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How useful is the fixed-scatterer approximation in pion physics?*

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We compare detailed dynamical calculations of pion-^{4}He elastic scattering with calculations carried out in the fixed-scatterer approximation. We consider pion (laboratory) energies of 51 to 260 MeV. The dynamical calculations are (essentially) parameter free and provide an excellent fit to the data. It is shown that by introducing an energy shift parameter into the fixed-scatterer approximation calculations it is possible to obtain good agreement with the data at low energies. At the higher energies (180–260 MeV) the quality of the (energy shifted) fixed-scatterer approximation fits to the data become systematically worse, with good agreement only at small angles. These results indicate that the use of a single parameter (an energy shift) in the fixed-scatterer approximation calculations is unable to simulate the effects of nuclear binding and Fermi motion. These effects are treated in detail in the dynamical calculations.

[NUCLEAR REACTIONS Pion-helium elastic scattering (50–260 MeV), fixed-scatterer approximation, role of Fermi motion.]

I. INTRODUCTION

The fixed-scatterer approximation (FSA) has been extensively applied in the study of pion-nucleus elastic scattering. This approximation can account for the main features of the differential cross sections; however, it has been shown that it fails badly at low energies ($\lesssim 50$ MeV).[1] This failure has been interpreted as being due to the neglect of binding effects and Fermi motion in the FSA calculations.[2]

It has been suggested that the FSA calculations could be improved by considering the energy variable ω in the off-shell π-nucleon T matrices, $\langle \vec{p}\,|T(\omega)|\vec{p}\,'\rangle$ as an adjustable parameter.[3] Indeed in most multiple scattering theories used to describe pion-nucleus elastic scattering the value of ω is not precisely specified. Various choices for this energy parameter have been suggested.[3,4]

In the covariant theory we have developed,[5] the energy parameter in the fundamental π-nucleon T matrices is precisely determined from the four-momenta of the pion and the (bound) target nucleon. The treatment of the Fermi motion and binding effects which we have used causes the π-N T matrices to be evaluated at energies that are significantly below those usually used in the FSA calculations.[2] Since the π-N T matrices are strongly energy dependent, this feature can lead to large differences between the FSA and dynamical calculations. (See Appendix A.)

In this work we are interested in determining to what extent the FSA calculations can be modified such that a good fit to the data may be obtained. If a simple modification were possible, various economies, particularly in time of computation, could be achieved. The study of pion-helium scattering is particularly useful in this investigation, since in this case, the dynamical theory is able to provide a good fit to the experimental data with no free parameters.

We will demonstrate in the next section that, while the use of a simple energy shift in the evaluation of the FSA amplitudes is able to provide a good fit to the data at 51, 110, and 150 MeV, the fits at 180 and 220 MeV are significantly worse than those obtained in the complete dynamical analysis. The major failing of the FSA is in fitting the data at the larger angles and this failing may directly be related to the neglect of the momenta of the struck particles.

At 260 MeV the *energy shifted* FSA calculation and the (parameter free) dynamical calculations fit the data up to about 40°; however, beyond 40° neither calculation provides a good fit to the data. (The deficiencies of the dynamical calculation, particularly at 260 MeV, may be attributed to higher-order terms in the optical potential arising from true pion absorption and/or correlations.[6])

II. RESULTS OF DYNAMICAL CALCULATIONS AND MODIFIED FSA CALCULATIONS

The details of the dynamical calculations have been presented previously.[2,5] The data for 51 MeV pion elastic scattering from helium[7] have been compared to the theoretical predictions and it has been shown that the inclusion of "true pion absorption" enables one to obtain a good fit to both the π^+ and π^- data.[2] The results of our calculations[2] at

