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Three-body calculation of two-body threshold electrodisintegration of ^{3}He and ^{3}He†

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Threshold two-body electrodisintegration of ^{3}He and ^{3}H is investigated within the context of exact three-body theory. The calculations performed are based on the formalism of Gibson and Lehman. Careful consideration is given to the singularities of the disintegration Born amplitude for this case, since the momentum transfer is not zero, to assure validity of the numerical methods. Calculated results are compared with all the latest threshold ^{3}He electrodisintegration data which sample a range of scattered-electron angles, $92.6° \leq \theta_e \leq 180°$, and incident electron energies, 40 MeV $\leq E_0 \leq$ 120 MeV. Predictions are made for ^{3}H electrodisintegration for some of the same kinematics. The mechanism for the sharp rise as a function of excitation energy in the (e,e') cross section for $\theta_e \sim 90°$ due to the $^2S \rightarrow {}^2S$ monopole transition from Coulomb scattering is singled out by examination of the contributions to the Coulomb doublet amplitude. A similar analysis is carried out for the doublet and quartet transverse amplitudes where the $^2S \rightarrow {}^4P$ magnetic quadrupole transition dominates for excitation energies less than 20 MeV.

[NUCLEAR REACTIONS Electrodisintegration of ^{3}He and ^{3}H, Faddeev calculation, separable interactions.]

I. INTRODUCTION

Three-nucleon calculations involving continuum processes have not progressed as rapidly as bound-state calculations due to the difficulties in handling the Faddeev equations for two- and three-body breakup amplitudes. The formalism recently introduced by Gibson and Lehman[1,2] (and applied by them to ^{3}He photodisintegration) is convenient for finding bound to continuum amplitudes for transition operators which can be treated perturbatively. The electromagnetic, weak, and low energy pion interactions fall into this class.

One of the most useful aspects of the Gibson-Lehman approach is the separation of the matrix elements of the transition operator of a specific reaction from the three-nucleon dynamics. In practice, one need only calculate the transition to the appropriate asymptotic three-particle final state. For example, in two-body disintegration of ^{3}He, the state would be a noninteracting proton and deuteron. This Born amplitude is then inserted into an integral whose kernel involves the three-nucleon final-state interactions. This integral plus the Born term gives the full transition amplitude. Therefore, for a given three-nucleon excitation energy, the three-nucleon continuum problem need be solved only once to consider different disintegration processes. Only the Born amplitude for each process must be computed. In the remainder of this Introduction, we give a prose road map to provide the reader with an orientation to the mathematical development of this method for two-body electrodisintegration of ^{3}He and ^{3}H.

The formalism leading to the two-body breakup amplitude for a general breakup operator and a general nucleon-nucleon potential is developed in Sec. II. The two-body scattering wave function Eq. (10) is expressed in terms of a resolvent and NN potential. A noniterative derivation of the equations in the Gibson-Lehman approach is made by use of the Alt-Grassberger-Sandhas transition operators Eqs. (14)–(16).[3] The full two-body disintegration amplitude Eq. (23) is then written in terms of the Born amplitude and the matrix element of a disintegration operator which connects particle-plus-correlated-pair states. A diagrammatic representation of the amplitude equations (Fig. 1) is given which illustrates the separation of the reaction Born term from the final-state interactions.

Section III becomes specific as to the form of the

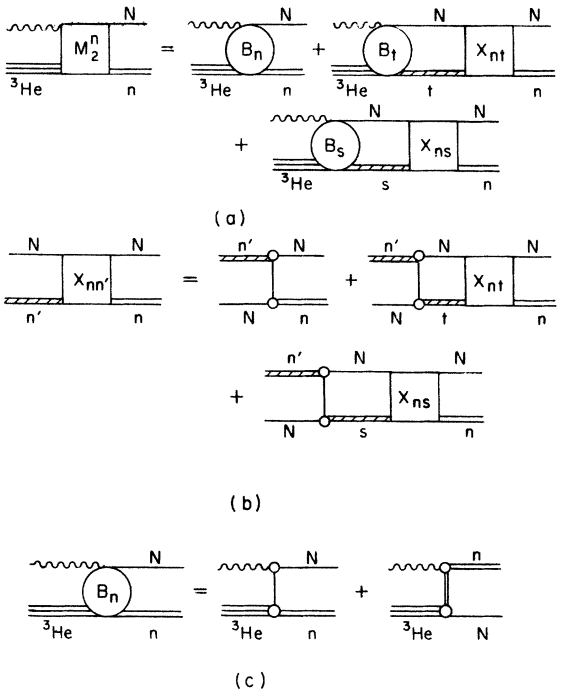

FIG. 1. Diagrammatic representation of the integral equations: (a) Eq. (32), the full amplitude; (b) Eq. (35), the off-shell transitions amplitude—the first term on the right-hand side is a one nucleon exchange amplitude; (c) Eqs. (A6) and (A7), the Born amplitude. The wavy line is the disintegration mechanism which for (e,e') is the virtual photon. Cross-hatched double lines indicate an off-shell correlated pair (plus nucleon).

transition operator for a NN separable potential Eqs. (26) and (27). The two-body transition amplitude Eq. (31) is symmetrized and written as an on-shell Born amplitude and an integral over the off-shell Born amplitudes and nucleon-pair scattering amplitudes. These latter are themselves expressed as integrals Eq. (35) over one-particle exchange amplitudes constructed from free-particle resolvents and pair form factors Eq. (36).

Next the standard electromagnetic current interaction with a single nucleon is introduced Eq. (37) and the Born amplitude for $^3\mathrm{He} \to p+d$ computed for the charge, current, and magnetic moment operators Eqs. (38)–(46). The triply differential inelastic electron scattering cross section, Eqs. (47) and (48), is given.

Beginning with Eq. (49) of Sec. III, an explicit form for the ^{3}He ground-state wave function is developed for the spin-dependent, s-wave separable potential of the Yamaguchi form. The actual potential and wave function parameters are given in Appendix B. A discussion of the spin and isospin dependence of the doublet and quartet pd final state completes the classification of possible transition matrix elements. The Coulomb Born term is given explicitly in Eqs. (64)–(68). The application section ends with the partial-wave expansion of the transition amplitude into angular momentum multipoles Eqs. (69)–(78). The mathematical machinery is now complete to compute the $^3\mathrm{He}(e,e')pd$ cross section.

In Sec. IV, our numerical results for the partial-wave amplitudes are given together with a comparison of the electron scattering cross sections of two recent experiments. A short discussion is given of the important features, particularly the dominance of the Coulomb-monopole transition in charge scattering. Cross section predictions for ^{3}H two-body electrodisintegration are made based on the charge independence of the assumed NN force. A concise list of conclusions we have reached in this study forms Sec. V.

In Appendix A, we treat the numerically important topic of the singularity structure of the amplitude. The integrals over intermediate momenta are performed by contour rotation to avoid the poles and cuts on the real momentum axis. All complex poles and cuts are located and plotted in order to find an optimum rotation angle. Examples of the sensitivity of this choice are given.

II. FORMALISM

In this section, we review the formalism of Gibson and Lehman[1,2] for the disintegration of a three-body nucleus by an interaction which can be treated perturbatively. The total Hamiltonian is written as

$$H_{\mathrm{tot}} = H + H', \tag{1}$$

where H' is the interaction to be treated perturbatively and H is the nuclear Hamiltonian composed of kinetic-energy and pair-interaction operators; specifically,

$$H = H_0 + V, \tag{2}$$

with

$$V = \sum_{\alpha=1}^{3} V_\alpha = V_1 + V_2 + V_3 \equiv V_{23} + V_{31} + V_{12}. \tag{3}$$

The nuclear Hamiltonian is assumed to have eigenstates corresponding to a three-body bound state, a scattering state of a particle plus bound pair, and a scattering state of three unbound particles. For these states, respectively, we have

$$H|\Psi_B\rangle = -E_B|\Psi_B\rangle, \quad E_B > 0, \tag{4}$$

$$H|\Psi_{\alpha n\vec{p}}\rangle = E^{(2)}_{\alpha n}|\Psi_{\alpha n\vec{p}}\rangle, \quad E^{(2)}_{\alpha n} = \frac{p_\alpha^{\,2}}{2m_\alpha} - \epsilon_{\alpha n}, \quad \epsilon_{\alpha n} > 0, \tag{5}$$

$$H|\Psi_{\alpha n\vec{p}\vec{k}}\rangle = E^{(3)}_{\alpha n}|\Psi_{\alpha n\vec{p}\vec{k}}\rangle, \quad E^{(3)}_{\alpha n} = \frac{p_\alpha^{\,2}}{2m_\alpha} + \frac{k_{\beta\gamma}^{\,2}}{2\mu_{\beta\gamma}}, \tag{6}$$

where n represents quantum numbers like spin and

isospin, $\vec{p}_\alpha$ is the relative momentum of particle α with respect to the center of mass of particles β and γ, $\epsilon_{\alpha n}$ is the two-body binding energy, the reduced masses m_α and $\mu_{\beta\gamma}$ are expressed in terms of the individual-particle masses as

$$m_\alpha = \frac{M_\alpha(M_\beta + M_\gamma)}{M_\alpha + M_\beta + M_\gamma}, \tag{7}$$

$$\mu_{\beta\gamma} = \frac{M_\beta M_\gamma}{M_\beta + M_\gamma}, \tag{8}$$

with $\alpha \neq \beta \neq \gamma \neq \alpha$ and each index permitted to have the values 1 to 3.

