Near threshold photoproduction of neutral pions from the deuteron*

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Photoproduction of low energy neutral pions from a single nucleon and a deuteron target is examined in detail. Features of a direct and two-step nuclear production mode are discussed for the deuteron and estimates for correction terms are given.

[NUCLEAR REACTIONS (γ, π^0), proton and ^{2}H targets; calculated total and differential cross section from 0 to 6 MeV above threshold.]

I. INTRODUCTION

Over the last few years there has been considerable activity in the field of near threshold pion photoproduction.[1,2] Most experiments involve production of charged pions and there is reasonable agreement between theory and experiment. Neutral pion production, on the other hand, has been much less studied even though two special features of π^0 photoproduction make this an interesting reaction. First, in contrast to charged pions, neutral pions can be produced coherently from nuclei. Second, the π^0 photoproduction cross section from a single nucleon is very small near threshold. The large static terms, which make up the well known Kroll-Ruderman[3] amplitude for charged pions, are absent for a neutral pion. The π^0 amplitude therefore depends entirely on small correction terms such as nonstatic, momentum dependent terms. This seems to make nuclear π^0 photoproduction an ideal probe for the fine details of the production amplitude. However, since the π^0 amplitude is so small near threshold, a two-step mechanism competes strongly with direct π^0 production from a single target nucleon. In this two-step production first a charged pion is produced on one nucleon and then charge exchanges to a π^0 on a second nucleon.

In a previous letter[4] we have estimated the coherent photoproduction cross section for ^{2}H, ^{3}H, and ^{3}He in the threshold limit. Two interesting features emerged from that estimate: First, as expected, the results are sensitive to details of the single nucleon production operator. Second, in each case it was found that the two nucleon charge exchange mechanism was as significant as the direct π^0 production process.[5] Clearly, this has important implications for the interpretation of nuclear π^0 photoproduction and in this paper we therefore examine these features in detail.

A basic ingredient is the single nucleon amplitude and we discuss the model used in Ref. 4 and compare its predictions with the available low energy data. We construct the general nuclear production operator in the impulse approximation and discuss the uncertainties inherent in this approach. For an application, we chose the simplest nuclear target, the deuteron. We calculate the production cross section for pion energies up to 6 MeV, the energy region in which experiments are now under way.[1] Additional contributions to the production process, which were neglected in Ref. 4, are estimated in the present work. The results of this paper are also relevant for estimating the validity of the approximations used in the calculations of Ref. 5.

Section II contains a description of our model for pion production from a single free nucleon, a modification of the effective chiral Lagrangian approach of Peccei.[6] The importance of the $\Delta(1232)$ isobar and ω meson contributions is studied and the resulting total and differential cross sections for $\gamma p \to \pi^0 p$ are then compared with the data. In Sec. III we construct the operator for nuclear photoproduction. Special attention is paid to the treatment of the momentum dependent terms, which include contributions from the nuclear Fermi motion. Section IV contains the calculation of the ^{2}H$(\gamma, \pi^0)^2$H cross section and results are presented for the total and differential cross sections. A discussion of our results is given in Sec. V and the Appendix contains details of the deuteron form factors which enter the nuclear photoproduction amplitude.

II. PHOTOPRODUCTION FROM A SINGLE NUCLEON

A. Formalism

This section contains a description of the model we use for the photoproduction from a single nucleon. Since this amplitude will be used to calculate photoproduction from nuclei described by nonrelativistic wave functions, we write it in a form appropriate for two-component Pauli spinors. In the two-body c.m. frame (denoted by an asterisk) the photoproduction amplitude has the general form[7]

$$\mathfrak{M}^*_{\gamma\pi} = \chi^\dagger[F_1\vec{\sigma}\cdot\vec{\epsilon}^* + F_2\vec{\sigma}\cdot\vec{k}^*\vec{\epsilon}^*\cdot\vec{q}^* + F_3\vec{\sigma}\cdot\vec{q}^*\vec{\epsilon}^*\cdot\vec{q}^* + iF_4\vec{q}^*\cdot(\vec{k}^*\times\vec{\epsilon}^*)]\chi\,, \qquad (1)$$

where $\vec{q}^*$, $\vec{k}^*$, and $\vec{\epsilon}^*$ are the pion momentum, photon momentum, and polarization vector, respectively. The transverse gauge condition, $\vec{\epsilon}^*\cdot\vec{k}^*=0$, has been imposed. The coefficients F_i are operators in the nucleon isospin space of the form

$$F_i = F_i^+\delta_{\beta 3} + F_i^-\tfrac{1}{2}[\tau_\beta, \tau_3] + F_i^0\tau_3\,, \qquad (2)$$

where β refers to the isospin components of the produced pion. The F_i are also functions of two kinematic variables which we choose to be k^* and $\vec{k}^*\cdot\vec{q}^*$.