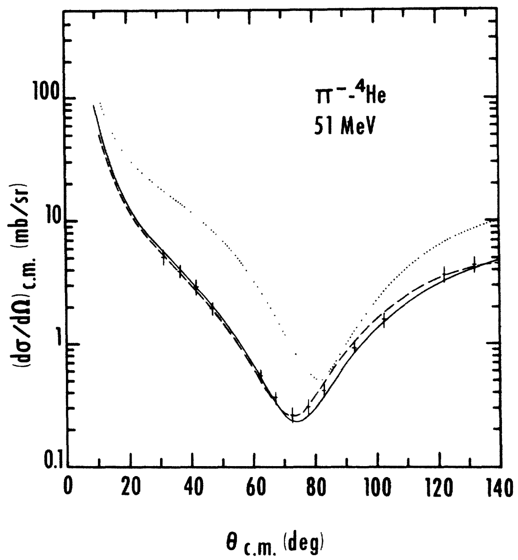

FIG. 1. The solid curve is the result of a dynamical calculation of π^--^{4}He scattering at 51 MeV (Ref. 2). The dotted curve is a standard FSA result, while the dashed curve is the modified FSA result with an energy shift parameter $\delta = 29$ MeV. All curves include the effects of "true pion absorption" as discussed in Ref. 2.

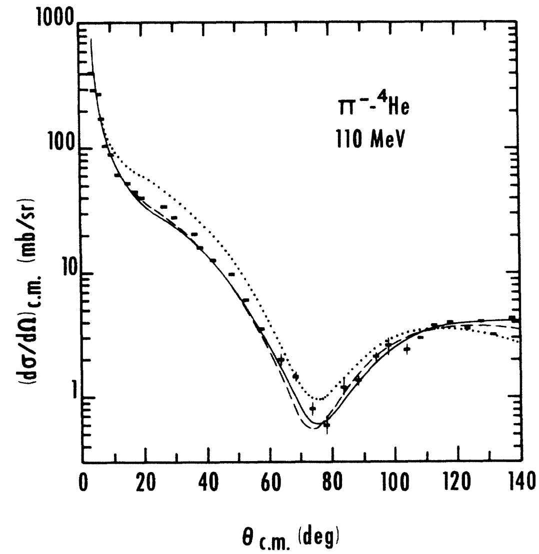

FIG. 2. The solid curve is the result of a dynamical calculation of π^--^{4}He scattering at $T_\pi = 110$ MeV. The dotted curve represents a standard FSA calculation while the dashed curve is the result of a modified FSA calculation with an energy shift parameter of $\delta = 32$ MeV.

51 MeV are again presented in Fig. 1 where they are compared with the usual and modified FSA results.[8] The solid line represents the result of the dynamical calculation. The dotted curve represents the results of a standard FSA calculation. We recall that in such a calculation the energy parameter is determined by considering a pion of momentum $\vec{k}_{lab}$ and kinetic energy T_{lab} incident on a free nucleon at rest. The s value for such a collision is $s = (T_{lab} + M_\pi + M_N)^2 - \vec{k}_{lab}{}^2$ and $\omega \equiv \sqrt{s} - (M_\pi + M_N)$.

In the modified FSA calculation we replace ω by $(\omega - \delta)$ where δ is an adjustable parameter. The choice of $\delta = 29$ MeV yields the FSA result shown as the dashed curve in Fig. 1. (Note that all curves of Fig. 1 include the effects of "true pion absorption."[2])

The data for 110, 180, 220, and 260 MeV π-helium scattering[9] have already been compared with the results of our covariant dynamical calculations.[6] We have repeated these calculations, and have improved our treatment of the effects of the Coulomb potential of the nucleus. The results for our *revised* calculation for 110, 150, 180, 220, and 260 MeV π^--^{4}He elastic scattering are shown in Figs. 2–6. None of the results shown in Figs. 2–6 contain any contribution from true pion absorption. (This contribution is most important at low energy where the imaginary part of the optical potential calculated from the leading term of the multiple scattering series is small.) Because of the improved treatment of the Coulomb effects we are able to eliminate a shift parameter Δ, which was introduced in our previous work.[10] The *solid* curves in Figs. 2–6 represent the results of the dynamical calculation and these calculations contain no free parameters.

