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Many-body exchange currents and the tri- and four-nucleon charge form factors

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We find that the three-body two-pion-exchange currents lead to an effect very similar to that of the two-body one-pion-exchange currents on the charge form factors of ^{3}H, ^{3}He, and ^{4}He. Most of this effect is, however, canceled by similar ρ-meson-exchange currents. The four-body three-meson-exchange currents are of negligible importance for the α-particle charge form factor.

[NUCLEAR REACTIONS ^{3}He, ^{3}H, ^{4}He(e,e); calculated charge form factors with 2-, 3-, and 4-body meson-exchange operators; pion exchange with/without ρ exchange; simple wave functions; effects of correlations, vertex factors.]

I. INTRODUCTION

Knowledge of meson-exchange current phenomena in electron-nucleus scattering has advanced rapidly over the last few years. For example, the effect of the two-body one-pion-exchange current contribution to the nuclear spin current in inelastic e-d scattering and the magnetic form factors of the three-nucleon ground states has been firmly established.[1-3]. That there is a beneficial effect due to the one-pion-exchange current on nuclear charge form factors as well seems to be generally recognized,[4-7] although there remains some uncertainty as to the form of that part (i.e., the charge component) of the one-pion-exchange current.[8]

The large effects associated with two-body one-meson-exchange currents suggest the question of how important the three- or many-body exchange currents might be. This question is particularly relevant now that calculations have been performed of nuclear charge form factors with account of two-body currents and including three-body two-meson-exchange forces in the construction of the nuclear wave functions.[7,9]. In this paper we address ourselves to this question and explore the effects of three- and four-body currents on the charge form factors of the three- and four-body nuclei. The models used for the exchange current operators are the same as those commonly used for the two-body one-meson-exchange current operators and for the three-body forces.[10] The reader who has accepted the models for the two-body currents and the three-body force will have to accept the three- and four-body currents that we derive below as well, as no additional level of assumption nor approximation is introduced.

One result of the present investigation is that the effects of the three-body and four-body pion-exchange currents on the charge form factors of the three- and four-nucleon nuclei are qualitatively similar to those of the two-body one-pion-exchange current, although their magnitude is somewhat smaller. On the other hand, we also consider ρ-meson exchange in addition to pion exchange and find large cancellations to occur between these exchange mechanisms. Hence the net effects of the combined pion and ρ-meson three- and four-body currents are rather small.

The quantitative importance of the three- and four-body currents as compared to the one-body currents grows rapidly for momentum transfers of $q^2 > 10$ fm^{-2}. At smaller values of momentum transfer they are completely insignificant. The sign of the most important three- and four-body current matrix elements is the same as that of the two-body current. Consequently they tend to amplify the beneficial two-body exchange current effect on nuclear charge form factors first pointed out by Kloet and Tjon[4] and later emphasized by Kümmel, Lührmann, and Zabolitzky.[11]

In Sec. II of this paper we present the three-nucleon two-meson-exchange current operators and the similar four-nucleon three-meson-exchange current operators. In Sec. III we study the effects of these operators using the simple harmonic oscillator model for the nuclear wave functions. The use of the simple wave functions allow us to present the exchange current matrix elements in simple closed form and to illustrate the general features of the matrix elements in a, as it turns out, qualitatively correct way. In Sec. IV we illustrate how correlations in the wave functions affect the exchange current effects. In Sec. V we give a summary of the conclusions.

II. THREE- AND FOUR-BODY EXCHANGE CURRENTS

The two-body pion-exchange current operator that leads to the most important effect on nuclear charge form factors at values of squared momentum transfer $q^2 \lesssim 20\ \mathrm{fm}^{-2}$ is the one first considered by Kloet and Tjon.[4] They obtained the exchange current expression by considering the lowest order limit of the charge component of the four-current caused by the "pair" diagram in Fig. 1a. This diagrammatic identification of the operator is associated with the use of the pseudoscalar model for the πN coupling. If the pseudovector coupling model is used, the exchange current operator arises as a relativistic correction to the part of the relativistic pion photoproduction amplitude with a positive energy nucleon intermediate state. Since at the level of the tree-graph approximation the use of pseudovector coupling is easier to justify,[12] we prefer that interpretation and the corresponding representation by the seagull diagram in Fig. 1(b). The exchange current operator has the form

$$\rho_2^\pi(\vec{k}_1, \vec{k}_2) = \frac{g^2}{8m^3}[F_1^s(q^2)\vec{\tau}^1\cdot\vec{\tau}^2 + F_1^V(q^2)\tau_3{}^2] \times \frac{\vec{\sigma}^1\cdot\vec{q}\vec{\sigma}^2\cdot\vec{k}_2}{\mu^2+k_2^2} + (1 \leftrightarrow 2)\,. \quad (1)$$

Here $F_1^{S,V}$ are the nucleon isoscalar and isovector vector form factors[13] $[F_1^S(0) = F_1^V(0) = 1]$, g is the πN coupling constant ($g^2/4\pi = 14.5$), m is the nucleon mass, and μ the pion mass. The momentum delivered to the nucleon which absorbs the photon is denoted $\vec{k}_1$ and that to the second nucleon $\vec{k}_2$. By momentum conservation we have $\vec{q} = \vec{k}_1 + \vec{k}_2$. The nucleon spin and isospin matrices are denoted $\vec{\sigma}$ and $\vec{\tau}$.

Consider now the three-body currents in Fig. 2. The vertex on the first nucleon in Figs. 2(a) and 2(b) is the same as that considered above and previously for the two-body current. The vertex on the intermediate nucleon in Fig. 2(a) represents an S-wave rescattering of the meson off the second nucleon, whereas the vertex on the intermediate nucleon in Fig. 2(b) represents a P-wave rescattering of the pion (visualized as being mediated by the Δ_{33} resonant intermediate state). The presence of a diagram of the type shown in Fig. 2(b), but with the order of emission and absorption of the exchanged mesons by the Δ reversed, should be inferred. We do not display such permuted diagrams explicitly in the subsequent figures.

The general three-body exchange current operator involves an off-shell pion-nucleon scattering amplitude for the intermediate nucleon with the Born terms omitted. This vertex is the same as that appearing in the three-body force. The main part of the S-wave rescattering amplitude can be viewed as being due to ρ-meson exchange between the pion and the nucleon. Compared to the P-wave rescattering contribution (through the Δ_{33} resonance) this is of small importance in the three-body force.[10] We shall show below that, similarly, only P-wave rescattering leads to an important many-body exchange current operator.

To describe the S-wave πN scattering amplitude we choose not to try to construct a dynamical model but rather to use the phenomenological Lagrangian of Koltun and Reitan[14]:

$$\mathfrak{L}_{\pi\pi NN} = -4\pi(\lambda_1/\mu)\chi^\dagger\psi^\dagger\vec{\phi}\cdot\vec{\phi}\psi\chi - 4\pi(\lambda_2/\mu^2)\chi^\dagger\psi^\dagger\vec{\tau}\cdot\vec{\phi}\times\vec{\pi}\psi\chi. \quad (2)$$

Here $\vec{\phi}$ is the pion isovector field operator and $\vec{\pi}$ it canonically conjugate momentum operator. The nucleon spinor is denoted ψ and the isospinor χ. The two coupling constants λ can be determined in terms of the S-wave πN scattering lengths as

$$\lambda_1 = \tfrac{1}{6}\mu[(m+\mu)/m](a_1 + 2a_3)\,, \quad (3a)$$

$$\lambda_2 = \tfrac{1}{6}\mu[(m+\mu)/m](a_1 - a_3)\,. \quad (3b)$$

With the values for the scattering lengths $a_1 = 0.17\ \mu^{-1}$ and $a_3 = -0.092\ \mu^{-1}$ given by Bugg, Carter, and Carter,[15] we obtain $\lambda_1 = 0.003$ and $\lambda_2 = 0.050$. The value for λ_2 would, if the isospin-$\frac{3}{2}$ interaction were viewed as ρ exchange, correspond to a ρN coupling constant value $g_\rho{}^2/4\pi = 0.77$, which is somewhat larger than the conventional values[16] ($g_\rho{}^2/4\pi = 0.55$).

