Analyzing-power measurements for the ^{3}He($\vec{t}$, d)^{4}He reaction*

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We report analyzing-power angular distributions for the ^{3}He($\vec{t}$, d)^{4}He reaction at bombarding energies of 9.02, 12.86, and 17.02 MeV, and an excitation function at 90° c.m. from 9.02 to 17.27 MeV. The angular distributions show marked deviations from the antisymmetric shape predicted by a simple particle-transfer model incorporating charge symmetry. Reaction mechanisms and violations of charge symmetry which might account for the data are discussed.

[NUCLEAR REACTIONS ^{3}He($\vec{t}$, d). Measured $A_y(\theta)$, $\theta_{c.m.}$ = 16°–159°, E = 9.02, 12.86, 17.02 MeV; $A_y(E)$, E = 9.02–17.27 MeV, θ = 90.0° c.m.]

I INTRODUCTION

Conzett has recently suggested[1] that analyzing-power measurements on reactions of the form $C'(\vec{C},A)B$, with charge-symmetric partners C and C' in the initial state can yield information on the reaction mechanism. In particular, the vector analyzing power A_y should be antisymmetric about 90° c.m. if the reaction proceeds via a simple isospin-conserving particle transfer and if C and C' are charge-symmetric.

In this paper, we present measurements and suggest an interpretation of A_y data for the reaction ^{3}He($\vec{t}$, d)^{4}He. In Sec. II, we describe our experiment, in which angular distributions of A_y were taken at 9.02, 12.86, and 17.02 MeV bombarding energy (corresponding to a range of excitation energies from 20.31 to 24.31 MeV in the ^{6}Li system). An excitation function at 90.0° c.m. was also measured, from 9.02 to 17.27 MeV. The A_y angular distributions show pronounced deviations from antisymmetry, and the magnitude of the deviations increases with energy. As we point out in Sec. III, the deviations from antisymmetry in A_y for ^{3}He($\vec{t}$, d)^{4}He are much larger than those observed for ^{4}He($\vec{d}$, t)^{3}He. But, as we show using the spin-dependent scattering matrix for this reaction, there is no reason why the deviations should be comparable.

The interpretation of the observed deviations from antisymmetry is somewhat complicated. To clarify the situation, we give in Sec. IV a brief review of analyzing-power symmetry theorems based on charge symmetry alone. In our discussion, we take charge symmetry to hold when C and C' are exact isospin multiplets *and* when the total isospin T is conserved in the reaction, irrespective of the reaction mechanism. We then present in Sec. V a simplified derivation of Conzett's[1] symmetry theorem for A_y, which is based both on charge symmetry and on a specific model of the reaction.

Because the result of Conzett is the one directly applicable to the ^{3}He($\vec{t}$, d)^{4}He reaction, the deviations from antisymmetry which we have observed are due, *a fortiori*, either to violations of charge symmetry, or to reaction mechanisms not included in Conzett's model, or to both. However, as we argue in our conclusion (Sec. VI), the size of the deviations measured in this experiment suggests that the effects are due primarily to the reaction mechanism. We believe that our results may be traced to the excitation of a state (or states) in ^{6}Li, and we propose ways in which currently available calculations might be used to test this hypothesis.

II. EXPERIMENT

Our measurements were carried out using the LASL polarized-triton source[2] and FN tandem Van de Graaff accelerator. The target was a 9.7-cm-diam gas cell, with an entrance window and 300° exit window of 2.5-μm Havar foil. The nominal pressure of the gas cell was 300 Torr of ^{3}He. Further details of the target system may be found in Ref. 3. Detector telescopes containing three silicon surface-barrier detectors were mounted at equal angles right and left of the incident beam. The lab scattering angles could be set to a relative accuracy of ±0.02°; the zero-angle settings for the left and right counters were known to ±0.05° (Ref. 3). The collimation system for the detectors consisted of a front slit 3.33 mm wide, two antiscatter slits, and a rear slit 3.33 mm wide and 11.4 mm high. The slit separation was 190 mm, and the

target-center-to-detector distance was 241 mm.

For a given energy and angle, yields in the left and right detectors were accumulated for a preset amount of integrated charge both with triton spin up and triton spin down at the target. The spin was flipped by reversing the spin-quantization axis in the polarized-triton source. The spin-up and spin-down data were then combined according to the formulas of Ref. 4 into geometric mean left and right yields $\bar{L}$ and $\bar{R}$. In terms of these quantities, the analyzing power is

$$A_y(\theta) = \frac{1}{p} \frac{\bar{L}(\theta) - \bar{R}(\theta)}{\bar{L}(\theta) + \bar{R}(\theta)}, \quad (1)$$

where p is the average beam polarization. In our experiment, p was computed as the arithmetic average of quench-ratio polarization measurements[5] taken before and after each spin-up and spin-down run. This method of data acquisition eliminates all first-order errors due to instrumental asymmetries and random beam-wander effects.[4]