In two-body electrodistintegration of ^{3}He and ^{3}H, only two-body disintegration amplitudes are needed:

$$A_2(\alpha, n, \vec{p}) = \langle \Psi^{(-)}_{\alpha n \vec{p}} | H' | \Psi_B \rangle, \tag{9}$$

where the superscript (−) denotes the outgoing state which asymptotically corresponds to an incoming wave, and the label for possible momentum dependence of H' has been suppressed. The two-body scattering state is obtained from

$$\langle \Psi^{(-)}_{\alpha n \vec{p}} | = \langle \Phi_{\alpha n \vec{p}} | \left[1 - \sum_\beta \bar{\delta}_{\alpha\beta} V_\beta G(z) \right], \tag{10}$$

where

$$\bar{\delta}_{\alpha\beta} = 1 - \delta_{\alpha\beta}, \quad \eta > 0, \quad z = E^{(2)}_{\alpha n} + i\eta,$$

$$G(z) = (H - z)^{-1}, \tag{11}$$

and $\langle \Phi_{\alpha n \vec{p}} |$ denotes an asymptotic scattering state composed of particle α moving freely relative to the $\beta\gamma$ bound pair:

$$H_\alpha | \Phi_{\alpha n \vec{p}} \rangle = E^{(2)}_{\alpha n} | \Phi_{\alpha n \vec{p}} \rangle, \tag{12}$$

with

$$H_\alpha = H_0 + V_\alpha. \tag{13}$$

Equation (10) can be written in terms of the Alt-Grassberger-Sandhas transition operators[3] $U_{\beta\alpha}(z)$ by means of

$$G(z) = \delta_{\beta\alpha} G_\beta(z) - G_\beta(z) U_{\beta\alpha}(z) G_\alpha(z), \tag{14}$$

where

$$U_{\beta\alpha} = -\bar{\delta}_{\beta\alpha} G_0^{-1}(z) - \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma(z) G_0(z) U_{\gamma\alpha} \tag{15}$$

or

$$U_{\beta\alpha} = -\bar{\delta}_{\beta\alpha} G_0^{-1}(z) - \sum_\gamma \bar{\delta}_{\gamma\alpha} U_{\beta\gamma} G_0(z) T_\gamma(z). \tag{16}$$

The significance of the $U_{\beta\alpha}(z)$ is that their on-shell matrix elements between states of the form given in Eq. (12) yield the S matrix, e.g., from an initial configuration characterized by $|\Phi_{\alpha n \vec{p}}\rangle$ to a final configuration $|\Phi_{\beta m \vec{p}'}\rangle$. In our case, they permit separation of the particular disintegration mechanism from the final-state, three-particle rescattering. When Eq. (14) is substituted into Eq. (10) and some operator algebra performed, Eq. (10) becomes

$$\langle \Psi^{(-)}_{\alpha n \vec{p}} | = \langle \Phi_{\alpha n \vec{p}} | \left[1 + \sum_\gamma U_{\alpha\gamma}(z) G_0(z) T_\gamma(z) G_0(z) \right], \tag{17}$$

where the two-body transition operator $T_\gamma(z)$ is defined by

$$T_\gamma(z) G_0(z) = V_\gamma G_\gamma(z), \tag{18}$$

$$G_\gamma(z) = (H_\gamma - z)^{-1}, \tag{19}$$

and

$$G_0(z) = (H_0 - z)^{-1}. \tag{20}$$

Therefore, the two-body disintegration amplitude becomes

$$A_2(\alpha, n, \vec{p}) = \langle \Phi_{\alpha n \vec{p}} | H' | \Psi_B \rangle + \sum_\gamma \langle \Phi_{\alpha n \vec{p}} | U_{\alpha\gamma}(z) G_0(z) T_\gamma(z) G_0(z) H' | \Psi_B \rangle, \tag{21}$$

where $z = E^{(2)}_{\alpha n} + i\eta$. The equations of Gibson and Lehman are recovered by defining the $X_{\beta\alpha}(z)$ amplitudes which lead to Lippmann-Schwinger type equations; in detail,

$$X_{\beta\alpha}(z) \equiv -G_0(z) U_{\beta\alpha}(z) G_0(z), \tag{22}$$

so

$$A_2(\alpha, n, \vec{p}) = \langle \Phi_{\alpha n \vec{p}} | H' | \Psi_B \rangle - \sum_\gamma \langle \Phi_{\alpha n \vec{p}} | G_0^{-1}(z) X_{\alpha\gamma}(z) T_\gamma(z) G_0(z) H' | \Psi_B \rangle \tag{23}$$

and

$$X_{\beta\alpha}(z) = \bar{\delta}_{\beta\alpha} G_0(z) - \sum_\gamma G_0(z) \bar{\delta}_{\beta\gamma} T_\gamma(z) X_{\gamma\alpha}(z) \tag{24}$$

or

$$X_{\beta\alpha}(z)=\bar{\delta}_{\beta\alpha}G_0(z)-\sum_{\gamma}X_{\beta\gamma}(z)T_{\gamma}(z)\bar{\delta}_{\gamma\alpha}G_0(z)\ . \qquad (25)$$

The X transition operators connect the so-called particle-plus-correlated-pair states, i.e., $\langle\Phi_{\alpha n\vec{p}}|G_0^{-1}(z)$.

The above constitutes a noniterative derivation of the Gibson-Lehman equations.[4] The usefulness of expressing the two-body disintegration amplitude in terms of the X transition operators is immediately apparent by inserting a complete set of states between the G_0 and H' operators in the second term on the right-hand side of Eq. (23): the final-state three-particle dynamics are separated from the disintegration process of the bound state due to H'. The continuum three-body problem need only be considered once for a given excitation energy to calculate all perturbative-type disintegration processes.

III. APPLICATION OF FORMALISM TO TWO-BODY ELECTRODISINTEGRATION OF ^{3}He AND ^{3}H

Application of Eqs. (23)–(25) to two-body electrodisintegration of ^{3}He and ^{3}H requires knowledge of $T_\alpha(z)$ and H'. We shall specify the two-nucleon interaction model first, then manipulate and simplify the equations as far as possible without an explicit H'. Finally, H' will be given, the partial-wave expansions defined, and the equations actually used written down.

The two-nucleon transition operator in the three-particle Hilbert space is taken to be separable, attractive, s-wave, spin dependent, but charge independent:

$$T_\alpha(z)=-\sum_{n=s}^{t}|g_{\alpha n}\rangle\tau_{\alpha n}(z)\langle g_{\alpha n}|(|SI\rangle\langle SI|)_{\alpha n}, \qquad (26)$$

where

$$\tau_{\alpha n}(z)=\frac{\lambda_n}{2\mu_\alpha}\left[1-\frac{\lambda_n}{2\mu_\alpha}\langle g_{\alpha n}|G_0(z)|g_{\alpha n}\rangle\right]^{-1}. \qquad (27)$$

The lower-case letters s and t denote singlet and triplet spin, respectively, for the interacting nucleon pair, while the upper-case letters $S(I)$ represent the total spin (isospin) of the three-nucleon system obtained by coupling the spin (isospin) of the noninteracting particle α to the spin (isospin) of the interacting pair $\beta\gamma$. The strength of the interaction is given by λ_n, and the form factors $|g_{\alpha n}\rangle$ determine its range. This form of the two-nucleon interaction corresponds to a separable, nonlocal potential which, if it can support a bound state of binding energy ϵ_n, has the two-nucleon bound-state wave function

$$\langle\chi_n|=N_2\langle g_n|G_0^{(2)}(-\epsilon_n),\quad \epsilon_n>0\ , \qquad (28)$$

where N_2 is chosen such that $\langle\chi_n|\chi_n\rangle=1$ and $G_0^{(2)}(z)$ is the free-particle resolvent for *two* particles. The states of Eq. (12) are

$$\langle\Phi_{\alpha n\vec{p}}|=N_2\langle g_{\alpha n}\vec{p}\,|G_0(p^2/2m_\alpha-\epsilon_{\alpha n}) \qquad (29)$$

with $\langle g_{\alpha n}\vec{p}\,|=\langle g_{\alpha n}|\langle\vec{p}\,|$. Then Eqs. (26) and (29) can be substituted into Eq. (23) to obtain

$$A_2(\alpha,n,\vec{p})=N_2\left[\langle g_{\alpha n}\vec{p}\,|G_0(z)H'|\Psi_B\rangle + \sum_{\beta=1}^{3}\sum_{n'=s}^{t}\int d^3p'\,\langle g_{\alpha n}\vec{p}\,|X_{\alpha\beta}(z)|g_{\beta n'}\vec{p}\,'\rangle\tau_{\beta n'}(z-p'^2/2m_\beta)\,\langle g_{\beta n'}\vec{p}\,'|G_0(z)H'|\Psi_B\rangle\right], \qquad (30)$$

where $\int d^3p'|\vec{p}\,'\rangle\langle\vec{p}\,'|=1$ has been used, the spin-isospin projection operator has been suppressed in the second term, and $z=E_{\alpha n}^{(2)}+i\eta$.

In our application of Eq. (30) to trinucleon electrodisintegration, the three nucleons will be taken as three identical particles of mass M. Therefore the amplitudes of interest must be symmetric in the three nucleons. We define symmetrized amplitudes as follows:

$$M_2^n(z,\vec{p})\equiv\left(\frac{1}{3}\right)^{1/2}\sum_{\alpha=1}^{3}A_2(\alpha,n,\vec{p}) \qquad (31)$$

$$=B_n(z,\vec{p})+\sum_{n'=s}^{t}\int d^3p'\,\langle\vec{p}\,|X_{nn'}(z)|\vec{p}\,'\rangle\,\tau_{n'}(z-3p'^2/4M)B_{n'}(z,\vec{p}\,'), \qquad (32)$$

where

$$B_n(z,\vec{p})=N_2\left(\frac{1}{3}\right)^{1/2}\sum_{\alpha=1}^{3}\langle g_{\alpha n}\vec{p}\,|G_0(z)H'|\Psi_B\rangle \qquad (33)$$

and

$$\langle \vec{p}\,|X_{nn'}(z)|\vec{p}\,'\rangle = \frac{1}{3}\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\langle g_{\alpha n}\vec{p}\,|X_{\alpha\beta}(z)|g_{\beta n'}\vec{p}\,'\rangle . \tag{34}$$

The amplitudes in Eqs. (32)–(34) are written in off-shell form to emphasize that z and $|\vec{p}\,|$ need not be related, nor is $|\vec{p}\,|$ necessarily equal to $|\vec{p}\,'|$. For two-body electrodisintegration of the trinucleons, the amplitude $M_2^t(z,\vec{p}\,)$ is required with $z = 3p^2/4M - \epsilon_d + i\eta$, where ϵ_d is the deuteron binding energy. The main ingredients in this amplitude are the half-off-shell three-particle transition amplitudes which satisfy

$$\langle \vec{p}\,|X_{mm'}(z)|\vec{p}\,'\rangle = \langle \vec{p}\,|Z_{nn'}(z)|\vec{p}\,'\rangle + \sum_{m=s}^{t}\int d^3p''\langle \vec{p}\,|X_{nm}(z)|\vec{p}\,''\rangle\,\tau_m(z-3p''^2/4M)\langle \vec{p}\,''|Z_{mn'}(z)|\vec{p}\,'\rangle , \tag{35}$$

where the one-particle exchange amplitude is

$$\langle \vec{p}\,|Z_{mn'}(z)|\vec{p}\,'\rangle = \frac{1}{3}\sum_{\alpha=1}^{3}\sum_{\beta=1}^{3}\bar{\delta}_{\alpha\beta}\langle g_{\alpha n}\vec{p}\,|G_0(z)|g_{\beta n'}\vec{p}\,'\rangle . \tag{36}$$

Equations (35) and (36) follow directly from Eqs. (24) and (25) with Eq. (34). The physics embodied in Eqs. (32) and (35) is more apparent if they are represented graphically as in Fig. 1.