Further, the dotted curves are results of stand-

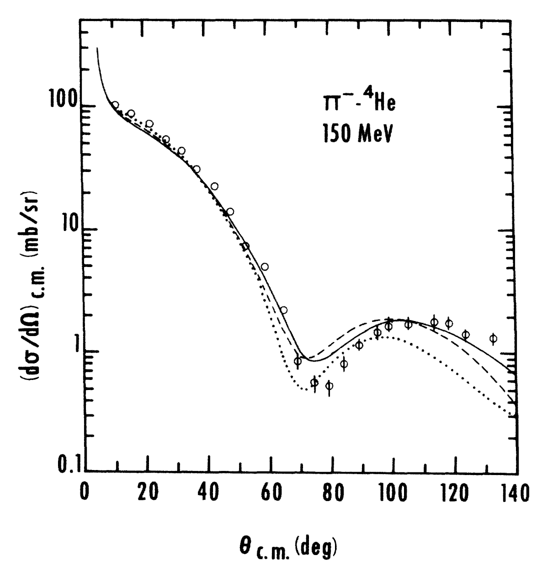

FIG. 3. Same as Fig. 2 with $T_\pi = 150$ MeV and $\delta = 32$ MeV.

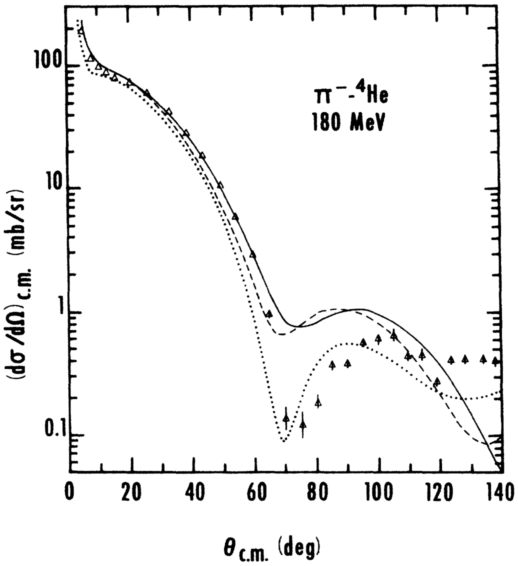

FIG. 4. Same as Fig. 2 with $T_\pi = 180$ MeV and $\delta = 32$ MeV.

ard FSA calculations as described above. Typically, in the first diffraction peak, the FSA result for the differential cross section tends to be larger than the experimental data below the (3, 3) resonance and is smaller than the data above the (3, 3) resonance. An explanation of this feature was presented in Ref. 2.

In Figs. 2–4, the dashed curves represent the

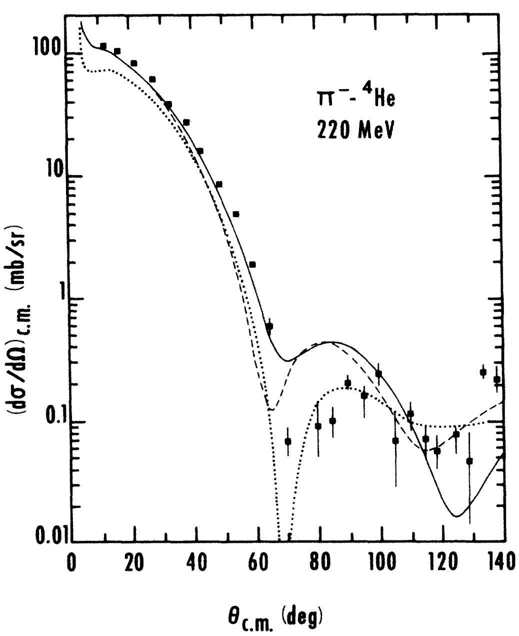

FIG. 5. Same as Fig. 2 with $T_\pi = 220$ MeV and $\delta = 36$ MeV.