The measured angular distributions and the 90° c.m. excitation function are shown in Figs. 1 and 2. Relative errors are shown where they exceed the size of the plotting symbols. The sources of relative and scale error are discussed at length in Ref. 3. The relative errors of the data are of order ±0.010–0.015, and the scale error is approximately ±2% of A_y. Multiple-scattering effects were computed from the formula for solid-target multiple-scattering corrections to A_y (Ref. 3), and found to be negligible. A tabulation of the data is available from the American Institute of Physics.[6]

III DEVIATIONS FROM ANTISYMMETRY

From the experimental data, we computed the quantity

$$D(\theta) = A_y(\theta) + A_y(\pi - \theta), \quad (2)$$

which gives the deviation from antisymmetry about 90° (c.m.). $D(\theta)$ was calculated for 5° c.m. intervals by a linear interpolation of the experimental data. The associated error $\Delta D(\theta)$ is

$$\Delta D(\theta) = \{[\Delta A_y(\theta)]^2 + [\Delta A_y(\pi - \theta)]^2\}^{1/2}. \quad (3)$$

The error due to the interpolation procedure was neglected. The calculated values of $D(\theta)$ are shown in Fig. 3.

The deviations $D(\theta)$ derived from the present experiment are much larger in magnitude than those found in the reaction $^4\mathrm{He}(\vec{d}, t)^3\mathrm{He}$ at $E_d = 32$ MeV (22.8 MeV excitation in ^{6}Li) by Dahme *et al.*[7] This difference is not necessarily surprising, since the scattering-matrix elements contribute in entirely different ways to the vector analyzing power for

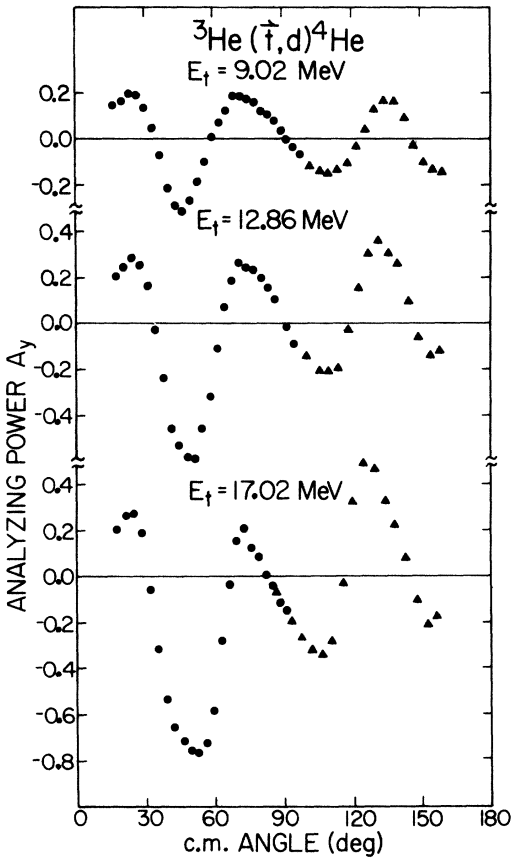

FIG. 1. Analyzing-power angular distributions $A_y(\theta)$ for the $^3\mathrm{He}(\vec{t}, d)^4\mathrm{He}$ reaction, measured at triton bombarding energies of 9.02, 12.85, and 17.02 MeV. The circles are data where the outgoing deuteron was detected; triangles are used where the recoil ^{4}He particle was detected. Relative errors are smaller than the plotting symbols.

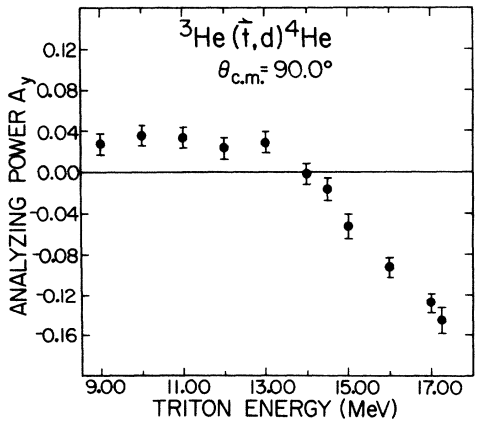

FIG. 2. Analyzing power $A_y(E)$ for the $^3\mathrm{He}(\vec{t}, d)^4\mathrm{He}$ reaction, measured at 90.0° c.m. angle for triton bombarding energies from 9.02 to 17.27 MeV. Only relative errors are shown.

