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Two-nucleon transfer reactions in the SU(6) boson model

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We suggest that two-nucleon transfer reactions be treated within the framework of the SU(6) boson model. We derive the corresponding intensity rules in the vibrational, SU(5), and rotational, SU(3), limit. We show that the failure of the pair vibrational model in accounting for the observed intensities is due to the neglecting of the finite dimensionality of the proton and neutron shells.

[NUCLEAR REACTIONS Two-nucleon transfer, boson model, intensity rules given in vibrational and rotational limit.]

Only a small fraction of the available data on two-nucleon transfer reactions (those in the spherical region) has up to now been interpreted in terms of a simple unified model, the pair vibrational model.[1] The remaining body of data, especially those in the transitional (spherical-deformed) region, remains to a large extent unexplained. Moreover, the pair vibrational model accounts only qualitatively for the observed cross sections. For example, ground state to ground state cross sections are predicted to increase in the ratio 1:2:3:4:... as one moves away from the closed shell, in contrast with experiment where the cross sections increase much slower, if at all. In this note we point out the following: (i) The SU(6) boson model recently proposed by us[2] provides a natural framework for a unified description of two-nucleon transfer reactions, including those in the spherical, deformed, and transitional regions and including states with $J^\pi = 2^+$ in addition to $J^\pi = 0^+$; (ii) the failure of the pair vibrational model in describing the data is due to the neglecting of the *finite* dimensionality of the proton (neutron) shells, and a crude (but simple) treatment of this effect brings the predictions in agreement with experiment.

To begin with, we note that, to the extent that the spectrum can be described by the SU(6) model, in order to calculate transfer strengths from the ground state of a given even-even nucleus to any $J^\pi = 0^+, 2^+$ collective state in the adjacent nucleus, one needs only to specify the form of the pair addition (removal) operator in terms of the basic creation and annihilation operators $s^\dagger(s)$ and $d^\dagger(d)$. In addition to proposing a description of the collective states in terms of a system of interacting bosons, we have identified[3] these bosons with fermion pairs coupled to $L=0$ (s) and $L=2$ (d). Then, by equating corresponding matrix elements in the boson and fermion spaces, it is possible to translate any operator in the fermion space into an operator in the boson space.[3,4] For the $L=0$ two-nucleon transfer operators (to which we restrict ourselves in this note) we obtain

$$\begin{aligned} P_{+\pi}^{(0)} &= \alpha_\pi s_\pi^\dagger(\Omega_\pi - N_\pi - n_{d\pi})^{1/2}, \\ P_{-\pi}^{(0)} &= \alpha_\pi(\Omega_\pi - N_\pi - n_{d\pi} + 1)^{1/2} s_\pi, \\ P_{+\nu}^{(0)} &= \alpha_\nu s_\nu^\dagger(\Omega_\nu - N_\nu - n_{d\nu})^{1/2}, \\ P_{-\nu}^{(0)} &= \alpha_\nu(\Omega_\nu - N_\nu - n_{d\nu} + 1)^{1/2} s_\nu. \end{aligned} \tag{1}$$

Here P_+ (P_-) denote addition (removal) operators, the superscript (0) denotes the transferred angular momentum and the subscript $\pi(\nu)$ refers to protons (neutrons). Finally, Ω_π (Ω_ν) are the effective proton (neutron) pair degeneracies, which, for the purposes of this note, we take equal to the degeneracies of the major shells [for example, $\Omega = \frac{1}{2}(82-50) = 16$ in the 50-82 shell], N_π (N_ν) are the proton (neutron) pair numbers, and $n_{d\pi}$ ($n_{d\nu}$) are the proton (neutron) d-boson numbers.

