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Folding model description of heavy ion inelastic scattering*

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We develop a macroscopic description of heavy ion inelastic scattering in the context of a folding model. Form factors for the single nuclear excitation of either the projectile or target are derived, as well as the mutual excitation form factor. The formalism is applied to several typical examples.

[NUCLEAR REACTIONS Heavy ion inelastic scattering; folding model for single and mutual excitation; applied to inelastic excitation of 2^+, 3^-, 5^- states in ^{208}Pb by ^{12}C at 96 MeV; $^{16}O + ^{12}C$ at 168 MeV, elastic, single, and mutual inelastic.]

I. INTRODUCTION

In the usual macroscopic description of the nuclear inelastic excitation of collective target states[1,2] one uses a transition form factor $F(r)$ obtained by deforming the optical potential $V(r)$ so that $F(r) \propto dV(r)/dr$. For heavy ion collisions a more physically motivated method has recently been explored,[3] in which one generates $F(r)$ by deforming the relevant nuclear density directly, starting from a folding model description of the optical potential. This method leads to a state dependence (L dependence) of $F(r)$, in contrast to the usual macroscopic description. When such an L-dependent intrinsic form factor is used, it has been shown[3,4] that target deformation lengths $\beta_L R$ extracted from heavy ion inelastic scattering data are consistent with those obtained from electromagnetic and proton scattering experiments.

In this paper, we present the formalism for inelastic scattering in the folding model. In addition to the case of target excitation previously examined,[3,4] we consider here the possibility of projectile excitation and mutual excitation.

In the first order folding model,[5] the optical potential $V(r)$ is obtained by convoluting the one-body densities ρ_1 and ρ_2 of the projectile and target with an effective nucleon-nucleon *amplitude* (a complex g matrix). Once ρ_1, ρ_2, and g have been chosen, the geometrical properties and complex strength of $V(r)$ are determined. In principle, the g matrix is calculable starting from a model for the nucleon-nucleon (NN) potential. At high energies, g is equivalent to the free space NN amplitude (the t matrix), and is hence well determined by NN data alone. For the low energy heavy ion collisions of interest here (5–15 MeV/particle), there are very sizable corrections to the approximation $g \approx t$, due to the interplay of off-shell propagation of the nucleons, the requirements of the Pauli principle,[6] and the effect of projection onto a single-channel framework. Rather than trying to evaluate g quantitatively, we prefer to adopt a simple phenomenological ansatz for g, the parameters of which are determined by a fit to heavy ion elastic scattering data in the folding model. Our model for $V(r)$ thus involves one complex strength parameter $\bar{f}(E)$. Since we hold the range of g fixed, there are no geometrical parameters to vary; this represents a simplification in comparison to a phenomenological analysis employing Woods-Saxon potential forms. The folding model has the additional advantage of possessing a clear physical interpretation as the first term in a systematic hole line expansion of the potential.[6,7] Since $V(r)$ is written in terms of densities ρ_1 and ρ_2, it is also clear how to proceed from $V(r)$ to a form factor for inelastic scattering via a deformation of ρ_1 or ρ_2 directly. In the same context, one appreciates the inadequacy of the usual prescription of taking a form factor proportional to $dV(r)/dr$ for inelastic scattering induced by a *composite* projectile.

In our approach, we use the folded potential to generate distorted waves for the inelastic scattering as well as the transition form factor via a density deformation. Studies of the *localization* properties of heavy ion elastic, inelastic, and one-particle transfer reactions[8] show that, at low energy (5–15 MeV/nucleon), these processes are

sensitive to the potential or form factor in essentially the *same* region of coordinate space. Thus we expect to get a good description of the distorted waves in the relevant region for inelastic scattering from a folded potential which fits elastic scattering.

The paper is organized as follows. In Sec. II, we present the formalism required to generate inelastic scattering form factors in the folding model. We consider both single and mutual excitation. In Sec. III, we consider various applications of the formalism. First we look at the ^{12}C + ^{208}Pb reaction, where we excite the low-lying collective 2^+, 3^-, and 5^- states in ^{208}Pb. We verify the need for an L-dependent intrinsic form factor (as given by the folding model), in order to obtain consistent deformation lengths $\beta_L R$. Using the same model, we predict the angular distribution of the projectile excitation (2^+ in ^{12}C). We then examine single and mutual inelastic scattering in the $^{16}O+^{12}C$ reaction.