The dynamical model used to evaluate the coefficients F_i is a modification of the chiral Lagrangian model of Peccei.[6] This model starts out with a chiral Lagrangian for πN scattering which includes an axial vector πNN coupling and a $\pi N\Delta$ interaction term. A gauge invariant Lagrangian appropriate for pion photoproduction is then obtained by means of the minimal substitution. The lowest order graphs contributing to photoproduction are shown in Figs. 1(a)–1(f). Reference 6 gives the explicit expression for the amplitude.[8] This approach provides a good description for charged pion production near threshold.[6] However, for $\gamma p\to\pi^0 p$ it yields a reduced threshold cross section, $a=k^*\sigma_T/q^*$, of 1.7 μb which lies far above the experimental value of 0.9 ± 0.2 μb.[9] Following the suggestion of Berends, Donnachie, and Weaver[10] we therefore add to the Lagrangian a term that takes ω-meson exchange into account [Fig. 1(g)]. The contribution to the pion photoproduction amplitude is

$$M_\omega = \bar{u}g_{\omega NN}\left[\gamma^\mu + i\sigma^{\mu\nu}(k-q)_\nu\frac{\kappa}{2M}\right]u \times \frac{1}{(k-q)^2 - m_\omega^2}\,\frac{g_{\omega\pi\gamma}}{m_\omega}g_{\mu\alpha\beta\gamma}\epsilon^\alpha k^\beta q^\gamma\,.$$

For the ω-nucleon coupling constant we take $g_{\omega NN}^2/4\pi=10$ and for the tensor coupling coefficient

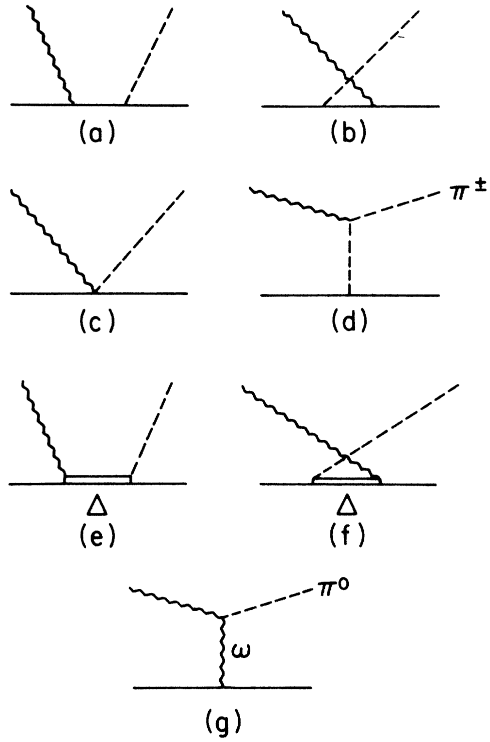

FIG. 1. Diagrams included in the elementary production amplitude.

$\kappa=-0.06$.[11] The $\omega\pi\gamma$ coupling constant $g_{\omega\pi\gamma}$ is related to the $\omega\to\pi\gamma$ decay width by

$$\Gamma(\omega\to\pi\gamma) = \frac{1}{3}\frac{g_{\omega\pi\gamma}^2}{4\pi}m_\omega\left[\frac{m_\omega^2 - m_\pi^2}{2m_\omega^2}\right]$$

and for $\Gamma(\omega\to\pi\gamma)=0.9$ MeV (Ref. 12) this yields $g_{\omega\pi\gamma}^2/4\pi=0.03$. The ω-exchange graph, Fig. 1(f), contributes only to neutral pion production, i.e., only to the + component of the F_i, Eq. (2). The exchange of other vector mesons is suppressed due to their smaller radiative widths. Also, the exchange of a photon, corresponding to the "Primakoff interaction"[13] yields only a negligibly small contribution.

In terms of the amplitude given by Eq. (1), the reduced differential cross section is

$$\frac{k^*}{q^*}\left(\frac{d\sigma}{d\Omega^*}\right) = \left[(4\pi)^2\left(1+\frac{k^*}{E^*}\right)\left(1+\frac{q_0^*}{E'^*}\right)\right]^{-1} \times\left\{f_1^2 + q^{*2}\sin^2\vartheta^*\left[\tfrac{1}{2}(f_2k^*)^2 + f_3^2\tfrac{1}{2}q^{*2} + 4f_1f_3 + f_2f_3\vec{k}^*\cdot\vec{q}^* + f_4^2k^{*2}\right]\right\}. \qquad (3)$$

The coefficients f_i are the isospin combinations $F_i^+ + F_i^0$ for $\gamma p\to\pi^0 p$ and $F_i^+ - F_i^0$ for $\gamma n\to\pi^0 n$, etc. Since the low energy model used here contains only pole terms the functions F_i and f_i are all real.

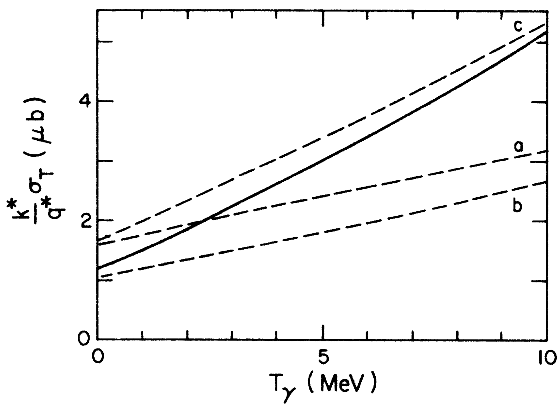

FIG. 2. Reduced total cross section for $\gamma+p\rightarrow\pi^0+p$. T_γ is the photon lab energy above threshold. a Cross section without ω and Δ contributions; b with ω, c with Δ contribution. Solid curve: full calculation.

