Beta decays and related processes in the $A = 14$ nuclei

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We present an analysis of the processes $^{14}C(^{14}O)\to{}^{14}N + e^- (e^+) + \bar{\nu}_e (\nu_e)$ and $\mu^- + {}^{14}N\to\nu_\mu + {}^{14}C$ (ground state) on the basis of an "elementary particle" treatment and of a "microscopic" treatment including the nucleons-only impulse approximation and the meson exchange. The elementary particle treatment in conjunction with data on $e^- + {}^{14}N \to e^- + {}^{14}N^*$ (2.31 MeV), conserved vector current, partially conserved axial-vector current, and a possible second-class axial current, yields 50 $sec^{-1} < [\Gamma(\mu^- + {}^{14}N\to\nu_\mu + {}^{14}C$ (ground state)$)]_{\text{stat. av.}} < 70$ sec^{-1} and 95 $sec^{-1} < [\Gamma(\mu^- + {}^{14}N\to\nu_\mu + {}^{14}C$ (ground state)$)]_{F=1/2} < 210$ sec^{-1}. The microscopic treatment leads to the conclusion that the nucleons-only-impulse-approximation and meson-exchange contributions to the $^{14}C(^{14}O)\to{}^{14}N + e^- (e^+) + \bar{\nu}_e (\nu_e)$ amplitude may well be opposite in sign and comparable in magnitude so that the destructive interference between them contributes significantly to the anomalously small values of $\Gamma(^{14}C(^{14}O)\to{}^{14}N + e^- (e^+) + \bar{\nu}_e (\nu_e))$ and $[\Gamma(\mu^- + {}^{14}N\to\nu_\mu + {}^{14}C$ (ground state)$)]_{\text{stat. av.}, F=1/2}$.

[RADIOACTIVITY $^{14}C\to{}^{14}N+e^-+\bar{\nu}_e$, $^{14}O\to{}^{14}N+e^++\nu_e$, $\mu^-+{}^{14}N\to\nu_\mu+{}^{14}C$ (ground state), theoretical analysis via "elementary particle" and "microscopic" treatments.]

I. INTRODUCTION

It has been known for some 40 years that the $^{14}C\to{}^{14}N+e^-+\bar{\nu}_e$ is "anomalously slow." Thus the angular momenta, parities, and isospins involved: $[0^+,1]\to[1^+,0]$ correspond to an allowed Gamow-Teller transition, while the high ft value ($10^{9.04}$) indicates a poor overlap of the ^{14}C and ^{14}N ground states with respect to the axial weak current at small momentum transfers.[1] More recently, anomalous slowness has also been observed or inferred in reactions closely related to $^{14}C\to{}^{14}N+e^-+\bar{\nu}_e$, i.e., $^{14}O\to{}^{14}N+e^++\nu_e$,[2] $\mu^-+{}^{14}N\to\nu_\mu+{}^{14}C$,[3] $\gamma+{}^{14}N\to\pi^++{}^{14}C$,[4] and $\pi^-+{}^{14}N\to\gamma+{}^{14}C$,[5] a situation again indicative of poor overlap of the ^{14}C (^{14}O) and ^{14}N ground states with respect to the axial weak current but now, at least in the latter three reactions, at significantly higher momentum transfers.

Such an anomaly in the value of $\langle^{14}N|$(Axial weak current)$_\lambda|^{14}C\rangle\equiv\langle^{14}N|A_\lambda|^{14}C\rangle$ is of very considerable interest in the sense that terms in A_λ usually omitted in the evaluation of $\langle^{14}N|A_\lambda|^{14}C\rangle$ and relatively small in the evaluation of unhindered allowed matrix elements such as $\langle^6Li|A_\lambda|^6He\rangle$ and $\langle^{12}C|A_\lambda|^{12}B\rangle$ may become relatively large in $\langle^{14}N|A_\lambda|^{14}C\rangle$ and so conceivably even dominate the various reactions in the $A=14$ system. This circumstance could possibly make the $A=14$ system a laboratory for the investigation of, e.g., meson-exchange or second-class contributions to A_λ.

In the discussion below, we first give an "elementary-particle" (EP) treatment of the reactions

$$^{14}C\to{}^{14}N+e^-+\bar{\nu}_e\,, \tag{1}$$

$$^{14}O\to{}^{14}N+e^++\nu_e\,, \tag{2}$$

$$\mu^-+{}^{14}N\to\nu_\mu+{}^{14}C \quad [^{14}C\equiv{}^{14}C(\text{ground state})]\,, \tag{3}$$

including in our evaluation of $\langle^{14}N|A_\lambda|^{14}C\rangle$, $\langle^{14}N|A_\lambda^\dagger|^{14}O\rangle$, and $\langle^{14}C|A_\lambda^\dagger|^{14}N\rangle$ the contribution of any possibly present second-class part of A_λ. We then present a detailed "microscopic" treatment of $\langle^{14}N|A_\lambda|^{14}C\rangle$, where appropriate wave functions depending on the nucleon positions, spins, and isospins are used for $|^{14}C\rangle$ and $|^{14}N\rangle$ and where A_λ is decomposed into a nucleons-only-impulse-approximation contribution (NOIA) and a meson-exchange contribution (ME), i.e., $A_\lambda=\{A_\lambda\}^{\text{NOIA}}+\{A_\lambda\}^{\text{ME}}$, with the possibility kept in mind that $|\langle^{14}N|\{A_\lambda\}^{\text{ME}}|^{14}C\rangle|$ may be as large as or even larger than $|\langle^{14}N|\{A_\lambda\}^{\text{NOIA}}|^{14}C\rangle|$ (in contradistinction to the situation in unhindered allowed transitions where, e.g., $|\langle^6Li|\{A_\lambda\}^{\text{ME}}|^6He\rangle|\lesssim0.1|\langle^6Li|\{A_\lambda\}^{\text{NOIA}}|^6He\rangle|$. In this connection, we remind the reader that a certain number of rather drastic conclusions have been drawn regarding the structure of the ground states of the $A = 14$ systems on the basis of the small observed values of $|\langle {}^{14}\mathrm{N}|A_\lambda|{}^{14}\mathrm{C}\rangle|$ *and* the identification of A_λ with $\{A_\lambda\}^{\mathrm{NOIA}}$ [6]; clearly these conclusions will have to be modified if in fact $|\langle {}^{14}\mathrm{N}|\{A_\lambda\}^{\mathrm{ME}}|{}^{14}\mathrm{C}\rangle| \gtrsim |\langle {}^{14}\mathrm{N}|\{A_\lambda\}^{\mathrm{NOIA}}|{}^{14}\mathrm{C}\rangle|$.

II. EP TREATMENT

Before starting our calculations we list the experimental data relative to the decays to be considered. The nuclei ^{14}C, ^{14}N* (2.31 MeV), and ^{14}O form an isotriplet connected to the ^{14}N isosinglet by[7]

$$
\begin{aligned}
&{}^{14}\mathrm{C} \to {}^{14}\mathrm{N} + e^- + \bar\nu_e, \quad [(ft)_{e^-}]_{\mathrm{exp}} = 1.12\times 10^9 \text{ sec},\\
&\Delta^- \equiv M({}^{14}\mathrm{C}) - M({}^{14}\mathrm{N}) = 0.667 \text{ MeV},\\
&{}^{14}\mathrm{N}^*(2.31 \text{ MeV}) \to {}^{14}\mathrm{N} + \gamma,\\
&[\Gamma\gamma]_{\mathrm{exp}} = (7.6 \pm 0.8)\times 10^{-3} \text{ eV},\\
&{}^{14}\mathrm{O} \to {}^{14}\mathrm{N} + e^+ + \nu_e, \quad [(ft)_{e^+}]_{\mathrm{exp}} = 2.14\times 10^7 \text{ sec},\\
&\Delta^+ \equiv M({}^{14}\mathrm{O}) - M({}^{14}\mathrm{N}) = 5.657 \text{ MeV},
\end{aligned}
\tag{4}
$$

to be compared, for example, with the ^{12}B, ^{12}C*(15.11 MeV), and ^{12}N isotriplet connected to the ^{12}C isosinglet by

$$
\begin{aligned}
&{}^{12}\mathrm{B} \to {}^{12}\mathrm{C} + e^- + \bar\nu_e, \quad [(ft)_{e^-}]_{\mathrm{exp}} = 1.79\times 10^4 \text{ sec},\\
&\Delta^- \equiv M({}^{12}\mathrm{B}) - M({}^{12}\mathrm{C}) = 13.88 \text{ MeV},\\
&{}^{12}\mathrm{C}^*(15.11 \text{ MeV}) \to {}^{12}\mathrm{C} + \gamma, \quad [\Gamma\gamma]_{\mathrm{exp}} = 37.0 \pm 1.1 \text{ eV},\\
&{}^{12}\mathrm{N} \to {}^{12}\mathrm{C} + e^+ + \nu_e, \quad [(ft)_{e^+}]_{\mathrm{exp}} = 1.317\times 10^4 \text{ sec},\\
&\Delta^+ \equiv M({}^{12}\mathrm{N}) - M({}^{12}\mathrm{C}) = 16.833 \text{ MeV}.
\end{aligned}
\tag{5}
$$