The addition (removal) operators in (1) are written in terms of the individual creation (annihilation) operators for proton and neutron bosons. We have shown in Ref. 3 that the separate dependence on the proton and neutron variables can be removed in the case in which the combined proton-neutron system is invariant with respect to proton-neutron transformations (a variable called F spin in Ref. 3). Then the states of the system are labeled by the symmetry character of the wave functions, determined by the F-spin Young tableaux $[n_1, n_2]$, with the totally symmetric representations $[N]$ being the lowest in energy. In this simple case, corresponding to the SU(6) boson model of Ref. 2, the transfer operators $P_\pm$ of (1) can be written as

$$
\begin{aligned}
P_{+\pi}^{(0)} &= \alpha_\pi \left(\frac{N_\pi+1}{N+1}\right)^{1/2} s^\dagger \left(\Omega_\pi - N_\pi - \frac{N_\pi}{N} n_d\right)^{1/2}, \\
P_{-\pi}^{(0)} &= \alpha_\pi \left(\Omega_\pi - N_\pi - \frac{N_\pi}{N} n_d + 1\right)^{1/2} s \left(\frac{N_\pi}{N}\right)^{1/2}, \\
P_{+\nu}^{(0)} &= \alpha_\nu \left(\frac{N_\nu+1}{N+1}\right)^{1/2} s^\dagger \left(\Omega_\nu - N_\nu - \frac{N_\nu}{N} n_d\right)^{1/2}, \\
P_{-\nu}^{(0)} &= \alpha_\nu \left(\Omega_\nu - N_\nu - \frac{N_\nu}{N} n_d + 1\right)^{1/2} s \left(\frac{N_\nu}{N}\right)^{1/2}
\end{aligned}
\qquad (2)
$$

which is now expressed in terms of the operators $s^\dagger$ (s), $N=N_\pi+N_\nu$ and $n_d=5^{1/2}(d^\dagger d)^{(0)}$ of Ref. 2. We remark here that the factor $[\Omega_\pi - N_\pi - (N_\pi/N)n_d]^{1/2}$ in (2) is somewhat reminiscent of the Holstein-Primakoff[5] factor $(\Omega - N)^{1/2}$ introduced by Jolos,[6] although not identical to it because of the additional term $(N_\pi/N)n_d$. We also remark that this factor arises from the finite dimensionality of the shells $(\Omega \neq \infty)$ and it is the only (and major) difference between the transfer operators (2) and those used in the pair vibrational model.[1] As it will be shown below, it is the neglecting of this factor which is responsible for the failure of the pair vibrational model in describing the data in the spherical region.

Without further assumptions, using the operators (2), we can now calculate all $L=0$ two-nucleon transfer strengths between collective 0^+ states, in a major shell. Let in fact $|[N],\chi\rangle$ be any 0^+ collective state in the nucleus with $2N$ particles (or holes) outside the closed shells $(N=N_\pi+N_\nu)$. In the SU(6) boson model of Ref. 2, this state is characterized by the partition $[N]$ of SU(6) and by the quantum numbers χ needed to specify uniquely the state. Let $|[N+1],\chi'\rangle$ be any 0^+ collective state in the nucleus with $2N+2$ particles (or holes). The transfer intensity can be defined as

$$I(N \to N+1) = |\langle [N+1],\chi' \,||\, P_+^{(0)} \,||\, [N],\chi\rangle|^2 , \qquad (3)$$

where $P_+^{(0)}$ is equal to $P_{+\pi}^{(0)}(P_{+\nu}^{(0)})$ for two-proton (neutron) transfer, respectively. In calculating matrix elements of $P_+^{(0)}$ two simple situations may occur:

(i) In the *vibrational* limit, wave functions are characterized by the group chain $\mathrm{SU(6)} \supset \mathrm{SU(5)} \supset 0^+(5)$ and labeled by[2] $|[N](n_d)vn_\Delta LM\rangle$. In particular, ground state wave functions $|[N],n_d=0, v=0,n_\Delta=0,L=0,M=0\rangle$ are obtained by applying to the closed shell $|0\rangle$ the creation operator $(s^\dagger)$ N times, i.e., $s^{\dagger N}|0\rangle$. Using these wave functions and the definition (2) we obtain the ground state to ground state two-neutron transfer intensity

$$I^{\mathrm{VIB}}(N_\nu \to N_\nu+1) = \alpha_\nu^2(N_\nu+1)(\Omega_\nu - N_\nu) \qquad (4)$$

and a similar expression for two-proton transfer. Transitions to excited 0^+ states are forbidden in this limit since the wave functions of these states are of the form $s^{\dagger N-n_d} d^{\dagger n_d}|0\rangle$. From the structure of the boson Hamiltonian[3] it also follows that binding energies are quadratic functions of the number of proton (neutron) bosons N_π (N_ν),