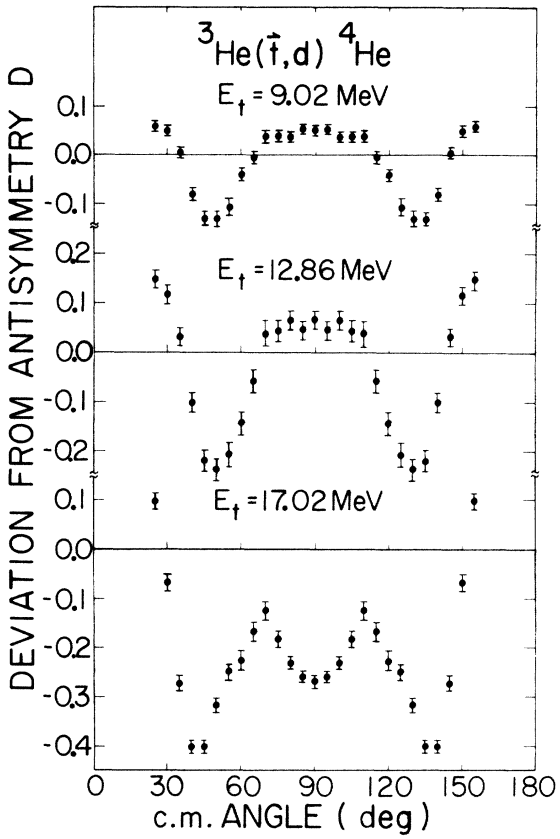

FIG. 3. Deviation of the analyzing power from exact antisymmetry, expressed in terms of the quantity $D(\theta)$ defined in Eq. (2). Only relative errors are shown. All the *physical* information is contained in the points between $\theta = 0°$ and $\theta = 90°$. The points beyond $\theta = 90°$ are, by construction, the image of points with $\theta \leq 90°$.

$^4\mathrm{He}(\vec{d},t)^3\mathrm{He}$. This can be shown explicitly from a knowledge of the spin-dependent scattering matrix, or M matrix, as it is often called.

Consider a reaction of the form

$$\mathrm{spin}\tfrac{1}{2} + \mathrm{spin}\tfrac{1}{2} \rightleftarrows \mathrm{spin1} + \mathrm{spin0}\,.$$

The entrance channel may be described by four possible spin projections, the exit channel by three. The spin-dependent observables in the entrance channel are best treated in the so-called "uncoupled basis," in which the four-dimensional spinors and operators for this channel are direct products of the two-dimensional spinors and operators appropriate to each particle. Hence, the columns of the M matrix are indexed by the spin projections

$$(\tfrac{1}{2})_1(\tfrac{1}{2})_2, (\tfrac{1}{2})_1(-\tfrac{1}{2})_2, (-\tfrac{1}{2})_1(\tfrac{1}{2})_2, (-\tfrac{1}{2})_1(-\tfrac{1}{2})_2,$$

where the subscript labels the particles and the quantity in parentheses is the spin projection for particle 1 or particle 2. The rows of the M matrix are labeled by the spin projections

$$(1)(0), (0)(0), (-1)(0)\,.$$

The M matrix for this reaction, then, has dimension 3×4, and has the form (see the Appendix to this paper and Ref. 8)

$$M = \begin{pmatrix} A & D & -C & B \\ -E & F & F & E \\ B & C & -D & A \end{pmatrix}, \tag{4}$$

where $A, B, \ldots, F$ are amplitudes for transitions between various spin configurations.

The analyzing power for $^3\mathrm{He}(\vec{t},d)^4\mathrm{He}$ is

$$\begin{aligned}\sigma A_y(\theta) &= \mathrm{Tr}(M\sigma_y I M^+) \\ &= 4\,\mathrm{Im}(AD^* + BC^* - EF^*)\,,\end{aligned} \tag{5}$$

where σ is the cross section, A_y is the vector analyzing power, and $\sigma_y I$ is the direct product of the 2×2 identity matrix and the 2×2 Pauli matrix σ_y. The vector analyzing power for $^4\mathrm{He}(\vec{d},t)^3\mathrm{He}$, on the other hand, is given by

$$\begin{aligned}\sigma A_y(\theta) &= \mathrm{Tr}(MM^+P_y) \\ &= 4\,\mathrm{Im}(DF^* + FC^* - AE^* - EB^*)\,,\end{aligned} \tag{6}$$

where P_y is the spin operator of rank 1 appropriate to the usual Madison-convention coordinate system. Hence, the observed deviations from antisymmetry in $^3\mathrm{He}(\vec{t},d)^4\mathrm{He}$ and $^4\mathrm{He}(\vec{d},t)^3\mathrm{He}$ would be expected to differ, for example, according to the sensitivity of the individual matrix elements to violations of charge symmetry.

