Coulomb excitation measurements of reduced $E2$ and $E4$ transition matrix elements in ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb

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The reduced $E2$ and $E4$ transition matrix elements were measured for ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb via Coulomb excitation with α particles at energies below those where nuclear effects are expected to contribute significantly. Charge deformation parameters β_2^c and β_4^c were extracted from these matrix elements by assuming the charge distribution to have a deformed Fermi distribution form with quadrupole and hexadecapole components. Values of $\langle 0^+ \| (E2) \| 2^+ \rangle$ of 1.929, 2.161, 2.238, 2.341, and 2.402 eb were found for ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb, respectively, with $\lesssim 1\%$ uncertainty. The values of $\langle 0^+ \| M(E4) \| 4^+ \rangle$ for these same respective nuclei are $0.21^{+0.16}_{-0.20}$, $0.16^{+0.10}_{-0.15}$, $0.16^{+0.14}_{-0.26}$, $0.12^{+0.12}_{-0.13}$, and $0.19^{+0.14}_{-0.19}$ eb^2.

NUCLEAR REACTIONS ^{156,158}Dy(α,α'), ^{162,164}Er(α,α'), ^{168}Yb(α,α'), E_α=12–13 MeV; measured Coulomb excitation cross sections relative to elastic cross sections at θ_{1ab}=150°; deduced $\langle 0^+ \| M(E2) \| 2^+ \rangle$, $\langle 0^+ \| M(E4) \| 4^+ \rangle$, using rotational model; extracted charges deformation parameters β_2^c, β_4^c from rotational model and Fermi charge distribution. Enriched targets.

I. INTRODUCTION

Studies of yrast bands of nuclei in the rare-earth region have indicated some very interesting structural properties. For example, in recent years (heavy ion, xn) and (α, xn) reactions have been used to map out level energies of high spin states throughout the region. Sayer, Smith, and Milner[1] have compiled level energies, and have displayed their systematic behaviors for most of the even-even nuclei. Some of these nuclei show anomalously spaced levels ("backbending"[2]) when compared with the $E_I = \hbar^2/2\mathscr{I}(I+1)$ relationship predicted by the simple rotational model. To date, the commonly accepted explanation of the backbending behavior is that a second band crosses the ground band,[3,4] typically at $I^\pi \sim 14^+$. The complete nature of the second band, which has a larger moment of inertia than the ground band, is not well understood.

Very recently, Lee *et al.*[5] have shown that Coulomb excitation with heavy projectiles offers much promise as a probe to study and interpret the interactions between the ground band and crossing bands. They show that a xenon beam on a rare-earth target can Coulomb excite states with spins greater than the value where the bands cross. Transition probabilities extracted from such experiments can be directly compared with predictions which use either the simple rotational model or current theoretical descriptions of the band crossing phenomenon.

However, crucial to both the extraction of experimental excitation probabilities to high spin states as well as to the basis for theoretical calculations is a precise knowledge of the $E2$ and $E4$ transition reduced matrix elements to the 2^+ and 4^+ members of the ground band. The multiple excitations to high spin states are mainly via $E2$ excitations, but $E4$ excitations are known to significantly contribute. Eichler *et al.*[6] and Guidry *et al.*[7] have found that for argon projectiles on actinide targets the calculated γ ray yields for the I = 8–14 states are quite sensitive to the choices of sign and magnitude of the $E4$ matrix elements used. Thus, a precise knowledge of the hexadecapole effect, via these $E4$ transition reduced matrix elements, is then needed in the analysis of heavy ion Coulomb excitation experiments. In fact, for such experiments where sensitivity to the sign and magnitudes of these matrix elements connecting states to high spins occurs this technique is a useful tool to eliminate the sign-magnitude ambiguity in the value of the $E4$ matrix element from Coulomb-excitation experiments with light ion projectiles.

Also, current theoretical calculations[8-13] of ground-state deformations have shown that even a small hexadecapole component in the nuclear shape can have important effects on nuclear properties. These include asymmetries, prolate-oblate ground-state shape transition regions, and stability against β decay, α decay, and spontaneous fission. Indeed, many of the rare-earth and actinide nuclei have been found experimentally, in accordance with theory, to have sizable hexadecapole components. These have been measured by Coulomb excitation with α particles, electron scattering, Coulomb-nuclear interference measurements, and α-particle scattering at energies above the Coulomb barrier.

