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Consequences of isospin mixing for compound nucleus reactions

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Consequences of isospin mixing are explored for compound nucleus reactions and statistical cross section fluctuations. This is done in the framework of a theory of transport phenomena. Formulas differing from previously published results are given for average compound nucleus cross sections, and for the correlation function of cross section fluctuations. The extraction of isospin mixing from evaporation spectra is discussed. It is pointed out that external mixing, which so far has been considered only in the theory of isobaric analog resonances, may dominate internal or bound-state mixing. Hence, Coulomb matrix elements deduced from highly excited compound nuclei only represent upper limits.

[NUCLEAR REACTIONS Isospin mixing in compound nucleus reactions.]

I. INTRODUCTION

The detection of isospin impurities in nuclear levels is an interesting subject of nuclear spectroscopy. An impressive variety of experimental and theoretical techniques is used.[1] The subject has recently been revived by very precise measurements of isospin impurities for bound or quasibound states in ^{12}C and ^{16}O (see Refs. 2–5). The deduced matrix elements H_c which are due to isospin breaking forces were found to be as large as 200 keV. For heavier nuclei they are somewhat smaller.[6]

In the present paper we investigate the possibility of extracting isospin mixing matrix elements for nuclear states at high excitation energy where the natural width of the levels is larger than their mean spacing ($\Gamma \gg D$). In this case one might expect that isospin mixing is essentially complete so that isospin selection rules lose their significance. However, several experiments contradict this conjecture.[7-13] For example,[8,9] in the reaction $^{28}\mathrm{Si}(d,\alpha)^{26}\mathrm{Al}$ the compound nucleus ^{30}P was excited and the isospin "forbidden" reaction $^{28}\mathrm{Si}(d,\alpha)^{26}\mathrm{Al}$ (0.23, 0^+, $T=1$) turned out to be clearly suppressed as compared to the isospin "allowed" transitions to the $T=0$ states of ^{26}Al. A similar conclusion is drawn from the available studies of evaporation spectra.[14-17] Some of these results prompted the introduction of isospin into the Hauser-Feshbach formalism of statistical nuclear reactions.[18,19] In the analysis of Ref. 19 isospin mixing was neglected altogether; the authors of Ref. 18 allowed for isospin mixing in the compound nucleus. Based on this formalism and the experiments of Ref. 17 average mixing matrix elements H_c have been estimated.[20]

In the present paper, we use general results on the statistical theory of nuclear reactions[21,22] to reformulate the problem of isospin mixing in the domain of strongly overlapping resonances. Our formulation shows that the quantity of spectroscopic interest, the Coulomb matrix element H_c, which produces what is called "internal" mixing in the theory of isobaric analog resonances,[23] always appears in conjunction with "external" isospin mixing. The latter one is due to the coupling of the compound nucleus levels to the particle decay channels and seems to be the dominant effect. Since this point has not been treated in the current literature we feel that a brief presentation of the formulas is useful. This is given in Sec. II. In Sec. III, the formulas are interpreted, and internal versus external mixing is considered. Specific examples of the theory are discussed in Sec. IV, and in Sec. V the theory is applied to experimental data.

II. AVERAGE CROSS SECTIONS AND FLUCTUATIONS IN THE CASE OF ISOSPIN MIXING

We consider a reaction leading from channel α to channel β via compound nucleus formation neglecting direct reactions in all channels. It is assumed that the compound nucleus has two classes[24] of levels, called class 1 and class 2, that have different isospin T_m with $T_1 < T_2 = T_0 + \frac{1}{2}$. The reaction partners a, A in all channels are assumed to have pure isospin T_a, T_A. The reaction of a with A leads to the formation of the compound nucleus in a state of class $m = 1$ or 2 with a probability governed by the transmission coefficients $\tau_{\alpha m}$. The transmission coefficients can be expressed by average properties of the compound nucleus via

$$\tau_{\alpha m}=\frac{4x_{\alpha m}}{(1+x_{\alpha 1}+x_{\alpha 2})^2}\ , \tag{2.1}$$