$$E_B^{\mathrm{VIB}}(N_\nu) = E_{0\nu} + A_\nu N_\nu + B_\nu \tfrac{1}{2} N_\nu(N_\nu - 1) , \qquad (5)$$

where $E_{0\nu}, A_\nu, B_\nu$ are constants characteristic of each major shell. Thus, this limit is characterized by two-particle separation energies $S_2(N_\nu) = E_B(N_\nu+1) - E_B(N_\nu)$, which are linear functions of N_ν,

$$S_2^{\mathrm{VIB}}(N_\nu) = A_\nu + B_\nu N_\nu \qquad (6)$$

and by transfer intensities with a bell-shaped behavior, symmetric with respect to the middle of the shell, as shown in Fig. 1(a). This behavior may be compared with that predicted by the pairing vibrational model,[1] $S_2^{\mathrm{PV}}(N_\nu) = A_\nu$, $I^{\mathrm{PV}}(N_\nu \to N_\nu+1) = \alpha_\nu^2(N_\nu+1)$, also shown in Fig. 1(a). The available data appear to be in agreement with (4) and (6) and not with the predictions of the pair vibrational model. An example is shown in Fig. 2.

(ii) In the *rotational* limit, wave functions are characterized by the group chain $\mathrm{SU(6)} \supset \mathrm{SU(3)}$ and labeled by[2] $|[N](\lambda,\mu)KLM\rangle$. In particular, the ground state wave functions are given by $|[N],(\lambda = 2N, \mu=0), K=0, L=0, M=0\rangle$. In this case, the ground state to ground state transfer intensities cannot in general be calculated analytically. An analytic expression (accurate within few percent) can only be given if one replaces the operator n_d in (2) by its expectation value in the ground state $\langle n_d\rangle$. We then obtain for the ground state two-neutron transfer intensity

$$I^{\mathrm{ROT}}(N_\nu \to N_\nu+1) = \alpha_\nu^2(N_\nu+1)\left(\frac{2N+3}{3(2N+1)}\right) \times \left(\Omega_\nu - N_\nu - \frac{4}{3}\frac{(N-1)}{(2N-1)} N_\nu\right) \qquad (7)$$

and a similar expression for two-proton transfer. Equation (7) has been obtained by using the identity

$$
\begin{aligned}
\langle n_s\rangle &= \langle [N](2N,0)L=0 \,|\, s^\dagger s \,|\, [N](2N,0)L=0\rangle \\
&= \langle [N](2N,0)L=0 \,|\, s^\dagger \,|\, [N-1](2N-2,0)L=0\rangle \\
&\quad \times \langle [N-1](2N-2,0)L=0 \,|\, s \,|\, [N](2N,0)L=0\rangle \\
&= \frac{N(2N+1)}{3(2N-1)} ,
\end{aligned}
\qquad (8)
$$

where the notation for the SU(3) representations is the same as in Ref. (2). The expected behavior of the two-neutron transfer intensities in the SU(3) limit, Eq. (7), is shown in Fig. 1(b). Comparing the previous Eq. (4) with (7), one can see that, in going from the vibrational to the rotational limit, one expects a drop of a factor $\simeq 3$ in the two-nu-