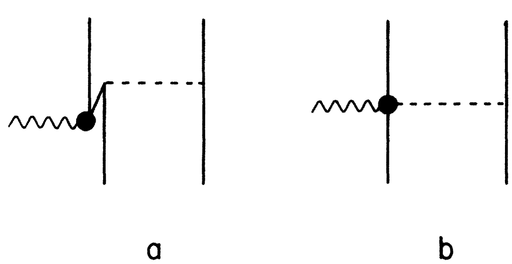

FIG. 1. Diagrammatic representation of pion exchange current in (a) the pseudoscalar coupling model and (b) the pseudovector coupling model.

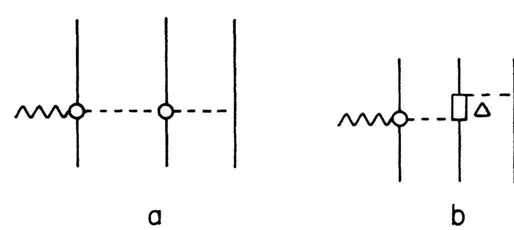

FIG. 2. Three-body two-meson-exchange currents with pion rescattered by an intermediate nucleon in (a) the S wave and (b) the P wave as represented by a Δ resonance.

Using the phenomenological Lagrangian (2), we may now construct the three-body charge operator associated with the Feynman diagram in Fig. 2(a) if the first $\gamma\pi NN$ vertex and the final pion absorption vertex are described in the same way as in the derivation of the two-body charge operator (1). We obtain

$$\rho_3^S(\vec{k}_1,\vec{k}_2,\vec{k}_3) = -\frac{\pi g^2}{m^3\mu}\frac{\vec{\sigma}^1\cdot\vec{q}\,\vec{\sigma}^2\cdot\vec{k}_3}{(\mu^2+p^2)(\mu^2+k_3^2)} \times\{\lambda_1[F_1^S\vec{\tau}^1\cdot\vec{\tau}^2+F_1^V\tau_3^2]-i\frac{\lambda_2}{\mu}(\omega_p+\omega_{k_3}) \times[F_1^S\vec{\tau}^1\cdot\vec{\tau}^2\times\vec{\tau}^3+F_1^V(\vec{\tau}^2\times\vec{\tau}^3)_3]\}. \tag{4}$$

Here we denote the momentum and energy of the first exchanged meson $\vec{p}$ and ω_p. The momenta imparted to each of the three nucleons are denoted $\vec{k}_1$, $\vec{k}_2$, and $\vec{k}_3$. By momentum conservation $\vec{q}=\vec{k}_1+\vec{k}_2+\vec{k}_3$. The momentum and energy of the final pion are then $\vec{k}_3$ and ω_{k_3}.

More than 90% of the ground-state configuration of the three- and four-nucleon systems is assumed to be a spatially symmetric S state. For this wavefunction component the part in (4) which contains λ_2 has no matrix element because of the antisymmetric isospin operators. Consequently the three-body operator (4) will be rather unimportant, as the remaining part is multiplied by the very small coupling constant λ_1.

To construct the three-body charge operator that corresponds to P-wave pion rescattering through the Δ_{33} resonance (which we treat in the sharp resonance approximation), we employ the $\pi N\Delta$ coupling

$$\mathcal{L}_{\pi N\Delta}=\frac{f_\Delta}{\mu}\chi_a^*\vec{\psi}^\dagger\cdot\vec{\nabla}\phi_a\psi\chi+\text{H.c.}, \tag{5}$$

with $\vec{\psi}$ and $\vec{\chi}$ being the vector-spinor and isovector spinor describing the Δ resonance. For f_Δ we use the value obtained from the decay width ($f_\Delta^2/4\pi=0.35$). Treating the other two vertices in Fig. 2(b) as before, we derive the three-body charge operator

$$\rho_3^\Delta(\vec{k}_1,\vec{k}_2,\vec{k}_3)=\frac{g^2}{9m^3}\left(\frac{f_\Delta}{\mu}\right)^2\frac{\vec{\sigma}^1\cdot\vec{q}\,\vec{\sigma}^3\cdot\vec{k}_3}{(m_\Delta-m)(\mu^2+p^2)(\mu^2+k_3^2)}\{F_1^S[(\vec{p}\cdot\vec{k}_3)\vec{\tau}^1\cdot\vec{\tau}^3+\tfrac{1}{4}(\vec{\sigma}^2\cdot\vec{p}\times\vec{k}_3)\vec{\tau}^1\cdot\vec{\tau}^2\times\vec{\tau}^3] +F_1^V[(\vec{p}\cdot\vec{k}_3)\tau_3^3+\tfrac{1}{4}(\vec{\sigma}^2\cdot\vec{p}\times\vec{k}_3)(\vec{\tau}^2\times\vec{\tau}^3)_3]\}+\text{permutations}. \tag{6}$$

We may construct in a similar way the four-body charge operator due to two successive P-wave rescatterings illustrated by the diagram in Fig. 3. The result is (we exhibit only the isoscalar part)

$$\rho_4^\Delta(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)=\frac{8g^2}{81m^3}\left(\frac{f_\Delta}{\mu}\right)^4F_1^S\frac{\vec{\sigma}^1\cdot\vec{q}\,\vec{\sigma}^4\cdot\vec{k}_4}{(m_\Delta-m)^2(\mu^2+p_1^2)(\mu^2+p_2^2)(\mu^2+k_4^2)} \times\{\vec{p}_1\cdot\vec{p}_2\vec{p}_2\cdot\vec{k}_4\vec{\tau}^1\cdot\vec{\tau}^4+\tfrac{1}{4}(\vec{p}_1\cdot\vec{p}_2)(\vec{\sigma}^3\cdot\vec{p}_2\times\vec{k}_4)\vec{\tau}^1\cdot\vec{\tau}^3\times\vec{\tau}^4+\tfrac{1}{4}(\vec{p}_2\cdot\vec{k}_4)(\vec{\sigma}^2\cdot\vec{p}_1\times\vec{p}_2)\vec{\tau}^1\cdot\vec{\tau}^2\times\vec{\tau}^4 +\tfrac{1}{16}(\vec{\sigma}^2\cdot\vec{p}_1\times\vec{p}_2)(\vec{\sigma}^3\cdot\vec{p}_2\times\vec{k}_4)(\vec{\tau}^1\times\vec{\tau}^2)\cdot(\vec{\tau}^3\times\vec{\tau}^4)\}. \tag{7}$$

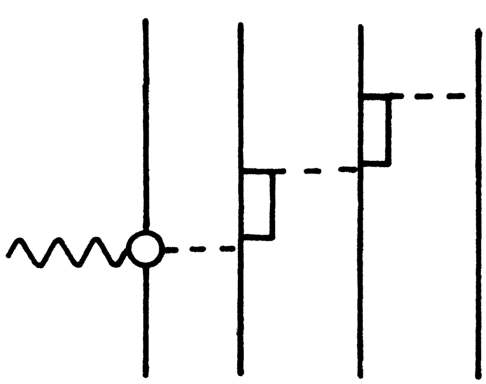

FIG. 3. Four-body exchange current involving a double P-wave rescattering of the exchanged pion.