II. FORMALISM

If we assume a local, energy-dependent effective nucleon-nucleon amplitude $g(r,E)$, the first order optical potential $V(R,E)$ describing the collision of two heavy ions may be written in a folding model as[5,6]

$$V(R,E)=\int \rho_1(\vec{r}')\rho_2(\vec{r}'')g(\vec{R}+\vec{r}''-\vec{r}',E)d\vec{r}'d\vec{r}'', \quad (1)$$

where ρ_1 and ρ_2 are the projectile and target densities, respectively, and g is taken to have the form[5,6]

$$g(\vec{r},E)=-\frac{2\pi\hbar^2}{M}\bar{f}(E)N\exp(-r^2/r_0^2) \quad (2)$$

Here, $\bar{f}(E)$ is the spin-isospin averaged forward scattering effective amplitude evaluated at the incoming energy per nucleon E, M is the nucleon mass, and $N=(\pi r_0^2)^{-3/2}$ is a normalization constant. The coordinates are defined in Fig. 1(a). The range r_0 is chosen to be 1.4 fm, consistent with the range of the spin-isospin-independent component of the free nucleon-nucleon t matrix.[9] We neglect the modification of the range r_0 in the nuclear medium. Note that the equivalent range μ^{-1} of a Yukawa interaction $e^{-\mu r}/\mu r$ which has the same volume integral and root-mean-square radius would be $\mu^{-1}\approx 0.5$ fm. This corresponds to the intermediate range part of the NN force; note that single-pion exchange does not contribute to $g(r,E)$.

The excitation of collective nuclear states is naturally described in a folding model approach,[3,4,10] since the densities of the nuclei appear explicitly. Such collective states can be described by a deformed density obtained by introducing a surface vector[1,2]

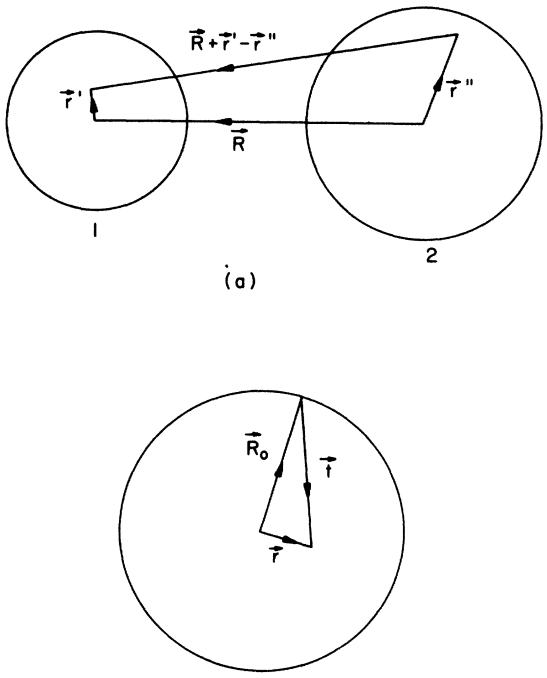

FIG. 1. Part (a) defines the coordinates used in constructing the folded potential. The circles labeled 1 and 2 represent the projectile and target densities. Part (b) defines coordinates used in deforming the density.

$$\vec{R}_0\to\vec{R}_0\left(1+\sum_{LM}\alpha_{LM}\tilde{Y}_{LM}\right), \quad (3)$$

where the $\tilde{Y}_{LM}$'s are defined in the body-fixed coordinate system. Writing $\rho(\vec{r})$ as $\rho(\vec{R}_0+\vec{t})$ [see Fig. 1(b)], we have

$$\rho(\vec{r})=\rho(\vec{R}_0+\vec{t})\to\rho\left(\vec{t}+\vec{R}_0+\vec{R}_0\sum_{LM}\alpha_{LM}\tilde{Y}_{LM}\right). \quad (4)$$

The deformed density can be explained in a Taylor series about the point $\alpha_{LM}=0$, and to first order in α_{LM} we get

$$\rho(\vec{r})\to\rho(\vec{R}_0+\vec{t})+R_0\sum_{LM}\alpha_{LM}Y_{LM}\frac{\partial\rho}{\partial r}\bigg|_{\alpha_{LM}=0}. \quad (5)$$