IV. GEOMETRIC SYMMETRY THEOREMS

Before further discussion of the present experiment, we review the so-called "geometric" symmetry theorems, which are derived solely from considerations of charge (or mirror) symmetry. The adjective "geometric" is used because these theorems rest on assumptions about quantum-mechanical symmetries and not on any dynamical models. As we shall see in Sec. V, these theorems do not apply directly to the $^3\mathrm{He}(\vec{t},d)^4\mathrm{He}$ polarization observables, and it is important to see just why these theorems are inapplicable.

In a reaction of the form $B(A,C)D$, the c.m. cross section σ, the analyzing powers T_{kq}, and the polarizations t_{kq} satisfy the following conditions

$$\sigma[\theta;B(A,C)D] = \sigma[\pi-\theta;B(A,D)C]\,, \tag{6a}$$

$$T_{kq}[\theta;B(\vec{A},C)D] = (-)^q T_{kq}[\pi-\theta;B(\vec{A},D)C]\,, \tag{6b}$$

$$t_{kq}[\theta;B(A,\vec{C})D] = (-)^q t_{kq}[\pi-\theta;B(A,D)\vec{C}]\,. \tag{6c}$$

Here an arrow denotes a particle whose polarization is under consideration, and we have followed

the Madison convention in choosing the coordinate system in which the T_{kq} and t_{kq} are measured. It is to be emphasized that these equations follow simply from the choice of a particular c.m. coordinate frame, and have nothing to do with properties of the particles A,B,C,D.

The symmetry theorems for the cross section σ (due to Barshay and Temmer[9]) and for the analyzing power T_{kq} and polarizations t_{kq} (due to Simonius[10] and Bilen'kii *et al.*[11]) give relations of the form

$$\sigma[\theta;B(A,C)C'] = \sigma[\theta;B(A,C')C], \tag{7a}$$

$$T_{kq}[\theta;B(\vec{A},C)C'] = T_{kq}[\theta;B(\vec{A},C')C], \tag{7b}$$

$$t_{kq}[\theta;B(A,\vec{C})C'] = t_{kq}[\theta;B(A,\vec{C}')C], \tag{7c}$$

where the exit-channel particles C and C' are members of an isospin multiplet. Naively, these equations follow because, in the isospin formalism, C and C' are identical. This intuitive expectation can be verified by applying full-fledged isospin invariance (as in Ref. 9) or the weaker assumption of mirror symmetry (Ref. 10).

If we combine Eqs. (6a)–(6c) and (7a)–(7c), we arrive at the symmetry relations

$$\sigma[\theta;B(A,C)C'] = \sigma[\pi-\theta;B(A,C)C'], \tag{8a}$$

$$T_{kq}[\theta;B(\vec{A},C)C'] = (-)^q T_{kq}[\pi-\theta;B(\vec{A},C)C'], \tag{8b}$$

$$t_{kq}[\theta;B(A,\vec{C})C'] = (-)^q t_{kq}[\pi-\theta;B(A,C)\vec{C}']. \tag{8c}$$

We emphasize that these theorems are based on the isospin symmetry of the particles and conservation of isospin in the reaction. The assumption of definite isospin symmetry, in turn, constrains the allowable space-spin configurations of the particles and leads to symmetry relations for the observables. Because these symmetry theorems are based on the allowed symmetries of quantum-mechanical wave functions, they are model-independent. Deviations from Eqs. (8a)–(8c) must be laid to violations of charge symmetry either in the nature of the isospin partners C and C' (Ref. 12), or in the reaction, or both.

V. A MODEL-DEPENDENT SYMMETRY THEOREM

The geometric symmetry theorem for the cross section [Eq. (8a)] is valid whether the isospin partners C and C' are in the exit or entrance channels. And, if the vector polarization P_y ($\sim it_{11}$) could be shown to satisfy

$$P_y[\theta;B(A,\vec{C})C'] = -P_y[\pi-\theta;B(A,\vec{C})C'] \tag{9}$$

instead of Eq. (8c), the polarization-asymmetry equality[13] would lead immediately to a relation for A_y ($\sim iT_{11}$) analogous to Eq. (8b):

$$A_y[\theta;C'(\vec{C},A)B] = -A_y[\pi-\theta;C'(\vec{C},A)B]. \tag{10}$$

However, Eq. (9) cannot be shown to hold true solely on the basis of charge symmetry, because the polarization measurement distinguishes the otherwise-identical particles C and C'. Hence there appears to be no model-independent way of deriving Eq. (10). Conzett circumvents this difficulty by using both charge symmetry and a "zeroth-order" model of the direct-reaction mechanism.[14] The mathematical proof is given in detail in Ref. 1. Here we present a simple derivation of Eq. (10) based on Conzett's model.

Consider a reaction of the form $C'(C,A)B$ where C and C' are isospin partners, and suppose we observe particle A at a c.m. angle θ. Conzett assumes that only two processes contribute to the total direct-reaction amplitude:

$$C + C' \equiv (A \oplus n_1) + C' \rightarrow A + (C' \oplus n_1)$$
$$\equiv A + B$$

and

$$C + C' \equiv C + (A \oplus n_2) \rightarrow A + (C \oplus n_2)$$
$$\equiv A + B,$$

where n_1 and n_2 are the transferred particles or clusters. These two amplitudes are shown schematically in Fig. 4.