To this point, the formalism has been developed to include all two-body disintegration processes of a trinucleon caused by a perturbative Hamiltonian H'. In our case, H' is taken to be the electron-nucleon Hamiltonian developed by McVoy and Van Hove.[5] This involves a reduction to the nonrelativistic limit of the nuclear electromagnetic current for interactions with a single nucleon. This gives for the jth nucleon

$$H_j' = -\frac{4\pi e^2}{q_\mu{}^2}\langle u_f|F_{1N}(q_\mu{}^2)e^{-i\bar{q}\cdot\bar{x}_j}|-\frac{F_{1N}(q_\mu{}^2)}{2M}(\vec{p}_j\cdot\vec{\alpha}\,e^{-i\bar{q}\cdot\bar{x}_j}+e^{-i\bar{q}\cdot\bar{x}_j}\vec{\alpha}\cdot\vec{p}_j)$$
$$-\frac{1}{2M}\left[F_{1N}(q_\mu{}^2)+\kappa_N F_{2N}(q_\mu{}^2)i\vec{\sigma}_j\cdot(\vec{q}\times\vec{\alpha})e^{-i\bar{q}\cdot\bar{x}_j}+\frac{q_\mu{}^2}{8M^2}(F_{1N}(q_\mu{}^2)+2\kappa_N F_{2N}(q_\mu{}^2))\right]e^{-i\bar{q}\cdot\bar{x}_j}|u_i\rangle , \tag{37}$$

where $\bar{x}_j$ and $\bar{p}_j$ are four-vectors denoting the nucleon position and momentum, $\vec{\sigma}_j$ is the nucleon spin operator, $\vec{\alpha}$ is the electron's Dirac operator which operates on the free-electron spinors $|u_f\rangle$ and $|u_i\rangle$, $q_\mu{}^2<0$ is the exchanged four-momentum squared, F_{1N} and F_{2N} are the nucleon Dirac and Pauli form factors, κ_N is the nucleon static anomalous magnetic moment, $\bar{q}\cdot\bar{x}_j$ is a four-vector scalar product, and M is the nucleon mass.

To find the electron-nucleus interaction operator, the operator in Eq. (37) is summed over the three nucleons. This implies the assumption that the nucleons behave in a nucleus as free nucleons.

This form of H' is put into Eq. (33) to find the Born amplitude $B_n(z,\vec{p}\,)$ to obtain [note that we suppress the variable $\vec{q}$, i.e., $B_n(z,\vec{p}\,)\equiv B_n(z,\vec{p},\vec{q})$]:

$$B_n(z,\vec{p}\,) = \left(\frac{1}{3}\right)^{1/2}\sum_{\alpha=1}^{3}N_2\langle g_{\alpha n}\vec{p}\,|G_0(z)\sum_{j=1}^{3}H_j'|\Psi_B\rangle \tag{38}$$

$$= -\frac{4\pi e^2}{q_\mu{}^2}\left(\frac{1}{3}\right)^{1/2}\sum_{\alpha=1}^{3}(\langle u_f|u_i\rangle Q - \langle u_f|\vec{\alpha}\,|u_i\rangle\cdot\vec{J}) , \tag{39}$$

where

$$\vec{J} = \vec{J}^{\rm el} + \vec{J}^{\rm mag} , \tag{40}$$

$$Q = F_{\rm ch}^p(1+q_\mu{}^2/8M^2)\left\langle f\left|\sum_{j=1}^{3}e^{-i\bar{q}\cdot\bar{x}_j}\tfrac{1}{2}(1+\tau_3)_j\right|\Psi_B\right\rangle + F_{\rm ch}^n(1+q_\mu{}^2/8M^2)\left\langle f\left|\sum_{j=1}^{3}e^{-i\bar{q}\cdot\bar{x}_j}\tfrac{1}{2}(1-\tau_3)_j\right|\Psi_B\right\rangle , \tag{41}$$

$$\vec{J}^{\rm el} = \left\langle f\left|\sum_{j=1}^{3}(F_{\rm ch}^p/2M)(\vec{p}_j\,e^{-i\bar{q}\cdot\bar{x}_j}+e^{-i\bar{q}\cdot\bar{x}_j}\vec{p}_j)\tfrac{1}{2}(1+\tau_3)_j\right|\Psi_B\right\rangle$$
$$+\left\langle f\left|\sum_{j=1}^{3}(F_{\rm ch}^n/2M)(\vec{p}_j e^{-i\bar{q}\cdot\bar{x}_j}+e^{-i\bar{q}\cdot\bar{x}_j}\vec{p}_j)\tfrac{1}{2}(1-\tau_3)_j\right|\Psi_B\right\rangle , \tag{42}$$

$$\vec{J}^{\rm mag} = (i/2M)F_{\rm mag}^p\left\langle f\left|\sum_{j=1}^{3}e^{-i\bar{q}\cdot\bar{x}_j}\vec{\sigma}_j\times\vec{q}\,\tfrac{1}{2}(1+\tau_3)_j\right|\Psi_B\right\rangle + (i/2M)F_{\rm mag}^n\left\langle f\left|\sum_{j=1}^{3}e^{-i\bar{q}\cdot\bar{x}_j}\vec{\sigma}_j\times\vec{q}\,\tfrac{1}{2}(1-\tau_3)_j\right|\Psi_B\right\rangle . \tag{43}$$

In these equations

$$|f\rangle = N_2 G_0(z)|g_{\alpha n}\vec{p}\rangle , \tag{44}$$

τ_3 is the nucleon isospin operator, and F_{ch} and F_{mag} are the nucleon charge and magnetic form factors, defined by

$$F_{ch} = F_1 + (q_\mu{}^2/4M^2)\kappa F_2 , \tag{45}$$

$$F_{mag} = F_1 + \kappa F_2 . \tag{46}$$

[The superscript n on F_{ch} and F_{mag} refers to the neutron and should not be confused with the subscript on $B_n(z,\vec{p})$.] The analytical fit to the nucleon form factors given by Janssens[6] *et al.* is used for the calculations. Ultimately, the charge form factor of the neutron is taken to be zero, but it is retained in the derivations. In this manner, the ^{3}H cross sections may be obtained from the ^{3}He cross sections merely by exchanging neutron and proton labels.

In place of a single term for $B_n(z,\vec{p})$, we now have three terms: Q, $\vec{J}^{el}$, and $\vec{J}^{mag}$, which are kept separate because of their differing dependence on electron and nucleon spins. Transitions caused by Q will be called Coulomb transitions and those caused by $\vec{J}^{el}$ and $\vec{J}^{mag}$ will be called electric-current and magnetic-moment transitions, respectively.

It may be seen from Eqs. (31) and (32) that the total amplitude $M_2^n(z,\vec{p})$ has the same structure as $B_n(z,\vec{p})$. Therefore, corresponding to the Born Q and $\vec{J}$ there are rescattered Q's and $\vec{J}$'s from which the cross section can be calculated. Summing and averaging over electron spins give the result for the coincidence cross section

$$\frac{d^3\sigma}{dE_f\, d\Omega_e\, d\Omega_p} = \frac{\sigma_m}{(2\pi)^3} M\, |\vec{p}_p|\, E_f{}^2 \frac{F^2}{|1 - M\vec{p}_p\cdot\vec{p}_d/M_d p_p{}^2|} , \tag{47}$$

$$\sigma_m = \left(\frac{e^2\cos\tfrac{1}{2}\theta}{2E_i \sin^2\tfrac{1}{2}\theta}\right)^2 ,$$

$$F^2 = Q^2 + J^2\tan^2(\tfrac{1}{2}\theta) - \tfrac{1}{2}\sec^2(\tfrac{1}{2}\theta)(Q^*\vec{J} + \vec{J}^*Q)\cdot(\hat{K}_i + \hat{K}_f)$$

$$+ \tfrac{1}{2}\sec^2(\tfrac{1}{2}\theta)(\vec{J}\cdot\hat{K}_i\,\vec{J}^*\cdot\hat{K}_f + \vec{J}\cdot\hat{K}_f\,\vec{J}^*\cdot\hat{K}_i) , \tag{48}$$

where the electron is scattered through the angle θ to a final energy E_f, $\vec{p}_p$ and $\vec{p}_d$ are the laboratory momenta of the outgoing proton and deuteron, and M_d is the deuteron mass (see Fig. 2).

To solve these equations and calculate a cross section, explicit forms of the wave functions are needed. For the ground-state wave function Ψ_B we included both the spatially symmetric S and the mixed symmetry S' components. The relatively small S' component in the ground state was retained both because of large expected interference terms in the analysis of coincidence data and because it was necessary in order to maintain orthogonality with the final nuclear state. The D-state component of the ground state was neglected both because of the greatly increased complexity it would add to the problem, and because the Coulomb transitions which dominate most of the kinematic regions we studied do not allow interference between the S and D states.

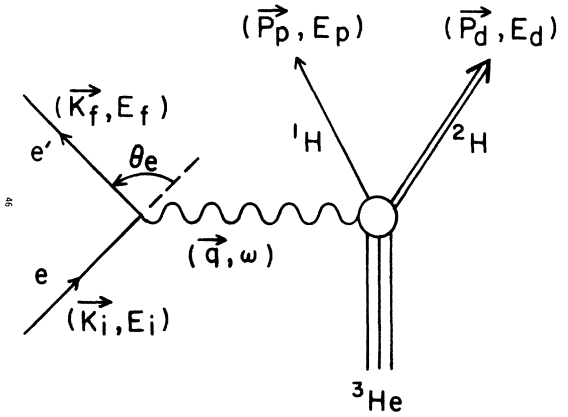

FIG. 2. Kinematics of the electrodisintegration of ^{3}He. The momentum and energy transfer to the three-nucleon system are $q = \vec{K}_i - \vec{K}_f$, $\omega = E_i - E_f$. The relative momentum between proton and deuteron in the final state is $\vec{p} = \frac{2}{3}\vec{p}_p - \frac{1}{3}\vec{p}_d$.