B. Results

Using the model outlined above, we obtain a reduced total cross section for $\gamma p\rightarrow\pi^0 p$ of

$$k^*\sigma_T/q^*=1.1\ \mu\text{b}\,,$$

in agreement with the experimental value.[9] The value predicted for the neutron is 0.19 μb. Figure 2 shows the reduced total cross section for the proton over the first 10 MeV above threshold. Also shown are the effects of including the ω and Δ. Close to threshold, it is important to include the ω meson, but as the energy increases the Δ becomes the more important contribution. The reduced differential cross section at 4 MeV above threshold is shown in Fig. 3. While the ω and Δ contributions lead to appreciable changes in magnitude, the strong backward peaking of the differential cross section is preserved.

Unfortunately there are no π^0 data in the near threshold region. The lowest available data[14,15] are for a photon laboratory energy of 160 MeV, i.e., 15 MeV above threshold. In Fig. 4 the differential cross section obtained from our model is compared with this measurement. Theoretical and experimental[14,15] values for the total cross section are shown in Fig. 5. Given the experimental uncertainties, there is reasonable agreement between experiment and the model predictions. It is clear, however, that better data are needed to allow for a more rigid test. We have also compared the model cross section for the reaction $\gamma p\rightarrow\pi^+ n$ with low energy data. The prediction for the reduced total cross section at threshold is 210 μb [expt.: 194 ± 7 μb (Ref. 9) and 196 ± 7 μb (Ref. 16)]; 15 MeV above threshold we obtain 200 μb [expt. 192 μb (Ref. 17)]. Thus, the effective Lagrangian model agrees reasonably well with the available low energy data and this model can therefore be used with some confidence in nuclear photoproduction calculations near threshold.

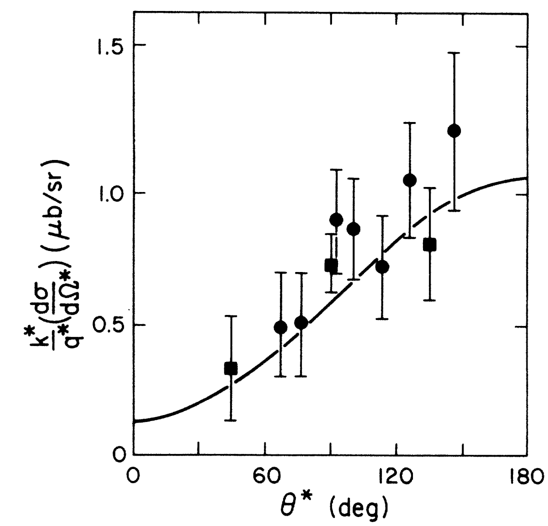

FIG. 4. Reduced differential cross section for $\gamma+p\rightarrow\pi^0 p$ at $T_\gamma=15$ MeV. Experimental values: ■ Ref. 18, ● Ref. 19.

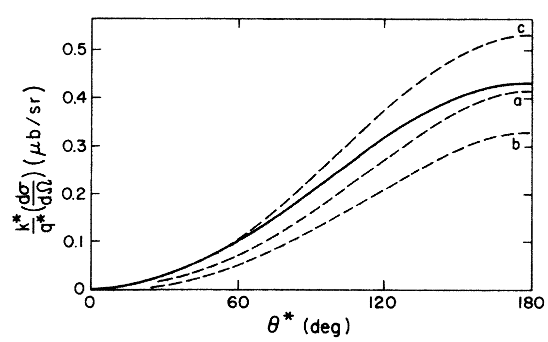

FIG. 3. Reduced differential cross section for $\gamma+p\rightarrow\pi^0+p$ at $T_\gamma=4$ MeV. Labeling of curves as in Fig. 2.

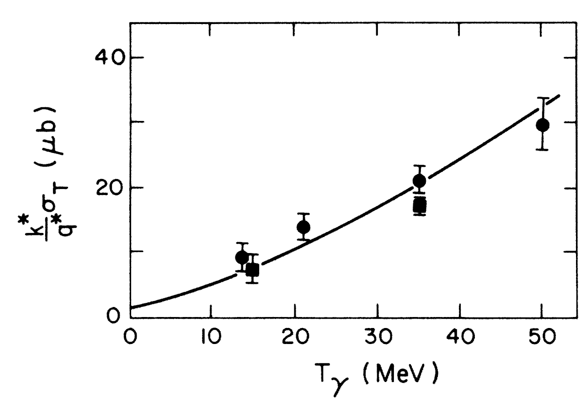

FIG. 5. Reduced total cross for $\gamma+p\rightarrow\pi^0+p$. Experimental values: ■ Ref. 18, ● Ref. 19.

III. NUCLEAR PHOTOPRODUCTION OPERATOR

In this section we discuss the construction of the general nuclear photoproduction operator, including its momentum dependent terms. We use the standard impulse approximation, that is, the free single nucleon amplitude, Eq. (1), is used for each bound target nucleon.