where

$$x_{\alpha m}=\pi^2\overline{(V_m^\alpha)^2}/D_m\ . \tag{2.2}$$

Here, $\overline{(V_m^\alpha)^2}$ is the mean square matrix element[21,22] coupling a state of class m to channel α, and D_m is the mean level spacing in class m. From these definitions it follows that the isospin dependent transmission coefficients $\tau_{\alpha m}$ are related through a vector coupling coefficient[25] to the usual ones τ_α which are calculated with an isospin independent optical model potential, i.e.,

$$\tau_{\alpha m}=(T_A,\ T_A^{(z)},\ T_a,\ T_a^{(z)}\,|\,T_m,\ T_A^{(z)}+T_a^{(z)})^2\tau_\alpha\ . \tag{2.3}$$

Upon formation of the compound nucleus, isospin mixing causes a diffusion process in which class 1 levels decay into class 2 levels and vice versa. The diffusion is controlled by two mechanisms: (i) the mean coupling $H_c^2=\overline{V_{12}{}^2}$ of the levels of classes 1 and 2, and (ii) the coupling of the levels of, say, class 1 to any open channel α which also couples to class 2 (virtual emission into the continuum and reabsorption into the other class of levels). This diffusion process influences the value of the correlation function $\langle S_{\alpha\beta}(E)S_{\alpha\beta}^*(E+\epsilon)\rangle$, where $S_{\alpha\beta}$ is the S matrix element and $\langle\ \rangle$ denotes the average over an energy interval large compared to the decay width of the compound nucleus. With the averaging techniques of Refs. 21 and 22 one finds[26] for $\alpha\neq\beta$

$$\langle S_{\alpha\beta}(E)S_{\alpha\beta}^*(E+\epsilon)\rangle=(\tau_{\alpha 1},\tau_{\alpha 2})\begin{pmatrix} N_1+z+2\pi i\epsilon/D_1 & -z \\ -z & N_2+z+2\pi i\epsilon/D_2\end{pmatrix}^{-1}\begin{pmatrix}\tau_{\beta 1}\\ \tau_{\beta 2}\end{pmatrix}. \tag{2.4}$$

For $\epsilon=0$, this result follows directly from Eq. (5.22) of Ref. 22. All quantities of interest, like average cross sections and autocorrelation functions of cross sections, can be expressed in terms of the correlation functions (2.4); see Ref. 22 and Eqs. (2.8) and (2.9), below.

In Eq. (2.4), N_m is the number of decay channels open to levels of class m,

$$N_m=\sum_\gamma \tau_{\gamma m}\ , \tag{2.5}$$

and the mixing parameter z is

$$z=4\pi^2\,\frac{\overline{V_{12}^2}}{D_1D_2}+\sum_\gamma \tau_{\gamma 1}\,x_{\gamma 2}\ . \tag{2.6}$$

The matrix in Eq. (2.4) is readily inverted to give

$$\begin{pmatrix} N_1+z+2\pi i\epsilon/D_1 & -z \\ -z & N_2+z+2\pi i\epsilon/D_2\end{pmatrix}^{-1}=\begin{pmatrix} N_2+z+2\pi i\epsilon/D_2 & z \\ z & N_1+z+2\pi i\epsilon/D_1\end{pmatrix}$$
$$\times[(N_1+z+2\pi i\epsilon/D_1)(N_2+z+2\pi i\epsilon/D_2)-z^2]^{-1}\ . \tag{2.7}$$

The set of formulas (2.4) through (2.7) allows for the calculation of average compound nucleus cross sections[27]

$$\sigma_{\alpha\beta}\equiv\langle\sigma_{\alpha\beta}(E)\rangle=\langle S_{\alpha\beta}(E)S_{\alpha\beta}^*(E)\rangle \tag{2.8}$$

and of autocorrelation functions[22,28]

$$\begin{aligned}C_{\alpha\beta}(\epsilon)&=\langle\sigma_{\alpha\beta}(E)\sigma_{\alpha\beta}(E+\epsilon)\rangle-\sigma_{\alpha\beta}{}^2\\&=|\langle S_{\alpha\beta}(E)S_{\alpha\beta}^*(E+\epsilon)\rangle|^2\ .\end{aligned} \tag{2.9}$$