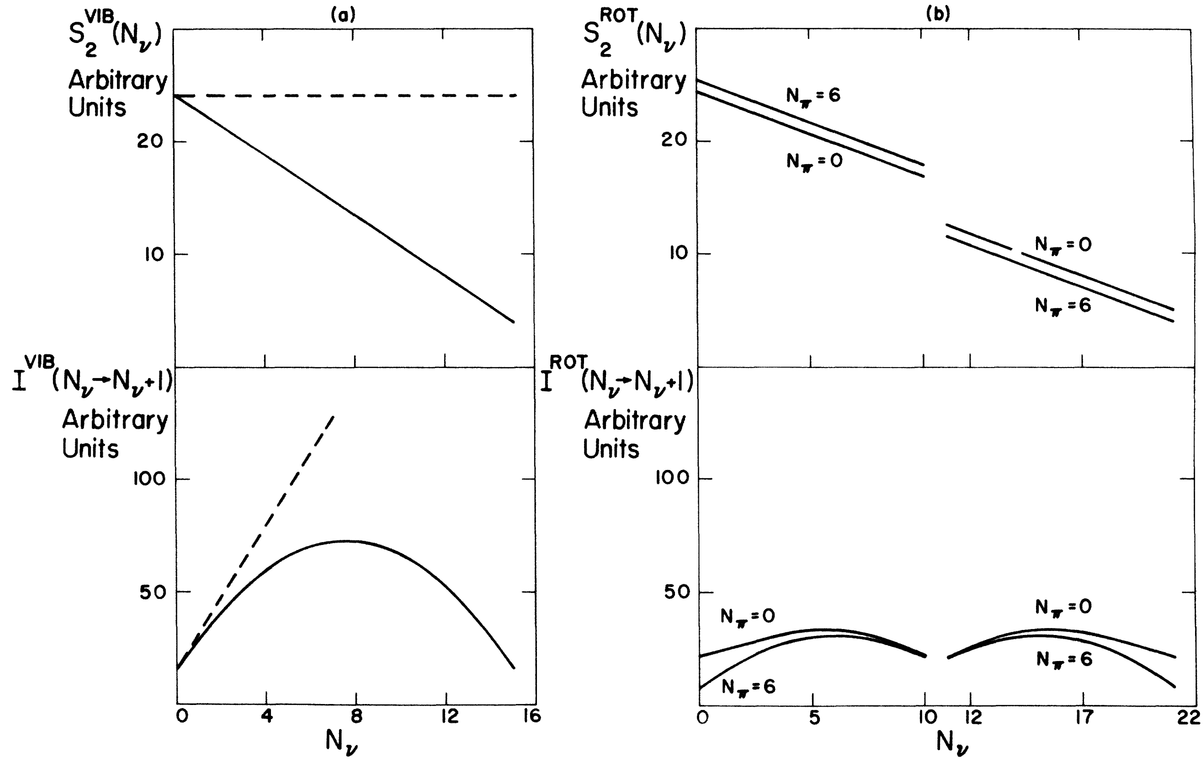

FIG. 1. (a) Schematic behavior of the two-neutron separation energies $S_2^{VIB}(N_\nu)$ and of the ground state to ground state transfer intensities $I^{VIB}(N_\nu \to N_\nu+1)$ (both in arbitrary units) in the *vibrational* limit of the SU(6) boson model. The results of the pair vibrational model (dashed lines) are shown for comparison. (b) Schematic behavior, for fixed proton number N_π, of the two-neutron separation energies $S_2^{ROT}(N_\nu)$ and of the ground state to ground state transfer intensities $I^{ROT}(N_\nu \to N_\nu+1)$ (both in arbitrary units) in the *rotational* limit of the SU(6) boson model.

cleon transfer intensities. This drop has been observed in the Sm and Gd isotopes. In Fig. 3 we show the experimental data in Sm superimposed to the SU(6) predictions, Eqs. (4) and (7). The remarkable feature of the boson model appears to be its ability to describe quantitatively the spherical-deformed transition. Next, we note that from the structure of the SU(3) solutions it follows that binding energies in the rotational limit are still given by a quadratic function of N_π, N_ν but that this function is somewhat different from (5). For fixed proton number N_π

$$E_B^{ROT}(N_\nu) = E_{0\nu} + A_\nu N_\nu + B_\nu \tfrac{1}{2} N_\nu(N_\nu - 1) + \kappa(4N^2 + 6N)\,, \tag{9}$$

where κ is the strength of the boson $Q\cdot Q$ interaction, given in Ref. 2, and $N = N_\pi + N_\nu$. The last term in (9) is the contribution of the deformation energy, $4N^2 + 6N$ being the eigenvalue of the quadratic Casimir operator of SU(3) in the state ($\lambda = 2N$, $\mu = 0$). From (9) we can calculate the separation energies $S_2^{ROT}(N_\nu) = E_B(N_\nu + 1) - E_B(N_\nu)$. Noting that $N = N_\pi + N_\nu$ when $0 \le N_\nu < \frac{1}{2}\Omega_\nu$ and that $N = N_\pi + \Omega_\nu - N_\nu$ when $\frac{1}{2}\Omega_\nu \le N_\nu < \Omega_\nu$, we obtain