Since we work in the threshold region, we expand the functions F_i, Eq. (1) and (2), in powers of $\vec{k}^*\cdot\vec{q}^*$ and keep only the lowest term. Eq. (1) then becomes

$$\begin{aligned} M^*_{\gamma\pi} &= (c_1 + c_2\vec{k}^*\cdot\vec{q}^*)\vec{\sigma}\cdot\vec{\epsilon}^* + c_3\vec{\sigma}\cdot\vec{k}^*\vec{\epsilon}^*\cdot\vec{q}^* \\ &\quad + ic_4\vec{q}^*\cdot(k^*\times\epsilon^*) \\ &\equiv \vec{\epsilon}^*\cdot\vec{J}^*_5 , \end{aligned} \tag{4}$$

where the coefficients c_i are now functions only of the energy and have the isospin structure given in Eq. (2). For single nucleons, the expanded form of the operator, Eq. (4), yields cross sections that differ by no more than 5% over the first 10 MeV from the cross sections obtained from the exact expression, Eq. (1).

The photoproduction operator will be used in the photon-nucleus c.m. frame. To take nuclear Fermi motion properly into account, the operator in Eq. (4) must be transformed into a general frame in which the initial nucleon has momentum $\vec{p}$. It is also convenient to write the transformed operator in the transverse gauge. Before making the transformation, explicit gauge invariance of the amplitude is restored by redefining $M^*_{\gamma\pi}$, Eq. (4), as (cf. Adler[8])

$$M^*_{\gamma\pi} = \epsilon^*\cdot\vec{J}^*_5 - (\vec{\epsilon}^*\cdot\vec{k}^*)(\vec{J}^*_5\cdot\vec{k})/k^{*2} . \tag{5}$$

We now perform the transformation by using

$$\begin{aligned} \vec{q}^* &= \vec{q} - \xi(\vec{k}+\vec{p}) , \\ \vec{k}^* &= \vec{k} - \xi(\vec{k}+\vec{p}) , \quad \vec{\epsilon}^* = \vec{\epsilon} , \end{aligned} \tag{6}$$

where $\xi = k/(M+k)$ and terms of order M^{-2} have been neglected. The result is

$$\begin{aligned} M_{\gamma\pi} = \{&c_1 + c_2[\vec{q}\cdot\vec{k} - (\vec{q}+\vec{k})\cdot(\vec{k}+\vec{p})\xi]\}[\vec{\sigma}\cdot\vec{\epsilon} + \xi\vec{\sigma}\cdot\vec{k}\vec{\epsilon}\cdot\vec{p}/k^2] \\ &+ c_3\{\vec{\sigma}\cdot\vec{k}[\vec{\epsilon}\cdot\vec{q} - \xi\vec{\epsilon}\cdot\vec{p}(1-\vec{q}\cdot\vec{k}/k^2)] - \xi\vec{\sigma}\cdot(\vec{p}+\vec{k})\vec{\epsilon}\cdot\vec{q}\} \\ &+ ic_4\{\vec{q}\cdot(\vec{k}\times\vec{\epsilon}) - \xi[\vec{p}\cdot(\vec{k}\times\vec{\epsilon}) + \vec{q}\cdot(\vec{p}+\vec{k})\times\vec{\epsilon}]\}, \end{aligned} \tag{7}$$

The gauge condition $\vec{\epsilon}\cdot\vec{k}=0$ has been imposed in the new frame.

As stated above, the c_i are still functions of the energy. In approximating the photoproduction amplitude for a bound nucleon by the free single nucleon amplitude, no definite prescription is implied as to which energy to use for the nuclear case. This ambiguity inherent to the impulse approximation has been widely discussed in connection with pion-nucleus scattering. The most common choice is to evaluate the c_i at the total energy available in the two-body—in our case the πN—subsystem. We will also use this prescription and evaluate the energy for the c_i assuming that the nucleons have momentum $-\vec{k}/A$, where A is the mass number of the target.

The nuclear operator, Eq. (4), is general and can be applied to charged and neutral pion production as well as to radiative pion capture.[18] We would like to emphasize three features of this operator. First, nuclear Fermi motion is explicitly introduced by transforming the momenta from the two-body to the general nuclear frame using Eq. (6). Second, by transforming a manifestly gauge invariant amplitude, Eq. (5), we are free to choose the convenient transverse gauge for the nuclear operator, Eq. (7). Finally, the form of the nuclear operator in Eq. (7) does not depend on a particular dynamical model. The model dependence is contained entirely in the coefficients c_i. An alternative to our approach would be to obtain these coefficients, e.g., from the experimentally determined multipoles for photoproduction from a single nucleon.[19]