In the EP treatment,[8] the matrix elements of the polar and axial hadronic weak currents taken between the nuclear states are expressed in a model-independent way in terms of form factors characteristic of the transition in question. Thus in the case of the $^{14}\mathrm{C} \to {}^{14}\mathrm{N} + e^- + \bar\nu_e$ decay

$$\langle {}^{14}\mathrm{N}(p_2, \xi)|V_\lambda(0)|{}^{14}\mathrm{C}(p_1)\rangle = \sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\,\xi_\kappa \frac{q_\rho}{2m_p}\frac{Q_\eta}{2M} F_M^-(q^2), \tag{6a}$$

$$\langle {}^{14}\mathrm{N}(p_2, \xi)|A_\lambda(0)|{}^{14}\mathrm{C}(p_1)\rangle = \sqrt{2}\left[\xi_\lambda F_A^-(q^2) + \frac{q_\lambda q\cdot\xi}{m_\pi^2} F_P^-(q^2) - \frac{Q_\lambda}{2M}\frac{q\cdot\xi}{2m_p} F_E^-(q^2)\right], \tag{6b}$$

$$
\begin{aligned}
V_\lambda(x) &= V_\lambda^{(\mathrm{I})}(x) + V_\lambda^{(\mathrm{II})}(x),\\
A_\lambda(x) &= A_\lambda^{(\mathrm{I})}(x) + A_\lambda^{(\mathrm{II})}(x),
\end{aligned}
\tag{6c}
$$

$$F^-_{M,A,P,E}(q^2) = F^{(\mathrm{I})-}_{M,A,P,E}(q^2) - F^{(\mathrm{II})-}_{M,A,P,E}(q^2), \tag{6d}$$

where $q_\lambda \equiv (p_1 - p_2)_\lambda$, $Q_\lambda \equiv (p_1 + p_2)_\lambda$, $M \equiv \frac{1}{2}[M({}^{14}\mathrm{C}) + M({}^{14}\mathrm{N})]$, $F^-_{M,A,P,E}(q^2)$ are, respectively, the nuclear weak magnetism, axial, pseudoscalar, and weak electricity (or pseudotensor) form factors, and ξ is the polarization four-vector of the spin-one ^{14}N. In this case, where the initial and final states are not members of the same isomultiplet, both $V_\lambda^{(\mathrm{I})}$ and $V_\lambda^{(\mathrm{II})}$ contribute to $F_M^{(-)}$ and both $A_\lambda^{(\mathrm{I})}$ and $A_\lambda^{(\mathrm{II})}$ contribute to $F_A^{(-)}(q^2)$, $F_P^{(-)}(q^2)$, and $F_E^{(-)}(q^2)$. However, if conserved vector current (CVC) holds, $V_\lambda^{(\mathrm{II})} = 0$, and $F_M^-(q^2) = F_M^{(\mathrm{I})-}(q^2)$ is simply related to the corresponding electromagnetic form factors.[8]

We proceed to calculate the $(ft)_{e^-}$ value and the shape factor $S^-(E_e)$ of the electron momentum spectrum. We obtain from Eqs. (6a) and (6b)

$$(ft)_{e^-} = \ln 2 \Big/ \frac{3(G\cos\theta_c)^2}{(2\pi)^3} m_e^5 [\sqrt{2}\,F_A^-(0)]^2 \times \left(\frac{\int_{m_e}^{\Delta^-} S^-(E_e) F_-(Z, E_e) p_e E_e (\Delta^- - E_e)^2\, dE_e}{\int_{m_e}^{\Delta^-} F_-(Z, E_e) p_e E_e (\Delta^- - E_e)^2\, dE_e}\right), \tag{7a}$$

$$S^-(E_e) \cong 1 + \frac{4}{3m_p}\left(E_e - \frac{1}{2}\Delta^- - \frac{1}{2}\frac{m_e^2}{E_e}\right)\frac{F_M^-(0)}{F_A^-(0)} + \frac{1}{3m_p}\left(\Delta^- - \frac{m_e^2}{E_e}\right)\frac{F_E^-(0)}{F_A^-(0)}, \tag{7b}$$

where, in spite of the anomalously small value of $|F_A^-(0)|(\approx 10^{-3})$, and the anticipated considerably larger values of $|F_M^-(0)|$, $|F_E^-(0)|$, and $|F_P^-(0)|$ (≈ 0.1), we neglect terms $\sim [F_M^-(0)/F_A^-(0)]^2$, $[F_E^-(0)/F_A^-(0)]^2$, and $F_P^-(0)/F_A^-(0)$ on the basis, respectively, of the even smaller values of $\Delta^-/2m_p$ $(= 3.5\times 10^{-4})$ and $(m_e\Delta^-)/m_\pi^2$ (2×10^{-5}). The small value of $\Delta^- = i^{-1}q_4$ also justifies the evaluation of the various form factors at $q^2 = 0$.

For the case of $(ft)_{e^+}$ and the shape factor $S^+(E_e)$ of the positron momentum spectrum we have, making similar approximations and remembering that here $\Delta^+ \gg m_e$,

$$(ft)_{e^+} = \ln 2 \Big/ \frac{3(G\cos\theta_c)^2}{(2\pi)^3} m_e^5 [\sqrt{2}\,F_A^+(0)]^2 \times\left(1 + \frac{\Delta^+}{3m_p}\frac{F_E^+(0)}{F_A^+(0)}\right), \tag{8a}$$

$$S^+(E_e) \cong 1 + \frac{4}{3m_p}(E_e - \tfrac{1}{2}\Delta^+)\frac{F_M^+(0)}{F_A^+(0)} + \frac{\Delta^+}{3m_p}\frac{F_E^+(0)}{F_A^+(0)}, \tag{8b}$$

$$F^+_{M,P,A,E}(q^2) = F^{(\mathrm{I})+}_{M,P,A,E}(q^2) + F^{(\mathrm{II})+}_{M,P,A,E}(q^2), \tag{8c}$$

where

$$F^{\mathrm{I}(+)}_{M,P,A,E}(q^2) = F^{\mathrm{I}(-)}_{M,P,A,E}(q^2), \tag{9a}$$

$$F^{\mathrm{II}(+)}_{M,P,A,E}(q^2)=F^{\mathrm{II}(-)}_{M,P,A,E}(q^2)\,, \quad (9b)$$

only to the extent that the $|^{14}\mathrm{O}\rangle$, $|^{14}\mathrm{N}^*\rangle$, $|^{14}\mathrm{C}\rangle$ states are precisely members of the same isotriplet. It is to be emphasized that the anomalously small values of $|F_A^-(q^2)|$ and $|F_A^+(q^2)|$ indicate that any small admixtures to the $|^{14}\mathrm{O}\rangle$, $|^{14}\mathrm{N}^*\rangle$, and $|^{14}\mathrm{C}\rangle$ states which arise, e.g., from different internal Coulomb interactions in the three cases, and which spoil the isotriplet property, have a relatively large effect on $F_A^\pm(0)$; thus it is not expected that Eqs. (9a) and (9b) hold for these form factors. On the other hand, the small values of $|F_A^\pm(0)|$ and the resultant large values of $|F_M^\pm(0)/F_A^\pm(0)|$ allow us to neglect the Coulomb correction factors in Eqs. (7b) and (8b) for $S^\pm(E_e)$. Thus, evaluating $F_A^-(0)$ from Eqs. (7a) and (7b) with neglect of the term $(4\Delta^-/3m_p)F_M^-(0)/F_A^-(0)\cong 8\times10^{-4}\times F_M^-(0)/F_A^-(0)$ [remember that $\int_0^{\Delta^-}(E_e-\frac{1}{2}\Delta^-)\times E_e^2(\Delta^--E_e)^2\,dE_e=0$] and the term $(\Delta^-/3m_p)F_E^-(0)/F_A^-(0)\cong 2\times10^{-4}F_E^-(0)/F_A^-(0)$ yields $|F_A^-(0)|\cong 0.95\times10^{-3}$, while a similar evaluation of $F_A^+(0)$ yields $|F_A^+(0)|\cong 7.2|F_A^-(0)|\cong 6.9\times10^{-3}$. It is to be noted in this connection that $(\Delta^+/3m_p)F_E^+(0)/F_A^+(0)\cong 2\times10^{-3}\times F_E^+(0)/F_A^+(0)$ is not expected to be appreciably larger than $(\Delta^-/3m_p)F_E^-(0)/F_A^-(0)\cong 2\times10^{-4}F_E^-(0)/F_A^-(0)$ since, in first approximation, $|F_A^+(0)|\cong 7.2|F_A^-(0)|$, and since we expect $F_E^{(+)}(0)\cong F_E^-(0)$ from Eqs. (9a) and (9b).