When this deformed target density is then folded with the spherical projectile density and the effective interaction, one gets

$$V(\vec{R},E)=\int\rho_1(\vec{r}')\rho_2(\vec{r}'')g(\vec{R}+\vec{r}''-\vec{r}',E)d\vec{r}'d\vec{r}''$$
$$+R_0\sum_{LM}\alpha_{LM}\int\rho_1(\vec{r}')[d\rho_2(\vec{r}'')/dr'']\tilde{Y}_{LM}$$
$$\times g(\vec{R}+\vec{r}''-\vec{r}',E)d\vec{r}'d\vec{r}'', \quad (6)$$

where we take $R_0 \equiv 1.2A^{1/3}$ to agree with the convention for electron scattering. The first term of Eq. (6) is just the elastic scattering optical potential of Eq. (1). The second term is the inelastic transition operator (form factor) to be used in a distorted-wave Born approximation (DWBA) calculation of inelastic scattering.[1,2]

The form factor $\mathcal{F}(\vec{R})$ is needed in the space-fixed coordinate system and hence $\tilde{Y}_{LM}$ must be rotated into the space-fixed frame[1]:

$$\tilde{Y}_{LM} = \sum_{\mu} D^{L}_{\mu M} Y_{L\mu} . \tag{7}$$

For an axially symmetric nucleus $M = 0$ and $\alpha_{LM} = \beta_L \delta_{MO}$. Then

$$\mathcal{F}(\vec{R}) = R_0 \sum_{LM} \beta_L D^{L}_{MO} \int Y_{LM}(\hat{r}'') \rho_1(r') \frac{d}{dr''} \rho_2(r'') \times g(\vec{R} + \vec{r}'' - \vec{r}') d\vec{r}' d\vec{r}'' . \tag{8}$$

This can be written as

$$\mathcal{F}(\vec{R}) = \sum_{LM} \beta_L R_0 D^{L}_{MO} I_{LM}(\vec{R}) . \tag{9}$$

In order to calculate the six-dimensional integral I_{LM}, we introduce the Fourier transforms of the densities and effective amplitude.

$$I_{LM}(\vec{R}) = \int d\vec{r}' \int d\vec{r}'' \left[\int e^{-i\vec{q}\cdot\vec{r}'} \rho_1(q) d\vec{q} \right] \times \left[\int e^{-i\vec{q}'\cdot\vec{r}''} \tilde{\rho}_2^{LM}(q') d\vec{q}' \right] \times \left[\int e^{-i\vec{q}''\cdot(\vec{R}+\vec{r}''-\vec{r}')} g(q'') d\vec{q}'' \right] , \tag{10}$$

where

$$\rho(q) \equiv \frac{1}{(2\pi)^3} \int d\vec{r} e^{i\vec{q}\cdot\vec{r}} \rho(r) \tag{11}$$

and

$$\tilde{\rho}^{LM}(q) \equiv \frac{1}{(2\pi)^3} \int d\vec{r}\, e^{i\vec{q}\cdot\vec{r}} \rho'(r) Y_{LM}(\hat{r}) \tag{12}$$

$$= \frac{i^L}{2\pi^2} \int r^2 dr\, j_L(qr) \rho'(r) Y_{LM}(\hat{q})$$

$$\equiv \frac{i^L}{2\pi^2} Y_{LM}(\hat{q}) \bar{\rho}^L(q) . \tag{13}$$

We have made use of the expansion

$$e^{i\vec{q}\cdot\vec{r}} = 4\pi \sum_{LM} i^L j_L(qr) Y^*_{LM}(\hat{r}) Y_{LM}(\hat{q}) , \tag{14}$$

where $j_L(qr)$ is a spherical Bessel function. We now perform the trivial $\vec{r}'$ and $\vec{r}''$ integrations in Eq. (10) to obtain

$$I_{LM}(\vec{R}) = (2\pi)^6 \int d\vec{q}\, \rho_1(q) \bar{\rho}_2^{LM}(-q) g(q) e^{-i\vec{q}\cdot\vec{R}}$$

$$= 4(2\pi)^5 Y_{LM}(\hat{R}) \int q^2 dq\, \rho_1(q) \bar{\rho}_2^{L}(q) \times g(q) j_L(qR) . \tag{15}$$

Hence we have the following form factor $\mathcal{F}_{LM}(\vec{R})$ for a transition to a state of angular momentum L:

$$\mathcal{F}_{LM}(\vec{R}) = 4(2\pi)^5 Y_{LM}(\hat{R}) \beta_L R_0 D^{L}_{MO} \times \int q^2 dq\, \rho_1(q) \bar{\rho}_2^{L}(q) g(q) j_L(qR) . \tag{16}$$

Since ρ_2 describes the target density, $\mathcal{F}_{LM}(\vec{R})$ describes the excitation of a collective state in the target. In order to describe projectile excitation, the subscripts 1 and 2 must be interchanged in Eq. (16). Note that unless $\rho_1 = \rho_2$, these two form factors are different. Also, because of the presence of the Bessel function in Eq. (16), $\mathcal{F}_{LM}(\vec{R})$ has a nontrivial L dependence. These points will be expanded upon later.