IV. π^0 PHOTOPRODUCTION FROM THE DEUTERON

Using Eq. (7), we now evaluate the deuteron photoproduction amplitude in the γ-^{2}H c.m. frame with the kinematics defined in Fig. 6. The final state interaction of the pion is described by a single rescattering of the pion, Fig. 6b. Therefore, the full ^{2}H$(\gamma,\pi^0)^2$H amplitude has two terms

$$\mathfrak{M} = \mathfrak{M}_{\text{dir}} + \mathfrak{M}_{\text{res}} .$$

The direct term $\mathfrak{M}_{\text{dir}}$, Fig. 6a, is given by

$$\mathfrak{M}_{\text{dir}} = 2\int d\vec{p}\psi^\dagger(\vec{p}-\tfrac{1}{2}\vec{q})M_{\gamma\pi}(1)\,\psi(\vec{p}-\tfrac{1}{2}\vec{k}) , \tag{8}$$

and the rescattering amplitude $\mathfrak{M}_{\text{res}}$, Fig. 6b, is

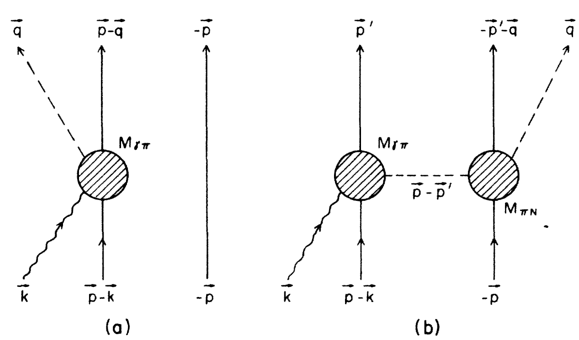

FIG. 6. Diagrams considered for π^0 production from ^{2}H: (a) direct production, (b) rescattering production.

$$\mathfrak{M}_{res} = \frac{2}{(2\pi)^3} \int\int d\vec{p}\, d\vec{p}\,' \psi^\dagger(\vec{p} - \tfrac{1}{2}\vec{q}) \times \frac{M_{\pi N}(2) M_{\gamma\pi}(1)}{(\vec{p} - \vec{p}\,')^2 - \Delta^2 - i\epsilon} \psi(\mathrm{p} - \tfrac{1}{2}\vec{k}) , \tag{9}$$

where Δ is defined in the Appendix, Eq. (A1). The arguments 1, 2 of the operators M in Eqs. (8) and (9) label the nucleons and ψ is the deuteron wave function

$$\psi(\vec{p}) = \chi_{S=1} \eta_{I=0} \varphi(\vec{p}) .$$

Here φ is taken to be the S-state Hulthén wave function

$$\varphi(\vec{p}) = \left(\frac{\alpha\beta\,(\alpha+\beta)^3}{\pi^2} \right)^{1/2} (\alpha^2 + p^2)^{-1} (\beta^2 + p^2)^{-1} ,$$

$\alpha = 0.23\ \mathrm{fm}^{-1}$, $\beta = 1.43\ \mathrm{fm}^{-1}$, and χ and η are the spin and isospin wave functions.

In the rescattering operator, $M_{\pi N}$, we keep only the s-wave part of the interaction of the slow pion with the nucleon and approximate the scattering amplitudes by the scattering lengths, $a^\pm$

$$M_{\pi N} = M^+_{\pi N} \delta_{\alpha\beta} + M^-_{\pi N} \tfrac{1}{2}[\tau_\alpha, \tau_\beta] ,$$

$$M^\pm_{\pi N} = 4\pi(1 + m_\pi/M) a^\pm ,$$

where[20]

$$a^+ = -0.014 \pm 0.005 m_\pi^{-1}$$

$$a^- = 0.087 \pm 0.005 m_\pi^{-1} .$$

The interpretation of the rescattering contribution, Eq. (9), to π^0 production becomes clear if we look at its isospin structure

$$\eta^\dagger_{I=0} M_{\pi N}(2) M_{\gamma\pi}(1) \eta_{I=0} = M^+_{\pi N}(2) M^+_{\gamma\pi}(1) - 2 M^-_{\pi N}(2) M^-_{\gamma\pi}(1) . \tag{10}$$

The first term, involving the + components, corresponds to photoproduction of a neutral pion which then elastically scatters off the other nucleon. The second term, on the other hand, describes production of an intermediate charged pion, $M^-_{\gamma\pi}$, which then charge-exchanges via $M^-_{\pi N}$. As seen in Sec. II, the charged pion photoproduction amplitude is an order of magnitude larger than the π^0 production amplitude. Second, $M^-_{\pi N}$ is about a factor 6 larger than $M^+_{\pi N}$. Finally, Eq. (10) shows that there is an additional factor of 2 enhancing the charge exchange process. These three facts lead to the conclusion that the dominant rescattering process goes through an intermediate charged pion.

The nuclear amplitude, resulting from Eqs. (8) and (9) is of the form

$$\mathfrak{M} = \chi^\dagger_{S=1} [d_1 \vec{\sigma}(1) \cdot \vec{\epsilon} + d_2 \vec{\sigma}(1) \cdot \vec{k}\, \vec{\epsilon} \cdot \vec{q} + d_3 \vec{\sigma}(1) \cdot \vec{q}\, \vec{\epsilon} \cdot \vec{q} + i d_4 \vec{q} \cdot (\vec{k} \times \vec{\epsilon})] \chi_{S=1} . \tag{11}$$

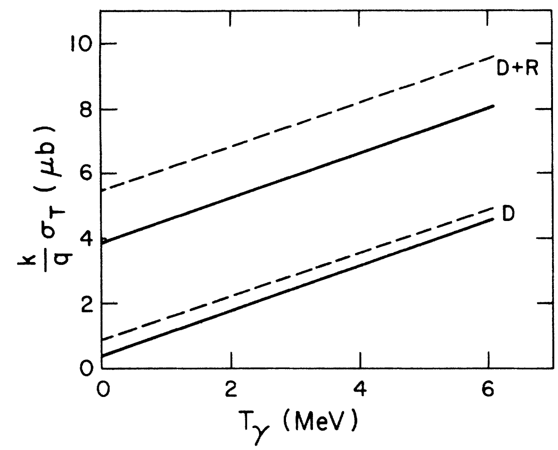

FIG. 7. Reduced total cross section for $\gamma + {}^2\mathrm{H} \rightarrow \pi^0 + {}^2\mathrm{H}$. Top curves (D+R): cross section for direct and rescattering production. Dashed curve shows cross section when internal nucleon motion the target is neglected.. Bottom curves (D): cross section for direct production only.