In the first case, particle C is stripped to yield particle A at the angle θ. The M matrix describing this process is denoted by

$$M^{(1)}[\theta;C'(C,A)B].$$

In the second process, particle C' is stripped, producing particle B at the angle $\pi-\theta$, and particle A recoils at the angle θ. The M matrix for this process is

$$M^{(2)}[\pi-\theta;C'(C,B)A].$$

However $M^{(1)}$ is defined in a coordinate system with the y axis parallel to $\vec{k}_i \times \vec{k}_f$ (see Fig. 4), while $M^{(2)}$ is referred to a system in which the y axis is parallel to $\vec{k}_i \times (-k_f)$. Since we wish to add the elements of $M^{(1)}$ and $M^{(2)}$ coherently to obtain the total reaction amplitude, we must describe them in the same coordinate system. A rotation about the z axis ($\vec{k}_i$) accomplishes this change, and it can be shown[15] that, in this common coordinate system, the elements of $M^{(2)}$ become

$$(-)^m M^{(2)}_{\mu\nu}[\pi-\theta;C'(C,B)A], \tag{11}$$

where

$$m = \mu - \nu = (m_C + m_{C'}) - (m_A + m_B) \tag{12}$$

equals the difference in the spin projections for the entrance and exit channels.

If charge symmetry holds, the amplitudes for transferring n_1 and n_2 must be equal, since they

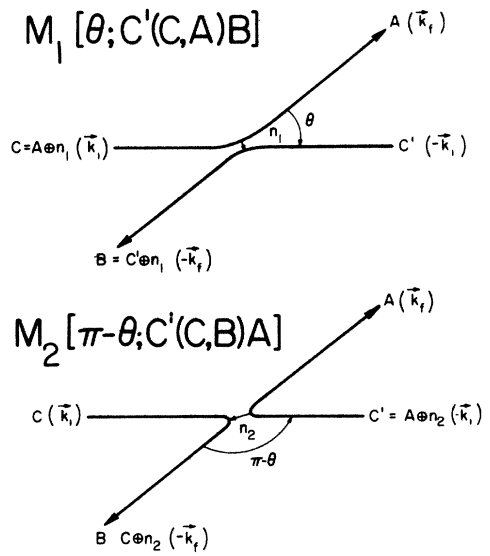

FIG. 4. Schematic diagram of the two components of the direct-reaction amplitude for a simple particle-transfer reaction. The symbols and M-matrix amplitudes are defined in the text.

are isospin partners. Hence

$$M^{(1)}_{\mu\nu}[\theta;C'(C,A)B] = M^{(2)}_{\mu\nu}[\theta;C'(C,B)A]\,. \tag{13}$$

If the direct reaction amplitude is

$$M = M^{(1)} + M^{(2)}\,,$$

then Eq. (11) and Eq. (13) together yield

$$M_{\mu\nu}[\theta;C'(C,A)B] = M^{(1)}_{\mu\nu}[\theta;C'(C,A)B] + (-)^m M^{(1)}_{\mu\nu}[\pi-\theta;C'(C,A)B]\,. \tag{14}$$

It is apparent from Eq. (14) that, *in this model*, every M matrix element is either symmetric or antisymmetric about 90°. Thus the symmetry for a particular analyzing power $T_{kq}(\theta)$ depends on which matrix elements are selected by the spin operator τ_{kq} in the observable

$$\sigma T_{kq}(\theta) = \mathrm{Tr}(M\tau_{kq}M^+)\,.$$

For the ^{3}He$(\vec{t},d)^4$He reaction, the form of the M matrix is given explicitly by Eq. (4). In Table I, we have written out the M-matrix elements in the form of Eq. (14), e.g.,

$$A(\theta) = a(\theta) + a(\pi-\theta)\,,$$
$$\cdots$$
$$D(\theta) = d(\theta) - d(\pi-\theta)\,,$$
$$\cdots$$

where the + or − sign is determined as in Eq. (12). If we write a G for every symmetric (even-parity) M-matrix element and a U for antisymmetric matrix elements, the symmetry relation for A_y can be determined schematically from Eq. (5):

TABLE I. M-matrix elements for the ^{3}He$(t,d)^4$He reaction, assuming a two-component direct-reaction amplitude. The rows are labeled by the exit-channel spin projection $\mu = m_d$; the columns are labeled by the spin projection $\nu = m_t + m_{^3\mathrm{He}}$ in the entrance channel. The matrix elements are related by $\tilde{X}(\theta) \equiv X(\pi-\theta)$. The expression for the M-matrix elements is derived in the text [Eq. (14)].