Since $g(r)$ involves the interaction of a nucleon in the projectile with one in the target, it is possible to have a one-step process (first order in g) that inelastically excites both the target and projectile. It is straightforward to expand both densities and obtain a form factor $\mathcal{F}_{L_1M_1, L_2M_2}$ for such a mutual excitation:

$$\mathcal{F}_{L_1M_1L_2M_2}(\vec{R}) = \beta^{(1)}_{L_1} R^{(1)}_0 \beta^{(2)}_{L_2} R^{(2)}_0 \times D^{L_1}_{M_1 0} D^{L_2}_{M_2 0} I_{L_1M_1L_2M_2}(\vec{R}) , \tag{17}$$

where the superscript 1 (2) refers to projectile (target) and

$$I_{L_1M_1L_2M_2}(\vec{R}) = \int d\vec{r}' d\vec{r}'' \frac{d}{dr'} \rho_1(r') \frac{d}{dr''} \rho_2(r'') \times Y_{L_1M_1}(\hat{r}') Y_{L_2M_2}(\hat{r}'') g(\vec{R} + \vec{r}'' - \vec{r}') . \tag{18}$$

Introducing Fourier transforms and integrating over $\vec{r}'$ and $\vec{r}''$ as before we get

$$I_{L_1M_1L_2M_2}(R) = 8\pi^2 i^{L_1+L_2} \int d\vec{q}\, Y_{L_1M_1}(\hat{q}) Y_{L_2M_2}(\hat{q}) \bar{\rho}_1^{L_1}(q) \bar{\rho}_2^{L_2}(q) g(q) e^{-i\vec{q}\cdot\vec{R}}$$

$$= 32\pi^3 \sum_{\mathcal{L}M} i^{L_1+L_2+\mathcal{L}} \int d\vec{q}\, j_{\mathcal{L}}(qR) \bar{\rho}_1^{L_1}(q) \bar{\rho}_2^{L_2}(q) g(q) Y_{L_1M_1}(\hat{q}) Y_{L_2M_2}(\hat{q}) Y^*_{\mathcal{L}M}(\hat{q}) Y_{\mathcal{L}M}(\hat{R}) . \tag{19}$$

Integrating over $\Omega_{\hat{q}}$, and defining $\hat{L}_1 \equiv (2L_1+1)^{1/2}$, we get the final result

$$\mathcal{F}_{L_1M_1,\,L_2M_2}(\vec{R}) = \frac{4(2\pi)^3}{\sqrt{4\pi}} \beta^{(1)}_{L_1} R^{(1)}_0 \beta^{(2)}_{L_2} R^{(2)}_0 D^{L_1}_{M_1 0} D^{L_2}_{M_2 0} \sum_{\mathcal{L}M} i^{L_1+L_2} \begin{pmatrix} L_2 & L_1 & \mathcal{L} \\ -M_2 & M_1 & M \end{pmatrix} \begin{pmatrix} L_2 & L_1 & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix} \hat{L}_1 \hat{L}_2 \hat{\mathcal{L}}\, Y_{\mathcal{L}M}(\hat{R})$$

$$\times \int q^2 dq\, j_{\mathcal{L}}(qR) \bar{\rho}^{L_1}_1(q) \bar{\rho}^{L_2}_2(q) g(q) \,. \tag{20}$$

We note that there is a sum of terms in the mutual excitation form factor. The factor $\begin{pmatrix} L_2 & L_1 & \mathcal{L} \\ 0 & 0 & 0 \end{pmatrix}$ ensures that $L_1 + \mathcal{L}$ must vector couple to L_2 and that $L_1 + L_2 + \mathcal{L}$ must be even.

The matrix elements of these form factors sandwiched between distorted waves and collective wave functions are then calculated in the usual manner[1,2] and angular distributions are obtained. The complications arising in the mutual excitation calculation are discussed in Ref. 11 in an Austern-Blair model[12] approach.