III. INTERPRETATION, INTERNAL AND EXTERNAL MIXING

In order to demonstrate the physical picture behind Eqs. (2.4) to (2.9), it is useful to introduce a slightly different notation. The mean *decay width* $\Gamma_m^\uparrow$ of levels in class m is customarily defined by

$$\Gamma_m^\uparrow=N_mD_m/2\pi\ . \tag{3.1}$$

The spreading width for internal mixing $\Gamma_{m,\,\mathrm{int}}^\downarrow$ of levels of class m with those of class $n\neq m$ is due to the matrix elements $\overline{V_{mn}{}^2}$. In our case, we have

$$\Gamma_{1,\,\mathrm{int}}^\downarrow=2\pi\overline{V_{12}{}^2}/D_2,\quad \Gamma_{2,\,\mathrm{int}}^\downarrow=2\pi\overline{V_{12}{}^2}/D_1\ . \tag{3.2}$$

It is also useful to introduce a *spreading width for external mixing* $\Gamma_{m,\,\mathrm{ext}}^\downarrow$ of levels of class m with those of class $n\neq m$ due to virtual transitions to the channels. This quantity is defined by

$$\Gamma_{m,\,\mathrm{ext}}^\downarrow=D_m\Big(\sum_\gamma\tau_{\gamma 1}x_{\gamma 2}\Big)\Big/2\pi\ . \tag{3.3}$$

We note that because of the simplicity of the case of two classes of levels considered here the definition of $\Gamma^{\downarrow}_{m,\mathrm{ext}}$ is different from the general definition in Ref. 22.

We combine $\Gamma^{\downarrow}_{m,\mathrm{int}}$ and $\Gamma^{\downarrow}_{m,\mathrm{ext}}$ to define the total spreading width

$$\Gamma^{\downarrow}_{m}=\Gamma^{\downarrow}_{m,\mathrm{int}}+\Gamma^{\downarrow}_{m,\mathrm{ext}}\,. \tag{3.4}$$

The distinction between internal and external mixing is very similar to the one introduced by Robson in the theory of isolated analog resonances.[23] The width $\Gamma^{\downarrow}_{m,\mathrm{int}}$ is due to the internal mixing between the bound states embedded in the continuum introduced by the isospin-breaking Coulomb interaction V. The width $\Gamma^{\downarrow}_{m,\mathrm{ext}}$ arises from the coupling of the compound nuclear states to those decay channels which have either isotopic spin. Such channels are, for instance, channels in which one of the two fragments is a proton. This decomposition corresponds to the two mechanisms described below Eq. (2.3).

It will be shown below that the quantities of physical interest are the ratios $\Gamma^{\downarrow}_{m}/\Gamma^{\uparrow}_{m}$. This is not surprising; the ratio $\Gamma^{\downarrow}_{m}/\Gamma^{\uparrow}_{m}$ determines which fraction of the states of class m mixes with quasibound states in the other class, and which fraction decays into the open channels before such mixing takes place. This ratio incorporates the two criteria for isospin mixing which have been discussed separately in the literature.[29,30]

(a) *Static criterion.* Isospin mixing for states in class 1 is small if the ratio $(\overline{V_{12}^2})^{1/2}/D_2$ is small. This criterion can be derived from time-independent perturbation theory.

(b) *Dynamic criterion.* Isospin mixing is small if the mean life time $\hbar/\Gamma^{\uparrow}_{m}$ of compound states in class m is small compared to the time $\hbar/(\overline{V_{12}^2})^{1/2}$ it takes to mix the levels.