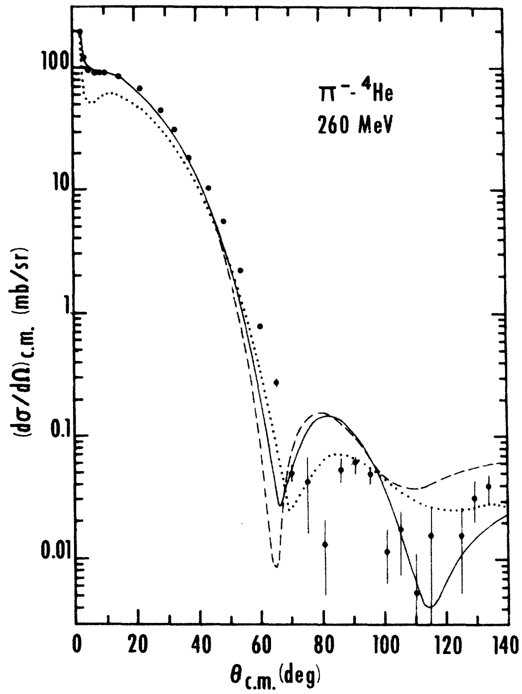

FIG. 6. Same as Fig. 2 with $T_\pi = 260$ MeV and $\delta = 36$ MeV.

FSA results with $\delta = 32$ MeV, while in Figs. 5 and 6, $\delta = 36$ MeV. It may be seen that at 51, 110, and 150 MeV (Figs. 1–3) the dashed curve provides a good fit to the data. At 180 and 220 MeV the dashed curve only fits the experimental data at small angles. At 260 MeV, as remarked previously, both the dynamical calculation and the energy shifted ($\delta = 36$ MeV) FSA calculation fit the data only up to 40°. We have no simple explanation of the small variation with energy of the parameter δ.

One may remark that at angles larger than 40° the shifted FSA results are somewhat closer to the data points than the curve obtained from the covariant calculation. This fact is not significant as the FSA fails badly in fitting the data at the smaller angles. Because of the use of a logarithmic scale the larger angles tend to be overemphasized in the figures. A good theory must fit the data at small angles since most of the cross section is concentrated at these angles.

It is worth noting that the inclusion of Fermi motion in the (covariant) dynamical calculation affects our results in two ways. First, there is the effect on the value of the energy available for the two-body π-N collision. As discussed previously, taking into account the kinetic energy of the "spectator" nucleus *reduces* the energy available for the π-N collision.[2] (This feature may also be seen in the "three-body" prescription of Landau and Thomas.[3]) Second, we note that the angular distribution of the scattered pion is affected by the motion of the struck nucleons. The angular distribution due to the fundamental π-N amplitude is usually specified in the center-of-mass frame of the π-N system. In the FSA, where the nucleons are at rest in the laboratory, for each pion momentum, a single Lorentz transformation will take one from the center-of-mass frame of the pion-*nucleus* system to the pion-nucleon c.m. frame. This is in contrast to the situation in the dynamical calculation where there is a distribution of momenta for the target nucleons. Each elementary π-N collision requires a different Lorentz transformation to go from the π-*nucleus* c.m. frame to the π-*nucleon* c.m. frame. The fact that the shape of the FSA angular distribution often provides a poor representation of the data is probably due to the neglect of the momentum of the struck nucleons in the fixed-scatterer approximation.

III. CONCLUSIONS

In this work we have investigated whether a simple modification of the FSA analysis, via the introduction of an energy shift parameter δ can lead to useful fits to the experimental data.[3] One is encouraged in this investigation by the situation at low energies, 51, 110, and 150 MeV, where such an energy shift does seem to effectively take into account some of the complicated dynamical effects due to nuclear binding and Fermi motion. It would appear that this simple modification of the FSA works quite well at low energies. This scheme, however, does not appear to be generally applicable, as may be seen from the analysis at 180 and 220 MeV.