As regards experimental verification of the shape factors in Eqs. (7b) and (8b), no sufficiently accurate measurements exist to our knowledge in the $^{14}\mathrm{C}\to{}^{14}\mathrm{N}+e^-+\bar\nu_e$ case where, because of the small value of (Δ^--m_e), it would be difficult to measure reliably any deviation of $S^-(E_e)$ from 1. On the other hand, in the $^{14}\mathrm{O}\to{}^{14}\mathrm{N}+e^++\nu_e$ case[2] measurement of $S^+(E_e)$ yields $(4/3m_p)[F_M^+(0)/F_A^+(0)]=(9.2\pm0.6)\times10^{-2}/\mathrm{MeV}$ which, with $|F_A^+(0)|=6.9\times10^{-3}$, corresponds to $|F_M^+(0)|=0.45\pm0.03$; this value is to be compared with the value predicted by CVC: $[(\Gamma_\gamma)_{\exp}m_p^2/\alpha E_\gamma^3]^{1/2}=0.28\pm0.02$. The discrepancy between these two values of $F_M^+(0)$, aside from any question of systematic experimental errors in the measurements of $S^+(E_e)$ and Γ_γ, may arise in part from the fact that in the calculation of $F_M^+(0)$ from the observed $S^+(E_e)$ we took $|F_A^+(0)|\cong 6.9\times10^{-3}$ and so neglected the term $(\Delta^+/3m_p)[F_E^+(0)/F_A^+(0)]$ in the expression for $(ft)_{e^+}$ [Eq. (8a)]. In fact, if as a numerical illustration, we assume that $F_E^+(0)/F_A^+(0)=3/|F_A^+(0)|$, Eq. (8a) for $(ft)_{e^+}$ yields $|F_A^+(0)|=4.5\times10^{-3}$ [$\cong 4.7|F_A^-(0)|$] so that $|F_M^+(0)|=\frac{3}{4}m_p\times[(9.2\pm0.6)\times10^{-2}/\mathrm{MeV}]\times(4.5\times10^{-3})=0.29\pm0.02$, which removes the discrepancy *vis-à-vis* the CVC prediction. However, this value of $|F_E^+(0)|$ appears to be much too large (see below) so that, granting the validity of CVC, the discrepancy in question is probably mostly of experimental origin.

We treat next the muon capture process

$$\mu^-+{}^{14}\mathrm{N}\to\nu_\mu+{}^{14}\mathrm{C}\quad[{}^{14}\mathrm{C}\equiv{}^{14}\mathrm{C}(\text{ground state})]\,, \quad (10)$$

whose rate (suitably averaged over the $F=\frac{3}{2}$ and $F=\frac{1}{2}$ hyperfine states) is, using Eqs. (6a) and (6b),[8,9]

$$\{\Gamma(\mu^-+{}^{14}\mathrm{N}\to\nu_\mu+{}^{14}\mathrm{C})\}_{\text{stat. av.}}=\frac{G^2(\cos\theta_c)^2}{2\pi^2}E_\nu^2\left(1-\frac{E_\nu}{m_\mu+M({}^{14}\mathrm{C})}\right)C({}^{14}\mathrm{N})\left(\frac{Z({}^{14}\mathrm{N})}{137}\,\frac{m_\mu M({}^{14}\mathrm{N})}{m_\mu+M({}^{14}\mathrm{N})}\right)^3[\sqrt{2}\,F_A^-(q_m^2)]^2$$
$$\times\left\{\frac{2}{3}\left[1+\frac{F_M^-(q_m^2)}{F_A^-(q_m^2)}\left(\frac{E_\nu}{2m_p}\right)\right]^2+\frac{1}{3}\left[1-\left(\frac{\tilde F_P^-(q_m^2)}{F_A^-(q_m^2)}+\frac{F_E^-(q_m^2)}{F_A^-(q_m^2)}\right)\left(\frac{E_\nu}{2m_p}\right)\right]^2\right\}$$
$$=8.9\times10^3\ \mathrm{sec}^{-1}\left\{2\left[F_A^-(q_m^2)+F_M^-(q_m^2)\left(\frac{E_\nu}{2m_p}\right)\right]^2+\left[F_A^-(q_m^2)-(\tilde F_P^-(q_m^2)+F_E^-(q_m^2))\left(\frac{E_\nu}{2m_p}\right)\right]^2\right\}, \quad (11)$$

where $\tilde F_P^-(q^2)\equiv -F_P^-(q^2)(2m_pm_\mu/m_\pi^2)$, $E_\nu=0.99m_\mu$, $q_m^2=(p_1-p_2)_m^2=(p_\mu-p_\nu)^2=0.98m_\mu^2$, $Z({}^{14}\mathrm{N})=7$, and $C({}^{14}\mathrm{N})$ is the correction factor arising from the nonpoint character of the charge distribution of $^{14}\mathrm{N}=0.82$.

To estimate the various form factors at $q^2=q_m^2$ we proceed as follows. We have

$$F_M^-(q^2)=\frac{F_M^-(q^2)}{F_M^-(0)}F_M^-(0)=(1.26)(\pm0.28)=\pm0.35\,, \quad (12a)$$

where the numerical values of $F_M^-(0)$ and $F_M^-(q_m^2)/F_M^-(0)$ have been calculated on the basis of CVC from the value of the radiative decay rate $[\Gamma({}^{14}\mathrm{N}^*\,(2.31\ \mathrm{MeV})\to{}^{14}\mathrm{N}+\gamma)]_{\exp}=(7.6\pm0.8)\times10^{-3}$ eV (see above) and from the values of the inelastic electron scattering cross section $[\sigma(e^-(p_i)+{}^{14}\mathrm{N}\to e^-(p_f)+{}^{14}\mathrm{N}^*(2.31\ \mathrm{MeV}))]_{\exp}$ for various $(p_i-p_f)^2=q^2$, these last yielding[6]

$$\frac{F_M^-(q^2)}{F_M^-(0)}=[1+0.58(q^2/m_\mu^2)]\,e^{-0.22q^2/m_\mu^2}\,. \quad (12b)$$

Further, NOIA, even after considerable augmentation by ME, suggests the relation[10]

$$F_M^-(q^2)\cong\{F_M^-(q^2)\}_{\text{orbit}}+\{F_M^-(q^2)\}_{\text{spin}}$$
$$\cong\{F_M^-(q^2)\}_{\text{orbit}}+\left(\frac{g_V+g_M}{g_A}\right)F_A^-(q^2)\,, \quad (13)$$

where $\{F_M^-(q^2)\}_{\text{orbit}}$ and $\{F_M^-(q^2)\}_{\text{spin}}$ are, respectively, the contributions to $F_M^-(q^2)$ arising from the $^{14}N^* \to {}^{14}N$ transition orbital magnetic moment and spin magnetic moment and where $g_A = 1.24$, $g_V = 1.00$, and $g_M = 1.79 - (-1.91) = 3.70$ are the neutron-proton axial, polar, and weak-magnetism coupling constants. Comparison of Eq. (13) with Eq. (12b), and recognition of the fact that $F_A^-(0) \cong 0$ implies that $F_A^-(q^2)$ is approximately proportional to q^2 for relatively small q^2, results in the identification (for $q^2 \lesssim q_m^2$)

$$\{F_M^-(q^2)\}_{\text{orbit}} \cong F_M^-(0) e^{-0.22 q^2/m_\mu^2} = \frac{F_M^-(q^2)}{1 + 0.58 q^2/m_\mu^2}, \tag{14a}$$

$$\{F_M^-(q^2)\}_{\text{spin}} \cong 0.58 (q^2/m_\mu^2) F_M^-(0) e^{-0.22 q^2/m_\mu^2} = \frac{0.58 (q^2/m_\mu^2) F_M^-(q^2)}{1 + 0.58 (q^2/m_\mu^2)}, \tag{14b}$$

$$\begin{aligned} F_A^-(q^2) &\cong \left(\frac{g_A}{g_V + g_M}\right) \{F_M^-(q^2)\}_{\text{spin}} \\ &\cong \left(\frac{g_A}{g_V + g_M}\right) 0.58 \left(\frac{q^2}{m_\mu^2}\right) F_M^-(0) e^{-0.22 q^2/m_\mu^2} \\ &= \left(\frac{g_A}{g_V + g_M}\right) \frac{0.58 (q^2/m_\mu^2) F_M^-(q^2)}{1 + 0.58 q^2/m_\mu^2}, \end{aligned} \tag{14c}$$

whence, using also Eq. (12a),

$$F_A^-(q_m^2) \cong 0.096 F_M^-(q_m^2) = \pm 0.034. \tag{14d}$$

Thus $|F_A^-(q_m^2)|$ is about 35 times larger than $|F_A^-(0)|$ but still some 10 times smaller than $|F_M^-(q_m^2)|$.

Continuing, we have[10]

$$\tilde{F}_P^-(q^2) \cong \left(\frac{F_A^-(q^2)}{1 + q^2/m_\pi^2}\right) \times \left\{1 + m_\pi^2 \left[\frac{d}{d(q^2)} \ln\left(\frac{F_A^-(q^2)}{F_D^-(q^2)}\right)\right]_{q^2=0}\right\} \times \left(\frac{2 m_p m_\mu}{m_\pi^2}\right), \tag{15a}$$

where $F_D^-(q^2)$, the form factor associated with $\partial A_\lambda / \partial x_\lambda$ multiplied by $(1 + q^2/m_\pi^2)$, satisfies

$$\lim_{q^2 \to 0} \left(\frac{F_D^-(q^2)}{F_A^-(q^2)}\right) \cong 1 \quad \text{(Ref. 10)}. \tag{15b}$$

We can, therefore, remembering Eq. (14c), parametrize $F_D^-(q^2)$ as

$$F_D^-(q^2) = F_A^-(q^2) e^{-a 0.22 q^2/m_\mu^2}, \tag{15c}$$

so that

$$\tilde{F}_P^-(q_m^2) \cong \left(\frac{F_A^-(q_m^2)}{1 + q_m^2/m_\pi^2}\right) \left(1 + \frac{a\, 0.22 m_\pi^2}{m_\mu^2}\right) \left(\frac{2 m_p m_\mu}{m_\pi^2}\right) = F_A^-(q_m^2) \begin{bmatrix} 9.0, & a = 1; \\ 6.5, & a = 0, \\ 4.0, & a = -1 \end{bmatrix}. \tag{15d}$$