We see that these two criteria combined appear in $\Gamma^{\downarrow}_{m,\mathrm{int}}/\Gamma^{\uparrow}_{m}$. The ratio $\Gamma^{\downarrow}_{m,\mathrm{ext}}/\Gamma^{\uparrow}_{m}$ is similarly patterned, but refers to external mixing. With the definitions just given, the ratio z/N_m can be expressed in the form

$$z/N_m=\Gamma^{\downarrow}_{m}/\Gamma^{\uparrow}_{m}\,. \tag{3.5}$$

The denominator appearing on the right-hand side of Eq. (2.7) can be written in the form

$$\left|(\epsilon-i\lambda_1)(\epsilon-i\lambda_2)\right|^{-2}. \tag{3.6}$$

Therefore, the autocorrelation function is characterized by two Lorentzians, and by two associated correlation lengths λ_1, and λ_2 given by

$$\lambda_{1,2}=\tfrac{1}{2}\left(\Gamma^{\uparrow}_{1}+\Gamma^{\downarrow}_{1}+\Gamma^{\uparrow}_{2}+\Gamma^{\downarrow}_{2}\right)\pm\tfrac{1}{2}\left[(\Gamma^{\uparrow}_{1}+\Gamma^{\downarrow}_{1}-\Gamma^{\uparrow}_{2}-\Gamma^{\downarrow}_{2})^2+4\Gamma^{\downarrow}_{1}\Gamma^{\downarrow}_{2}\right]^{1/2}. \tag{3.7}$$

This shows explicitly how the unperturbed correlation lengths $\Gamma^{\uparrow}_{1}$ and $\Gamma^{\uparrow}_{2}$ are modified by the presence of the spreading widths $\Gamma^{\downarrow}_{1}$ and $\Gamma^{\downarrow}_{2}$. At present there seem to be a few experiments[19,38] where the analysis indicates the presence of two correlation lengths, associated with the existence of two classes of states. That analysis, however, did not incorporate isospin mixing.

In the expressions for cross sections given below, it is not the quantity z which appears but rather the renormalized quantity

$$\tilde{z}=z(1+z/N_1+z/N_2)^{-1}\,. \tag{3.8}$$

This is not surprising as $\tilde{z}$ incorporates the effects of all higher order terms in the isospin-breaking interaction, while z contains only the terms of lowest order. The expressions given below can be greatly simplified by defining, in analogy to Eq. (3.5),

$$\tilde{z}/N_m=\tilde{\Gamma}^{\downarrow}_{m}/\Gamma^{\uparrow}_{m}\,. \tag{3.9}$$

This quantity is, for $m=2$, the "fractional mixing parameter" μ introduced by several authors.[31] It describes the fraction of strength taken away from the levels of class 2 due to their mixing with the levels of class 1. The quantity $\tilde{z}/N_1$ is related to the opposite way of mixing, called "upward" mixing in Ref. 32. "Upward" mixing was introduced in order to improve the original theory[18] in which only "downward" mixing was taken into account. The present formulation takes care of both.

IV. SPECIFIC EXAMPLES

In this section we wish to apply the formalism outlined so far to some selected cases.

(i) *Isospin allowed reactions.* For an isospin allowed reaction $\alpha\rightarrow\beta$ which has nonvanishing transmission coefficients $\tau_{\alpha m}$, and $\tau_{\beta m}$ for just one class of levels, say $m=1$, the cross section reads

$$\sigma_{\alpha\beta}(\text{allowed})=\tau_{\alpha 1}(1-\tilde{\Gamma}^{\downarrow}_{1}/\Gamma^{\uparrow}_{1})(1/N_1)\tau_{\beta 1}\,. \tag{4.1}$$

In the limit of no mixing $z\rightarrow 0$, this expression becomes the usual Hauser-Feshbach formula

$$\sigma_{\alpha\beta}(\text{allowed})\approx\tau_{\alpha 1}(1/N_1)\tau_{\beta 1}\,, \tag{4.2}$$

where the levels of class 2 do not intervene any longer. In the limit of very strong mixing, $z\rightarrow\infty$, Eq. (4.1) tends towards

$$\sigma_{\alpha\beta}(\text{allowed})\approx\tau_{\alpha 1}\frac{1}{N_1+N_2}\tau_{\beta 1}\,, \tag{4.3}$$

which is the Hauser-Feshbach formula without distinction between the levels of class 1 and 2—an obvious result since the distinction between the classes is no longer meaningful. For arbitrary z, the interpretation of Eq. (4.1) is also obvious:

Only a fraction $(1-\tilde{\Gamma}_1^{\downarrow}/\Gamma_1^{\uparrow})$ of the levels of class 1 is available for decay back into the channels, each channel β being populated with the probability $\tau_{\beta 1}/N_1$.

