, J. R. O'Fallon, T. J. Richards, R. J. Ott, J. Trischuk, and J. Va'vra, Phys. Rev. Lett. 27, 1813 (1971).
[28]J. M. Eisenberg, in *Proceedings of the International Conference on Few Body Problems in Nuclear and Particle Physics, Los Angeles, 1972,* edited by I. Slaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1973).
[29]F. Myhrer, Nucl. Phys. A241, 524 (1975).