Finally, $F_E^-(q_m^2) = F_E^{(\text{I})-}(q_m^2) - F_E^{(\text{II})-}(q_m^2)$ where $F_E^{(\text{I})-}(q_m^2)$ can be as large as $F_M^-(q_m^2)$, and $F_E^{(\text{II})-}(q_m^2)$, which equals zero if $A_\lambda^{(\text{II})}$ vanishes, can also be as large as $F_M^-(q^2)$ if $A_\lambda^{(\text{II})}$ is present to the extent indicated by the Sugimoto *et al.* and Calaprice *et al.* experiments.[11,12,10] It is therefore not unreasonable to set

$$-F_M^-(q_m^2) \lesssim F_E^-(q_m^2) \lesssim F_M^-(q_m^2). \tag{16}$$

Thus, combining Eqs. (15d), (16), and (14d) with Eq. (11),

$$\begin{aligned} &2[F_A^-(q_m^2)]^2 \left[1 + \frac{F_M^-(q_m^2)}{F_A^-(q_m^2)} \left(\frac{E_\nu}{2m_p}\right)\right]^2 \\ &\leqslant \frac{\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}}{8.9 \times 10^3 \text{ sec}^{-1}} \\ &\leqslant 2[F_A^-(q_m^2)]^2 \left[1 + \frac{F_M^-(q_m^2)}{F_A^-(q_m^2)} \left(\frac{E_\nu}{2m_p}\right)^2\right] \\ &\quad + [F_A^-(q_m^2)]^2 \left[1 - \left(\frac{\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)}{F_A^-(q_m^2)}\right) \left(\frac{E_\nu}{2m_p}\right)\right]^2, \\ &-6 \lesssim \{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)] / F_A^-(q_m^2)\} \lesssim 20, \end{aligned} \tag{17a}$$

or, numerically [using Eq. (14d)],

$$50 \text{ sec}^{-1} \leqslant \{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}} \leqslant 70 \text{ sec}^{-1}, \tag{17b}$$

with the lower and upper limits corresponding, respectively, to $\{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)] / F_A^-(q_m^2)\} = 20$ and $\{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)] / F_A^-(q_m^2)\} = -6$. Equations (11) and (15d)–(17b) show that $\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}$ is fairly sensitive to the value of $\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2) = \tilde{F}_P^-(q_m^2) + F_E^{(\text{I})-}(q_m^2) - F_E^{(\text{II})-}(q_m^2)$ and so is fairly sensitive to the magnitude of any nonvanishing $A_\lambda^{(\text{II})}$.

The result in Eq. (17b) combined with a preliminary measurement of the rate of muon capture by ^{14}N to the $[2^+, 1]$ 7 MeV excited state of ^{14}C, viz.[13]:

$$[\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C^*(7 \text{ MeV}))]_{\text{exp}} = (6.0 \pm 1.5) \times 10^3 \text{ sec}^{-1}, \tag{18a}$$

yields

$$\frac{\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}}{[\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C^* (7 \text{ MeV}))]_{\text{exp}}} \approx \frac{1}{100}. \tag{18b}$$

The small value of this ratio shows clearly that $\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}$ is "anomalously slow."

With regard to a direct measurement of $\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}$ we first note that

$$\frac{\{\Gamma(\mu^- + {}^{14}N \to \nu_\mu + {}^{14}C)\}_{\text{stat. av.}}}{[\Gamma(\mu^- + {}^{14}N \to \nu_\mu + \text{All})]_{\text{exp}} + \Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu)} \approx \frac{60 \text{ sec}^{-1}}{6 \times 10^4 \text{ sec}^{-1} + 45 \times 10^4 \text{ sec}^{-1}} = \frac{1}{8500},$$

$$\frac{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}}}{[\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + \mathrm{All})]_{\exp}} \approx \frac{60\ \mathrm{sec}^{-1}}{6\times 10^4\ \mathrm{sec}^{-1}} = \frac{1}{1000}, \quad (19)$$

so that a rather intense slow muon beam would be required. This beam would be stopped in gaseous $({}^{14}\mathrm{N})_2$ and one would have to detect the 400 keV recoil ${}^{14}\mathrm{C}$ ions, in delayed coincidence with the stopping muons, in anticoincidence with any nuclear γ rays (with an extremely high anticoincidence efficiency), and with sufficient discrimination against $\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{13}\mathrm{C}$(ground state)$+n$, $\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{12}\mathrm{C}$(ground state)$+n+n$, $\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{12}\mathrm{B}$(ground state)$+p+n$, etc. We call this experiment to the attention of nuclear spectroscopists in full realization that a "tour de force" of technique would be required.

We proceed to append formulas[14] for (i) the capture rates of the μ^- from the individual $F=\frac{3}{2}$ and $F=\frac{1}{2}$ hyperfine states, and (ii) the angular distributions of the decay e^- from the μ^- and the recoil ${}^{14}\mathrm{C}$. The formulas for the capture rates are

$$\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=3/2} = (8.9\times 10^3\ \mathrm{sec}^{-1})[G_P(q_m^2)]^2, \quad (20\mathrm{a})$$

$$\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2} = (8.9\times 10^3\ \mathrm{sec}^{-1})[3G_A(q_m^2) - G_P(q_m^2)]^2, \quad (20\mathrm{b})$$

where

$$G_P(q_m^2) \equiv [\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2) + F_M^-(q_m^2)]\left(\frac{E_\nu}{2m_p}\right), \quad (20\mathrm{c})$$

$$G_A(q_m^2) \equiv F_A^-(q_m^2) + F_M^-(q_m^2)\left(\frac{E_\nu}{2m_p}\right), \quad (20\mathrm{d})$$

so that

$$\begin{aligned}\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}} &= \tfrac{2}{3}\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=3/2} + \tfrac{1}{3}\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2} \\ &= (8.9\times 10^3\ \mathrm{sec}^{-1})\{2[G_A(q_m^2)]^2 + [G_A(q_m^2) - G_P(q_m^2)]^2\}, \quad (20\mathrm{e})\end{aligned}$$

in agreement with the $\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}}$ of Eq. (11). Numerically one has [using Eqs. (20a)–(20e), (14d), (15d), and (16)]

$$\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=3/2} = \begin{cases} 1\ \mathrm{sec}^{-1} : \{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)]/F_A^-(q_m^2)\} = -6 \\ 30\ \mathrm{sec}^{-1} : \{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)]/F_A^-(q_m^2)\} = 20, \end{cases} \quad (20\mathrm{f})$$

$$\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2} = \begin{cases} 210\ \mathrm{sec}^{-1} : \{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)]/F_A^-(q_m^2)\} = -6 \\ 95\ \mathrm{sec}^{-1} : \{[\tilde{F}_P^-(q_m^2) + F_E^-(q_m^2)]/F_A^-(q_m^2)\} = 20, \end{cases} \quad (20\mathrm{g})$$

in agreement with Eqs. (20e) and (17b). Thus, if the rate of conversion from the $F=\frac{3}{2}$ hyperfine state to the $F=\frac{1}{2}$ hyperfine state is large compared to the sum of the rates of muon decay and muon capture from the $F=\frac{3}{2}$ hyperfine state to all possible final nuclear states, i.e., if $\Gamma([\mu^-\,{}^{14}\mathrm{N}]_{F=3/2} \to [\mu^-\,{}^{14}\mathrm{N}]_{F=1/2}) \gg \Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu) + \{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + \mathrm{All})\}_{F=3/2} \approx 5\times 10^5\ \mathrm{sec}^{-1}$, the quantity relevant to the proposed measurement of the rate of $\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C}$ is $\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2}$ and not $\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}}$. In this case the branching ratios in Eq. (19) will be replaced by

$$\frac{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2}}{[\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + \mathrm{All})]_{\exp} + \Gamma(\mu^- \to e^- + \bar{\nu}_e + \nu_\mu)} \approx \frac{150\ \mathrm{sec}^{-1}}{6\times 10^4\ \mathrm{sec}^{-1} + 45\times 10^4\ \mathrm{sec}^{-1}} = \frac{1}{3400},$$

$$\frac{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2}}{[\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + \mathrm{All})]_{\exp}} \approx \frac{150\ \mathrm{sec}^{-1}}{6\times 10^4\ \mathrm{sec}^{-1}} = \frac{1}{400}. \quad (20\mathrm{h})$$