Here the momentum of the pion emerging from the photon vertex is denoted $\vec{p}_1$, and that of the second pion is denoted $\vec{p}_2$. By momentum conservation $\vec{q}=\sum_i\vec{k}_i$, where $\vec{k}_i$ is the momentum delivered to the ith nucleon.

The four-body charge operators obtained when one or both of the rescatterings are due to S-wave interactions are of negligible importance compared to those due to two successive P-wave rescatterings considered above, and we shall not deal with them here.

It is known that the inelastic one-pion-exchange interaction between a NN and a $N\Delta$ state is strongly damped by the similar interaction due to ρ-meson exchange.[17,18] We therefore expect the unmodified pion-exchange charge operators above which involve intermediate Δ's to overestimate the three- and four-body effects. Therefore we find it necessary to consider the ρ-meson-exchange currents as well.

We first construct the two-body charge operator associated with ρ-meson exchange. The diagrammatic representation of that exchange current is given in Fig. 4(c). To describe the $\gamma\rho NN$ vertex we first write down the relativistic Born term for the $\gamma N \to \rho N$ reaction amplitude including the four-point vertex necessitated by the derivative in the ρNN tensor coupling and gauge invariance. Then we separate out that component of the operator which is associated with the positive energy intermediate state and which is singular in the soft photon or soft meson limit, and take the static limit of the remainder. The resulting charge operator is

$$\rho_2^{\rho}(\vec{k}_1, \vec{k}_2) = \frac{g_\rho^{\,2}(1+\kappa)^2}{8m^3}\left[F_1^S \vec{\tau}^1 \cdot \vec{\tau}^2 + F_1^V \tau_3^{\,2}\right] \times \frac{\vec{\sigma}^1 \cdot \vec{\sigma}^2 \vec{q} \cdot \vec{k}_2 - \vec{\sigma}^1 \cdot \vec{k}_2 \vec{\sigma}^2 \cdot \vec{q}}{m_\rho^{\,2} + k_2^{\,2}} + (1 \to 2)\,. \tag{8}$$

Here g_ρ is the ρN coupling constant, $g_\rho \kappa$ the ρN tensor coupling constant, and m_ρ the ρ-meson mass. For the ρ-coupling constants we use the values $g_\rho^{\,2}/4\pi = 0.55$ and $\kappa = 6.66$ given by Höhler and Pietarinen.[16] The approximations involved in the derivation of (8) are similar to those used in obtaining the two-body pion-exchange charge operator (1) (i.e., neglect of nonlocal operators and terms of the order q/m compared to 1).

In addition we have omitted some unimportant terms from the charge operator (8). Among these is a group of isovector operators which have zero matrix elements for wave functions that separate into a product of spatial and spin-isospin factors. The contribution due to the four-point $\gamma\rho NN$ vertex necessitated in principle by gauge invariance is of this type. Furthermore, we have left out terms of less than second order in $(1+\kappa)$; specifically, those involving the nucleon anomalous moment

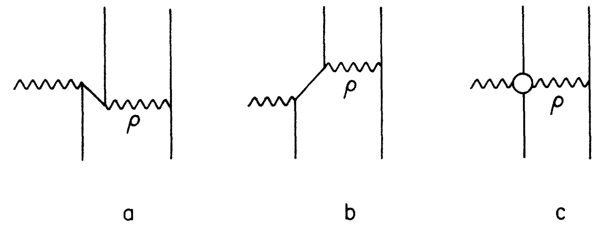

FIG. 4. Two-body ρ-meson-exchange currents. The pair diagram (a), the relativistic correction to the forward-going nucleon diagram (b), and an explicit seagull term in the relativistic Lagrangian, contribute in their static limits to the two-body ρ-exchange operator represented by (c).

form factor F_2 and those associated with the time components of the ρ-meson field. This approximation we motivate by the fact that $1+\kappa$ is large ($\simeq 7.66$).

It is worth noting that both the "pair" diagram in Fig. 4(a) and the positive energy nucleon intermediate state diagram Fig. 4(b) contribute to the ρ-meson exchange operators. In fact, the latter diagram is the more important one, being of order $(1+\kappa)^2$, whereas the pair diagram considered by Gari and Hyuga[21] is only of order $(1+\kappa)$.

We now turn to the three- and four-body charge operators associated with π and ρ-exchange. The additional three-body diagrams are shown in Fig. 5. There are seven four-body diagrams in addition to that in Fig. 3. We treat the $\gamma\rho NN$ vertex as in the construction of the two-body operator and use the usual model for the coupling of the ρ-meson with a nucleon and an isobar[16]

$$\mathcal{L}_{\rho N\Delta} = \frac{g_{\rho\Delta}}{2m}\, \chi_a^* \vec{\psi}^\dagger \cdot (\vec{\nabla} \times \vec{\rho}_a)\psi\chi + \text{H.c.} \tag{9}$$

The coupling constant $g_{\rho\Delta}$ is related to the ρNN coupling constant by the static quark model as $g_{\rho\Delta} = \frac{6}{5}\sqrt{2}(1+\kappa)g_\rho$. The combined pion and ρ-meson three- and four-body exchange charge operators are then

$$\rho_3^{\Delta}(\vec{k}_1, \vec{k}_2, \vec{k}_3) = \frac{g^2}{36m^3}\left(\frac{f_\Delta}{\mu}\right)^2 \frac{1}{m_\Delta - m}(F_1^S \tau_a^1 + F_1^V \delta_{3a})\vec{\sigma}^1 \cdot \overleftrightarrow{\Phi}_{\gamma\Delta}(\vec{q}, \vec{p}_1) \cdot \overleftrightarrow{T}_{ab}{}^2 \cdot \overleftrightarrow{\Phi}_{\Delta}(\vec{p}_2) \cdot \vec{\sigma}^3 \tau_b^{\,3}, \tag{10}$$

$$\rho_4^{\Delta}(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) = \frac{g^2}{162m^3}\left(\frac{f_\Delta}{\mu}\right)^4 \frac{1}{(m_\Delta - m)^2} F_1^S \tau_1^a \vec{\sigma}^1 \cdot \overleftrightarrow{\Phi}_{\gamma\Delta}(\vec{q}, \vec{p}_1) \cdot \overleftrightarrow{T}_{ab}{}^2 \cdot \overleftrightarrow{\Phi}_{\Delta\Delta}(\vec{p}_2) \cdot \overleftrightarrow{T}_{bd}{}^3 \cdot \overleftrightarrow{\Phi}_{\Delta}(\vec{p}_3) \cdot \vec{\sigma}^4 \tau_d^{\,4}. \tag{11}$$

Here we have only exhibited the isoscalar part of ρ_4. The spin-isospin tensors $\overleftrightarrow{T}^i$ in (10) and (11) are defined as

$$T^i_{ab,jk} = 4\delta_{ab}\delta_{jk} - \epsilon_{abc}\tau_c^i \epsilon_{jkl}\tau_l^i\,. \tag{12}$$

The meson exchange tensor functions $\overleftrightarrow{\Phi}$ are defined as

$$\overleftrightarrow{\Phi}_{\gamma\Delta}(\vec{q}, \vec{p}) = \frac{\vec{q}\vec{p}}{\mu^2 + p^2} - \xi\frac{\vec{p}\vec{q} - \overleftrightarrow{1}\,\vec{p}\cdot\vec{q}}{m_\rho^{\,2} + p^2},$$
$$\overleftrightarrow{\Phi}_{\Delta\Delta}(\vec{p}) = \frac{\vec{p}\vec{p}}{\mu^2 + p^2} + 2\xi'\frac{\vec{p}\vec{p} - \overleftrightarrow{1}\,p^2}{m_\rho^{\,2} + p^2}, \tag{13}$$
$$\overleftrightarrow{\Phi}_{\Delta}(\vec{p}) = \overleftrightarrow{\Phi}_{\gamma\Delta}(\vec{p}, \vec{p}).$$