d-^{4}He spin projections	μ \ ν	t-^{3}He spin projections $(\frac{1}{2})(\frac{1}{2})$ 1	$(\frac{1}{2})(-\frac{1}{2})$ 0	$(-\frac{1}{2})(\frac{1}{2})$ 0	$(-\frac{1}{2})(-\frac{1}{2})$ −1
(1)(0)	1	$a+\tilde{a}$	$d-\tilde{d}$	$-(c-\tilde{c})$	$b+\tilde{b}$
(0)(0)	0	$-(e-\tilde{e})$	$f+\tilde{f}$	$f+\tilde{f}$	$e-\tilde{e}$
(−1)(0)	−1	$b+\tilde{b}$	$c-\tilde{c}$	$-(d-\tilde{d})$	$a+\tilde{a}$

$$\sigma A_y = \mathrm{Tr}(M\sigma_y I M^+)$$
$$= 4\,\mathrm{Im}(AD^* + BC^* - EF^*)$$
$$= 4\,\mathrm{Im}(GU^* + GU^* - UG^*)$$

Since the product of a symmetric and an antisymmetric quantity is antisymmetric, it follows that

$$A_y(\theta) = -A_y(\pi-\theta)\,. \tag{15}$$

Symmetry relations for other spin-dependent observables could be exhibited in this same fashion.

The assumption of a two-component direct-reaction amplitude is central to the derivation of Eq. (15), since it is that assumption which gives a definite symmetry to each M-matrix element. Thus, deviations from Eq. (15) may arise from violations of charge symmetry, or from a reaction mechanism more complicated than the simple two-component model described by Conzett.[14]

VI. DISCUSSION AND CONCLUSIONS

As we have shown in Secs. IV and V, deviations from antisymmetry in the angular distributions of $A_y(\theta)$ must be interpreted differently for reactions of the form $B(\vec{A},C)C'$ and $C'(\vec{C},A)B$.

In the former case, the deviations are due to violations of charge or mirror symmetry only, arising either from isospin impurities in the particles themselves or isospin-nonconservation in the reaction. For example, attempts have been made to fit A_y data for ^{4}He$(\vec{d},t)^3$He using a zero-range distorted-wave Born approximation (DWBA) calculation with differing neutron- and proton-transfer amplitudes.[7] However, the fits obtained in this way were not as satisfactory as those given by similar calculations for the cross section,[16] possibly because exchange effects were neglected.[17] Schütte *et al.*[18] have suggested that isospin-mixing through coupled-channel effects would better explain the deviations from antisymmetry for A_y (and

from symmetry in the cross section σ). They performed a cluster-model calculation which included virtual coupling to the mixed-isospin states of ^{5}He $+ p$ and ^{5}Li $+ n$, and found reasonable agreement between their calculated values and the data of Dahme *et al.*[7]

For reactions of the form $C'(\vec{C},A)B$, deviations from antisymmetry in A_y may arise not only from charge-symmetry violations, but also from reaction mechanisms more complicated than the model proposed by Conzett. The deviations $D(\theta)$ observed in the present experiment are much larger than those seen in ^{4}He$(\vec{d},t)^3$He at comparable excitation energies in ^{6}Li (~0.5 compared to ~0.1). Hence, we believe that a resonance reaction in the intermediate state is a plausible mechanism, because charge asymmetry alone would evidently not lead to such large values of $D(\theta)$. Such a resonance reaction might proceed through one or more of the broad excited states of the ^{6}Li nucleus.

One model whose effects can be calculated to some extent involves isospin mixing in ^{6}Li due to overlapping $T=0$ and $T=1$ levels. This mechanism was first suggested by Murakami,[19] and later invoked by Nocken *et al.*[20] to explain deviations from cross-section symmetry seen in the ^{3}H(^{3}He, d)^{4}He reaction at low bombarding energies. They demonstrated that interference between two broad states at $E_x \approx 16.2$ MeV ($J^\pi, T = 2^-, 1$) and $E_x \approx 17$ MeV ($J^\pi, T = 1^-, 0$) could produce the resonant behavior seen in the maximal cross-section asymmetry. A similar mixing of two broad levels at higher energies might account for the deviations $D(\theta)$ observed in the present experiment. Indeed, resonating-group calculations for ^{3}He$+t$ elastic scattering by Thompson and Tang[21] predict a pair of levels with $T=0$ and $T=1$ at 21.3 and 22.3 MeV, respectively, in ^{6}Li—near the middle of the energy range covered by the present data. This prediction tends to support the compound-nucleus model, since it indicates that the cluster configuration in the ^{3}He$+t$ entrance channel would favor the formation of broad intermediate states in ^{6}Li. The rapid change of $A_y(90.0°)$ as a function of energy above 14 MeV (see Fig. 2) lends additional credence to this hypothesis.