Explicitly, the form of the ground state is

$$\Psi_B = \psi^s\,\xi^a + (\psi'\xi'' - \psi''\,\xi') , \tag{49}$$

where

$$\xi^a = \frac{1}{\sqrt{2}}[\chi'\eta'' - \chi''\eta'] , \tag{50}$$

$$\xi' = \frac{1}{\sqrt{2}}[\chi'\eta'' + \chi''\eta'] , \tag{51}$$

$$\xi'' = \frac{1}{\sqrt{2}}[\chi'\eta' - \chi''\eta''] . \tag{52}$$

The ψ's are functions of momentum only, and the ξ contain the spin (χ) and isospin (η) dependence. The first term on the right-hand side of Eq. (49) forms the completely symmetric part of the ground state (ψ^s is symmetric), and the terms in parentheses together form the S' part of the ground state. $\chi'(1,2\,3)$ is the spin-$\frac{1}{2}$ function obtained by first coupling the spins of particles 2 and 3 to spin 0, and $\chi''(1,2\,3)$ is the spin-$\frac{1}{2}$ function obtained by first coupling the spins of particles 2 and 3 to spin 1. The η are the corresponding isospin functions. The symmetry classifications of the various functions are indicated by their superscripts: s indicates symmetry to an exchange of any two particles; a indicates antisymmetry to an exchange of

any two particles; and the single and double prime indicate, respectively, antisymmetry and symmetry to an exchange of particles 2 and 3.

The functions ψ are further decomposed as follows:

$$\psi^s = g(1,23) + g(2,31) + g(3,12)\,, \tag{53}$$

$$\psi' = \tfrac{3}{2}[h(3,12) - h(2,31)]\,, \tag{54}$$

$$\psi'' = -h(1,23) + \tfrac{1}{2}[h(3,12) + h(2,31)]\,, \tag{55}$$

and

$$g(1,23) = v(1,23) + u(1,23)\,, \tag{56}$$

$$h(1,23) = v(1,23) - u(1,23)\,, \tag{57}$$

$$v(p,k) = N_3 \frac{g_t(k)a_t(p)}{K^2 + \frac{3}{4}p^2 + k^2}\,, \tag{58}$$

$$u(p,k) = N_3 \frac{g_s(k)a_s(p)}{K^2 + \frac{3}{4}p^2 + k^2}\,. \tag{59}$$

The notation $g(1,23)$ indicates that g is a function of $\vec{p}_1$, $\vec{k}_{23}$, and the permutations are defined accordingly. N_3 is the ground-state normalization constant, and K^2 is defined in terms of the three-nucleon binding energy B_3 as

$$K^2 = MB_3\,. \tag{60}$$

$a_s(p)$ and $a_t(p)$ are solutions of two coupled one-dimensional integral equations[7] [similar to Eq. A4)], and are parametrized in the form

$$a_i(p) = N_i(1 + \bar{\alpha}_i p^2 + \bar{\beta}_i p^4 + \bar{\gamma}_i p^6)^{-1}\,, \tag{61}$$

where $i = s$ or t. u, v, g, and h are each symmetric to exchange of particles 2 and 3. $g_n(k) \equiv \langle \vec{k} | g_n \rangle = 1/(k^2 + \beta_n^2)$ in our parametrization.

The final state in Born approximation consists of a deuteron plus a plane-wave nucleon. The deuteron wave function is of the form [see Eq. (28)]:

$$\chi_t(k) = N_2 g_t(k)(k^2 + \gamma^2)^{-1}\,, \tag{62}$$

where γ is found from the calculated deuteron binding energy ϵ_d by

$$\gamma^2 = M\epsilon_d\,. \tag{63}$$

See Appendix B for potential and wave function parameters. The spin-isospin of the final state is determined by the requirement that a deuteron be formed. This allows only the spin doublet ($\chi''\eta'$) and quartet ($\chi^s\eta'$) as physical final states, and they may rescatter through one other intermediate state, the singlet ($\chi'\eta''$).

Since both the doublet and quartet are possible physical final states, amplitudes for both must be calculated. The total cross section has contributions from both. The singlet amplitude is not a possible physical final state, and contributes only as an intermediate state through the rescattering part of Eq. (32); i.e., through $B_{n'}(z, \vec{p}')$.

Now that the initial and final states have been given explicit form, they may be substituted into Eqs. (41) to (43). Once the spin and isospin projections are made, the Born matrix elements may be given in a form appropriate for calculation. This gives for the Coulomb matrix elements

$$Q_d = \frac{F_{ch}^p}{\sqrt{2}}(1 + q_\mu^2/8M^2)\int d^3k \frac{g_t(k)}{K^2 + \frac{3}{4}p^2 - zM}\left\{[\psi_{\rm I}^s - \psi_{\rm I}''] + \left[\psi_{\rm II}^s - \psi_{\rm II}'' - \frac{1}{\sqrt{3}}\psi_{\rm II}'\right]\right\}$$
$$+ \frac{F_{ch}^n}{\sqrt{2}}(1 + q_\mu^2/8M^2)\int d^3k \frac{g_t(k)}{K^2 + \frac{3}{4}p^2 - zM}\left\{\psi_{\rm II}^s - \psi_{\rm II}'' + \frac{1}{\sqrt{3}}\psi_{\rm II}'\right\}\,, \tag{64}$$

$$Q_a = 0\,, \tag{65}$$

$$Q_s = \frac{F_{ch}^p}{\sqrt{2}}\left(1 + \frac{q_\mu^2}{8M^2}\right)\int d^3k \frac{g_s(k)}{K^2 + \frac{3}{4}p^2 - zM}\left[\left(-\tfrac{1}{3}\psi_{\rm I}^s - \tfrac{1}{3}\psi_{\rm I}''\right) + \left(-\tfrac{5}{3}\psi_{\rm II}^s - \tfrac{5}{3}\psi_{\rm II}'' + \frac{1}{\sqrt{3}}\psi_{\rm II}'\right)\right]$$
$$+ \frac{F_{ch}^N}{\sqrt{2}}\left(1 + \frac{q_\mu^2}{8M^2}\right)\int d^3k \frac{g_s(k)}{K^2 + \frac{3}{4}p^2 - zM}\left[\left(-\tfrac{2}{3}\psi_{\rm I}^s - \tfrac{2}{3}\psi_{\rm I}''\right) + \left(-\tfrac{1}{3}\psi_{\rm II}^s - \tfrac{1}{3}\psi_{\rm II}'' - \frac{1}{\sqrt{3}}\psi_{\rm II}'\right)\right]\,. \tag{66}$$

The functions $\psi_{\rm I}$ and $\psi_{\rm II}$ are defined by

$$\psi_{\rm I} = \psi(\vec{k}, \vec{p} - \tfrac{2}{3}\vec{q})\,, \tag{67}$$

$$\psi_{\rm II} = \psi(\vec{k} - \tfrac{1}{2}\vec{q}, \vec{p} + \tfrac{1}{3}\vec{q})\,. \tag{68}$$

All the terms which depend on the form factor of the proton correspond to the virtual photon interacting with the proton, and likewise for the neutron. In the Born approximation, all the terms which depend on the wave functions $\psi_{\rm I}$ correspond to scattering from the nucleon which is ejected; the terms which depend on $\psi_{\rm II}$ correspond to scat-

tering from a nucleon which remains a part of the residual pair. Similar expressions were found for $\vec{J}^{el}$ and $\vec{J}^{mag}$.

The equations as given above completely define the problem of two-body electrodisintegration of the trinucleon. One further step is necessary for ease of calculation and to provide further insight. The amplitudes are expanded into partial waves. Partial-wave expansion eliminates the vector dependence of $\langle \vec{p}|X_{nn'}(z)|\vec{p}'\rangle$, making Eq. (35) easier to solve; and, by obtaining the transition results multipole by multipole, greater insight is given into the dynamics of the process.

Following Gibson and Lehman,[1] $\langle \vec{p}|X_{nn'}(z)|\vec{p}'\rangle$ and $\langle \vec{p}|Z_{nn'}(z)|\vec{p}'\rangle$ are expanded in Legendre polynomials

$$\langle \vec{p}|X_{nn'}(z)|\vec{p}'\rangle = \sum_{L=0}^{\infty}(2L+1)X^L_{nn'}(p,p',z)P_L(\hat{p}\cdot\hat{p}'), \quad (69)$$

$$\langle \vec{p}|Z_{nn'}(z)|\vec{p}'\rangle = \sum_{L=0}^{\infty}(2L+1)Z^L_{nn'}(p,p';z)P_L(\hat{p}\cdot\hat{p}'). \quad (70)$$

When this is done, Eq. (35) becomes

$$X^L_{nn'}(p,p';z) = Z^L_{nn'}(p,p';z) + 4\pi\sum_{m=s}^{t}\int_0^{\infty} p''^2dp'' Z^L_{nm}(p',p'';z)\tau_m(z-3p''^2/4M)X^L_{nm}(p,p'';z), \quad (71)$$

and $Z^L_{nn'}(p,p';z)$ has the explicit form

$$Z^L_{nn'}(p,p';z) = C_{nn'}\int_{-1}^{+1} dx \frac{P_L(x)g_n(q^2)g_{n'}(q'^2)}{p^2+p'^2+pp'x-Mz}, \quad (72)$$

where

$$q^2 = \tfrac{1}{4}p^2 + p'^2 + pp'x. \quad (73)$$

$$q'^2 = p^2 + \tfrac{1}{4}p'^2 + pp'x. \quad (74)$$

The $C_{nn'}$ are

$$C^d_{nn'} = \begin{bmatrix} C_{tt} & C_{ts} \\ C_{st} & C_{ss} \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix}, \quad (75)$$

$$C^q_{nn} = -\tfrac{1}{2}, \quad (76)$$

for doublet and quartet, respectively. A similar partial-wave decomposition is made for $B_n(z,\vec{p})$,

$$B_n(z,\vec{p}) = \sum_{L=0}^{\infty}(2L+1)B^L_n(z,p)P_L(\hat{p}\cdot\hat{q}), \quad (77)$$

where $\vec{q}$ is the momentum transfer due to the disintegration process. Then Eq. (32) has the partial-wave form

$$M^n_2(z,\vec{p}) = \sum_{L=0}^{\infty}(2L+1)P_L(\hat{p}\cdot\hat{q})\left[B^L_n(z,p) + 4\pi\sum_{m=s}^{t}\int_0^{\infty} p'^2dp' X^L_{nm}(p,p';z)\tau_m(z-3p'^2/4M)B^L_m(z,p')\right]. \quad (78)$$

The process of calculation proceeds as follows: Eqs. (71) and (72) are used to generate the three-particle half-off-shell partial-wave amplitudes. As a separate calculation, the components of $B^L_n(z,p)$ (Q and $\vec{J}$) are found. [See Eqs. (64) to (68) for the explicit form of Q.] Equation (78) is then used to combine the results into rescattered amplitudes, and Eq. (47) gives the cross section.