III. RESULTS

A. Introduction

DWBA calculations[13] utilizing the optical potentials of Eq. (1) and the form factor of Eq. (8) were first performed for the reaction ^{58}Ni(^{16}O, ^{16}O′)^{58}Ni* (2⁺) at three energies.[3] It was found that folding model fits were comparable to those obtained by using the deformed optical potential (DOP) approach.[14] The more interesting results are obtained when one looks for L-dependent form factor effects and differences between target and projectile excitation which arises from the folded form factor. The shape of the DOP form factor is just given by $dV_{opt}(R)/dR$ and has no L dependence. As noted previously, the presence of the spherical Bessel function in Eq. (16) leads to an L-dependent shape of the folded form factor. Experimental evidence for this L-dependence has been given.[4]

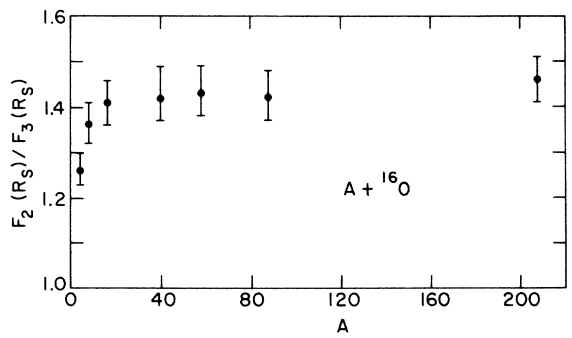

FIG. 2. Ratio of intrinsic form factors for the excitation of the first 2⁺ and 3⁻ states in ^{16}O for a variety of projectiles. The ratio is taken at the strong absorption radius R_S defined as $R_S = 1.5(A^{1/3}+B^{1/3})$, where A and B are the projectile and target mass numbers, respectively. The error bars are determined from the values of the ratio at $R_S \pm 0.5$ fm.

For inelastic excitation of the 2⁺ and 3⁻ collective states in ^{16}O, the ratio of the form factors F_2/F_3 at the strong absorption radius R_S is shown in Fig. 2 as a function of projectile mass A. For A greater than the mass of the target, this ratio is approximately constant at 1.4. Thus the relative cross sections for 2⁺ and 3⁻ excitation are changed by about a factor of 2 with respect to the prediction of the deformed optical potential. Thus, L-dependent form factor effects are enhanced by using a *heavy* projectile to excite a target. For a point projectile with zero range forces there is no L dependence, and the folding model form factors reduce to the usual prescription.

In order to maximize the differences that arise in the folded form factor between projectile and target excitation, it is best to study the case of a very light projectile incident on a very heavy target. The projectile excitation for such a system has usually not been measured because it is difficult to resolve experimentally the projectile excitation peak in the often dense excitation spectrum of the heavy target. However, the excitation of the 4.43 MeV 2⁺ state in ^{12}C should be measurable on a ^{208}Pb target. The elastic scattering and excitation of the 2⁺, 3⁻, and 5⁻ states on ^{208}Pb by a 98 MeV ^{12}C beam has been measured.[15] In Sec. III B, we present folding model fits to this data and compare them with DOP calculations,[15] again supporting the L-dependent form factor shape. We then show how these two approaches lead to large differences in the predicted projectile excitation angular distribution.

B. ^{12}C + ^{208}Pb

The elastic scattering data[15] and a corresponding Woods-Saxon optical model fit[15] are displayed in Fig. 3 along with the folding model result.[16] It was found[5,6] that in order to get adequate agreement with elastic heavy ion data, the prescription of Eq. (2) for the effective interaction required a phenomenological value of $\bar{f}$ rather than the free space value. By confronting a wide range of heavy ion elastic scattering data at different energies, a