However, the resonating-group calculations[21] which predicted these levels were based only on elastic-scattering cross sections. Now that analyzing-power data are available for ^{3}He$+\vec{t}$ elastic scattering,[3] it would be useful to try to fit the ^{3}He $+\vec{t}$ cross section and analyzing power simultaneously with a resonating-group calculation (as was recently done for d-α elastic scattering[22]) to see if one still predicts a $T=0$, $T=1$ level pair in the energy range of the present data. Once the resonating group calculations have provided a set of starting phases and levels in this energy range, it would be possible to test the compound-nucleus model for the ^{3}He$(\vec{t},d)^4$He reaction in detail using a multichannel, multilevel R-matrix calculation.[23]

APPENDIX: CONSTRUCTION OF THE M MATRIX FOR SPIN $\frac{1}{2}$ + SPIN $\frac{1}{2}$ → SPIN 1 + SPIN 0

The M matrix for an arbitrary reaction is a sum of parity-invariant direct products of spinors in the entrance and exit channels. Each term in the sum has the form[24]

$$A(\Phi\cdot\hat{p}_i)(\Psi\cdot\hat{p}_j)^\dagger , \tag{A1}$$

where A is a complex function of the energy E and scattering angle θ; Ψ and Φ are entrance- and exit-channel spinors, respectively; and $\hat{p}_j$ and $\hat{p}_i$ are unit momentum vectors in the entrance and exit channels, respectively. To construct the M matrix, then, we need to find an appropriate set of basis spinors and discard all the products $(\Phi\cdot\hat{p}_i)(\Psi\cdot\hat{p}_j)^\dagger$ which are not parity-invariant.

It is convenient to choose spinors in the uncoupled representation since the calculation of polarization observables for the individual particles is simplest in this scheme. For the $\frac{1}{2}+\frac{1}{2}\rightarrow 1+0$ reaction, the uncoupled spinors in the $1+0$ channel are the familiar spherical deuteron spinors[8]

$$\chi_+ = \begin{pmatrix}1\\0\\0\end{pmatrix},\quad \chi_0 = \begin{pmatrix}0\\1\\0\end{pmatrix},\quad \chi_- = \begin{pmatrix}0\\0\\1\end{pmatrix}. \tag{A2}$$

The spherical-basis spinors χ_+, χ_0, χ_- are related to the Cartesian-basis spinors χ_x, χ_y, χ_z by the equations

$$\begin{aligned}\chi_+ &= -\frac{1}{\sqrt{2}}(\chi_x + i\chi_y),\\ \chi_0 &= \chi_z,\\ \chi_- &= \frac{1}{\sqrt{2}}(\chi_x - i\chi_y).\end{aligned} \tag{A3}$$

For the $1+0$ channel, then, we use the Cartesian-basis spinors

$$\begin{aligned}\chi_{x'} &= \frac{1}{\sqrt{2}}(-\chi_+ + \chi_-) = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\0\\1\end{pmatrix},\\ \chi_{y'} &= \frac{i}{\sqrt{2}}(\chi_+ + \chi_-) = \frac{i}{\sqrt{2}}\begin{pmatrix}1\\0\\1\end{pmatrix},\\ \chi_{z'} &= \chi_0 = \begin{pmatrix}0\\1\\0\end{pmatrix}\end{aligned} \tag{A4}$$

where (x',y',z') are the exit-channel coordinates.

The two spin-$\frac{1}{2}$ particles can combine to form the singlet state

$$\phi_1 = \frac{1}{\sqrt{2}}[\alpha(1)\beta(2) - \beta(1)\alpha(2)] \tag{A5}$$

and the triplet states

$$\begin{aligned}\phi_2 &= \alpha(1)\alpha(2)\,,\\ \phi_3 &= \frac{1}{\sqrt{2}}[\alpha(1)\beta(2) + \beta(1)\alpha(2)]\,,\\ \phi_4 &= \beta(1)\beta(2)\,,\end{aligned} \tag{A6}$$

where α and β are spin-up and spin-down eigenfunctions, respectively, and 1 and 2 are particle labels. The singlet state has a scalar character, since it is unchanged by any linear transformation of the coordinate system.[25] The triplet states, on the other hand, have the same pseudovector character as the deuteron spinors of Eq. (A2).

To obtain a matrix representation of the spin-$\frac{1}{2}$ + spin-$\frac{1}{2}$ channel, we choose a basis set in the uncoupled representation as follows:

$$\begin{aligned}\alpha(1)\alpha(2) &= \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\,,\quad \beta(1)\alpha(2) = \begin{bmatrix}0\\0\\1\\0\end{bmatrix}\,,\\ \alpha(1)\beta(2) &= \begin{bmatrix}0\\1\\0\\0\end{bmatrix}\,,\quad \beta(1)\beta(2) = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\,.\end{aligned} \tag{A7}$$

The scalar singlet state, in this representation, is

$$\Psi_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}\,. \tag{A8}$$

The triplet states are

$$\Psi_+ = \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\,,\quad \Psi_0 = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}\,,\quad \Psi_- = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\,, \tag{A9}$$

where the notation is chosen to be analogous to that for the spherical basis set for spin-1 particles.