As regards the formulas for the angular distributions one has[14]

$$\{\mathfrak{D}(\hat{p}_e)\}_{F=3/2} = 1 - \tfrac{1}{3}(\tfrac{5}{9}P_\mu)\hat{p}_\mu\cdot\hat{p}_e, \quad (20\mathrm{i})$$

$$\{\mathfrak{D}(\hat{p}_e)\}_{F=1/2} = 1 - \tfrac{1}{3}(\tfrac{1}{9}P_\mu)\hat{p}_\mu\cdot\hat{p}_e, \quad (20\mathrm{j})$$

$$\{\mathfrak{D}(\hat{p}_e)\}_{\text{stat. av}} = \tfrac{2}{3}\{\mathfrak{D}(\hat{p}_e)\}_{F=3/2} + \tfrac{1}{3}\{\mathfrak{D}(\hat{p}_e)\}_{F=1/2} = 1 - \tfrac{1}{3}(\tfrac{11}{27}P_\mu)\hat{p}_\mu\cdot\hat{p}_e, \quad (20\mathrm{k})$$

$$\{\mathfrak{D}(\hat{p}_{14_\mathrm{C}})\}_{F=3/2} = 1 + \tfrac{27}{20}(\tfrac{5}{9}P_\mu)\hat{p}_\mu\cdot\hat{p}_{14_\mathrm{C}}, \quad (20\mathrm{l})$$

$$\{\mathfrak{D}(\hat{p}_{14_\mathrm{C}})\}_{F=1/2} = 1 - 3(\tfrac{1}{9}P_\mu)\hat{p}_\mu\cdot\hat{p}_{14_\mathrm{C}}, \quad (20\mathrm{m})$$

$$\begin{aligned}\{\mathfrak{D}(\hat{p}_{14_\mathrm{C}})\}_{\text{stat. av.}} &= \frac{2}{3}\frac{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=3/2}}{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}}}\{\mathfrak{D}(\hat{p}_{14_\mathrm{C}})\}_{F=3/2} + \frac{1}{3}\frac{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{F=1/2}}{\{\Gamma(\mu^- + {}^{14}\mathrm{N} \to \nu_\mu + {}^{14}\mathrm{C})\}_{\text{stat. av.}}}\{\mathfrak{D}(\hat{p}_{14_\mathrm{C}})\}_{F=1/2}, \\ &= 1 + \frac{\frac{1}{2}[G_P(q_m^2)]^2 - \frac{1}{9}[3G_A(q_m^2) - G_P(q_m^2)]^2}{2[G_A(q_m^2)]^2 + [G_A(q_m^2) - G_P(q_m^2)]^2} \times P_\mu\hat{p}_\mu\cdot\hat{p}_{14_\mathrm{C}}, \quad (20\mathrm{n})\end{aligned}$$

where $\hat{p}_\mu$, $\hat{p}_e$, and $\hat{p}_{14_\mathrm{C}}$ are unit vectors in the directions of the incoming μ^- momentum, decay e^- momentum, and recoil ${}^{14}\mathrm{C}$ momentum, and P_μ is what the μ^- residual polarization would be if the

^{14}N magnetic moment did not further depolarize the μ^- ($\frac{5}{9}P_\mu$ and $\frac{1}{9}P_\mu$ are, respectively, the residual polarizations in the $F=\frac{3}{2}$ and $F=\frac{1}{2}$ hyperfine states). We emphasize that a reasonably accurate measurement of $\mathfrak{D}(\hat{p}_e)$ would permit one to decide whether or not appreciable $F=\frac{3}{2}\to F=\frac{1}{2}$ conversion has taken place; thus, with $P_\mu \cong 0.20$, $\mathfrak{D}(\hat{p}_e)_{F=1/2} = 1-0.007\hat{p}_\mu\cdot\hat{p}_e$, and $\{\mathfrak{D}(\hat{p}_e)\}_{\text{stat. av.}} = 1-0.027\hat{p}_\mu\cdot\hat{p}_e$. Further, again with $P_\mu\cong 0.20$, $\{\mathfrak{D}(\hat{p}_{14_C})\}_{F=1/2}=1-0.067\hat{p}_\mu\cdot\hat{p}_{14_C}$ while $\{\mathfrak{D}(\hat{p}_{14_C})\}_{\text{stat. av.}}=1-0.065\hat{p}_\mu\cdot\hat{p}_{14_C}$ for $\{[\tilde{F}_P^-(q_m^2)+F_E^-(q_m^2)]/F_A^-(q_m^2)\}=-6$ and $\{\mathfrak{D}(\hat{p}_{14_C})\}_{\text{stat. av.}}=1+0.018\hat{p}_\mu\cdot\hat{p}_{14_C}$ for $\{[\tilde{F}_P^-(q_m^2)+F_E^-(q_m^2)]/F_A^-(q_m^2)\}=20$. The sensitivity of $\{\mathfrak{D}(\hat{p}_{14_C})\}_{\text{stat. av.}}$ to $\{[\tilde{F}_P^-(q_m^2)+F_E^-(q_m^2)]/F_A^-(q_m^2)\}$ is particularly to be noted.

In concluding this section we make a brief comment regarding the radiative pionic reactions $\gamma+{}^{14}\text{N}\to\pi^+ + {}^{14}\text{C}$ and $\pi^- + {}^{14}\text{N}\to\gamma+{}^{14}\text{C}$. The rates of these reactions for very low velocity pions are proportional to $[F_A^-(q^2\cong m_\pi^2)]^2=(0.051)^2$ [Eq. (14c)] whence, in particular, the predicted value for the $\{[(\pi^-)_{1s\text{ orbit}}\to\gamma]/[(\pi^-)_{1s\text{ orbit}}\to\text{All}]\}$ branching ratio comes out a factor of 10 smaller than the measured value of $(3\pm2)\times10^{-5}$.[5] It therefore follows that most of the observed $(\pi^-)_{\text{All } n,l \text{ orbits}}+{}^{14}\text{N}\to\gamma+{}^{14}\text{C}$ comes from the $(\pi^-)_{2p}$ orbit.[5]

III. NOIA AND ME MICROSCOPIC TREATMENT

A. Calculation

We have already briefly described in the Introduction the essential features of a microscopic calculation of $\langle{}^{14}\text{N}|A_\lambda|{}^{14}\text{C}\rangle$ where appropriate wave functions depending on the positions, spins, and isospins of the various nucleons are used for ^{14}C and ^{14}N and where $A_\lambda=(\vec{A},A_4)$ is decomposed according to

$$\vec{A}=(\{\vec{A}\}^{\text{NOIA}}+\{\vec{A}\}^{\text{ME}}) = -g_A\sum_{j=1}^{14}\tau_j^{(+)}\vec{\sigma}_j e^{-i\vec{q}\cdot\vec{r}_j} - g_A\sum_{j=1,k=1}^{14}(\overrightarrow{\text{ME}})_{jk},$$

$$\vec{q}=\vec{p}_e+\vec{p}_{\bar\nu},\quad |\langle\vec{q}\cdot\vec{r}_j\rangle|\ll 1, \tag{21}$$

$$(\overrightarrow{\text{ME}})_{jk}\equiv\overrightarrow{\text{ME}}(\vec{r}_j,\vec{r}_k;\vec{\sigma}_j,\vec{\sigma}_k;\vec{\tau}_j,\vec{\tau}_k), \tag{22}$$

with a similar expression for A_4. Further,

$$\begin{aligned}|{}^{14}\text{C}\rangle &= C_{00}\psi_{L=0,S=0,T=-T_z=1}(\ldots\vec{r}_i,(\sigma_z)_i,(\tau_z)_i,\ldots) + C_{11}\psi_{L=1,S=1,T=-T_z=1}(\ldots\vec{r}_i,(\sigma_z)_i,(\tau_z)_i,\ldots),\\ |{}^{14}\text{N}\rangle &= N_{01}\psi_{L=0,S=1,T=T_z=0}(\ldots\vec{r}_i,(\sigma_z)_i,(\tau_z)_i,\ldots) + N_{10}\psi_{L=1,S=0,T=T_z=0}(\ldots\vec{r}_i,(\sigma_z)_i,(\tau_z)_i,\ldots)\\ &\quad + N_{21}\psi_{L=2,S=1,T=T_z=0}(\ldots\vec{r}_i,(\sigma_z)_i,(\tau_z)_i,\ldots),\end{aligned} \tag{23}$$

in an obvious notation. The wave functions of Eq. (23) describe the ^{14}C and ^{14}N ground states as an ^{16}O double closed shell with two $1p$ holes, the two holes interacting with each other through an effective potential whose tensor component is relatively quite significant.[1] For reasons of simplicity, we neglect any contributions to $|{}^{14}\text{C}\rangle$ and $|{}^{14}\text{N}\rangle$ which arise from excited configurations, e.g., from the configuration that corresponds to an ^{16}O double closed shell with four $1p$ holes and two $2s1d$ nucleons.[14a]

Equations (22) and (23) yield[1,15,5,6]

$$\begin{aligned}\{F_A^-(0)\}^{\text{NOIA}} &= \frac{1}{\sqrt2}\langle{}^{14}\text{N}|\{A_z\}^{\text{NOIA}}|{}^{14}\text{C}\rangle\\ &=\frac{1}{\sqrt2}\langle{}^{14}\text{N}|-g_A\sum_{j=1}^{14}\tau_j^{(+)}(\sigma_z)_j|{}^{14}\text{C}\rangle\\ &= g_A\left(N_{01}C_{00}-\frac{1}{\sqrt3}N_{10}C_{11}\right),\end{aligned} \tag{24}$$

the first equality being the consequence of a comparison with Eq. (6b). Thus $\{F_A^-(0)\}^{\text{NOIA}}$ is indeed small if N_{21} is dominant and if $N_{01}\cong(1/\sqrt3)N_{10}C_{11}/C_{00}$. However, no available nuclear physics calculations can yield values of C_{00}, C_{11}, N_{01}, and N_{10} of sufficient precision to allow one to conclude that $|\{F_A^-(0)\}^{\text{NOIA}}|$ is as small as $|[F_A^-(0)]_{\text{exp}}|\cong10^{-3}$ and, in any case, such a conclusion would also involve the (injustifiable) neglect of $\{F_A^-(0)\}^{\text{ME}}=(1/\sqrt2)\langle{}^{14}N|\{A_z\}^{\text{ME}}|{}^{14}\text{C}\rangle=(1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1,k=1}^{14}\{\text{ME}_z\}_{jk}|{}^{14}\text{C}\rangle$. We therefore consider setting $N_{01}=(1/\sqrt3)N_{10}C_{11}/C_{00}$ as a constraint to be satisfied by acceptable ^{14}C and ^{14}N wave functions[6] as quite unrealistic and instead calculate $(1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1}^{14}\tau_j^{(+)}(\sigma_z)_j|{}^{14}\text{C}\rangle + (1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1}^{14}\{\text{ME}_z\}_{jk}|{}^{14}\text{C}\rangle = g_A(N_{01}C_{00}-(1/\sqrt3)N_{10}C_{00}) + (1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1,k=1}^{14}\{\text{ME}_z\}_{jk}|{}^{14}\text{C}\rangle$ with the values of N_{01}, N_{10}, C_{00}, and C_{11} taken from the literature[15,5,6]; these values, in general, specify N_{01} as close to but not as exactly equal to $(1/\sqrt3)N_{10}C_{11}/C_{00}$. Thus we shall see whether $(1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1,k=1}^{14}\{\text{ME}_z\}_{jk}|{}^{14}\text{C}\rangle$ is comparable in magnitude to $(1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1}^{14}\tau_j^{(+)}(\sigma_z)_j|{}^{14}\text{C}\rangle$, and if it is, whether it can have the opposite sign so that $\{F_A^-(0)\}^{\text{NOIA}}$ and $\{F_A^-(0)\}^{\text{ME}}$ interfere destructively.