IV. RESULTS AND DISCUSSION

A. Amplitudes for proton plus deuteron breakup

As pointed out at the end of the previous section, the transition amplitudes for inelastic electron scattering are constructed by inserting the Born amplitudes for the charge, current, and magnetic moment operators, the partial-wave expansions of Eqs. (41)–(43), into Eq. (78) to form the full amplitude for each multipole: $M^L(z,p,q)$. After integration over the nuclear angular distribution the Coulomb form factor is

$$|F^L_{\text{Coul}}(q^2,\omega)|^2 = \pi(2L+1)Mp\,|M^L_{\text{Coul}}(z,p,q)|^2, \quad (79)$$

and the transverse is

$$|F^L_{\text{Trans}}(q^2,\omega)|^2 = \frac{(2L+1)Mp}{2\pi^2}|M^L_{\text{Trans}}(z,p,q)|^2. \quad (80)$$

The electron scattering cross section is then

$$\frac{d^2\sigma}{d\Omega_e dE_f} = \sigma_0\sum_L\left[|F^L_{\text{Coul}}|^2 + (\tfrac{1}{2}+\tan^2\tfrac{1}{2}\theta)|F^L_{\text{Trans}}|^2\right] \quad (81)$$

where

TABLE I. Coulomb doublet amplitudes ($fm^{3/2}$).

ω (MeV)	q (fm^{-1})	L	Born (on shell)	Triplet (on shell)	Triplet (off shell)	Singlet (off shell)	Full amplitude	$\lvert F^L(q^2,\omega)\rvert^2$ (MeV^{-1})
6	0.637	0	+3.45	−0.058 − i0.131	−3.55 + i0.137	−0.869 + i0.054	−1.03 + i0.060	0.101(−1)
		1	+0.024	+0.000 + i0.004	−0.006 − i0.004	+0.011 − i0.000	+0.029 − i0.000	0.248(−4)
		2	+0.026	+0.000 − i0.039	+0.001 + i0.039	+0.000 + i0.000	+0.027 + i0.000	0.355(−4)
8	0.630	0	+3.01	−1.68 − i2.62	−1.44 + i2.64	−0.444 + i0.244	−0.557 + i0.256	0.807(−2)
		1	+0.086	+0.001 + i0.006	−0.007 − i0.014	+0.034 − i0.003	+0.114 − i0.010	0.841(−3)
		2	+0.092	+0.008 − i0.145	−0.003 + i0.150	+0.002 + i0.000	+0.099 + i0.005	0.106(−2)
12	0.614	0	+2.30	−3.02 − i1.49	+0.737 + i0.996	−0.197 + i0.701	−0.178 + i0.206	0.256(−2)
		1	+0.145	+0.009 − i0.004	−0.022 − i0.003	+0.051 + i0.041	+0.183 + i0.035	0.358(−2)
		2	+0.129	+0.039 − i0.150	−0.040 + i0.162	+0.005 + i0.005	+0.133 + i0.018	0.311(−2)
20	0.582	0	+1.41	−2.02 − i0.075	+0.961 − i0.522	−0.383 + i0.693	−0.030 + i0.097	0.526(−3)
		1	+0.124	+0.009 − i0.007	−0.014 − i0.013	+0.004 + i0.061	+0.123 + i0.041	0.263(−2)
		2	+0.094	+0.033 − i0.067	−0.039 + i0.075	−0.001 + i0.009	+0.087 + i0.017	0.201(−2)

$$\sigma_0=\left[\frac{e^2\cos\frac{1}{2}\theta}{2E_i\sin^2\frac{1}{2}\theta}\right]^2\left(1+\frac{2E_i\sin^2\frac{1}{2}\theta}{M_{\mathrm{He}}}\right)^{-1}. \tag{82}$$

Representative partial-wave amplitudes are tabulated in Tables I and II. Table I has kinematics appropriate to $E_0=90$ MeV, $\theta_e=92.6°$ and gives amplitudes of the charge operator. The Coulomb Born term is an on-shell spin triplet in the interacting s-wave pair, and is taken to be real. The triplet rescattering amplitude can be further decomposed into its on-shell and off-shell components.[1] The on-shell part is related to the elastic N-d doublet scattering amplitude $X^L_{tt}(p,p;z)$ and the Born transition amplitude $B^L_t(z,p,q)$, viz.,

$$i\tfrac{8}{3}\pi^2N_2{}^2pX^L_{tt}(p,p;z)B^L_t(z,p,q).$$

The on-shell X^L_{tt} is computed from Eq. (71) using the single-nucleon exchange $Z_{tt}(p,p;z)$ as the inhomogeneous term and $Z_{tt}(p,p'';z)$ and $Z_{ts}(p,p'';z)$ under the integrals. The singlet rescattering amplitude is purely off shell.

TABLE II. Transverse amplitudes ($fm^{3/2}$).

ω (MeV)	q (fm^{-1})	L	Born	Triplet	Singlet	Full amplitude	$\lvert F^L(q^2,\omega)\rvert^2$ (MeV^{-1})
				Doublet			
6	0.587	0	−23.776	+25.661 − i0.119	+6.331 − i0.402	+8.212 − i0.522	0.831(−4)
		1	−0.731	+0.103 + i0.026	−0.107 + i0.002	−0.734 + i0.029	0.199(−5)
		2	−0.129	−0.006 − i0.000	−0.004 − i0.000	−0.141 − i0.000	0.122(−6)
8	0.573	0	−21.235	+22.425 − i0.277	+3.045 − i1.703	+4.232 − i1.979	0.536(−4)
		1	−1.356	+0.074 + i0.119	−0.347 − i0.030	−1.630 + i0.151	0.197(−4)
		2	−0.494	−0.026 − i0.027	−0.021 − i0.002	−0.542 − i0.029	0.361(−4)
12	0.557	0	−16.849	+16.603 + i3.588	+1.147 − i5.342	+0.900 − i1.753	0.140(−4)
		1	−1.448	+0.144 + i0.140	−0.467 − i0.429	−1.771 − i0.290	0.349(−4)
		2	−0.715	+0.005 − i0.066	−0.041 − i0.047	−0.752 − i0.114	0.104(−5)
				Quartet			
6	0.578	0	−4.209	+2.721 + i0.949	0	−1.488 + i0.949	0.382(−5)
		1	−5.271	+3.745 + i1.244	0	+9.016 + i1.244	0.305(−3)
		2	+0.224	−0.021 − i0.000	0	+0.203 − i0.000	0.253(−6)
8	0.573	0	−1.614	+1.270 + i1.847	0	−0.344 + i1.847	0.864(−5)
		1	+9.392	+1.648 + i5.931	0	+11.040 + i5.931	0.115(−2)
		2	+0.886	−0.088 − i0.074	0	+0.798 − i0.074	0.786(−5)
12	0.557	0	+0.465	−0.118 + i1.291	0	+0.349 + i1.291	0.644(−5)
		1	+9.773	−1.608 + i5.203	0	+8.165 + i5.203	0.101(−2)
		2	+1.308	−0.031 + i0.205	0	+1.277 − i0.205	0.302(−5)

The following features may be noted from Table I about the doublet Coulomb two-body breakup amplitudes. All amplitudes are zero at threshold except the s wave whose full amplitude has a finite real part. We note that a great deal of cancellation goes on among the s-wave Born and triplet amplitudes at the lower excitation energies, leaving the singlet off-shell rescattered amplitude as the governing term.

The $L>0$ partial waves have less of a rescattering correction to the Born amplitude than the s wave. The singlet rescattering is usually more important than the total triplet rescattering because of internal cancellation in the triplet between on-shell and off-shell components. This feature was also present for the photodisintegration calculation of Ref. 1.

Table II has kinematics appropriate to $E_0 = 60.63$ MeV, $\theta_e = 180°$. The amplitudes are dominated by the magnetic-moment operator. Two spin states of the nucleon-plus-deuteron final state contribute to the cross section, the $S = \frac{1}{2}$ doublet, and the $S = \frac{3}{2}$ quartet amplitudes. Note the quartet has no singlet amplitude. The transverse triplet has not been separated into on- and off-shell components.

We see from Table II that at threshold the doublet magnetic dipole ($^2S \to {}^2S$) competes with the quartet magnetic quadrupole ($^2S \to {}^4P$), but the quartet $M2$ soon dominates all other multipoles by two orders of magnitude. The dominance of $^2S \to {}^4P$ in the transverse and $^2S \to {}^2S$ in the longitudinal low energy two-body breakup can be traced to the constructive (or destructive) interference in each multipole of the two diagrams that make up the Born term. The relative sign between the two diagrams in Fig. 1(c) is determined by the multipole order L and the parity of the transition operator.

At the excitation energies treated in our calculations the Born amplitudes become dominant for increasing values of L. This effect can be seen in Table III, in which the charge amplitudes for the coincidence reaction $^3\mathrm{He}(e, e'p)d$ are listed for each partial wave at approximately 60 MeV excitation energy.[9] The fully symmetric ground state wave function was used in this calculation.

TABLE III. Coulomb amplitude for quasielastic scattering $^3\mathrm{He}(e, e'p)d$ (arbitrary units).