Pursuing the analogy further, we write

$$\begin{aligned}\Psi_+ &= -\frac{1}{\sqrt{2}}(\Psi_x + i\Psi_y)\,,\\ \Psi_0 &= \Psi_z\,,\\ \Psi_- &= \frac{1}{\sqrt{2}}(\Psi_x - i\Psi_y)\,,\end{aligned} \tag{A10}$$

so that

$$\begin{aligned}\Psi_x &= \frac{1}{\sqrt{2}}(-\Psi_+ + \Psi_-)\,,\\ \Psi_y &= \frac{i}{\sqrt{2}}(\Psi_+ + \Psi_-)\,.\end{aligned} \tag{A11}$$

Hence, we find that the spinors for the spin-$\frac{1}{2}$ + spin-$\frac{1}{2}$ channel, in the uncoupled representation, are

$$\begin{aligned}\Psi_0 &= \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\-1\\0\end{bmatrix}\,,\quad \Psi_y = \frac{i}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}\,,\\ \Psi_x &= \frac{1}{\sqrt{2}}\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}\,,\quad \Psi_z = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}\,,\end{aligned} \tag{A12}$$

where x,y,z are the entrance-channel coordinates.

To construct the M matrix, we now need to choose coordinate systems for the entrance and exit channels. Following Ref. 8, we choose the z and z' axes along the entrance- and exit-channel wave vectors $\vec{k}_i$ and $\vec{k}_f$, respectively, the y and y' axes along $\vec{k}_i\times\vec{k}_f$, and the x and x' axes parallel to $(\vec{k}_i\times\vec{k}_f)\times\vec{k}_i$ and $(\vec{k}_i\times\vec{k}_f)\times\vec{k}_f$, respectively. The unit momentum vectors in these directions are

$$\begin{aligned}\hat{p}_x &= \frac{(\vec{k}_i\times\vec{k}_f)\times\vec{k}_i}{k_i^{\,2}k_f}\,,\quad p_{x'} = \frac{(\vec{k}_i\times\vec{k}_f)\times\vec{k}_f}{k_i k_f^{\,2}}\,,\\ \hat{p}_y &= \frac{\vec{k}_i\times\vec{k}_f}{k_i k_f}\,,\quad \hat{p}_{y'} = \hat{p}_y\,,\\ \hat{p}_z &= \frac{\vec{k}_i}{k_i}\,,\quad \hat{p}_{z'} = \frac{\vec{k}_f}{k_f}\,.\end{aligned} \tag{A13}$$

Bearing in mind that Ψ_0 is a scalar, that Ψ_x, Ψ_y, Ψ_z, $\chi_{x'}$, $\chi_{y'}$, $\chi_{z'}$, $\hat{p}_y$, and $\hat{p}_{y'}$ are pseudovectors and that $\hat{p}_x$, $\hat{p}_{x'}$, $\hat{p}_z$, and $\hat{p}_{z'}$ are vectors, one readily finds that the M matrix comprises the following parity-invariant terms:

$$\begin{aligned}M = {} & a\chi_{y'}\Psi_0^\dagger + b\chi_{y'}\Psi_y^\dagger + c\chi_{x'}\Psi_x^\dagger + d\chi_{x'}\Psi_z^\dagger\\ & + e\chi_{z'}\Psi_x^\dagger + f\chi_{z'}\Psi_z^\dagger\,.\end{aligned} \tag{A14}$$

Substituting for the χ_j and $\Psi_k^\dagger$ and performing the implied matrix multiplication, we obtain

$$M=\frac{1}{2}\begin{pmatrix} b+c & -(d+ia) & -(d-ia) & b-c \\ -\sqrt{2}e & \sqrt{2}f & \sqrt{2}f & \sqrt{2}e \\ b-c & d-ia & d+ia & b+c \end{pmatrix}.$$

If we now set

$$A\equiv\tfrac{1}{2}(b+c),$$
$$B\equiv\tfrac{1}{2}(b-c),$$
$$C\equiv\tfrac{1}{2}(d-ia),$$
$$D\equiv-\tfrac{1}{2}(d+ia),$$
$$E\equiv\tfrac{1}{2}\sqrt{2}\,e,$$
$$F\equiv\tfrac{1}{2}\sqrt{2}f,$$

we arrive at the M matrix displayed in Eq. (4) of the text:

$$M=\begin{pmatrix} A & D & -C & B \\ -E & F & F & E \\ B & C & -D & A \end{pmatrix}. \qquad \text{(A15)}$$

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