The coefficients d_i contain contributions from both the direct and rescattering terms. Their specific forms for the deuteron are given in the Appendix.

Averaging over initial and summing over final spins, one obtains for the reduced cross section

$$\frac{k}{q}\left(\frac{d\sigma}{d\Omega}\right) = [(4\pi)^2 (1 + k/2M_D)(1 + q_0/2M_D)]^{-1} \times \tfrac{1}{6} [4 |d_1|^2 + q^2 \sin^2\vartheta (2 |d_2|^2 k^2 + 2 |d_3|^2 q^2 + 4\, \mathrm{Re}\, d_2 d_3^* \vec{k} \cdot \vec{q} + 4\, \mathrm{Re}\, d_1 d_3^* + 3k^2 |d_4|^2)] . \tag{12}$$

Figure 7 shows our ${}^2\mathrm{H}(\gamma, \pi^0){}^2\mathrm{H}$ reduced total cross section, both with and without the rescattering contribution for the first 6 MeV above threshold. As in the single nucleon case, the variation of the reduced cross section remains linear as the photon energy is increased above production threshold. Clearly, the rescattering contribution is very important and changes the magnitude of the cross section significantly while the slope remains essentially unchanged. The reduced differential cross section at a photon energy 4 MeV above threshold is shown in Fig. 8. In addition to changing the magnitude of the cross section, the rescattering term also changes the shape by producing an even more pronounced backward peaking of the differential cross section.

Figure 7 also shows the cross section obtained by neglecting the internal Fermi motion of the target nucleons. The effects are sizable, especially at threshold, where they amount to a 35% increase for the full calculation. Inspection of the amplitudes shows that near threshold this increase is

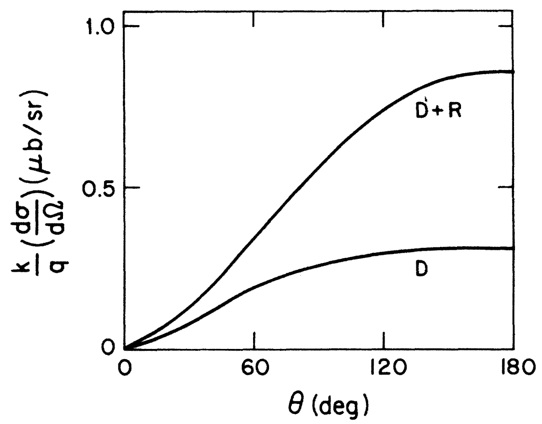

FIG. 8. Reduced differential cross section for $\gamma + {}^2H \to \pi^0 + {}^2H$ at $T_\gamma = 4$ MeV. Labeling as in Fig. 7.

mainly due to a large change in the direct production amplitude. This fact is reflected in the increase of the direct production cross section by about 90%. At higher energies, however, the rescattering amplitude also changes if the internal nucleon motion is neglected.

The coefficients, c_i, in the nuclear photoproduction operator, Eq. (4), are functions of the energy. We have tested the dependence of our results on the choice for this energy. Instead of using the kinematics appropriate for photoproduction from a deuteron target, we evaluated the c_i at an energy corresponding to threshold production on a single nucleon. The change in the full calculation is less than a percent over the entire energy range. The direct production cross section changes by 10% near threshold.

The final state interaction of the produced pion was described by a single rescattering, Fig. 6b, keeping only the s-wave term of the πN amplitude $M_{\pi N}$. An estimate of p-wave rescattering is easily obtained. Neglecting terms of order (m_π/M), the p-wave amplitude for the rescattering vertex in Fig. 6(b) becomes

$$M_{\pi N}^{\pm} = (\vec{p} - \vec{p}\,') \cdot \vec{q} 4\pi (1 + m_\pi/M) b^{\pm} ,$$

where[21]

$$b^+ = 0.21 m_\pi^{-3} ,$$

$$b^- = -0.175 m_\pi^{-3} .$$

Since the rescattering contribution is due to production of an intermediate charged pion, it is sufficient to keep the dominant $\vec{\sigma} \cdot \vec{\epsilon}$ part of the production operator $M_{\gamma\pi}$ for charged pions. Using the form factors defined in the Appendix, the p-wave contribution can then be easily incorporated into Eq. (11) by the substitution

$$d_1 \to d_1 + 8\pi (1 + m_\pi/M)(b^+ c_1^+ - 2b^- c^-) \vec{q} \cdot \vec{y} F_3(\vec{y}) .$$

This leading contribution amounts to less than 3% at the highest energy considered here. Clearly multiple s-wave rescattering is also possible, but this effect should not alter our results since these terms are scaled down by a factor of the order $a_{\pi N} \langle 1/r \rangle \sim 0.05$.