In this paper we present values of the $E2$ and $E4$ transition reduced matrix elements, $\langle 0||M(E\lambda)||I_f=\lambda\rangle$, $\lambda=2, 4$, for ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb that we have obtained by Coulomb excitation via the (α, α') reaction. These nuclei are interesting because they are deformed and are the farthest from the main line of stability against β decay. Their low-energy (<2 MeV) level structures have several collective $I^\pi=2^+$ and 3^- states, and several of them, ^{158}Dy, ^{162,164}Er, show anomalous behavior in their yrast bands. The fabrication of targets of these isotopes suitable for our studies is a formidable task, as each is $\leq$0.14% isotopically abundant, except ^{164}Er which has a 1.6% natural abundance.

II. EXPERIMENTAL TECHNIQUES

α particles were obtained in the tandem Van de Graaff accelerator at the Oak Ridge National Laboratory. A 20 cm long position-sensitive, gas-flow proportional counter, mounted in the focal plane of an Enge split-pole magnetic spectrograph, was used to detect the elastically and inelastically scattered α particles at the laboratory angle of 150°. The efficiency and linearity calibrations of the detector spectrograph system were made by measuring the yields and positions of the main α groups in the decay of ^{244}Cm as a function of magnetic field strength. This detector was found to be uniformly efficient, to $\leq$1%, across most of its length.

Isotopically enriched spectrograph targets of ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb were prepared for these experiments using the 180° Oak Ridge sector isotope separator.[14] These targets were prepared using the direct beam deposition method at a beam energy of ~200 eV. Target thicknesses of 25 μg/cm^2 on 80 μg/cm^2 carbon foils have been estimated. In the direct beam deposition method the 40 keV ion beam is first isotopically analyzed in the separator magnet and is then decelerated by a focusing lens system to an energy of 200 eV. This effectively filters out all low energy ion beam contaminants (e.g. ions formed by gas scattering, charge exchange, high voltage sparking) by acting as a high pass filter. Mass assays given in Table I were obtained for these targets, which were all prepared using *natural abundance* separator feed material. It is to be noted that the preparation of high purity targets of these isotopes by previous methods has been either impossible or prohibitively expensive due to the limited single stage enrichment of most separators and to the expense of pre-enriched isotope separator feed material.

We chose beam energies by requiring the target-projectile surface separation to be ~7 fm (for $R_0=1.2$ fm) at closest approach. This separation distance was judged to be safe in view of several studies[15,16] of the onsets of Coulomb-nuclear interferences.

Two of our particle spectra are shown in Fig. 1. Our resolution varied with target thickness but was at best 18 keV and at worst 30 keV. All peak intensities were found by manually stripping the spectrum and checking the results by a computer fitting routine. Statistical uncertainties in the 2^+ and 4^+ intensities were typically <0.8% and $\leq$3% respectively. Additional systematic uncertainties of ~0.5% for each of the uncertainties in beam energy and scattering angle were folded in with the statistical uncertainty. Uncertainties of ~0.5% correspond to $\Delta E\approx 15$ keV and $\Delta\theta\approx 0.6$, which are in fact upper limits on ΔE and $\Delta\theta$ in our experiment. One experiment was performed on each target at $\theta=150°$ from which our cross sections were extracted. However, these data were checked against earlier and subsequent experiments examining vibrational excitations, where we used $\theta_{lab}=90°$.

III. ANALYSIS AND DISCUSSION

The differential cross sections for inelastic scattering (relative to elastic scattering) were compared with those calculated with the aid of the computer code AROSA,[17] which has a quantal treatment of the Coulomb excitation process incorporated in it. The

TABLE I. The natural abundances of the targets and their assayed purities of each target are given.

Isotope	Feed (natural abundance) charge assay (%)	Single pass target assay (%)
^{156}Dy	0.0524	99.53
^{158}Dy	0.0902	98.91
^{162}Er	0.136	$\geq$ 99.26
^{164}Er	1.56	99.56
^{168}Yb	0.14	> 99.00