$$S_2^{ROT}(N_\nu) = A_\nu + B_\nu N_\nu + 8\kappa(N_\pi + N_\nu) + 10\kappa\,, \quad 0 \le N_\nu < \tfrac{1}{2}\Omega_\nu \tag{10}$$

and

$$S_2^{ROT}(N_\nu) = A_\nu + B_\nu N_\nu + 8\kappa(N_\nu - \Omega_\nu - N_\pi) - 2\kappa\,, \quad \tfrac{1}{2}\Omega_\nu \le N_\nu < \Omega_\nu\,. \tag{11}$$

The behavior of $S_2^{ROT}(N)$ as a function of N is shown schematically in Fig. 1(b). The discontinuity at the middle of the shell arises from the fact that there one has to shift from a description in terms of particles to one in terms of holes. In contrast to the vibrational limit, transitions to higher 0^+ states are not forbidden in the rotational limit. However, because the SU(3) quantum numbers of the pair transfer operator $s^\dagger$ (s) are (2,0), two-nucleon transfer intensities in the SU(3) limit satisfy selection rules. In fact, starting from a ground state representation $(\lambda, \mu) = (2N, 0)$ one can reach only SU(3) representations $(2N+2, 0)$ and $(2N-2, 2)$. As a consequence, for example, (p,t) and (t,p) reactions leading to excited 0^+ states have

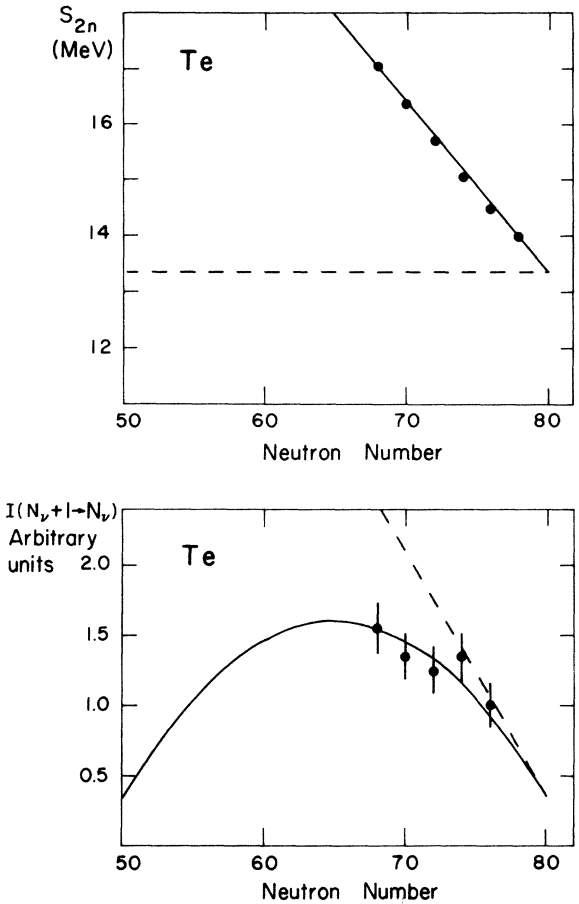

FIG. 2. Two-neutron separation energies S_{2n} and ground state to ground state (p,t) transition intensities (Ref. 7), $I(N_\nu+1\rightarrow N_\nu)$, in the Te isotopes. The full lines show the behavior predicted by the vibrational limit of the SU(6) boson model, the dashed lines that predicted by the pairing vibrational model.