The coefficients ξ and ξ' for the ρ-meson contributions are

$$\xi = \frac{g_{\rho\Delta} g_\rho (1+\kappa)}{2 g f_\Delta}\left(\frac{\mu}{m}\right) \simeq 1.80\,,$$

$$\xi' = -\tfrac{1}{2}\left(\frac{g_{\rho\Delta}}{2 f_\Delta}\right)^2\left(\frac{\mu^2}{m^2}\right) \simeq 0.73\,. \tag{14}$$

The operators (10) and (11) contain parts which lead in configuration space to δ functions of the relative separations of the nucleon pairs 2,3 and 3,4. It may well be argued that the wave-function correlations will as least eliminate the contributions from these δ-function terms. The δ-function components may be removed from the operators by subtracting from the tensor $\Phi_{\Delta\Delta}$ a term $\frac{1}{3}(1-4\xi')\overleftrightarrow{1}$ and from the tensor Φ_Δ a term $\frac{1}{3}(1+2\xi)\overleftrightarrow{1}$.

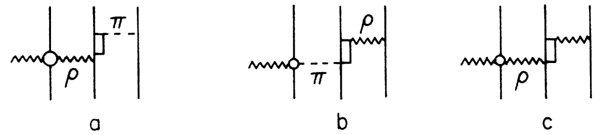

FIG. 5. Three-body currents involving ρ-meson as well as pion exchange.

III. THREE- AND FOUR-BODY NUCLEI

For the evaluation of the matrix elements of the many-body operators derived in the previous section we use the general formula for the matrix element of an N-body operator in an A-body nucleus:

$$\langle\rho_N\rangle = \frac{A!}{N!\,(A-N)!}\int d(1)d(2)\ldots d(A)\psi^\dagger(1\ldots A)\int\left(\prod_{j=1}^{N}\frac{d^3k_j}{(2\pi)^3}e^{i\vec{k}_j\cdot\vec{r}_j}\right)\delta\Big(\vec{q}-\sum_{m=1}^{N}\vec{k}_m\Big)\rho_N(\vec{k}_1,\ldots,\vec{k}_N)\,\psi(1\ldots A),$$

Here ψ is the nuclear wave function and $\vec{k}_j$ is the momentum delivered to the jth nucleon. The symbol $d(j)$ denotes the differential of all coordinates of the jth nucleon, including the position $\vec{r}_j$. If the harmonic oscillator model is used for ψ, the center of mass correction is trivially taken into account in the usual way.[19]

We proceed now to the calculation of the matrix elements of the many-body operators under consideration for the three- and four-nucleon nuclei. We shall only consider the dominant spatially symmetric S-state configuration in the ground-state wave functions. For this configuration the wave function factorizes into a totally symmetric spatial function φ and a totally antisymmetric spin-isospin vector ϕ_0. For ϕ_0 one may use the usual explicit form,[20,5] but in fact the spin-isospin matrix elements may be obtained directly once the total spin and isospin and the corresponding z components are known.

Below we give the spin-isospin expectation values of the charge operators presented in the previous section.

For the two-body pion- and ρ-exchange charge operators (1) and (8) we obtain the spin-isospin matrix elements

$$\phi_0^\dagger\rho_2^\pi\phi_0 = -\frac{g^2}{8m^3}\left(F_1^S \pm \tfrac{1}{3}F_1^V\right)\frac{\vec{q}\cdot\vec{k}_2}{\mu^2+k_2^2}\,, \tag{15}$$

$$\phi_0^\dagger\rho_2^\rho\phi_0 = -\frac{2g_\rho^{\,2}(1+\kappa)^2}{8m^3}\left(F_1^S \pm \tfrac{1}{3}F_1^V\right)\frac{\vec{q}\cdot\vec{k}_2}{m_\rho^2+k_2^2}\,. \tag{16}$$

For the α particle the F_1^V term in the bracket should be omitted.

For the three-body charge operator due to S-wave πN rescattering (4) we obtain the radial operator

$$\phi_0^\dagger\rho_3^S\phi_0 = \frac{\pi g^2\lambda_1}{m^3\mu}\left(F_1^S \pm \tfrac{1}{3}F_1^V\right)\frac{\vec{q}\cdot\vec{k}_3}{(\mu^2+p^2)(\mu^2+k_3^2)}\,. \tag{17}$$

The spin-isospin matrix elements of the three-body pion- and ρ-exchange operators with the Δ intermediate state [(6),(10)] can be combined into the following expression:

$$\phi_0^\dagger\rho_3^\Delta\phi_0 = -\frac{g^2}{6m^3}\left(\frac{f_\Delta}{\mu}\right)^2\frac{1}{m_\Delta - m}\left(F_1^S \pm \tfrac{1}{3}F_1^V\right)$$
$$\times\left(\vec{q}\cdot\vec{k}_3\,\vec{p}\cdot\vec{k}_3 - \tfrac{1}{3}k_3^2\,\vec{q}\cdot\vec{p}\right)$$
$$\times\left(\frac{1}{\mu^2+p^2}-\frac{\xi}{m_\rho^2+p^2}\right)\left(\frac{1}{\mu^2+k_3^2}-\frac{\xi}{m_\rho^2+k_3^2}\right). \tag{18}$$

The ρ-meson exchange coefficient ξ is defined in (14). For the α particle the F_1^V term in Eqs. (17) and (18) should again be dropped.

The spin-isospin matrix element of the four-body operator (11) is

$$\phi_0^\dagger\rho_4^\Delta\phi_0 = \frac{g^2}{162m^3}\left(\frac{f_\Delta}{\mu}\right)^4\frac{1}{(m_\Delta - m)^2}F_1^S$$
$$\times\left[\frac{1}{\mu^2+p_3^2}-\frac{\xi}{m_\rho^2+p_3^2}\right]\Phi_{\gamma\Delta}(\vec{q},\vec{p}_1)_{ij}$$
$$\times\,\Phi_{\Delta\Delta}(\vec{p}_2)_{kl}U_{ijkl}\,. \tag{19}$$

Here the tensor U is defined as

$$U_{ijkl} = 4\delta_{ij}(3p_{3k}p_{3l} - p_3^2\delta_{kl}) - 4\delta_{ik}(3p_{3j}p_{3l} - p_3^2\delta_{jl})$$
$$-\,8\delta_{jk}(3p_{3i}p_{3l} - p_3^2\delta_{il})\,. \tag{20}$$

The calculation of the radial matrix elements of the operator above is very simple if the harmonic oscillator model is used for the radial wave function. With that model it is possible to express the charge form factors of the three- and four-body nuclei in the following convenient form:

$$F(^3\text{He})=\tfrac{1}{4}(3F_1^S+F_1^V)e^{-q^2/6\alpha^2}(1+\delta_{II}+\delta_{III}),$$
$$F(^3\text{H})=\tfrac{1}{2}(3F_1^S-F_1^V)e^{-q^2/6\alpha^2}(1+\delta_{II}+\delta_{III}), \qquad (21)$$
$$F(^4\text{He})=F_1^S e^{-3q^2/16\alpha^2}(1+\tfrac{3}{2}\delta_{II}+3\delta_{III}+\delta_{IV}).$$

Here α is the oscillator parameter $(m\omega_0)^{1/2}$, which for the three-body nuclei we take to be 0.59 fm^{-1} and for the α particle to be 0.7 fm^{-1}. In (21) we have neglected the difference between the vector form factors F_1 and the electric form factors G_E which properly ought to be associated with the impulse approximation expressions.