V. SUMMARY

The small $\gamma N \to \pi^0 N$ cross section implies that in nuclei a two-step π^0 production, proceeding through an intermediate charged pion, can compete with the one-step direct production on a single target nucleon. In order to investigate the relative importance of these two production modes—"direct" and "rescattering"—and to keep nuclear structure complications to a minimum, we calculated in this paper the coherent π^0 photoproduction cross section for the deuteron.

One of the ingredients needed for calculating nuclear pion photoproduction is the single nucleon amplitude. We adopted the effective Lagrangian approach of Peccei and obtain with this model good agreement with the available low energy data.

We use the impulse approximation to construct the nuclear photoproduction operator. This general operator includes contributions due to the Fermi motion of the target nucleons. The form of the operator, Eq. (7), is independent of the specific model chosen here; i.e., all model dependence is contained in the coefficients c_i.

The coherent π^0 cross section for the deuteron is evaluated for photon energies up to 6 MeV above threshold. We find that the rescattering mechanism, which involves an intermediate charged pion, increases the cross section significantly. Thus, given the importance of this production mode, it will therefore be difficult to extract information about the direct (γ, π^0) amplitude on nuclei. For charged pion production, of course, such rescatterings are not so important and represent only small final state interactions.

The contribution from the internal motion of the target nucleons was found to be important (35%), but no new qualitative features were introduced by this correction.

The low energy coefficients c_i, Eq. (7), are only weakly dependent on the total energy E. Consequently, the coherent π^0 cross section was found to vary by less than a percent when the single nucleon threshold values were used. This insensitivity also justifies our "frozen" nucleus prescription for the evaluation of the parameter E.

Corrections due to p-wave πN scattering or multiple s-wave scattering were estimated and found to be small. However, since a major portion of the production process is due to the charge exchange scattering, it is clear that changes in the values of the πN scattering length will directly affect the production cross section.

The Hulthén wave function used here is a pure S state. We therefore have no estimate for contributions from the D state. We expect this effect to be quite small.[22]

In our production operator, Eq. (7), we have kept only terms linear in the pion momentum. This is justified for low energy on-shell pions. The off-shell intermediate pion in Fig. 6(b) need not have small momentum and terms in the production operator of higher power in the pion momentum,[23] such as the quadratic term proportional to F_3 in Eq. (4),[4] could in principle be important. In practice, these terms must be evaluated using (model dependent) off-shell form factors at the interaction vectices, which greatly reduce their contributions. In the chiral Lagrangian model, the term explicitly quadratic in the pion momentum in Eq. (4) is mainly due to the pion pole diagrams, Fig. 1(d). This means that in addition to the effect of form factors, this term will also be damped by the pion propagator when the pion is far off shell. A complete quantitative discussion of off shell effects along with possible binding corrections or other many-body effects is clearly outside the scope of this paper. As in π-^{2}H scattering, a full calculation using, e.g., the Faddeev three-body technique, which does not use the impulse approximation, would be the most appropriate way to improve the calculation when accurate enough data become available.

The importance of the rescattering term is due to the small single nucleon π^0 photoproduction amplitude near threshold. As the energy of the incident photon is increased, the π^0 amplitude grows rapidly and in the vicinity of the $\Delta(1232)$ resonance becomes as large as the charged pion amplitude. In this region, which has been studied by several authors,[24-26] the rescattering terms are still important, but their influence on the cross sections is not as pronounced as in the threshold region.

APPENDIX: DIRECT AND RESCATTERING AMPLITUDES

The coefficients d_i in Eq. (11) contain a contribution from direct production, Fig. 6(a), and from the rescattering process, Fig. 6(b):

$$d_i = d_i^{(\text{dir})} + d_i^{(\text{res})}\ .$$

A. Direct production amplitude $d_i^{(\text{dir})}$

We define the form factors

$$g(\vec{Q}) = \int d\vec{p}\,\varphi(\vec{p})\,\varphi(\vec{p}-\tfrac{1}{2}\vec{Q}),$$

$$g'(\vec{Q}) = \frac{1}{Q}\int d\vec{p}\,\varphi(\vec{p})\,\varphi(\vec{p}-\tfrac{1}{2}\vec{Q})\,\vec{p}\cdot\vec{Q}\,,$$

where $\vec{Q}=\vec{q}-\vec{k}$ and $\varphi(p)$ is the Hulthén wave function. The $d_i^{(\text{dir})}$ obtained from Eq. (8) can then be written as

$$d_1^{(\text{dir})} = 2\{g[c_1^+ + c_2^+\vec{q}\cdot\vec{k}(1-\tfrac{1}{2}\xi) - \tfrac{1}{2}c_2^+\xi k^2] - g'c_2^+\xi(q^2-k^2)/Q\}\,,$$

$$d_2^{(\text{dir})} = 2\{g'[c_1^+\xi/Q + c_2^+\xi\vec{k}\cdot\vec{q}/Q + c_3^+\vec{k}\cdot\vec{q}/(k^2Q)] + gc_3^+(1-\tfrac{1}{2}\xi)\}\,,$$