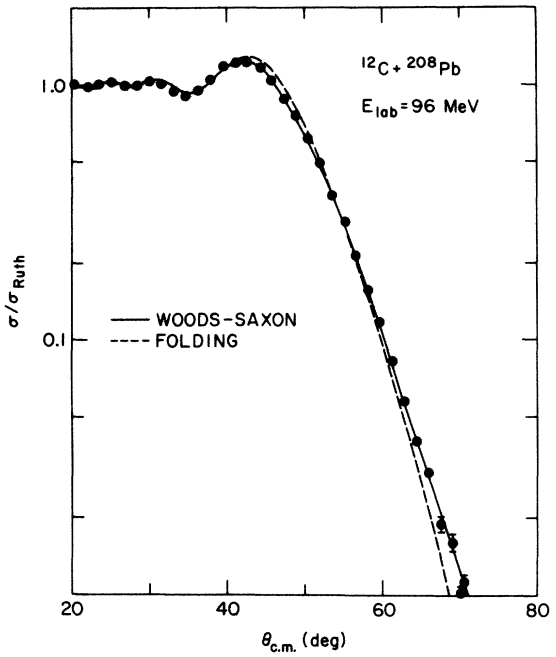

FIG. 3. Elastic scattering for $^{12}C + ^{208}Pb$ at $E_{lab} = 96$ MeV. The data and solid curve are from Ref. 15.

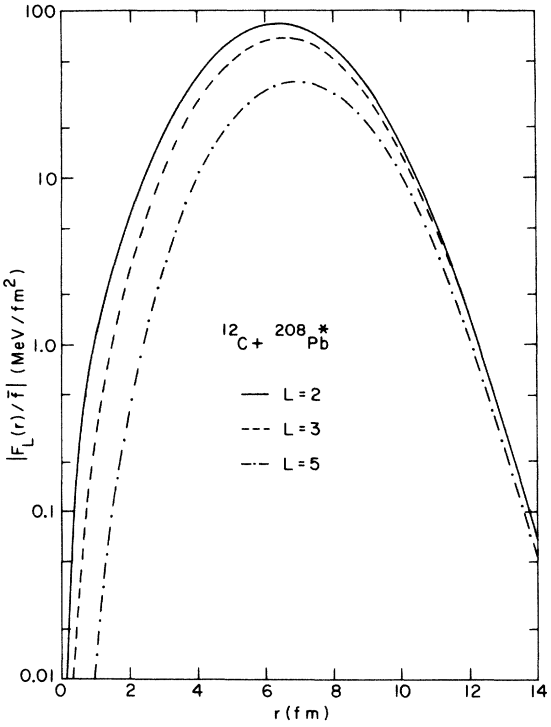

FIG. 4. The intrinsic folding model form factors for the excitation of the lowest-lying 2^+, 3^-, and 5^- states in ^{208}Pb by ^{12}C.

smoothly energy-dependent $\bar{f}(E)$ has been constructed. In all calculations presented here, these smooth values have been used with no searches performed. The value of $\bar{f}$ used in Fig. 3 is $(1.74 + 0.75i)$ fm. The fit is adequate but certainly not as good as the four-parameter Woods-Saxon fit. The ^{12}C density was obtained from the charge density of Friar and Negele,[17] corrected for the finite proton form factor. The ^{208}Pb density was derived from Hartree-Fock wave functions.[18]

Figure 4 displays the intrinsic form factors $F_L(r)$ for three different multipole excitations in ^{208}Pb. Here we define F_L by

$$\mathfrak{F}_L(r) \equiv \beta_L R F_L(r) . \tag{21}$$

In this way, the trivial L dependence of the form factor due to the different deformation lengths $(\beta_L R)$ has been removed. One can define a strong absorption radius[19] by $R_S \equiv 1.5(A^{1/3} + B^{1/3})$, where A and B are the projectile and target mass numbers, respectively. It has been shown that elastic and inelastic heavy ion reactions are sensitive to the radial region around R_S.[8,19] For $^{12}C + ^{208}Pb$, we have $R_S = 12.3$ fm. In the 10–14 fm region one sees that there are small but non-negligible differences in the F_L's, mostly in their magnitude. Since the DOP form factor is L independent, one sees that different β_L's must arise when the two methods are used to fit the inelastic data. The results are shown in Fig. 5. The data and dashed