The first term in the brackets in Eqs. (21) represents the impulse approximation result and the coefficients δ_{II}, δ_{III}, and δ_{IV} in the other terms represent the contributions to the form factor of the two-, three-, and four-body currents.

The enhancement factor δ_{II} due to the π-, and ρ-meson-exchange two-body charge operators (1) and (8) may be written in the form of an integral as

$$\delta_{II}=-\frac{1}{6}\left(\frac{g^2}{4\pi}\right)\left(\frac{q}{m}\right)^2 \times\frac{1}{m}\int_0^\infty dk\,[S_1(q,k;\mu)+\eta S_1(q,k;m_\rho)]. \qquad (22)$$

Here we define the function S_1 as

$$S_1(q,k;\mu)=\frac{1}{\pi\alpha^2}\frac{k^4}{\mu^2+k^2}e^{-k^2/2\alpha^2}\left(\frac{\pi\alpha^2}{qk}\right)^{1/2} \times\left[I_{1/2}\left(\frac{qk}{2\alpha^2}\right)-I_{5/2}\left(\frac{qk}{2\alpha^2}\right)\right] \cdot \qquad (23)$$

The coefficient η for the ρ-meson contribution is

$$\eta=2(g_\rho/g)^2(1+\kappa)^2\approx 4.45\,. \qquad (24)$$

The enhancement factor δ_{III} can only be reduced to a double numerical integral. For the contribution due to the three-body two-pion-exchange operator (17) arising from S-wave rescattering we obtain the enhancement factor

$$\delta_{III}^S=\frac{\lambda_1}{6}\left(\frac{g^2}{4\pi}\right)\left(\frac{q}{m}\right)^2 \times\frac{1}{m\mu}\int_0^\infty dk\int_0^\infty dp\,S_1(q,p;\mu)S_1(p,k;\mu). \qquad (25)$$

The three-body operator (18) which is associated with the Δ_{33} intermediate state leads to the enhancement factor

$$\delta_{III}^\Delta=-\frac{8}{27}\left(\frac{g^2}{4\pi}\right)\left(\frac{f_\Delta^2}{4\pi}\right)\left(\frac{q}{m}\right)^2\frac{1}{m(m_\Delta-m)} \times\int_0^\infty dp\int_0^\infty dk[S_1(q,p;\mu)-\xi S_1(q,p;m_\rho)] \times[S_2(p,k;\mu)-\xi S_2(p,k;m_\rho)]. \qquad (26)$$

Here the auxiliary function S_2 is defined as

$$S_2(p,k;\mu)=\frac{1}{\pi\alpha^2}\frac{k^4}{k^2+\mu^2}e^{-k^2/2\alpha^2}\left(\frac{\pi\alpha^2}{pk}\right)^{1/2}I_{5/2}\left(\frac{pk}{2\alpha^2}\right). \qquad (27)$$

The fact that the function S_2 in the integrand in (26) only contains a modified Bessel function of order $\frac{5}{2}$ is owing to the fact that the intermediate Δ resonance must be in a relative D state together with the third nucleon (Fig. 5).

The enhancement factor δ_{IV} due to the charge operator (19) caused by two successive meson rescatterings through the Δ intermediate state has the form

$$\delta_{IV}^\Delta=-\frac{128}{81}\left(\frac{g^2}{4\pi}\right)\left(\frac{f_\Delta^2}{4\pi}\right)^2\left(\frac{q}{m}\right)^2\left(\frac{\alpha}{\mu}\right)^4\frac{1}{m(m_\Delta-m)^2} \times\int_0^\infty dp_1dp_2dk[S_1(q,p_1;\mu)-\xi S_1(q,p_1,m_\rho)] \times[S_2(p_1,p_2;\mu)-\xi' S_2(p_1,p_2;m_\rho)] \times[S_2(p_2,k;\mu)-\xi S_2(p_2,k;m_\rho)]. \qquad (28)$$

The coefficients ξ and ξ' are given in Eq. (14). The effect on (28) of removing δ functions from the operator (19) is to subtract $\frac{1}{2}S_2(p_2,k,0)$ from $S_2(p_2,k,\mu)$ and $2S_2(p_2,k,0)$ from $S_2(p_2,k,m_\rho)$.

We now turn to a presentation of the numerical results. We first consider the enhancement factor δ_{II} due to the two-body charge operators (15) and (16). In Fig. 6 we plot δ_{II} as defined for the three-body nuclei in Eqs. (21) and (22) as a function of momentum transfer. The dashed curve is the result obtained with neglect of the ρ meson (i.e., $\eta=0$). The relatively large effect of the ρ meson in increasing δ_{II} is associated with the presence of the modified Bessel function of order $\frac{5}{2}$ in the function S_2, as that Bessel function enhances the effect of high-momentum components or short-range effects. With the conventional value 3.7 for the ρ-tensor coupling constant κ rather than the value 6.6 given in Ref. 16, the ρ-meson-exchange coefficient η would be $\simeq 1$ rather than 4.45, leaving a far less notable ρ-exchange effect. In the earlier work of Gari and Hyuga[21] on the ρ-exchange contribution to the two-body current in the deuteron, the effect appeared smaller because of the omission of the effect of the positive energy nucleon diagram in Fig. 4(b).

In Fig. 7 we plot the enhancement factor $\frac{3}{2}\delta_{II}$ relevant for the α particle as defined in (21). Again, the dashed curve is the result obtained with omission of ρ exchange. Evidently, the two-body exchange charge operator has a relatively smaller effect in the case of the α particle than in the case of the three-body nuclei.

The enhancement factor δ_{III} due to the three-body charge operators (18) associated with the Δ resonance intermediate state has also been plotted in Figs. 6 and 7. (For the α particle in Fig. 7 the relevant quantity $3\delta_{III}$ is shown.) The dashed curves are the results obtained with pion exchange alone. These results show that if it were not for ρ exchange the effect of the three-body current would be only 2–3 times less than that of the two-body current.

The effect of ρ exchange in cutting down the two-pion-exchange three-body effect is dramatic—especially in the case of the more dense α particle where the short-range ρ-exchange effect cancels most of the pion exchange effect. These results do, however, in some ways overestimate the ρ-exchange effect in that if hadronic form factors were introduced at the meson vertices, the ρ-exchange effect would be reduced more than the pion-exchange effect. Also, we have used the very large values for the ρ-coupling constants given in Ref. 16. Reducing the ρ-tensor coupling constant κ from 6.6 to 3.7 would reduce the ρ-exchange effect by a factor 2.6. Finally, short-range correlations in a more realistic wave function would affect the short-range ρ-exchange effect more than the long-range pion-exchange effect.

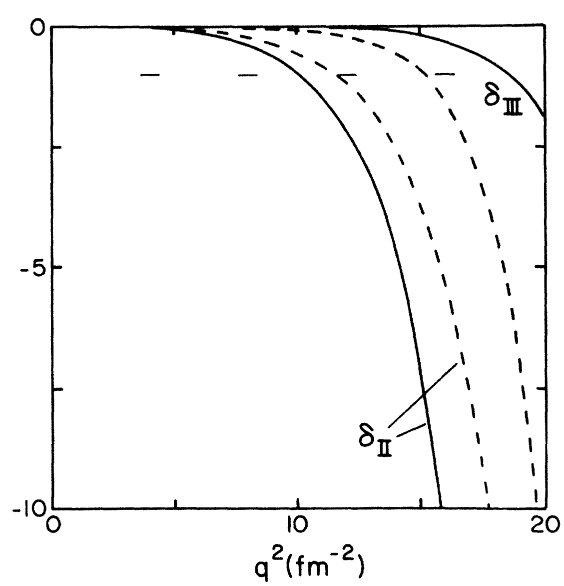

FIG. 6. Enhancement factors due to the two- and three-body exchange currents [Eqs. (22) and (26)] in the harmonic oscillator model of ^3He and ^3H with $\alpha = 0.59$ fm^{-1}. The dashed curves give the result obtained with omission of ρ-exchange effects; the solid curves include ρ exchange.