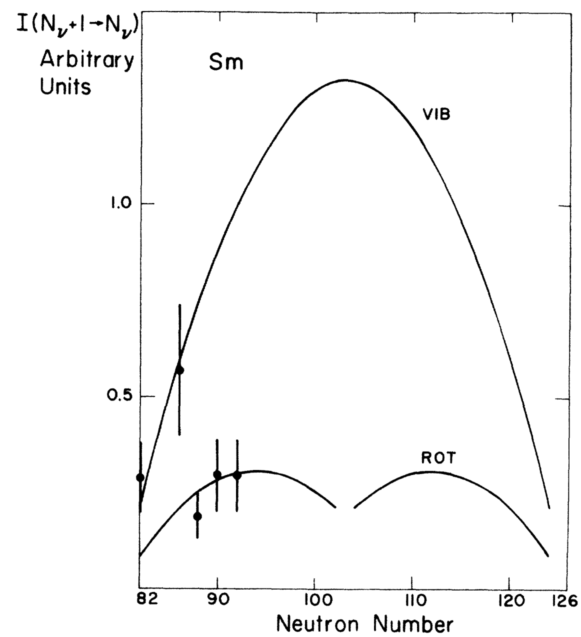

FIG. 3. Ground state to ground state (t,p) intensities, $I(N_\nu\rightarrow N_\nu+1)$, in the Sm isotopes (Ref. 8). The curves show the vibrational, Eq. (4), and rotational, Eq. (7), limits of the SU(6) boson model.

asymmetrical behavior, since in the SU(3) limit the first excited 0_2^+ state of a nucleus with $N+1$ pairs is described by the SU(3) quantum numbers $(2N-2,2)$, while that of the nucleus with $N-1$ pairs is described by $(2N-6,2)$, or vice versa. As mentioned above, by adding a pair addition (or removal) operator to $(2N,0)$ one can reach $(2N-2,2)$ but not $(2N-6,2)$. Since N is related to the number either of a particle or of hole pair, the detailed features of the asymmetry will depend on the particular neutron and proton shells in question. In general, (t,p) reactions to excited 0_2^+ states are allowed if $0\leq N_\nu<\Omega_\nu\frac{1}{2}$, while being forbidden if $\frac{1}{2}\Omega_\nu\leq N_\nu<\Omega_\nu$. Conversely, (p,t) reactions to excited 0_2^+ states are allowed if $\frac{1}{2}\Omega_\nu\leq N_\nu<\Omega_\nu$ and forbidden if $0\leq N_\nu<\Omega_\nu\frac{1}{2}$. Asymmetries have been observed in the Sm isotopes[8,9] and here they appear to be in agreement with the predictions of the SU(6) model.[10] They have also been observed in the actinides[11] but a comparison with theory in this region must await the results of detailed calculations.

In conclusion we emphasize the importance of the cutoff factor in (2). This factor arises from the Pauli principle and it is neglected in the pairing vibrational model. We also stress that the remarkable feature of the SU(6) boson model is its ability to cover the transitional as well as the vibrational and rotational regions.

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[1]For a review of the pair vibrational model see R. Broglia, O. Hansen, and C. Riedel, Advan. Nucl. Phys. 6, 287 (1973).
[2]A. Arima and F. Iachello, Phys. Rev. Lett. 35, 1069 (1975); Ann. Phys. (N.Y.) 99, 253 (1976).
[3]A. Arima, T. Ohtsuka, F. Iachello, and I. Talmi, Phys. Lett. 66B, 205 (1977).
[4]A. Arima, T. Ohtsuka, F. Iachello, and I. Talmi (unpublished); A. Arima, in Proceedings of the Colloque Franco-Japonais de Spectroscopie Nucléaire, Tokyo, 1976 (unpublished), p. 90.
[5]T. Holstein and H. Primakoff, Phys. Rev. 58, 1098 (1940).
[6]R. V. Jolos, Phys. Lett. 30B, 390 (1969).
[7]N. Hintz and R. Seltz, Univ. of Minnesota, Annual Report 1975 (unpublished), p. 91.
[8]J. H. Bjerregaard, O. Hansen, O. Nathan, and S. Hinds, Nucl. Phys. 86, 145 (1966).
[9]P. Debenham and N. M. Hintz, Nucl. Phys. A195, 385 (1972).
[10]O. Scholten, private communication to F. Iachello.
[11]J. V. Maher, J. R. Erskine, A. M. Friedman, R. H. Siemssen, and J. P. Schiffer, Phys. Rev. C 5, 1380 (1972); B. B. Back, E. R. Flynn, O. Hansen, R. F. Casten, and J. D. Garrett, Nucl. Phys. A217, 116 (1973).