For the autocorrelation functions we obtain from Eqs. (2.4), (2.7), and (2.9) in case of no mixing, i.e., $z=0$, the well known Lorentzian form[26]

$$C_{\alpha\beta}(\text{allowed},\ \epsilon)=\left(\tau_{\alpha 1}\frac{1}{N_1}\tau_{\beta 1}\right)^2 |1+i\epsilon/\Gamma_1^{\uparrow}|^{-2}. \tag{4.4}$$

The same result is obtained as long as

$$\Gamma_1^{\downarrow}/\Gamma_1 \ll 1. \tag{4.5}$$

As discussed below, this condition should be fulfilled very often.

(ii) *Isospin forbidden reactions.* The cross section for an isospin forbidden reaction $\alpha\to\beta$, where the channel α has nonvanishing transmission coefficients only with states of class 1 and channel β only with states of class 2, turns out to be

$$\sigma_{\alpha\beta}(\text{forbidden})=\tau_{\alpha 1}(\tilde{\Gamma}_1^{\downarrow}/\Gamma_1^{\uparrow})(1/N_2)\tau_{\beta 2}. \tag{4.6}$$

The interpretation of this expression is obvious.

In the analysis of compound nucleus cross sections the expression $\tau_{\alpha m}\tau_{\beta n}$ is usually calculated and compared to the measured $\sigma_{\alpha\beta}$. The analysis of an isospin-allowed and of an isospin-forbidden reaction as e.g., in Refs. 8, 9, and 13, then yields a suppression factor f of the forbidden with respect to the allowed transition. From Eqs. (4.1) and (4.6) f is seen to be

$$f=\Gamma_2^{\downarrow}/(\Gamma_2^{\uparrow}+\Gamma_2^{\downarrow}). \tag{4.7}$$

It may perhaps by surprising that f, the supression factor for reaction feeding primarily states of class *one*, should only depend on the widths $\Gamma_2^{\uparrow}$ and $\Gamma_2^{\downarrow}$ of states of class *two*. This, however, is due to the definition of f. A physically more appealing definition is obtained if one were to calculate the expressions $\tau_{\alpha 1}(1/N_1)\tau_{\beta 1}$ and $\tau_{\alpha 2}(1/N_2)\tau_{\beta 2}$. This leads to another suppression factor $\bar{f}$ with

$$\bar{f}=\frac{\Gamma_1^{\downarrow}}{\Gamma_1^{\uparrow}}\ \frac{\Gamma_2^{\uparrow}}{\Gamma_2^{\uparrow}+\Gamma_2^{\downarrow}}, \tag{4.8}$$

which is obviously more reasonable. However, in order to keep things simple we follow common usage and work with the factor f.

To estimate f, it is useful to have first an estimate of $\Gamma_{2,\text{ext}}^{\downarrow}/\Gamma_2^{\uparrow}$. The latter quantity can be estimated in the limit of strong absorption in all channels, i.e., the case in which the isospin-independent transmission coefficients τ_α are all unity. This implies $(x_{\alpha 1}+x_{\alpha 2})=1$ for all channels α. Let us also suppose that proton channels are the only open channels with dual isospin, and that the levels of class 2 can decay only into these channels in an isospin-allowed fashion. Then we have from Eqs. (2.1), (2.3), (3.1), and (3.3),

$$\begin{aligned}\Gamma_{2,\text{ext}}^{\downarrow}&=(T_0,T_0,\tfrac{1}{2},-\tfrac{1}{2}\mid T_0-\tfrac{1}{2},T_0-\tfrac{1}{2})^2\Gamma_2^{\uparrow}\\&=[2T_0/(2T_0+1)]\Gamma_2^{\uparrow}.\end{aligned} \tag{4.9}$$

Here, T_0 is the isospin of the residual nucleus C that remains after proton emission, while $T_2=T_>=T_0+\frac{1}{2}$. We consider (4.9) an upper limit for $\Gamma_{2,\text{ext}}^{\downarrow}$, since it is based on the assumption[33] $(x_{\alpha 1}+x_{\alpha 2})=1$.