We proceed with a description of our calculation of $(1/\sqrt2)\langle{}^{14}\text{N}|-g_A\sum_{j=1}^{14}\{\text{ME}_z\}_{jk}|{}^{14}\text{C}\rangle$. We use the wave functions of Eq. (23) with the various $\psi_{L,S}$ constructed fro. ı single-nucleon harmonic oscillator (HO) wave functions appropriate to the configuration $(1s)^4(1p)^{10}=(1s)^4(1p)^{12}(1p)^{-2}$; the HO energy quantum is taken as $\hbar\omega=14$ MeV. As regards the explicit form of $\{\vec{A}\}^{\text{ME}}=-g_A\sum_{j=1,k=1}^{14}(\overrightarrow{\text{ME}})_{jk}$, we use the expressions of Chemtob and Rho[16]:

$$\{\vec{A}\}^{\rm ME}=-g_A\sum_{j=1,k=1}^{14}(\vec{\rm ME})_{jk}=-g_A\sum_{j=1,k=1}^{14}\frac{1}{2}\Big\{([\vec\sigma_j\times\vec\sigma_k]\,g_{\rm I}(r_{jk})+\vec{S}^{(\times)}_{jk}g_{\rm II}(r_{jk}))(\vec\tau_j\times\vec\tau_k)^{(+)}$$
$$+([\vec\sigma_j-\vec\sigma_k][h_{\rm I}^{(0)}(r_{jk})+h_{\rm I}^{(\tau)}(r_{jk})P_{jk}^{(\tau)}+h_{\rm I}^{(\sigma)}(r_{jk})P_{jk}^{(\sigma)}]$$
$$+\vec{S}^{(-)}_{jk}[h_{\rm II}^{(0)}(r_{jk})+h_{\rm II}^{(\tau)}(r_{jk})P_{jk}^{(\tau)}+h_{\rm II}^{(\sigma)}(r_{jk})P_{jk}^{(\sigma)}])(\tau_j^{(+)}-\tau_k^{(+)})$$
$$+([\vec\sigma_j+\vec\sigma_k]\,j_{\rm I}(r_{jk})+\vec{S}^{(+)}_{jk}j_{\rm II}(r_{jk})+\vec\Sigma_{jk}\,j_{\rm III}(r_{jk}))(\tau_j^{(+)}+\tau_k^{(+)})\Big\}\,, \tag{25a}$$

where

$$(\vec\tau_j\times\vec\tau_k)^{+}\equiv\tfrac{1}{2}\{(\vec\tau_j\times\vec\tau_k)^{(1)}+i(\vec\tau_j\times\vec\tau_k)^{(2)}\}=i(\tau_j^{(+)}\tau_k^{(3)}-\tau_k^{(+)}\tau_j^{(3)})\,,$$
$$P_{jk}^{(\tau)}\equiv\tfrac{1}{2}(1+\vec\tau_j\cdot\vec\tau_k);\quad P_{jk}^{(\sigma)}\equiv\tfrac{1}{2}(1+\vec\sigma_j\cdot\vec\sigma_k)\,,$$
$$\vec{S}^{*}_{jk}\equiv(\vec\sigma_j*\vec\sigma_k)\cdot\hat r_{jk}\hat r_{jk}-\tfrac{1}{3}\vec\sigma_j*\vec\sigma_k;\quad *\equiv\times,+,-;\quad \hat r_{jk}\equiv\frac{\vec r_{jk}}{|\vec r_{jk}|}\equiv\frac{\vec r_j-\vec r_k}{|\vec r_j-\vec r_k|}\,,$$
$$\vec\Sigma_{jk}\equiv\frac{i}{3}([\vec\sigma_j\cdot\hat r_{jk}][\vec\sigma_k\times\hat r_{jk}]+[\vec\sigma_j\times\hat r_{jk}][\vec\sigma_k\cdot\hat r_{jk}]) \tag{25b}$$

and where the radial functions $g_{\rm I}$, $g_{\rm II}$, $h_{\rm I}^{(0)}$, $h_{\rm I}^{(\tau)}$, $h_{\rm I}^{(\sigma)}$, $h_{\rm II}^{(0)}$, $h_{\rm II}^{(\tau)}$, $h_{\rm II}^{(\sigma)}$, $j_{\rm I}$, $j_{\rm II}$, and $j_{\rm III}$ are obtained by considering the various diagrams of Fig. 1 which correspond to pion exchange. This gives[16]

$$g_{\rm I}(r_{jk})=a_{\rm I}\begin{pmatrix}Y_0(m_\pi r_{jk})\\ K_0(m_\pi r_{jk})-\dfrac{K_1(m_\pi r_{jk})}{m_\pi r_{jk}}\end{pmatrix},\quad h_{\rm I}^{(n)}(r_{jk})=b_{\rm I}^{(n)}\begin{pmatrix}Y_0(m_\pi r_{jk})\\ K_0(m_\pi r_{jk})-\dfrac{K_1(m_\pi r_{jk})}{m_\pi r_{jk}}\end{pmatrix},$$
$$j_{\rm I}(r_{jk})=c_{\rm I}\begin{pmatrix}Y_0(m_\pi r_{jk})\\ K_0(m_\pi r_{jk})-\dfrac{K_1(m_\pi r_{jk})}{m_\pi r_{jk}}\end{pmatrix},\quad n=0,\tau,\sigma\,, \tag{25c}$$
$$g_{\rm II}(r_{jk})=a_{\rm II}\begin{pmatrix}Y_2(m_\pi r_{jk})\\ K_2(m_\pi r_{jk})\end{pmatrix},\quad h_{\rm II}^{(n)}(r_{jk})=b_{\rm II}^{(n)}\begin{pmatrix}Y_2(m_\pi r_{jk})\\ K_2(m_\pi r_{jk})\end{pmatrix},\quad j_{\rm II,III}(r_{jk})=c_{\rm II,III}\begin{pmatrix}Y_2(m_\pi r_{jk})\\ K_2(m_\pi r_{jk})\end{pmatrix};$$
$$Y_0(x)\equiv e^{-x}/x\,,\quad Y_2(x)\equiv(1+3/x+3/x^2)e^{-x}/x\,,$$

$K_{0,1,2}(x)$ are Bessel functions of the second kind, $Y_0(m_\pi r_{jk})$ and $Y_2(m_\pi r_{jk})$ arise from diagrams involving intermediate states with Δ, ρ, and nucleon-antinucleon, $[K_0(m_\pi r_{jk})-K_1(m_\pi r_{jk})/m_\pi r_{jk}]$ and $K_2(m_\pi r_{jk})$ arise from nucleon-recoil diagrams, and the numerical values of the coefficients $a_{\rm I}$, $a_{\rm II}$, $b_{\rm I}^{(n)}$, $b_{\rm II}^{(n)}$, $c_{\rm I}$, $c_{\rm II}$, and $c_{\rm III}$ (all essentially proportional to the square of the pion-nucleon coupling constant $f_{\pi NN}{}^2$) are calculated using $f_{\pi NN}{}^2=0.080$. Thus, performing the hole→particle transformation and using a spherical basis for $\vec{S}^{*}_{jk}$ and $\vec\Sigma_{jk}$, we have

$$\{F_A^-(0)\}^{\rm ME}=\frac{1}{\sqrt2}\langle{}^{14}{\rm N}|\{A_z\}^{\rm ME}|{}^{14}{\rm C}\rangle=\frac{1}{\sqrt2}\langle{}^{14}{\rm N}|-g_A\sum_{j=1,k=1}^{14}\{{\rm ME}_z\}_{jk}|{}^{14}{\rm C}\rangle\,,$$
$$=\frac{1}{\sqrt2}\sum_{L,S,L',S'}N_{LS}C_{L'S'}\langle[1p]^2,L,S,J=1,T=T_z=0|-g_A\sum_{j=1,k=1}^{14}\{{\rm ME}_z\}_{jk}|[1p]^2,L',S',J=0,T'=T'_z=1\rangle$$
$$=-\sum_{L,S,L',S'}N_{LS}C_{L'S'}(h^{(1)}_{LS;L'S'}+h^{(2)}_{LS;L'S'})\,;$$

$$N_{LS}=N_{01},N_{10},N_{21};\quad C_{L'S'}=C_{00},C_{11}, \tag{26}$$

where in $h^{(2)}_{LS;L'S'}$ each of the two holes occupies a different single-nucleon orbital in ^{14}C and in ^{14}N, while in $h^{(1)}_{LS;L'S'}$ only one of the two holes occupies a different single-nucleon orbital in the two nuclei. General expressions for $h^{(1)}_{LS;L'S'}$ and $h^{(2)}_{LS;L'S'}$ (which on the basis of angular momentum selection rules predict $h^{(1)}_{01;11}=h^{(1)}_{10;00}=h^{(2)}_{01;11}=h^{(2)}_{10;00}=h^{(2)}_{21;11}=0$) will be presented elsewhere.[17] These expressions can be straightforwardly applied to the $1p$ shell[17] with integrals over the radial functions calculated by means of the Moshinsky transformations.[18] Numerical results for the contributions to $\{F_A^-(0)\}^{\rm ME}$ arising from the various diagrams [and so for the total $\{F_A^-(0)\}^{\rm ME}$] and also for the $\{F_A^-(0)\}^{\rm NOIA}$ are listed in Table II using values for N_{LS} and $C_{L'S'}$ given in the literature[15,5,6] and recorded in Table I.