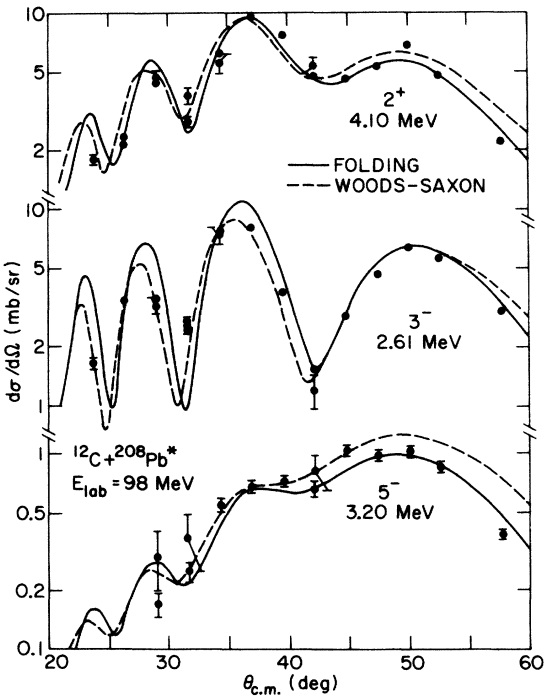

FIG. 5. The angular distributions and calculations for the excitation of three states in ^{208}Pb by ^{12}C. The data and dashed curves are from Ref. 15. The deformation lengths used in both calculations are in Table I.

TABLE I. Deformation lengths $\beta_L R$ (fm) extracted from inelastic scattering data.

	L	This work	Satchler *et al.*[a]	Other work
^{208}Pb	2	0.54	0.56	0.38–0.39[b]
	3	0.77	0.71	0.74–0.86[c]
	5	0.47	0.39	0.43–0.49[d]
^{12}C	2	1.6	•••	1.5 –1.8[e]

[a] See Ref. 15.
[b] See Refs. 22, 24, and 25.
[c] See Refs. 20–25.
[d] See Refs. 23 and 25.
[e] See Ref. 26.

curves are from Ref. 15. The deformation lengths derived from these fits are displayed and compared with values from other experiments in Table I. It is apparent that the folding model leads to deformation lengths that are in generally better agreement with other independent analyses. This lends further support to the predicted L dependence demonstrated in Ref. 4. The oscillations in the 20°–40° region are better reproduced by the DOP approach. This is probably due to the better elastic optical potential used in the DOP calculation. (These oscillations are sensitive to the phase of $\bar{f}$.) However, in the large angle region the folding model clearly does better, especially for the 5$^-$ excitation. The discrepancy in the 2$^+$ deformation length arises in both analyses and is not understood.[15]

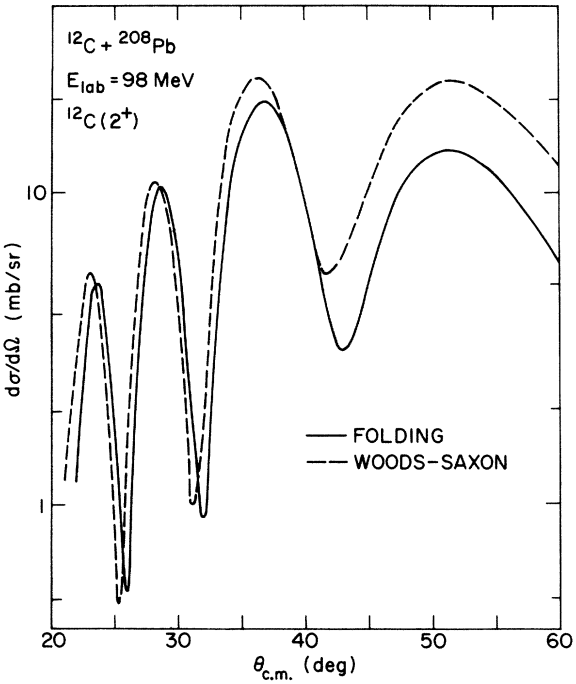

FIG. 7. The predicted angular distributions for the projectile excitation of the lowest-lying 2$^+$ state in ^{12}C by ^{208}Pb. The calculations utilize the form factors of Fig. 6 and the deformation length of Table I.