From arguments given in the theory of isobaric analog resonances one expects that internal mixing is smaller than external mixing. This implies together with Eq. (4.9)

$$\Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}\lesssim 1. \tag{4.10}$$

Note that for compound nuclei with neutron excess the levels of class 2 having the higher isospin $T_2=T_0+\frac{1}{2}$ have many fewer decay channels available than the levels of class 1, i.e.,

$$N_1 \gg N_2. \tag{4.11}$$

The reason is that levels of class 2 can decay via neutron emission only into the $|nA\rangle$ channels where A is the analog state of C. The hypotheses (4.10) and (4.11) imply the inequality (4.5).

(iii) *General case.* The cross section $\sigma_{\alpha\beta}$ reads

$$\begin{aligned}\sigma_{\alpha\beta}=\tau_{\alpha 1}\left(1-\frac{\tilde{\Gamma}_1^{\downarrow}}{\Gamma_1^{\uparrow}}\right)\frac{1}{N_1}\tau_{\beta 1}+\tau_{\alpha 1}\left(\frac{\tilde{\Gamma}_1^{\downarrow}}{\Gamma_1^{\uparrow}}\right)\frac{1}{N_2}\tau_{\beta 2}\\+\tau_{\alpha 2}\left(\frac{\tilde{\Gamma}_2^{\downarrow}}{\Gamma_2^{\uparrow}}\right)\frac{1}{N_1}\tau_{\beta 1}+\tau_{\alpha 2}\left(1-\frac{\tilde{\Gamma}_2^{\downarrow}}{\Gamma_2^{\uparrow}}\right)\frac{1}{N_2}\tau_{\beta 2}.\end{aligned} \tag{4.12}$$

Equation (4.12) contains four terms: Two of them correspond to allowed transitions in which isospin is not changed from $\alpha\to\beta$ and two of them describe forbidden transitions where it is changed. Furthermore, it can be seen that with increasing z the strengths of the allowed terms decrease in favor of the forbidden terms. This is a consequence of unitarity. In the limit of strong mixing, Eq. (4.12) attains the form

$$\sigma_{\alpha\beta}=(\tau_{\alpha 1}+\tau_{\alpha 2})\frac{1}{N_1+N_2}(\tau_{\beta 1}+\tau_{\beta 2}), \tag{4.13}$$

as it should.

In the general case the autocorrelation function is made up of a sum of interfering terms.[34]

V. APPLICATION TO EXPERIMENTAL RESULTS

(i) *Cross section fluctuations.* As explained in Sec. IV the quantity f of Eq. (4.7) can be extracted by comparing allowed and forbidden compound nucleus cross sections. In Ref. 8 this has been done for the reaction $^{28}\text{Si}(d,\alpha)^{26}\text{Al}$. Statistically fluctuating cross sections have been measured. The contribution of direct reactions was determined from the variances of the excitation functions.[35] Hauser-Feshbach calculations have been performed that yield the expressions formally called[27] $\tau_{\alpha m}\tau_{\beta n}$, and then from Eqs. (4.1) and (4.6) the factor f was determined. The result is given in Table I. We have reanalyzed the higher energy data of Ref. 9, performed the Hauser-Feshbach calculations, and obtained f (see Table I). For comparison, the quantity

$$f_{\max}^{\text{ext}} = 2T_0/(4T_0+1) \tag{5.1}$$

gives the maximum of f that is expected to arise from external mixing alone [see Eq. (4.7) and the discussion leading to Eq. (4.9)].