L	Born	Full amplitude
0	0.1571+01	0.7840+00+i0.3784+00
1	0.7888+00	0.7958+00+i0.1920+00
2	0.7527+00	0.6920+00+i0.1044+00
3	0.4277+00	0.4324+00+i0.6313−02
4	0.3152+00	0.3116+00+i0.1026−01
5	0.1914+00	0.1923+00+i0.3444−03
6	0.1271+00	0.1268+00+i0.9042−03

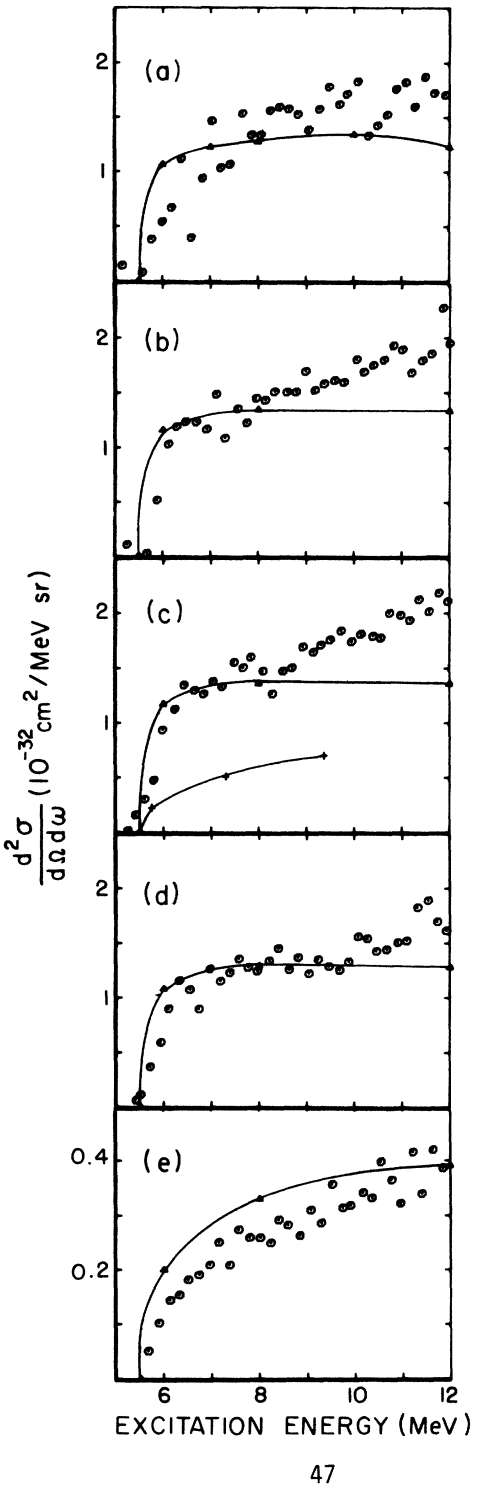

FIG. 3. Comparison of the $p+d$ breakup calculation with the data of Kan *et al.* (Ref. 8): (a) $E_0 = 60$ MeV, $\theta_e = 92.6°$; (b) $E_0 = 75$ MeV, $\theta_e = 92.6°$; (c) $E_0 = 90$ MeV, $\theta_e = 92.6°$ (lower curve for ^{3}H); (d) $E_0 = 110$ MeV, $\theta_e = 92.6°$; $E_0 = 120$ MeV, $\theta_e = 127.7°$. Statistical error bars have been omitted from the data points.

B. Cross sections

The two inelastic electron scattering experiments with which we want to compare our two-body calculations include three-body breakup. In principle only the interval between $\omega = 5.5$ and 7.7 MeV is pure two-body breakup in the experimental spectrum.

The kinematics of the measurement of Kan *et. al.*[8] ($\theta_e = 92.6°$ in most cases) stresses the contribution of the charge scattering to the breakup cross section. Figure 3 shows a comparison of the $p+d$ calculation with the measured cross section at five values of E_0. A decomposition of the calculated cross section into longitudinal and transverse partial waves is given in Table I (and Fig. 2 of Ref. 9 for the $E_0 = 90$ MeV curve). The rapid rise of the data at the two-body breakup threshold is accounted for in our calculation as due to the two-body Coulomb monopole transition $^2S \rightarrow {}^2S$. We did not include a final state Coulomb interaction between the proton and deuteron, which would tend to reduce the slope of the cross section at threshold. Although our calculation is only for $p+d$ breakup, the curves follow the data far beyond the three-nucleon breakup threshold at 7.7 MeV.

The kinematics of the measurement of Jones[10] and others ($\theta_e = 180°$) stresses the contribution of the magnetic moment operator. Figure 4 shows a comparison of the $p+d$ calculation with measured cross section at three values of E_0. A decomposition of the calculated cross section into partial waves is given in Table II (and Fig. 3 of Ref. 9 for $E_0 = 60$ MeV). We note that the $p+d$ calculation has a good fit to the data between the two- and three-body thresholds but, unlike the Coulomb breakup data, deviation of the two-body calculation begins immediately after the opening of the three-body channel.

By assuming charge symmetry of the two-nucleon force we are able to modify our calculation to yield the cross section for tritium $^3\mathrm{H}(e, e')nd$ by interchanging neutron and proton labels in the charge and magnetic moment form factors. The results are shown in Figs. 3(c) and 4 (curve 2). Since the threshold for $n+d$ breakup is 0.8 MeV higher in excitation energy than $p+d$, the theoretical curves should be shifted by this amount. We note the ^{3}H Coulomb monopole is depressed relative to the ^{3}He case. The enhanced monopole amplitude at threshold in ^{3}He arose from the constructive interference of the type I and type II Coulomb doublet amplitudes shown on the right in Fig. 1(c). In ^{3}H only the type II diagram (coupling to the charge of the dueteron) contributes, hence no constructive interference.

In 180° scattering the tritium cross section has approximately the same shape as that of ^{3}He, but with half the magnitude. In both cases the dominant transition is the magnetic quadrupole $^2S \rightarrow {}^4P$. The ratio of magnitudes can be understood in terms of $(\mu_p/\mu_n)^2 = 2.1$.

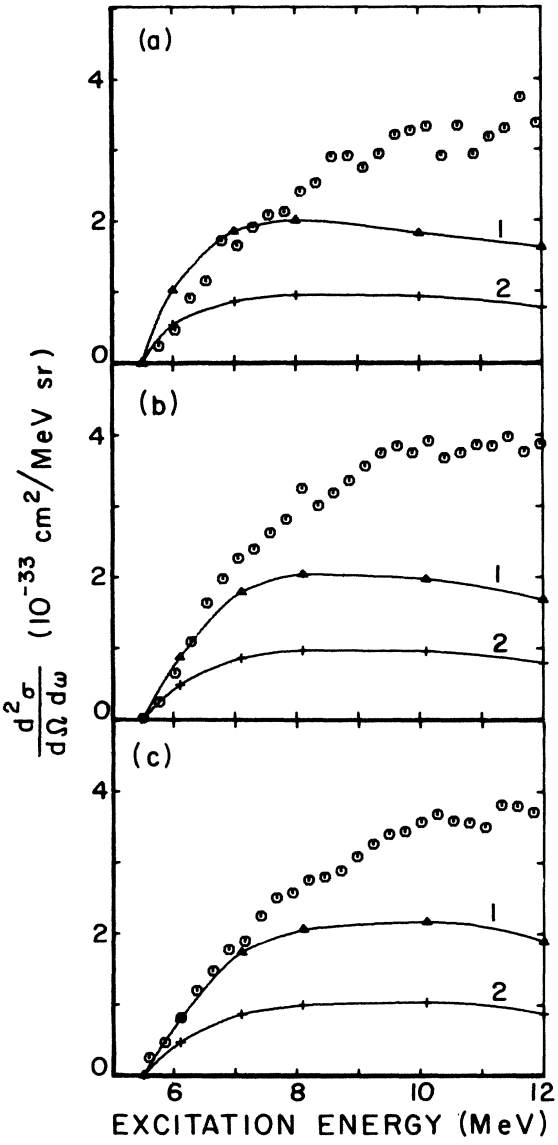

FIG. 4. Comparison of the $p+d$ breakup calculation with the $\theta_e = 180°$ data of Jones *et al.* (Ref. 10); (a) $E_0 = 60.63$ MeV; (b) $E_0 = 50.58$ MeV; (c) $E_0 = 40.44$ MeV. Statistical error bars have been omitted from the data points. Curve 1 for $^3\mathrm{He}(e,e')pd$, curve 2 for $^3\mathrm{H}(e,e')nd$.

V. CONCLUSIONS

The primary conclusions of this paper are:
(1) The Gibson-Lehman formalism for calculating bound-state disintegration with nonzero momentum transfer is a practical scheme for numerical com-

putation providing the singularities of the disintegration Born amplitude are considered and avoided in the momentum integrals. (2) The Coulomb scattering at the *pd* breakup threshold is dominated by the monopole transition $^2S \to {}^2S$, which causes a sharp rise in the (e, e') cross section as a function of excitation energy. However, a few MeV above threshold $C1$ and $C2$ transitions are equally important. (3) The transverse inelastic two-body form factor is dominated by the magnetic quadrupole $^2S \to {}^4P$ transition in the energy range threshold to 20 MeV. No sharp rise at threshold is predicted (or seen experimentally). (4) The ^{3}H cross sections are roughly half those of ^{3}He with the exception that the sharp rise at threshold in charge scattering is absent.

APPENDIX A: NUMERICAL METHODS—SINGULARITIES

The coupled integral equations for the half-off-shell nucleon-correlated-pair amplitudes, Eq. (71), are solved numerically by standard contour-rotation methods.[12] The variables p' and p'' are rotated from the real axis into the fourth quadrant: $p' \to p'e^{-i\varphi}$ and $p'' \to p''e^{-i\varphi}$. The contribution to the integral from the arc at infinity is zero, but the rotation angle φ is limited by a singularity in the inhomogenous term $Z^L_{nn'}(p, p'; 3p^2/4M - \epsilon_d)$ from the energy denominator, i.e., $p^2 + p'^2 + pp'x - Mz = 0$. To avoid this singularity, the rotation angle must be chosen such that

$$\varphi < \tan^{-1}(2\gamma/p)\,, \tag{A1}$$

where $\gamma^2 \equiv M\epsilon_d$.

Once the amplitudes $X^L_{tn'}(p, p'e^{-i\varphi}; 3p^2/4M - \epsilon_d)$ are available, $M^t_2(3p^2/4M - \gamma^2/M, \vec{p})$ is computed from them by rotating the p' integration in the integral terms of Eq. (78). This is useful because the bound-state pole of τ_t is avoided and the $X^L_{tn'}(p, p'e^{-i\varphi}; 3p^2/4M - \epsilon_d)$ amplitudes can be used directly. Again, there is no contribution along the arc at infinity. Nevertheless, this rotation is possible only if no singularities of τ_n or B^L_n interfere. The fact that τ_n has no such singularities is implicit in our statements above about the solution of Eq. (71). Unfortunately, the B^L_n are more complicated and they cause difficulties of two types. Firstly, the B^L_n depend on the ground-state spectator functions which have been fitted to an analytical form for ease of computation. Can this parametrization be analytically continued into the fourth quadrant? Secondly, $B^L_n(z, p', q)$ is obtained by an integration over d^3k. When $p' \to p'e^{-i\varphi}$, does this introduce singularities in the d^3k integration? For simplicity, these questions are answered for the case of spatially symmetric triton wave function. Inclusion of the mixed-symmetry component introduces nothing new.