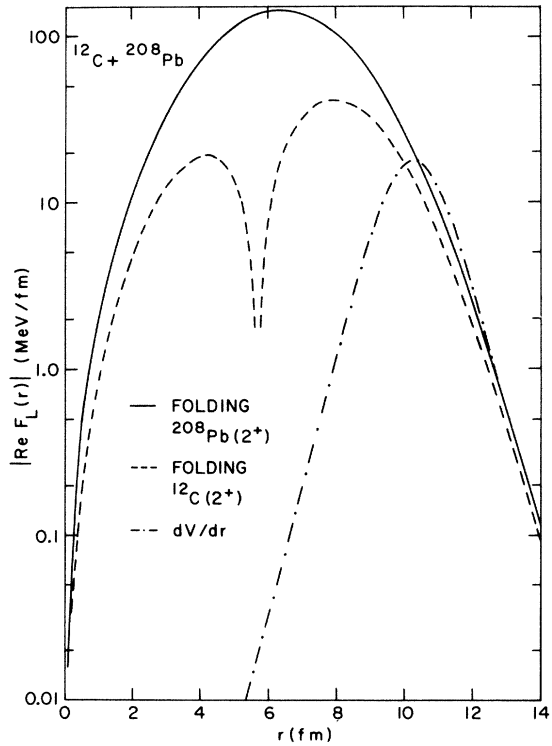

FIG. 6. A comparison of the intrinsic folding model form factors for projectile and target excitation in ^{12}C + ^{208}Pb. The DOP form factor (dV/dr) is also presented.

The form factor for projectile excitation along with the DOP form factor and the folded form factor for the ^{208}Pb 2$^+$ excitation are shown in Fig. 6. The differences between projectile and target excitation persist into the important peripheral region. It is clear that using the same deformation length for the projectile excitation in the two prescriptions will lead to a larger cross section for the DOP calculation. This is shown in Fig. 7, where the deformation length of Table I has been used.[4] Note that differences of up to a factor of 2 emerge in the cross section. Hence, even with the uncertainty of the deformation length (see Table I), experimental data would enable one to discriminate between the two predictions.

C. ^{16}O + ^{12}C

The mutual excitation of the first 2$^+$ state in ^{12}C and the first 3$^-$ state in ^{16}O was measured some years ago.[27] It was successfully analyzed using the Austern-Blair model.[11] However, this ap-

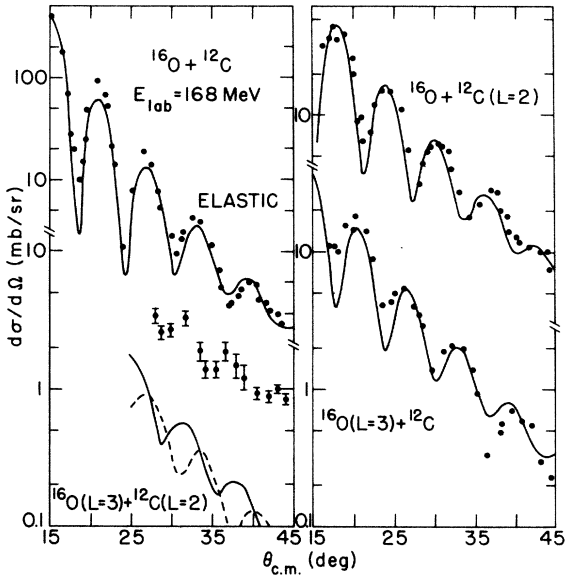

FIG. 8. The elastic scattering, projectile, target, and mutual excitation data for $^{16}O + ^{12}C$ at $E_{lab} = 168$ MeV are presented along with folding model results (solid line) and a DOP calculation (dashed line). The data are from Ref. 27. The deformation lengths used are in the text. For the mutual excitation, only the simultaneous excitation process (i.e., no sequential excitation) is calculated.

proach contains the contributions from the one-step (simultaneous) process and the two-step (sequential) process in a manner which does not allow for the analysis of their separate importance. We have calculated the simultaneous contribution to this mutual excitation in the folding model approach using Eq. (20). Figure 8 displays the results of a folding model calculation of the elastic scattering, projectile excitation, target excitation, and simultaneous mutual excitation. The DOP simultaneous excitation is also shown as a dashed line. In this case, it is clear that the simultaneous contribution to the mutual cross section is too small and hence the two-step sequential process must be included. A very similar conclusion was arrived at by Rickertsen[28] in his calculations for the mutual excitation of ^{12}C on ^{12}C.

The $\bar{f}$ used in the calculations shown in Fig. 8 was again extrapolated from smooth fits to other elastic data and was taken to be $(1.43 + 0.88i)$ fm. In order to fit data, the $\beta_2 R$ needed for ^{12}C was 2.1 fm and the $\beta_3 R$ needed for ^{16}O was 2.1 fm. The ^{12}C density used was the same as described in Sec. III B. For ^{16}O, we used the Hartree-Fock density of Negele,[29] applying a correction for center of mass motion. In both the $^{16}O + ^{12}C$ and $^{12}C + ^{12}C$ mutual excitation analyses, the Coulomb form factor contributions were neglected because of the relatively high energies and small charges.

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