We conclude from this investigation, that there is no simple modification of the FSA calculations that will replace a careful treatment of the effects of binding and Fermi motion. It also appears that investigating the role of higher-order corrections to the optical model *in the context of a fixed-scatterer calculation*, may not be particularly revealing.

Finally, we remark that unmodified FSA calculations at low energies will usually yield results that are in poor agreement with the data. This does not represent a fundamental problem in the understanding of the pion-nucleus interaction, but rather represents the limitations in the application of the fixed-scatterer approximation at low energies. When the extreme kinematic approximations used in the FSA are removed, the full dynamical calculations[2] are able to give a good account of the experimental data.

APPENDIX A

As noted in the introduction, the energy available in the fundamental π-N collision in a nucleus is quite different when this quantity is evaluated in the FSA and in the dynamical theory. For example, in the FSA, the struck nucleon is taken to be on its mass shell and at rest in the laboratory. If we denote the four-momentum of the incident pion as $(\omega_L, \vec{k}_L)$, then the four-momentum of the struck nucleon will be $(M_N, \vec{0})$. Consequently, we obtain

$$s_{\rm FSA} = (\omega_L + M_N)^2 - (\vec{k}_L)^2 \tag{A1}$$

$$= (T_L + M_\pi + M_N)^2 - \vec{k}_L{}^2 \tag{A2}$$

and

$$s_{\rm FSA}{}^{1/2} \simeq M_\pi + M_N + T_L - \frac{\vec{k}_L{}^2}{2(\omega_L + M_N)} + \cdots . \tag{A3}$$

On the other hand, in the dynamical calculation, the four-momentum of the struck nucleon is obtained from the analysis of a Feynman diagram and is given by $(M_A - E_{C,\vec{Q}_L}, \vec{Q}_L)$. Here M_A is the mass of the target nucleus and $E_{C,\vec{Q}_L} = (M_C{}^2 + \vec{Q}_L{}^2)^{1/2}$ is the energy of the "spectator" nucleus which has momentum $-\vec{Q}_L$ and mass M_C. In this case

$$s = (\omega_L + M_A - E_{C,\vec{Q}_L})^2 - (\vec{k}_L + \vec{Q}_L)^2 \tag{A4}$$

and

$$s^{1/2} \simeq M_\pi + M_A - M_C + T_L - \frac{\vec{Q}_L{}^2}{2M_C} - \frac{(\vec{k}_L + \vec{Q}_L)^2}{2(\omega_L + M_A - M_C - \vec{Q}_L{}^2/2M_C)} . \tag{A5}$$

By introducing $|E_{\rm sep}| = M_N + M_C - M_A$ we have

$$s^{1/2} \simeq M_\pi + M_N - |E_{\rm sep}| + T_L - \frac{\vec{Q}_L{}^2}{2M_C} - \frac{(\vec{k}_L + \vec{Q}_L)^2}{2(\omega_L + M_N)} + \cdots . \tag{A6}$$

Finally, we see from Eqs. (A3) and (A5), that the difference in the values of $s^{1/2}$ in the two theories is

$$s_{\rm FSA}{}^{1/2} - s^{1/2} \simeq |E_{\rm sep}| + \frac{\vec{k}_L \cdot \vec{Q}_L}{\omega_L + M_N} + \frac{\vec{Q}_L{}^2}{2M_C}\left(\frac{\omega_L + M_A}{\omega_L + M_N}\right) . \tag{A7}$$

For ^{4}He, $|E_{\rm sep}| \simeq 20$ MeV, while the last term is about 10 MeV. Therefore, we can understand why energy shifts of the order of 30 MeV are necessary when using the fixed-scatterer approximation to fit the data on pion-nucleus scattering.

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