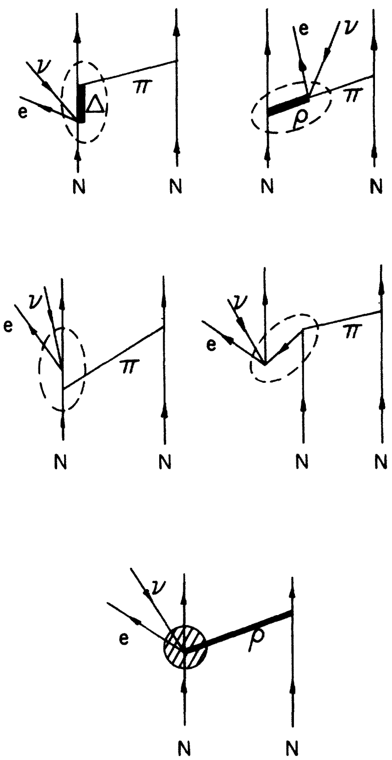

FIG. 1. Feynman diagrams considered in this paper.

B. Discussion

Inspection of Tables I and II suggests the following comments. First of all, while the sets of N_{01}, N_{10}, N_{21}, C_{00}, and C_{11} given in Table I describe ^{14}N and ^{14}C ground states roughly similar one to another, each having a dominant N_{21} and the same sign for $N_{01}C_{00}$ as for $N_{10}C_{11}$, the variation among them in the values of the N_{LS} and $C_{L'S'}$ induces a large variation in the values of $[N_{01}C_{00}-(1/\sqrt{3})N_{10}C_{11}]$ [Eq. (24)] so that the values of the $\{F_A^-(0)\}^{\text{NOIA}}$ oscillate wildly around zero. On the other hand, the corresponding values of the $\{F_A^-(0)\}^{\text{ME}}$ [Eq. (26)] are remarkably stable both in sign and in absolute magnitude and are larger numerically than those of $\{F_A^-(0)\}^{\text{NOIA}}$. This stability is perhaps rather surprising since the one-pion exchange diagrams considered yield relatively long range ($\approx 1/m_\pi$) expressions for $\{\overleftrightarrow{\text{ME}}\}_{jk}$ [Eqs. (25a)–(25c)] and a more sensitive dependence of $\{F_A^-(0)\}^{\text{ME}}$ on the exact values of the N_{LS} and $C_{L'S'}$ could well have occurred.

The major contributions to $\{F_A^-(0)\}^{\text{ME}}$ come from the $h^{(1)}_{LS;L'S'}$ associated with the Δ and ρ intermediate states, the contribution of the latter being considerably smaller than that of the former and also having the opposite sign. The contributions from the nucleon-antinucleon intermediate states are negligible, while the nucleon-recoil contributions are numerically unimportant and, on the basis of consistency arguments,[19] are actually omitted in the values given for $\{F_A^-(0)\}^{\text{ME}}$. Contributions to $\{F_A^-(0)\}^{\text{ME}}$ arising from ρ exchange are also omitted due to the short range of the corresponding radial functions $(m_\pi/m_\rho \ll 1)$; however, this exchange appears to play a not unimportant role in the description of the strong nucleon-nucleon interaction and should be included in a more elaborate investigation of $\{F_A^-(0)\}^{\text{ME}}$.

Due to the sensitivity of $\{F_A^-(0)\}^{\text{NOIA}}$ to the precise values of N_{01}, N_{10}, C_{00}, and C_{11} [Eq. (24)] and the uncertainty regarding the exact form of $\{\overleftrightarrow{\text{A}}\}^{\text{ME}}$ [Eq. (25a)], it is not possible to consider $F_A^-(0) = \{F_A^-(0)\}^{\text{NOIA}} + \{F_A^-(0)\}^{\text{ME}}$ as quantitatively reliable. However, the destructive interference between $\{F_A^-(0)\}^{\text{NOIA}}$ and $\{F_A^-(0)\}^{\text{ME}}$ visible, for example, in case (b), may well be real; such destructive interference is consistent with the fact that $\{\Gamma(\mu^- + {}^{14}\text{N} \to \nu + {}^{14}\text{C})\}_{\text{stat. av.}}$ calculated above according to the EP treatment [Eq. (17b)] is some 4 times smaller than the value calculated by Mukhopadhyay[3] using NOIA without inclusion of ME.

Some further light is thrown on the existence of this destructive interference by considerations of the sign of $\{F_A^+(0)\}^{\text{NOIA}} + \{F_A^+(0)\}^{\text{ME}}$ where, analogously to Eqs. (23), (24), and (26),

TABLE I. Wave functions found in the literature for ^{14}C and ^{14}N.

Label	Authors	Ref.	N_{01}	N_{10}	N_{21}	C_{00}	C_{11}
(a)	Cohen and Kurath	15	0.076	0.307	0.949	0.858	0.514
(b)	Vergados, in Baer *et al.*	5	0.163	0.247	0.956	0.798	0.603
(c)	Elliot	15	0.077	0.179	0.981	0.805	0.593
(d)	Visscher and Ferrell	15	0.173	0.355	0.920	0.764	0.646
(e)	Ensslin *et al.*	6	0.403	−0.068	0.913	−0.093	0.995

$$|^{14}\mathrm{O}\rangle = O_{00}\psi_{L=0,\,S=0,\,T=T_z=1}(\ldots \vec{r}_i, (\sigma_z)_i, (\tau_z)_i, \ldots) + O_{11}\psi_{L=1,\,S=1,\,T=T_z=1}(\ldots \vec{r}_i, (\sigma_z)_i, (\tau_z)_i, \ldots), \quad (27a)$$

$$\{F_A^+(0)\}^{\mathrm{NOIA}} = g_A\left(N_{01}O_{00} - \frac{1}{\sqrt{3}}N_{10}O_{11}\right), \quad (27b)$$

$$\{F_A^+(0)\}^{\mathrm{ME}} = -\sum_{L,S,L',S'} N_{LS}\,O_{L'S'}(h^{(1)}_{LS;L'S'} + h^{(2)}_{LS;L'S'}), \quad (27c)$$

and where O_{00} and O_{11} differ somewhat from C_{00} and C_{11} because $|^{14}\mathrm{O}\rangle$ and $|^{14}\mathrm{C}\rangle$ are not precisely members of the same isotriplet [see discussion after Eqs. (9a) and (9b)]. Invoking the previously mentioned experimental result,[2] we have

$$F_A^+(0) = \frac{(4/3m_p)F_M^+(0)}{[(9.2\pm 0.6)\times 10^{-2}/\mathrm{MeV}]} = \frac{F_M^+(0)}{65\pm 4}, \quad (28a)$$

whence, using Eq. (13),

$$F_A^+(0) \cong \left(1 - \frac{(g_V+g_M)/g_A}{65\pm 4}\right)^{-1}\left(\frac{\{F_M^+(0)\}_{\mathrm{orbit}}}{65\pm 4}\right) = \frac{\{F_M(0)\}_{\mathrm{orbit}}}{61\pm 4}. \quad (28b)$$

Thus, since

$$\{F_M^+(0)\}_{\mathrm{orbit}} = -|\{F_M^+(0)\}|_{\mathrm{orbit}} \quad (28c)$$

on the basis of NOIA calculations,[20] we conclude that

$$F_A^+(0) = -|F_A^+(0)|, \quad (28d)$$

i.e., [see Eq. (24)],

$$\langle^{14}\mathrm{N}|\sum_{j=1}^{14}\tau_j^{(-)}(\sigma_z)_j|^{14}\mathrm{O}\rangle = |\langle^{14}\mathrm{N}|\sum_{j=1}^{14}\tau_j^{(-)}(\sigma_z)_j|^{14}\mathrm{O}\rangle|. \quad (28e)$$

The result in Eq. (28c) should be reliable, since $\{F_M^+(0)\}^{\mathrm{NOIA}}_{\mathrm{orbit}}$ is not particularly sensitive to the N_{LS} and $O_{L'S'}$, and since $|\{F_M^+(0)\}^{\mathrm{ME}}_{\mathrm{orbit}}|$ can be estimated as several times smaller than $|\{F_M^+(0)\}^{\mathrm{NOIA}}_{\mathrm{orbit}}| \cong |\{F_M^+(0)\}_{\mathrm{orbit}}| \cong |F_M^+(0)| \cong 0.3$.