The spatially symmetric triton wave function has the form

$$\psi^s_0(\vec{k}, \vec{p}) = \sum_{i=1}^{3} \Psi^{(i)}\,, \tag{A2}$$

where

$$\Psi^{(1)} = N_3 \frac{g(k_{23})a(p_1)}{k_{23}^2 + \frac{3}{4}p_1^2 + K^2}\,, \tag{A3}$$

$a(p)$ is the spectator function, $g(k)$ is the s-wave two-nucleon separable-potential form factor, N_3 is the normalization constant, and $E_B = K^2/M$ is the three-nucleon binding energy.[7] The spectator function is obtained from the homogeneous integral equation

$$a(p) = 2\pi M\tau[-(E_B + 3p^2/4M)] \times \int p'^2 dp'\, Z^0(p, p', -E_B)a(p')\,, \tag{A4}$$

where subscripts on τ and Z^0 have been dropped since the spatially symmetric triton wave function is generated from a spin-independent two-nucleon interaction. Equation (A4) implies that as $p \to \infty$, $a(p) \to 0$ as p^{-6}. Furthermore, since Eq. (A4) is homogeneous, we can define $a(0) = 1$. This suggests that $a(p)$ may be parametrized as

$$a(p) = \frac{1}{1 + \bar{\alpha}p^2 + \bar{\beta}p^4 + \bar{\gamma}p^6}\,, \tag{A5}$$

where $\bar{\alpha}$, $\bar{\beta}$, and $\bar{\gamma}$ are real constants. Equation (A5) works very well and eliminates the need for continually solving Eq. (A4). Does Eq. (A5) still

TABLE IV. Results of computed $a(p)$ compared with empirical fit, for p rotated 45° into the fourth quadrant. $a(p) = |a|\exp(i\theta)$. θ is in radians.

$\lvert p\rvert$ (fm^{-1})	Empirical form $\lvert a\rvert$	Empirical form θ	Computed $\lvert a\rvert$	Computed θ
0.019	1.000	0.141 − 2	1.000	0.139 − 2
0.054	1.000	0.106 − 1	1.000	0.105 − 1
0.106	0.999	0.411 − 1	0.999	0.408 − 1
0.179	0.995	0.118	0.995	0.117
0.279	0.969	0.282	0.969	0.279
0.414	0.873	0.582	0.874	0.574
0.596	0.653	1.009	0.662	0.996
0.843	0.390	1.470	0.403	1.475
1.186	0.197	−1.186	0.197	−1.137
1.679	0.793 − 1	−0.562	0.724 − 1	−0.541
2.414	0.200 − 1	0.215	0.182 − 1	0.464 − 1
3.577	0.264 − 2	0.885	0.310 − 2	0.609
5.574	0.202 − 3	1.280	0.294 − 3	1.213
9.441	0.870 − 5	1.469	0.108 − 4	−1.489
18.588	0.150 − 6	1.544	0.152 − 6	−1.504
51.050	0.349 − 9	1.567	0.348 − 9	−1.562

fit the solution to Eq. (A4) when $p \to pe^{-i\varphi}$ *without* altering the a_i? Yes—this has been checked by solving Eq. (A4) for $p \to pe^{-i\varphi}$ and comparing with Eq. (A5).[13] This comparison was made for $0 \leq \varphi \leq 85°$. For $0 \leq \varphi \leq 45°$, the agreement is excellent. Beyond $\varphi = 45°$ the discrepancies grow until, at $\varphi = 85°$, the fit is poor. The results for $\varphi = 45°$ are given in Table IV.[14] In the actual computation, φ is always $<45°$, so Eq. (A5) is valid.

Two types of integral expressions occur in the evaluation of $B_n(z, \vec{p}, \vec{q})$ for electrodisintegration:

$$I_1^n(z, \vec{p}, \vec{q}) = \int d^3k \, \frac{g_n(k)}{k^2 + \frac{3}{4}p^2 - Mz} \, \psi_0^s(\vec{k}, \vec{p} - \tfrac{2}{3}\vec{q}) \tag{A6}$$

and

$$I_2^n(z, \vec{p}, \vec{q}) = \int d^3k \, \frac{g_n(k)}{k^2 + \frac{3}{4}p^2 - Mz} \, \psi_0^s(\vec{k} - \tfrac{1}{2}\vec{q}, \vec{p} + \tfrac{1}{3}\vec{q}) \, . \tag{A7}$$

when ψ_0^s is replaced by Eqs. (A2) and (A3), I_1^n and I_2^n can be written as

$$I_1^n(z, \vec{p}, \vec{q}) = N_3 \Big[\int d^3k \, \frac{g_n(k)}{(k^2 + \frac{3}{4}p^2 - Mz)} \, \frac{g(k) a(|\vec{p} - \frac{2}{3}\vec{q}|)}{[k^2 + \frac{3}{4}(\vec{p} - \frac{2}{3}\vec{q})^2 + K^2]}$$
$$+ 2 \int d^3k \, \frac{g_n(|\vec{k} + \frac{1}{2}\vec{p}|)}{(\vec{k} + \frac{1}{2}\vec{p})^2 + \frac{3}{4}p^2 - Mz} \, \frac{g(|\tfrac{1}{2}\vec{k} + \vec{p} - \frac{1}{2}\vec{q}|) a(|k + \frac{1}{3}\vec{q}|)}{(\vec{k} + \frac{1}{2}\vec{p})^2 + \frac{3}{4}(\vec{p} - \frac{2}{3}\vec{q})^2 + K^2} \Big] \tag{A8}$$

and

$$I_2^n(z, \vec{p}, \vec{q}) = N_3 \int d^3k \, \frac{g_n(k)}{[k^2 + \frac{3}{4}p^2 - Mz]} \, \frac{g(|k - \frac{1}{2}q|) a(|p + \frac{1}{3}q|)}{[\vec{k} - \frac{1}{2}\vec{q})^2 + \frac{3}{4}(\vec{p} + \frac{1}{3}\vec{q})^2 + K^2]}$$
$$+ \int d^3k \, \frac{g_n(|\vec{k} + \frac{1}{2}\vec{p}|)}{[(\vec{k} + \frac{1}{2}\vec{p})^2 + \frac{3}{4}p^2 - Mz]} \, \frac{g(|\tfrac{1}{2}\vec{k} + \vec{p}|) a(|\vec{k} - \frac{2}{3}\vec{q}|)}{[(\vec{k} + \frac{1}{2}\vec{p} - \frac{1}{2}\vec{q})^2 + \frac{3}{4}(\vec{p} + \frac{1}{3}\vec{q})^2 + K^2]}$$
$$+ \int d^3k \, \frac{g_n(k)}{[k^2 + \frac{3}{4}p^2 - Mz]} \, \frac{g(|\tfrac{1}{2}\vec{k} + \frac{3}{4}\vec{p} + \frac{1}{2}\vec{q}|) a(|\vec{k} + \frac{1}{2}\vec{p} - \frac{1}{3}\vec{q}|)}{[(\vec{k} + \frac{1}{2}\vec{q})^2 + \frac{3}{4}(\vec{p} + \frac{1}{3}\vec{q})^2 + K^2]} \, , \tag{A9}$$

where these particular forms lead to the easiest analysis of the singularities. To illustrate our method, we consider the first integral in Eq. (A8).

The first integral on the right-hand side of Eq. (A8) has four denominators in the integrand [remember $g_n(k) = (k^2 + \beta_n^2)^{-1}$ and $g(k) = (k^2 + \beta^2)^{-1}$], two of which depend on p, multiplied by the spectator function which depends on p and not k:

(a) $k^2 + \beta_n^2$

(b) $k^2 + \beta^2$

(c) $k^2 + \frac{3}{4}p^2 - Mz$

(d) $k^2 + \frac{3}{4}(\vec{p} - \frac{2}{3}\vec{q})^2 + K^2$

(e) $a^{-1}(|\vec{p} - \frac{2}{3}\vec{q}|)$.

In the complex k plane, (a) and (b) introduce simple poles at $\pm i\beta_n$ and $\pm i\beta$ which are of no concern for the k integration on the real axis. Denominator (c) introduces a cut in the complex k plane which is easily mapped out for $p \to pe^{-i\varphi}$. (Note: z always has a small positive imaginary part.) So far, as can be seen in Fig. 5, none of these singularities directly interfere with the k integration; however, (d) does. The envelope of the cut structure for (d) is illustrated in Fig. 5(d). This envelope is obtained from $x_p = \pm 1$, where $x_p \equiv \hat{p} \cdot \hat{q}$.