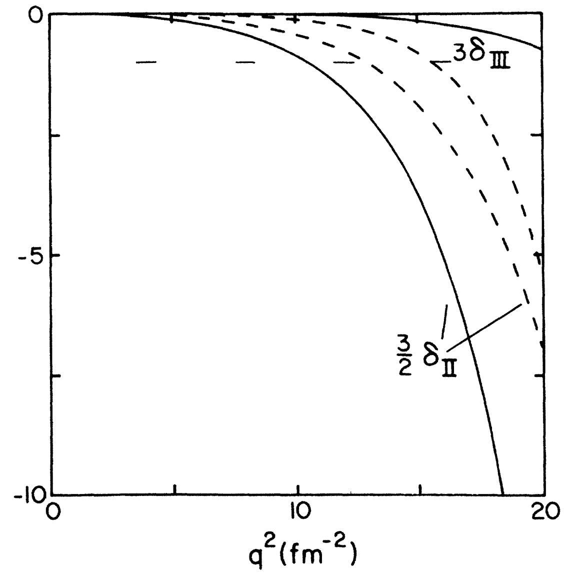

FIG. 7. Two- and three-body enhancement factors in the harmonic oscillator model of ^4He with $\alpha = 0.7$ fm^{-1}. The dashed curves are again the result without ρ exchange.

Before turning to a discussion of the four-body current effects we consider the remaining three-body charge operator (4) due to S-wave πN rescattering which leads to the enhancement factor δ_{III}^s [(25)]. This operator turns out to lead to a very small effect as shown in Table I. Compared with the three-body exchange current associated with P-wave πN rescattering through the Δ resonance, this operator is thus of negligible importance.

In Fig. 8 we plot the enhancement factor δ_{IV} for the α-particle charge form factor (21) caused by the four-body exchange charge operator (19) and given explicitly in (28). The curve a is the result with omission of the ρ-meson exchange effect, and the curve c the result of the combined π- and ρ-exchange four-body charge operator (28).

TABLE I. Enhancement factors (25) coming from the three-body charge operator (4) due to S-wave rescattering. The effect is inconsequential.

q^2/fm^{-2}	$\delta_{III}{}^s$
0	0
5	6.59×10^{-5}
10	8.34×10^{-4}
15	8.78×10^{-3}
20	8.85×10^{-2}

As mentioned before, the four-body operator (11) has δ-function-type interactions which have nonvanishing spin-isospin expectation values. [The δ-function parts of the three-body operator (10) have zero spin-isospin matrix elements.] The consequences of the removal of the δ-function terms on the enhancement factor δ_{IV} are also shown in Fig. 8: The curve b is the result for pion-exchange alone after removal of the δ-function term, and the curve d is the result for the combined π- and ρ-exchange effect after removal of the δ-function terms.

The result in Fig. 8 shows that the effect of the three-pion-exchange four-body charge operator is ≃5 times smaller than that of the corresponding two-pion-exchange three-body operator. The effect of the inclusion of the ρ-meson is essentially to wipe out the whole four-body exchange current effect. We may thus draw the conclusion that the three-body exchange effect is but a small correction to the two-body effect and that the four-body exchange effect is completely negligible. In the following section we shall show that the inclusion of hadronic form factors and short-range correlations does not change this conclusion.

IV. VERTEX FACTORS AND SHORT RANGE CORRELATIONS

In this section we shall show how the introduction of hadronic form factors and short-range correlations in the wave functions affect the previous results on the effects of two-, three-, and four-body exchange currents. Within the framework presented in Secs. II and III, it is simple to introduce hadronic form factors at the meson-nucleon vertices. With each pion and ρ-meson vertex in the operators considered in Sec. II we include a form factor

$$\Gamma(\vec{k}^2)=\frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{k}^2}\,, \tag{29}$$

where μ is the mass of the exchanged meson and $\vec{k}$ is its momentum. We chose the cutoff mass Λ as 1.4 GeV/c^2, which is an intermediate value between the value $2m$ corresponding to the vertex diagrams with $N\bar{N}$ intermediate states and the value ~1 GeV/c^2 which is the lowest limit for the $\pi\rho$ and $\pi\omega$ vertex triangle diagrams for π and ρ exchange, respectively.

The vertex functions are taken into account by the substitutions

$$S_{1,2}(q,k;\mu)\rightarrow S_{1,2}(q,k;\mu)\Gamma^2(k^2) \tag{30}$$

in the expressions (22), (25), (26), and (28) for the enhancement factors δ. In (30), μ is either the pion or ρ-meson mass, depending on which meson is being exchanged.

In Figs. 9 and 10 we show the enhancement factors obtained with the same wave-function models as before but with inclusion of the vertex factors. Comparing the results in Figs. 6 and 9, which show δ_{II} and δ_{III} for the three-body nuclei, indicates that the form factors tend to reduce the ρ-

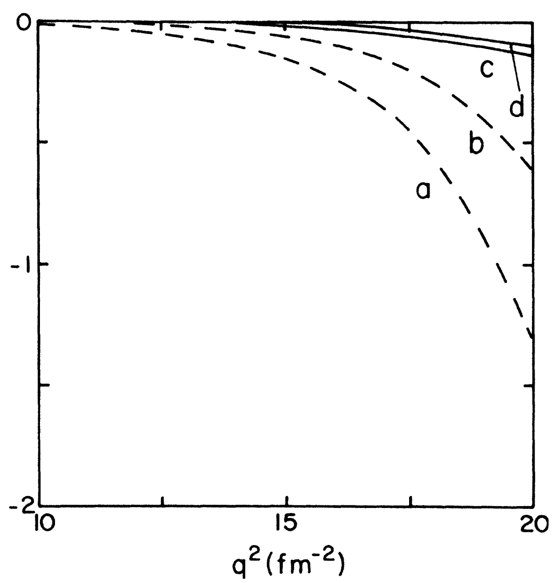

FIG. 8. Enhancement factor from the four-body exchange current. Curves (a) and (c) are the result of (28), (c) with ρ exchange and (a) without. Curves (b) and (d) are the result of removing δ functions from the four-body operator, as discussed below Eq. (28) in the text, (d) with ρ exchange and (b) without.