TABLE I. Isospin mixing in various compound nuclei. The quantities f, μ, and $f_{\max}^{\text{ext}}$ are defined by Eqs. (4.7), (5.6), and (5.1), respectively; T_0 is the isospin of the residual nucleus that remains after proton emission; $H_c^{\max}$ is an upper limit of the Coulomb matrix element (see text). This table is based on Table II of Ref. 36 except for the first two entries, which are from Refs. 8 and 9.

Compound nucleus	Excitation energy	T_0	f or μ	$f_{\max}^{\text{ext}}$	$H_c^{\max}$ (eV)
^{30}P	20.4	$\frac{1}{2}$	0.25 ± 0.05	0.33	
	25.4		0.06 ± 0.03		
^{49}V	20.5	2	0.20 ± 0.17	0.44	52
^{52}Cr	24.2	$\frac{5}{2}$	0.37 ± 0.07	0.45	110
^{55}Mn	21.8	3	0.33 ± 0.20	0.46	36
^{56}Fe	23.9	$\frac{5}{2}$	0.44 ± 0.22	0.45	110
^{60}Ni	23.3	$\frac{5}{2}$	0.37 ± 0.21	0.45	130
^{63}Cu	19.9	3	0.46 ± 0.21	0.46	27
^{64}Zn	17.6	$\frac{5}{2}$	0.70 ± 0.15	0.45	300
	19.0		0.55 ± 0.16		170
	20.5		0.44 ± 0.11		110
	22.0		0.34 ± 0.15		78
	23.5		0.40 ± 0.12		67
^{66}Zn	22.7	$\frac{7}{2}$	0.68 ± 0.10	0.47	51
^{69}Ga	17.4	4	0.41 ± 0.04	0.47	24
	18.4		0.45 ± 0.03		19
	20.4		0.39 ± 0.10		8.9
	22.4		0.34 ± 0.09		4.6
^{111}In	20.9	7	0.68 ± 0.14	0.48	0.18

(ii) *Evaporation spectra.* Most of the results collected in Table I are based on studies of evaporation spectra of (α,α'), (p,p'), (p,α'), and (α,p') reactions, all leading to the same compound nucleus. The principle is as follows. If isospin selection rules do not play any rôle, then the ratio of cross sections

$$R = \frac{\sigma(\alpha,\alpha')\sigma(p,p')}{\sigma(\alpha,p')\sigma(p,\alpha')} \tag{5.2}$$

is approximately unity. Within the framework of Sec. IV this may be expressed as

$$\begin{aligned} R_{\text{CM}} &= R(\text{comp mix}) \\ &= \left(\tau_\alpha \frac{1}{N_1+N_2} \tau_{\alpha'}\right)\left(\tau_p \frac{1}{N_1+N_2} \tau_{p'}\right) \\ &\quad \times \left(\tau_\alpha \frac{1}{N_1+N_2} \tau_{p'}\right)^{-1}\left(\tau_p \frac{1}{N_1+N_2} \tau_{\alpha'}\right)^{-1} \\ &\approx 1\,. \end{aligned} \tag{5.3}$$

The quantity R_{CM} is not exactly unity because the angular momenta involved in the four types of reactions may not be the same. If isospin is strictly conserved, then R is larger than unity since the (p,p') reaction can proceed through both classes of levels, while the other three reactions feed only class 1 levels. From Eq. (4.2) one obtains

$$\begin{aligned} R_{\text{NM}} &= R(\text{no mix}) \\ &= \left(\tau_{\alpha 1} \frac{1}{N_1} \tau_{\alpha' 1}\right)\left(\tau_{p1} \frac{1}{N_1} \tau_{p'1} + \tau_{p2} \frac{1}{N_2} \tau_{p'2}\right) \\ &\quad \times \left(\tau_{\alpha 1} \frac{1}{N_1} \tau_{p'1}\right)^{-1}\left(\tau_{p1} \frac{1}{N_1} \tau_{\alpha' 1}\right)^{-1} \\ &= R_{\text{CM}}[1+N_1/(4T_0^2N_2)]\,. \end{aligned} \tag{5.4}$$