If q or φ are large enough, the cut envelope ($x_p = 1$ branch) will move onto the real axis and interfere with k integration. This will not occur if φ is chosen such that

$$\varphi < \tan^{-1}(\sqrt{3}\, K/q) \, . \tag{A10}$$

In order to avoid the branch point $\sqrt{Mz}$ from (c), it is convenient to rotate the k integration into the fourth quadrant of the complex k plane. This rotation is limited by the cut (d). It is possible to show that the maximum allowed rotation angle for the k integration is

$$\delta = \sin^{-1}\{[\tfrac{1}{2}(1 + \cos\eta)]^{1/2}\} \, , \tag{A11}$$

where

$$\cos\eta = (K^2 + \tfrac{1}{3}q^2 + \tfrac{3}{4}p_0^2 \cos 2\varphi - p_0 q \cos\varphi)/|y| \, , \tag{A12}$$

$$|y| = ([K^2 + \tfrac{1}{3}q^2 + \tfrac{3}{4}p_0^2 \cos 2\varphi - p_0 q \cos\varphi]^2 + [\tfrac{3}{4}p_0^2 \sin 2\varphi - p_0 q \sin\varphi]^2)^{1/2} \, , \tag{A13}$$

and

$$p_0 = \frac{2\cos\varphi}{q} \, (K^2 + \tfrac{1}{3}q^2) \times \{1 - [1 - \tfrac{1}{3}q^2 \cos^2\varphi (K^2 + \tfrac{1}{3}q^2)]^{1/2}\} \, . \tag{A14}$$

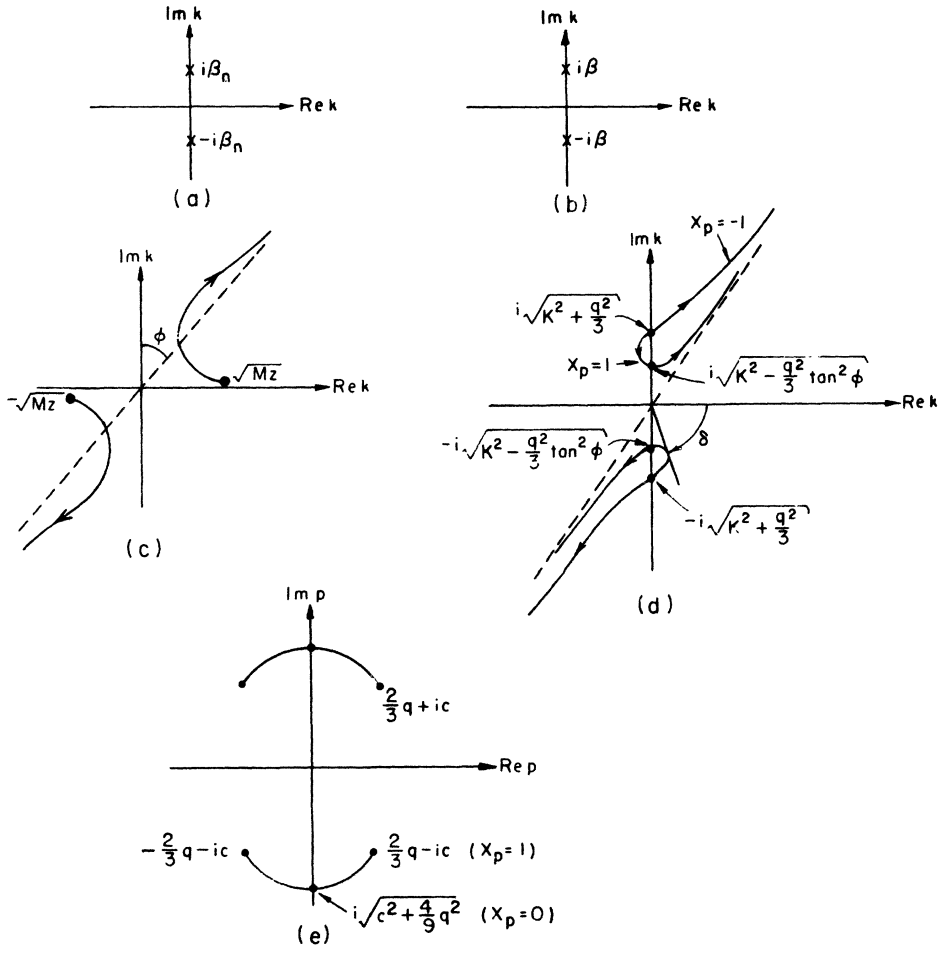

Fig. 5. Pictorial representation of the singularities discussed in Appendix A. The following denominators are responsible for the respective figures: (a) $k^2+\beta_n{}^2=0$; (b) $k^2+\beta^2=0$; (c) $k^2+\frac{3}{4}p^2-Mz=0$; (d) $k^2+\frac{3}{4}(\vec{p}-\frac{2}{3}\vec{q})^2=0$; and (e) $a^{-1}(|\vec{p}-\frac{2}{3}\vec{q}|)=0$. Arrows on the curves in (c) and (d) point in the direction of increasing p.

δ is shown in Fig. 5(d). Finally, the p rotation is limited by the fact that $a(p)$ has purely imaginary poles obtained from that value of p^2 corresponding to the two-body, i.e., nucleon-plus-bound-pair, binding energy of the model triton. In Eq. (A4), this is evident since $\tau(z)$ has a pole for z equal to the two-nucleon binding energy. $a(p)$ has many other singularities, but they lie farther away in the complex p plane. If these poles of $a(p)$ are denoted by $\pm ic$, then for the argument in (e) they become the source of branch points for the cuts illustrated in Fig. 5(e). Therefore, φ is restricted to

$$\varphi<\tan^{-1}(3c/2q) \tag{A15}$$

for $p\to pe^{-i\varphi}$.

The previous example illustrates the type of analysis carried out for all five terms in Eqs. (A8) and (A9). We found that $\varphi_{\max}$ cannot be greater than

$$\varphi_{\max}=\min\{\tan^{-1}\left[\left(\frac{2\gamma}{p},\ \frac{3c}{2q},\ \frac{\sqrt{3}K}{q},\ \left(\frac{1+12K^2/q^2}{3}\right)^{1/2},\ \frac{2\beta}{q},\ \sqrt{3}\right)\right], \tag{A16}$$

TABLE V. Limits of rotation determined by singularities. Restriction number refers to Eq. (A16) from left to right. Restrictions are for symmetric Tabakin parameters.

Restriction number	Angle (deg)
1	31.8
2	52.7
3	55.4
4	60.5
5	75.6
6	60.0

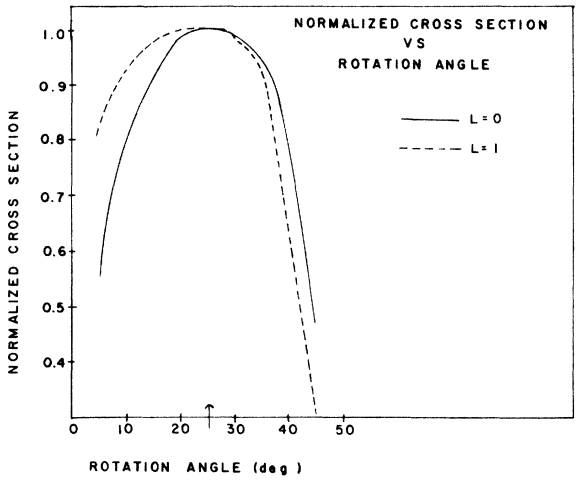

Fig. 6. The cross sections for $L=0$ and $L=1$ as a function of rotation angle. Both cross sections are normalized to unity at the optimal angle of rotation.

In practice, we always chose $\varphi = \frac{1}{2}\varphi_{max}$. Moreover, we found that it is either convenient or essential to rotate the k integration contour into the fourth quadrant as well. In practice, we chose $k \to ke^{-i\varphi}$, i.e., we rotated k the same as p. This was actually required in some cases, but not all; however, it was adequate for all. As p and q become larger, the restrictions in Eq. (A16) become tighter and the results more sensitive to the rotation angle. A typical case is shown in Table V, where the terms in Eq. (A16) are evaluated. Note that it is the restriction from the X equation which determines the size of φ.

To check the sensitivity of the results to variations of the contour rotation angle, we varied φ over a wide range for typical threshold kinematics: $E_i = 90$ MeV, $E_f = 80$ MeV, and $\theta_e = 92.6°$. The optimal choice of φ for this case is 26°. The ratios of the $l=0$ and $l=1$ cross sections for $5° \leq \varphi \leq 45°$ to those at $\varphi = 26°$ are shown in Fig. 6. This graph demonstrates that the results are insensitive to φ for ±5° about the optimum angle. For $l=2$, the band is even wider. Furthermore, when φ has its optimal value, the results are essentially insensitive to the integration mesh size. The angular integrals are performed with six-point Gaussian quadrature and the 0 to ∞ momentum integrals with 10-point Gegenbauer quadrature. When these quadratures are changed to 10 and 16 points, respectively, the results vary < 1%.

TABLE VI. S-wave potential parameters.

Interaction	Strength λ (fm^{-3})	Inverse range β (fm^{-1})
Triplet[a]	0.220	1.15
Singlet	0.148	1.15

[a] For the deuteron $\gamma = 0.232$ fm^{-1}.

APPENDIX B: POTENTIAL AND WAVE FUNCTION PARAMETERS

The nucleon-nucleon potential used in this calculation is of the separable spin-dependent type acting only in relative S waves,

$$\langle p' | v | p \rangle = -\frac{\lambda}{2\mu} g(p') g(p) .$$

The Yamaguchi form

$$g(p) = (p^2 + \beta^2)^{-1} \tag{B1}$$

is used with strength λ and range β parameters taken from Tabakin.[15] The numerical values for spin triplet and singlet are listed in Table VI.

The ^{3}He ground state wave function Ψ_B that results from solving the Faddeev equations with these potentials is of the form given by Eqs. (49)–(61). The binding energy B_3, normalization constant N_3, and spectator nucleon parameters are given in Table VII for the spin-dependent potential. This wave function was used in all calculations reported here except for the quasielastic scattering cross section[11] ^{3}He$(e, e'p)d$ given in Table III, which illustrates the convergence of the Born and full amplitude for increasing partial wave number. A fully symmetric ground state was generated by taking an average of the two nucleon spin-dependent potentials. The parameters of this wave function, which is of the form Eqs. (A2) and (A3), are listed on the last line of Table VII.

TABLE VII. ^{3}He ground state.

Wave function	B_3 (MeV)	N_3 (fm^{-1})	Spectator function $a(p)$ N_i	$\bar{\alpha}$	$\bar{\beta}$	$\bar{\gamma}$
Spin dependent	10.1	0.2268				
Triplet			1.00	4.225	1.930	0.1538
Singlet			0.4907	2.989	1.117	0.0984
Symmetric	9.33	0.3235	1.0	3.670	1.469	0.1620

†Based on work submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy in the Physics Department of the American University.

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[11]Coulomb amplitude for ^{3}He$(e,e'p)d$ at $E_i = 550$ MeV, $E_f = 443$ MeV, $\theta_e = 51.7°$, $\theta_p = -53°$. See Fig. 1 of Ref. 9.

[12]J. H. Hetherington and L. M. Schick, Phys. Rev. 137, B935 (1965); 156, 1647 (1967).

[13]To solve Eq. (A4) for $p \to pe^{-i\varphi}$, p' is also rotated $p' \to p'e^{-i\varphi}$ to avoid singularities from the Z° function. E_B is fixed at the value found by solving Eq. (A4) with p real.

[14]This result also justifies the same type of parametrization-rotation procedure in the work of Gibson and Lehman (see Appendices A and B in Refs. 1 and 2, respectively).

[15]F. Tabakin, Phys. Rev. 137, B75 (1965).