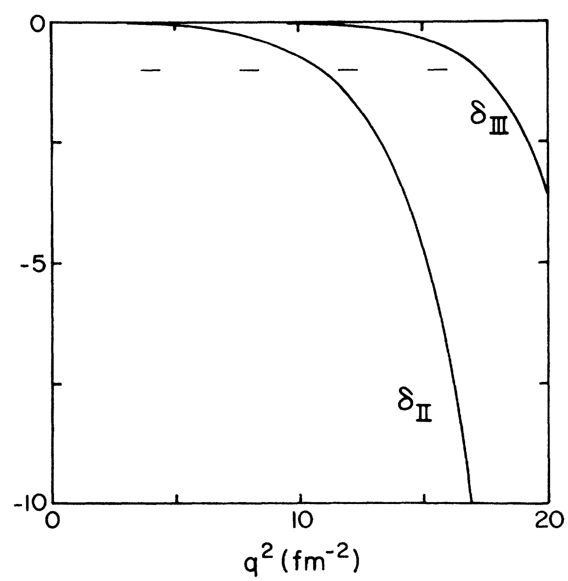

FIG. 9. Two- and three-body enhancement factors for three-body nuclei with inclusion of vertex factors in the operator. Compare Fig. 6.

meson effect considerably. The result for δ_{II} for $\pi+\rho$ exchange with form factors is very similar to the result for δ_{II} obtained with π exchange alone but no form factor. This result offers a partial explanation for why earlier exchange current calculations which only included bare pion exchange have been remarkably successful: the form factors serve to increase the effect of the three-body currents δ_{III} by roughly a factor of 2 over the value obtained with bare pion and ρ exchange, but it still remains roughly a factor of 3 smaller than the result obtained with bare pion exchange alone. The reason for this is the pure tensor character of the inelastic meson exchange interaction that creates the intermediate $N\Delta$ state. The tensor force enhances the importance of the short-range ρ-exchange interaction.

The results for the case of the α particle are very similar, as shown in Fig. 10, but the three-body current here has a relatively smaller effect than in the three-body nuclei.

In Fig. 10 the effect on the four-body current (34) as modified by form factors is also shown. This effect remains small despite the increase caused by the form factor.

In Fig. 11 we show the different contributions to the α-particle form factor compared to the empirical data of Frosch *et al.*[22] The model for the nucleon form factor used was that of Iachello, Jackson, and Lande.[23] The large effect of the two-body exchange current on the α-particle form factor has of course been noted before.[5-7] The relative smallness of the three-body exchange current effect is obvious. The effect of the four-body exchange current is too small to be seen on the scale of the figure.

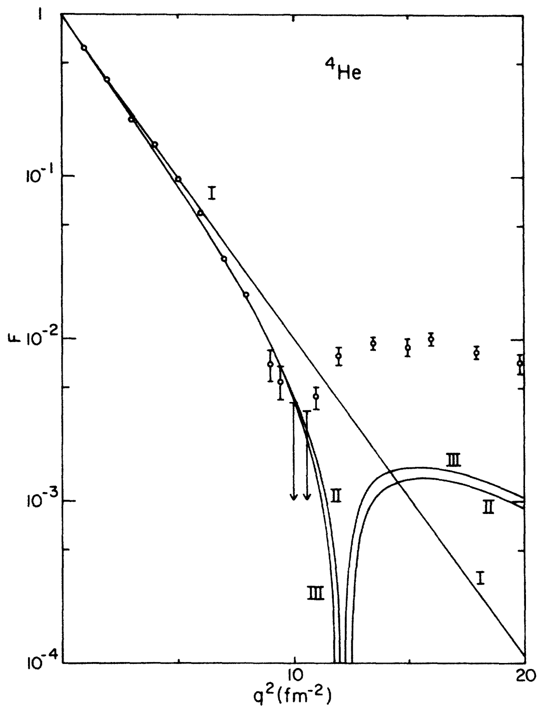

FIG. 11. Charge form factor of ^{4}He in the harmonic oscillator model ($\alpha = 0.7$ fm^{-1}). Curve I: impulse approximation; Curve II: with inclusion of two-body operators; Curve III: with inclusion also of three-body operators. The effect of four-body operators is invisible on the scale of the figure. The data are from Ref. 22.

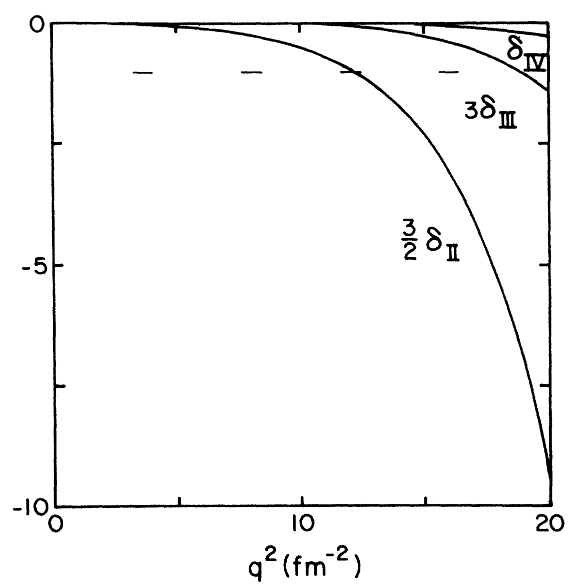

FIG. 10. Two-, three-, and four-body enhancement factors for ^{4}He with inclusion of vertex factors in the operator. Compare Fig. 7.

The combined exchange current effect obtained from two- and three-body currents as calculated above is still about 25% larger than what would have been the result if ρ-meson exchange had been neglected. This is a consequence of the large increase of the two-body effect caused by ρ exchange and which well overcomes the corresponding decrease of the three-body effect.

We finally turn to the discussion of how short-range correlations affect the results obtained above. We shall only treat short-range correlations in the three-body system. In order to have a compact expression for the matrix elements of the charge operators discussed in Sec. III for a general configuration space wave function, we have to construct their Fourier transforms. We define the Jacobi coordinates

$$\vec{r}=\vec{r}_2-\vec{r}_1\,,$$
$$\vec{\rho}=\vec{r}_3-\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\,. \quad (31)$$

The S-state wave function φ is then a function of r, ρ, and $z=\hat{r}\cdot\hat{\rho}$. The expression for the impulse approximation contribution to the charge form factor of ^{3}He is then

$$F_I(q^2)=\tfrac{1}{4}(3F_1^S+F_1^V)8\pi^2\int d\rho drdz\,\rho^2r^2j_0(\tfrac{2}{3}q\rho)\,\varphi^2. \quad (32)$$

The two-body pion- and ρ-meson-exchange current operators (15) and (16) give the form factor contribution

$$F_{II}(q^2)=-\tfrac{1}{36}(3F_1^S+F_1^V)\left(\frac{g^2}{4\pi}\right)\left(\frac{q}{m}\right)^2 8\pi^2\int d\rho drdz\,\rho^2r^2[j_0(\tfrac{2}{3}q\rho)+j_2(\tfrac{2}{3}q\rho)]$$
$$\times\frac{(\rho^2+\frac{1}{2}\rho rz)}{\xi^2}\left[\frac{\mu}{m}Y_1(\mu\zeta)+\eta\frac{m_\rho}{m}Y_1(m_\rho\zeta)\right]\varphi^2. \quad (33)$$

Here we use the notation

$$\zeta=(\rho^2+\rho rz+\tfrac{1}{4}r^2)^{1/2} \quad (34)$$

and

$$Y_1(x)=(1+x)e^{-x}/x\,. \quad (35)$$

The ρ-meson coefficient η is defined in Eq. (24).