Eq. (28d) [or Eq. (28e)] has several interesting consequences. First of all, while $\{F_A^+(0)\}^{\mathrm{NOIA}}$ and $\{F_A^-(0)\}^{\mathrm{NOIA}}$ may have somewhat different magnitudes and possibly even opposite sign [as follows from the sensitivity of $\{F_A^-(0)\}^{\mathrm{NOIA}}$ and $\{F_A^+(0)\}^{\mathrm{NOIA}}$ to the N_{LS}, C_{LS} and N_{LS}, $O_{L'S'}$—Eqs. (24), (27b), and Table II], it is likely that $\{F_A^+(0)\}^{\mathrm{ME}}$ and $\{F_A^-(0)\}^{\mathrm{ME}}$ have the same sign and similar magnitudes [as follows from the lack of sensitivity of $\{F_A^-(0)\}^{\mathrm{ME}}$ and $\{F_A^+(0)\}^{\mathrm{ME}}$ to the N_{LS}, $C_{L'S'}$ and N_{LS}, $O_{L'S'}$—Eqs. (26), (27c), and Table II].

TABLE II. Numerical values of the various contributions to $F_A^-(0)$. The labels (a), (b), (c), (d), and (e) correspond to the same wave functions as in Table I.

	$\{F_A^-(0)\}^{\mathrm{ME}}_{\Delta}$	$\{F_A^-(0)\}^{\mathrm{ME}}_{\rho}$	$\{F_A^-(0)\}^{\mathrm{ME}}_{\text{nucleon recoil}}$	$\{F_A^-(0)\}^{\mathrm{ME}}$ [Eq. (26)]	$\{F_A^-(0)\}^{\mathrm{NOIA}}$ [Eq. (24)]	$F_A^-(0)$
(a) From $h^{(2)}_{LS;L'S'}$	−0.009	−0.004	−0.002			
From $h^{(1)}_{LS;L'S'}$	−0.090	+0.020	−0.010			
Sum	−0.099	+0.016	−0.012	−0.083	−0.032	−0.115
(b) From $h^{(2)}_{LS;L'S'}$	−0.009	−0.005	−0.001			
From $h^{(1)}_{LS;L'S'}$	−0.096	+0.023	−0.014			
Sum	−0.105	+0.018	−0.015	−0.087	+0.055	−0.032
(c) From $h^{(2)}_{LS;L'S'}$	−0.009	−0.004	−0.002			
From $h^{(1)}_{LS;L'S'}$	−0.094	+0.022	−0.012			
Sum	−0.103	+0.018	−0.014	−0.085	+0.001	−0.084
(d) From $h^{(2)}_{LS;L'S'}$	−0.008	−0.005	−0.001			
From $h^{(1)}_{LS;L'S'}$	−0.094	+0.022	−0.012			
Sum	−0.102	+0.017	−0.013	−0.085	+0.000	−0.085
(e) From $h^{(2)}_{LS;L'S'}$	−0.007	−0.004	+0.001			
From $h^{(1)}_{LS;L'S'}$	−0.094	+0.022	−0.012			
Sum	−0.101	+0.018	−0.011	−0.083	+0.009	−0.072

Thus

$$F_A^-(0) - F_A^+(0) = \mp|F_A^-(0)| + |F_A^+(0)| \cong \{F_A^-(0)\}^{\rm NOIA} - \{F_A^+(0)\}^{\rm NOIA} = g_A\left(N_{01}(C_{00} - O_{00}) - (1/\sqrt{3})N_{10}(C_{11} - O_{11})\right), \quad (29a)$$

while on the basis of the experimental $(ft)_{e^\mp}$ (see above)

$$6\times10^{-3} \lesssim \mp|F_A^-(0)| + |F_A^+(0)| \lesssim 8\times10^{-3}. \quad (29b)$$

Equations (29a) and (29b) provide the appropriate constraint on the constants N_{LS}, C_{LS}, and O_{LS} which describe the ^{14}N, ^{14}C, and ^{14}O wave functions. Further, as seen from Table II, $\{F_A^-(0)\}^{\rm ME} \cong \{F_A^+(0)\}^{\rm ME}$ is negative and considerably larger numerically than $|F_A^-(0)|$ and $|F_A^+(0)|$. Thus, using Eqs. (28d) and (29b),

$$F_A^+(0) = -|F_A^+(0)| = \{F_A^+(0)\}^{\rm NOIA} + \{F_A^+(0)\}^{\rm ME} = \{F_A^+(0)\}^{\rm NOIA} - |\{F_A^+(0)\}^{\rm ME}|, \quad (30a)$$

$$F_A^-(0) = \mp|F_A^-(0)| = \{F_A^-(0)\}^{\rm NOIA} + \{F_A^-(0)\}^{\rm ME} \cong \{F_A^-(0)\}^{\rm NOIA} - |\{F_A^+(0)\}^{\rm ME}|, \quad (30b)$$

$$|F_A^-(0)| \cong 0.15|F_A^+(0)| \cong 1\times10^{-3}. \quad (30c)$$

Equations (30a), (30b), and (30c) show that the destructive interference between $\{F_A^-(0)\}^{\rm NOIA}$ and $\{F_A^-(0)\}^{\rm ME}$ is more complete than that between $\{F_A^+(0)\}^{\rm NOIA}$ and $\{F_A^+(0)\}^{\rm ME}$. Numerically, taking the average of the calculated values in Table II, i.e., taking $\{F_A^\pm(0)\}^{\rm ME} = -|\{F_A^\pm(0)\}^{\rm ME}| = -8.5\times10^{-2}$, we have

$$\{F_A^+(0)\}^{\rm NOIA} = (-7 + 85)\times10^{-3}, \quad (31)$$

$$\{F_A^-(0)\}^{\rm NOIA} = (\mp1 + 85)\times10^{-3}, \quad (32)$$

so that $\{F_A^+(0)\}^{\rm NOIA}$ and $\{F_A^-(0)\}^{\rm NOIA}$ have the same sign and differ in magnitude by about 10%.[21]

Finally, a comment is in order regarding a certain limitation on the above considerations. This limitation arises from the possible incompatibility between our $\{^{14}$N, ^{14}C, ^{14}O$\}$ wave functions and the $\{^{14}$N, ^{14}C, ^{14}O$\}$ wave functions appropriate to the nucleon-nucleon potential which involves the same types of pion-exchange diagrams (Fig. 1) which are used in the calculation of $\{\vec{A}\}^{\rm ME}$ [Eqs. (25a)–(25c)]. The effects of this incompatibility on the determination of $\{F_A^\mp(0)\}^{\rm ME}$ are, however, probably not too important in view of the above-noted insensitivity of the $\{F_A^\mp(0)\}^{\rm ME}$ to the N_{LS}, C_{LS}, and O_{LS} parameters in our ^{14}N, ^{14}C, and ^{14}O wave functions.

IV. CONCLUSIONS

We summarize our conclusions by emphasizing that our calculation of the meson-exchange (ME) contribution to the amplitude for ^{14}C (^{14}O) $\to$ ^{14}N $+ e^-(e^+) + \bar{\nu}_e(\nu_e)$ shows that this contribution may well be opposite in sign and of about the same magnitude as the nucleons-only-impulse-approximation (NOIA) contribution. Under these circumstances the coherent superposition of the two contributions results in anomalously small values for $\Gamma(^{14}{\rm C}(^{14}{\rm O}) \to {}^{14}{\rm N} + e^-(e^+) + \bar{\nu}_e(\nu_e))$. We also emphasize that our elementary-particle calculation of $\{\Gamma(\mu^- + {}^{14}{\rm N} \to \nu_\mu + {}^{14}{\rm C}\,(\text{ground state}))\}_{\rm stat.\,av.}$ predicts a value considerably smaller than that obtained by a NOIA calculation,[3] the discrepancy again indicating the presence of destructive interference between ME and NOIA.

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[20]We use

$$\{F_M^{\pm}(0)\}_{\text{orbital}} = -\frac{g_V}{\sqrt{2}}\left\langle {}^{14}\mathrm{N}\left|\sum_{j=1}^{14}\tau_j^{(-)}(\vec{r}_j\times\vec{p}_j)_z\right|{}^{14}\mathrm{O}\right\rangle$$

[see Eqs. (7a) and (6b) of Ref. (10)] with the matrix element found to be positive [Ref. (1)].

[21]This result is consistent with our picture of the respective roles of $\{F_A^{\pm}(0)\}^{\mathrm{NOIA}}$ and $\{F_A^{\pm}(0)\}^{\mathrm{ME}}$. In the conventional treatment ($\{F_A^{\pm}(0)\}^{\mathrm{ME}} \cong 0$) one has $N_{01} \cong (1/\sqrt{3})N_{10}O_{11}/O_{00}$ and $N_{01} \cong (1/\sqrt{3})N_{10}C_{11}/C_{00}$; thus, minute deviations $(O_{00}-C_{00})$ and $(O_{11}-C_{11})$ due to Coulomb distortion can induce an enormous ratio ($\cong 7$) for $|F_A^{+}(0)/F_A^{-}(0)|$. In the present work, the $\{F_A^{\pm}(0)\}^{\mathrm{NOIA}}$ are not so close to zero and correspondingly are more stable against fluctuations of the C_{LS} and O_{LS}. In particular, differences between O_{00}, O_{11} and C_{00}, C_{11} arising from Coulomb distortion do not induce large differences between $\{F_A^{+}(0)\}^{\mathrm{NOIA}}$ and $\{F_A^{-}(0)\}^{\mathrm{NOIA}}$.