$$d_3^{(\text{dir})} = -2g'c_3^+\xi/Q\,,$$

$$d_4^{(\text{dir})} = 2gc_4^+(1-\tfrac{1}{2}\xi)\,.$$

B. Rescattering amplitude

Using Eq. (10), the rescattering contribution, Eq. (9) can be written as

$$\mathfrak{M}^{(\text{res})} = \frac{2}{(2\pi)^3}\chi_{S=1}^\dagger \int\int d\vec{p}\,d\vec{p}\,'\varphi(\vec{p}\,'-\tfrac{1}{2}\vec{q})\,\varphi(\vec{p}-\tfrac{1}{2}\vec{k})[(\vec{p}-\vec{p}\,')^2-\Delta^2-i\epsilon]^{-1}[M_{\pi N}^+(2)M_{\gamma\pi}^+(1) - 2M_{\pi N}^-(2)M_{\gamma\pi}^-(1)]\,\chi_{S=1}\,, \quad \text{(B1)}$$

$$\Delta^2 = 2m_\pi[k(1+k/2M_D) - m_\pi - \epsilon_D - E_1 - E_2]\,.$$

ϵ_D is the deuteron binding energy, m_π the pion mass, and $E_{1,2}$ the nonrelativistic recoil energies of the nucleons. We first evaluated the leading form factor, G_1 (see below), exactly for $\vec{q}=0$ with

$$E_1 = -(\vec{p}+\vec{k})^2/2M - \epsilon_D + k^2/2M_D\,,$$

$$E_2 = p'^2/2M\,.$$

For the further calculations we then replaced $E_{1,2}$ by a constant closure value $E_1 = E_2 = P_0^2/2M$. P_0 was chosen to be 150 MeV, a value which again reproduced the exact value of G_1 at $\vec{q}=0$. This choice is in good agreement with a similar closure energy for low energy π-deuteron scattering.[27] With this approximation, all angular integrations can be done analytically.

The double integral in Eq. (B1) can be decomposed into the following types of integrands

$$I_i=(2\pi)^{-3}\int\int d\vec{p}\,d\vec{\delta}\,\varphi(\vec{p}-\vec{y})(\delta^2-\Delta^2-i\epsilon)^{-1}$$
$$\times\,\mathcal{O}_i\varphi(\vec{p}-\vec{\delta}),\ \vec{y}=\vec{k}+\vec{q}\,,$$
$$\mathcal{O}_1=1\,,\quad I_1=G_1(\vec{y})$$
$$\mathcal{O}_2=\vec{\epsilon}\cdot\vec{p}\,,\quad I_2=\vec{\epsilon}\cdot\vec{y}G_2(\vec{y})$$
$$\mathcal{O}_3=\vec{\delta}\cdot\vec{k}\,,\quad I_3=\vec{k}\cdot\vec{y}G_3(\vec{y})$$
$$\mathcal{O}_4=\vec{\delta}\cdot\vec{p}\,,\quad I_4=G_4(\vec{y})$$
$$\mathcal{O}_5=\vec{\delta}\cdot\vec{k}\vec{\epsilon}\cdot\vec{p},\quad I_5=\vec{\epsilon}\cdot\vec{k}G_5(\vec{y})+\vec{\epsilon}\cdot\vec{y}\vec{k}\cdot\vec{y}G_6(\vec{y})\,.$$

Collecting terms according to Eq. (11), the rescattering contribution to the production amplitude then becomes

$$d_1^{(\mathrm{res})}=2\{M^+_{\pi N}[c_1^+G_1+c_2^+(\vec{k}\cdot\vec{y}G_3-\xi(G_4+\vec{k}\cdot\vec{y}G_2$$
$$-\tfrac{1}{2}\vec{q}\cdot\vec{y}G_3-\tfrac{1}{2}\vec{q}\cdot\vec{k}G_1))-c_3^+\xi G_5]$$
$$-2M^-_{\pi N}[+\to-]\}$$
$$d_2^{(\mathrm{res})}=2\{M^+_{\pi N}[c_1^+\xi(G_2-\tfrac{1}{2}G_1)/k^2$$
$$+c_2^+\xi(\vec{k}\cdot\vec{y}G_6-\tfrac{1}{2}\vec{k}\cdot\vec{y}G_3)/k^2$$
$$+c_3^+(G_3-\xi G_2+\xi\vec{k}\cdot\vec{y}G_6/k^2-\xi G_6$$
$$+\tfrac{1}{2}\xi(G_1-\vec{k}\cdot\vec{y}G_3/k^2))]-2M^-_{\pi N}[+\to-]\}\,,$$
$$d_3^{(\mathrm{res})}=2\{M^+_{\pi N}[c_3^+(-\xi G_6+\tfrac{1}{2}\xi G_3)]-2M^-_{\pi N}[+\to-]\}\,,$$
$$d_4^{(\mathrm{res})}=2\{M^+_{\pi N}[c_4^+(G_3(1-\tfrac{1}{2}\xi)-\xi(G_2+\tfrac{1}{2}G_1))]$$
$$-2M^-_{\pi N}[+\to-]\}\,.$$

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