The three-body pion- and ρ-meson-exchange-current operator (18) gives the following contribution to the form factor of ^{3}He:

$$F_{III}(q^2)=-\tfrac{1}{162}(3F_1^S+F_1^V)\left(\frac{g^2}{4\pi}\right)\left(\frac{f_\Delta^2}{4\pi}\right)\left(\frac{q}{m}\right)^2\left(\frac{\mu}{m}\right)\left(\frac{\mu}{m_\Delta-m}\right)$$
$$\times 8\pi^2\int d\rho drdz\,\rho^2r^2[j_0(\tfrac{2}{3}q\rho)+j_2(\tfrac{2}{3}q\rho)]$$
$$\times\frac{2\rho rP_1(z)+4\rho^2P_2(z)}{\zeta^2}[Y_1(\mu\zeta)-\frac{m_\rho}{\mu}\xi Y_1(m_\rho\zeta)][Y_2(\mu r)-\left(\frac{m_\rho}{\mu}\right)^3\xi Y_2(m_\rho r)]\,. \quad (36)$$

Here we use the notation

$$Y_2(x)=\left(\frac{3}{x^2}+\frac{3}{x}+1\right)\frac{e^{-x}}{x}\,. \quad (37)$$

If vertex factors of the form (29) are introduced at the meson-baryon vertices, the expressions (33) and (36) are modified by the substitutions

$$Y_1(\mu\zeta)\to Y_1(\mu\zeta)-\frac{\Lambda}{\mu}Y_1(\Lambda\zeta)-\frac{\Lambda^2-\mu^2}{2\mu}\zeta e^{-\Lambda\zeta} \quad (38)$$

and

$$Y_2(\mu r)\to Y_2(\mu r)-\left(\frac{\Lambda}{\mu}\right)^3[Y_2(\Lambda r)+\frac{\Lambda^2-\mu^2}{2\Lambda^2}Y_1(\Lambda r)]. \quad (39)$$

In (38) and (39) μ is the mass of the exchanged meson (π or ρ).

As it is not possible to obtain a good fit to the empirical[24] charge form factor for ^{3}He at low momentum transfer with a Gaussian (oscillator) wave function, we shall use the Irving-type wave function[25] below:

$$\varphi=Nf\exp\left[-\tfrac{1}{2}\beta\left(\sum_{kl}r_{kl}^2\right)^{1/2}\right]$$
$$=Nf\exp\left[-\tfrac{1}{2}\beta(2\rho^2+\tfrac{3}{2}r^2)^{1/2}\right]. \quad (40)$$

Here N is a normalization constant and f a correlation factor for which we use the form

$$f=\prod_{k<l}[1-\exp(-\gamma^2r_{kl}^2)]^{1/2}. \quad (41)$$

In (40 and (41) we employ the notation $\vec{r}_{kl}=\vec{r}_k-\vec{r}_l$.

In Fig. 12 we show the results for the charge form factor of ^{3}He obtained with the wave function (40) with no correlations (i.e., setting $f=1$). Curve I is the impulse approximation result ($\beta=1.23fm^{-1}$). Curve II shows the effect of including the two-body pion- and ρ-meson-exchange effect (33) using the vertex form factor (29). Curve III shows the additional effect of the three-body pion- + ρ-meson-exchange currents (36). The relative magnitudes of the exchange current effects are similar to those in the α-particle case given in Fig. 11. The

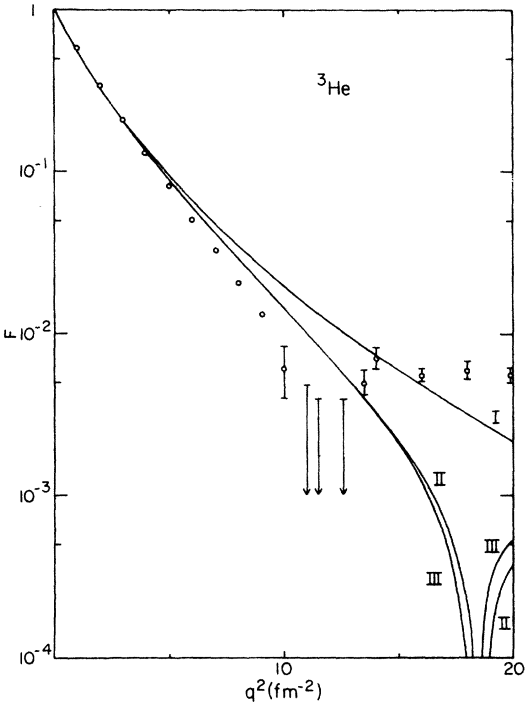

FIG. 12. Charge form factor of ^{3}He with uncorrelated wave function (40), $f=1$, $\beta=1.23$ fm^{-1}. Labeling of the curves is the same as in Fig. 11. The data are from Ref. 24.

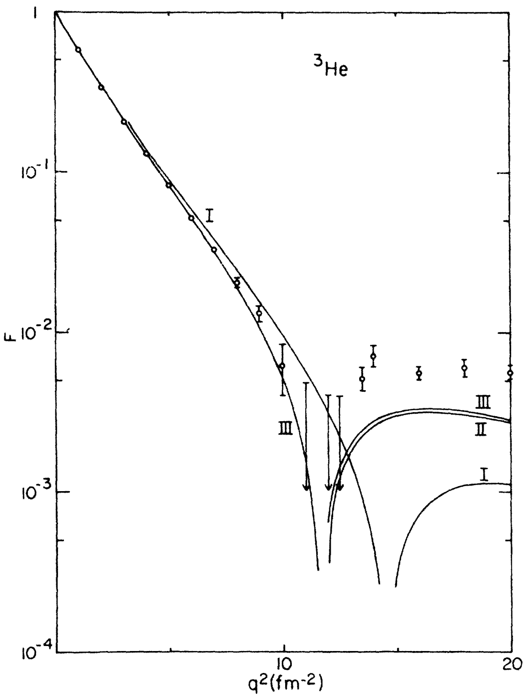

FIG. 13. Charge form factor of ^{3}He with correlated wave function (40), $\beta=1.38$ fm^{-1}, and $\gamma=1.15$ fm^{-1} [Eq. (41)]. The curves are labeled as in Fig. 11. The data are from Ref. 24.

data points in Fig. 12 are from Ref. 24. The model for the nucleon form factors used is that of Ref. 23.

The effect of including a correlation factor of the type (41) in the wave function is shown in Fig. 13. Here we have used the wave function parameters $\beta=1.38 fm^{-1}$ and $\gamma=1.15 fm^{-1}$. While the impulse approximation (one-body current) contribution to the form factor changes completely, the magnitude of the two-body effect is reduced by only ~35%, and the magnitude of the three-body effect by ~50%. Hence the importance of the exchange currents relative to that of the impulse approximation result is increased. The relative insensitivity of the exchange current effects to the wave-function details is related to the fact that they involve smaller momenta than the one-body currents. In fact, this circumstance is best demonstrated within the harmonic oscillator model, in which it is possible to obtain the asymptotic relation between the N-body exchange current form factor F_N and the impulse approximation form factor

$$F_N(q^2) \sim [F_I((q/N)^2)]^N \tag{42}$$

for large q for the pion-exchange currents if the vertex factors are neglected.[6]

V. CONCLUSIONS

We have found that pion exchange gives rise to appreciable three- and four-body exchange current effects on the charge form factors of the three- and four-nucleon systems. Inclusion of the ρ-meson-exchange effect enhances the two-body exchange current matrix elements but strongly reduces the effect of the three- and four-body currents. The effect of including hadronic form factors at the meson-nucleon vertices in the exchange diagrams reduces the ρ-meson-exchange effect much more than the pion-exchange effect, so that the net result is that the three- and four-body effects are considerably larger than when bare meson-nucleon couplings are used. The two-body exchange current effect for $\pi+\rho$ exchange with form factors is very similar to the previously considered bare pion exchange currents.[4-6]

The effect of wave-function correlations is to reduce the impulse approximation form factors strongly but to reduce the exchange effects only by a moderate amount. Hence wave-function correlations tend to increase the relative importance of the exchange currents, as may be seen by comparing Figs. 12 and 13. The net effect of the exchange current processes that we have considered is to increase the height of the secondary maximum in the tri- and four-nucleon form factors, hence reducing the discrepancy between the calculated and empirical values.

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