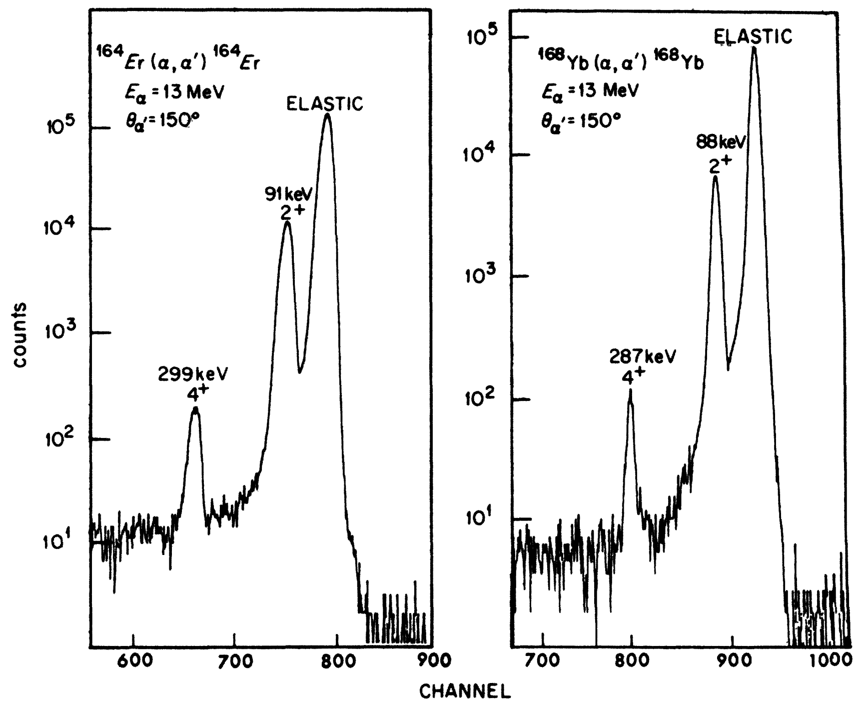

FIG. 1. Spectra of α particles scattered from ^{164}Er and ^{168}Yb.

0^+, 2^+, 4^+ ground band states were included. The comparison of quantal and semiclassical calculations by us and other groups have shown that truncating the $M_{if}(E\lambda)$ matrices to elements connecting only these levels has in most cases a small effect on the 2^+ and 4^+ excitation cross sections. Typical differences are $\lesssim 0.3\%$ for the 2^+ cross section and $\lesssim 0.8\%$ for the 4^+ cross section. Since these are well within our experimental uncertainties, they were not incorporated.

The calculations were performed by varying $M_{0\lambda}(E\lambda) = \langle 0^+ \| i^\lambda M(E\lambda) \| I^\pi = \lambda^+ \rangle$ while letting all other nonzero values of $M_{if}(E\lambda)$ which connect the three levels have values from rotational model predictions,

$$\frac{M_{if}(E\lambda)}{M_{0\lambda}(E\lambda)} = (2I_i + 1)^{1/2} \langle I_i 0 \lambda 0 | I_f 0 \rangle \, .$$

The values of M_{02} and M_{04} were then extracted by comparing experimental and calculated values of the ratios $d\sigma_{2^+}/d\sigma_{0^+}$, $d\sigma_{4^+}/d\sigma_{0^+}$, and $d\sigma_{4^+}/(d\sigma_{4^+})_{M_{04}=0}$ which are functions of M_{04} and M_{02}. To second order perturbation theory, this last ratio is quadratic in $M_{04}/M_{02}/M_{24}$. Positive values of M_{04} were taken as done in studies[18-21] of heavier isotopes of these nuclei. The motivation for this choice is that the charge deformation parameters, β_2^c and β_4^c, extracted from M_{02} and M_{04} when $M_{04} < 0$ are much farther from accordance with theoretical expectations than if $M_{04} > 0$ is taken.

Our values of $\langle 0^+ \| M(E2) \| 2^+ \rangle$ and $\langle 0^+ \| M(E4) \| 4^+ \rangle$ are given in Table II. The quadrupole and hexadecapole charge deformation parameters, β_2^c and β_4^c, were extracted as outlined[22] with $\gamma_0 = 1.1$ fm and $a = 0.6$ fm which are consistent with those from muonic x-ray studies.[23] The central density ρ_0 was found by fixing $\int \rho(\vec{r}) d\vec{r} = Ze$. Our values of β_2^c and β_4^c are compared with some other measurements and theoretical predictions in Ref. 22.

The accurate extraction of $M_{04}(E4)$ relies on a precise knowledge of the value of $M_{24}(E2)$. We have assumed the rotational model values of M_{24}. Ben-Zvi et al.[24] find quite good agreement with the rotational model from their mean life studies of the 2^+ and 4^+ states in ^{158}Dy and ^{164}Er. Riedinger et al.[25] have found the same true for ^{168}Yb. We feel that the use of the rotational model is then valid for our studies, especially in view of the uncertainties in experimental values of $M_{24}(E2)$ to date. The effect of stretching can be calculated from the formula given by Symons and Douglas.[26] The stretching parameter can be obtained from the B/A ratio, but in ^{152}Sm this was found to yield an α 2 to 3 times larger than the measured α as discussed by McGowan and Stelson.[27] Only for ^{156}Dy is even the B/A ratio sufficiently large to significantly alter M_{24}. For ^{156}Dy using this larger estimate for α yields a M_{04} that is larger than the value in Table II by two standard deviations. Thus in ^{156}Dy there is probably an effect of stretching which would increase the results in Table II by about one standard deviation. Since α is not known experimentally for ^{156}Dy, the value reported in Table II includes no stretching on M_{24}. As Shaw and Green-

TABLE II. Values of M_{02}, M_{04}, β_2^c, β_4^c, and $B(E2)$ extracted from the present work are given. Other less accurate measurements of $B(E2)$ values are given for comparison. The $|M_{02}|$ values are from the average values.

Nucleus	E_{2^+} (keV)	$\lvert M_{02}\rvert$ (e b)	M_{04} (e b^2)	β_2^c	β_4^c	This work $B(E2)$ (e^2b^2)	θ (deg)	Other studies $B(E2)$ (e^2b^2)	Ref.
^{156}Dy	138	1.929(7)	$0.21^{+0.16}_{-0.20}$	$0.287^{+0.011}_{-0.014}$	$0.01^{+0.05}_{-0.06}$	3.72(3)	150	3.79(30)	32
								3.74(23)	27
								3.74(30)	31
^{158}Dy	99	2.161(9)	$0.16^{+0.10}_{-0.15}$	$0.323^{+0.007}_{-0.010}$	$-0.01^{+0.03}_{-0.02}$	4.67(4)	150	4.67(40)	32
								4.56(27)	27
								4.73(23)	28
								4.41(25)	29
								4.76(24)	31
^{162}Er	101	2.238(7)	$0.16^{+0.14}_{-0.26}$	$0.320^{+0.009}_{-0.017}$	$-0.02^{+0.04}_{-0.07}$	5.06(4)	150	4.89(25)	32
						4.96(6)	90	5.82(50)	31
						5.01(3)	ave		
^{164}Er	91	2.341(9)	$0.12^{+0.12}_{-0.13}$	$0.335^{+0.008}_{-0.009}$	$-0.03(3)$	5.48(5)	150	5.20(35)	32
						5.47(5)	90	5.63(23)	30
						5.48(4)	ave	5.78(32)	31
^{168}Yb	88	2.402(8)	$0.19^{+0.14}_{-0.19}$	$0.325^{+0.009}_{-0.012}$	$-0.01(^{0.03}_{-0.05})$	5.81(5)	150	5.43(25)	32
						5.72(6)	90	5.68(95)	25
						5.77(4)	ave		

berg[28] suggest, it would be very useful to perform low-energy heavy ion (i.e., ^{12}C or ^{16}O) Coulomb excitation experiments where only the 0^+, 2^+, and 4^+ levels would be appreciably populated. Since the effect of $E4$ excitations is smaller for heavier ions than for light ions, then $P_4 \propto |M_{02}M_{24}|^2$. A precise knowledge of M_{02} from light ion experiments allows the accurate extraction of M_{24}.

Several measurements[29-33] of mean lives of the first 2^+ states for these nuclei have been reported along with one early Coulomb excitation study.[34] In Table II our $B(E2)$ values, which are our M_{02} values squared, are given along with those calculated from the 2^+ mean lives. For studies prior to 1965 on ^{164}Er, we have given the adopted $B(E2)$ value from the compilation of Stelson and Grodzins.[35] We attain good agreement with the averages of the less precise measurements. The worst agreement is perhaps for ^{168}Yb. There our data with $\theta_{\mathrm{lab}} = 90°$ yields a $B(E2;\ 0^+ \rightarrow 2^+)$ in good agreement with our value when $\theta_{\mathrm{lab}} = 150°$.

In summary, the fabrication of high purity targets has allowed us to study some nuclei of very low isotopic abundances. The results of our study show that the reduced $E4$ transition matrix elements for ^{156,158}Dy, ^{162,164}Er, and ^{168}Yb are small when the positive values of M_{04} are taken. Thus, the hexadecapole deformations of these nuclei are small and seem to be in accordance with theoretical expectations. Our M_{02} and M_{04} values should aid the experimental and theoretical efforts to understand neutron deficient systems. The systematics of the M_{02} and M_{04} values between $A = 152$ and 192 are given in the preceding paper[22] and compared with theoretical calculations.

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