With the help of Eq. (4.12) the general case can be worked out (see Appendix):

$$\begin{aligned} R/R_{\text{CM}} &= 1+[(1-\mu)N_1/N_2-\mu] \\ &\quad \times [(1-\mu N_2/N_1)2T_0+\mu]^{-2}\,. \end{aligned} \tag{5.5}$$

Here, the abbreviation

$$\mu = \tilde{\Gamma}_2^{\downarrow}/\Gamma_2^{\uparrow} \tag{5.6}$$

has been used. Equation (5.5) allows the determination of μ provided R_{CM} and the ratio N_1/N_2 may be calculated from level density expressions.[14-17] Since with the inequality (4.5) $f \approx \mu$ [see Eqs. (3.8), (3.9), and (4.7)], the results of the fluctuation experiments are directly comparable to the results from the evaporation spectra and they are all listed under the common heading "f or μ" in Table I. The theory on which Refs. 14–17 are based as well as its refinement in Ref. 36 is different from the present one. An argument why this arises is given in the Appendix. We have therefore recalculated μ from the quantities G and $G_{\max}$ given in Table II of Ref. 36 using Eq. (5.5),

$$G = R/R_{\text{CM}} \tag{5.7}$$

and

$$G_{\max} = 1+N_1/(4T_0^2N_2)\,. \tag{5.8}$$

(iii) *Discussion.* Inspecting the results in Table I, two different approaches for their interpretation may be taken. Firstly, these results can be explained without any internal mixing since the quantity $f_{\rm max}^{\rm ext}$ calculated from Eq. (5.1) is in only three cases out of 19 significantly smaller than f determined from experiment. Secondly, if f is used to estimate Coulomb matrix elements as in Refs. 20 and 36, the result has to be considered as an upper limit $H_c^{\rm max}$, because internal mixing is then taken as the only source for isospin breaking. Using the definition (5.6) and Eqs. (3.8) and (3.9), we express μ as

$$\mu = (N_2/z + N_2/N_1 + 1)^{-1} . \qquad (5.9)$$

Solving this for $z/N_2 = \Gamma_2^{\downarrow}/\Gamma_2^{\uparrow}$ and setting

$$\Gamma_2^{\downarrow} = 2\pi (H_c^{\rm max})^2/D_1 \qquad (5.10)$$

in analogy to Eq. (3.2), one gets

$$(H_c^{\rm max})^2 = \frac{1}{2\pi}\, \frac{\mu \Gamma_2^{\uparrow} D_1}{1 - \mu(N_2/N_1 + 1)} . \qquad (5.11)$$

Note that this expression is different from Eq. (9) of Ref. 36 even in the limit of negligible N_2/N_1, since in Ref. 36 (using our notation) $\Gamma_1^{\uparrow}$ appears instead of $\Gamma_2^{\uparrow}$. We have recalculated $H_c^{\rm max}$ from the values of μ in Table I and the information given in table II of Ref. 36. The level densities $1/D_1$, and $1/D_2$ are listed there as well as $\Gamma_1^{\uparrow}$. The decay width $\Gamma_2^{\uparrow}$ can then be found from the relation

$$\Gamma_2^{\uparrow} = \Gamma_1^{\uparrow} D_2 N_2/(D_1 N_1) \qquad (5.12)$$

[see Eq. (3.1)], since the ratio N_2/N_1 is given by Eq. (5.8).

In order to derive reliable Coulomb matrix elements in highly excited compound nuclei, external mixing has to be estimated from theory. This amounts to making a model for x in Eq. (2.6). The strong absorption model used for the estimates $f_{\rm max}^{\rm ext}$ quoted in Table I is certainly not realistic since the experimentally determined quantities f and μ are in many cases smaller than the theoretical ones.

Further, it is interesting to note that the upper limits of H_c in the case of highly excited nuclei are much smaller than the Coulomb matrix elements found at low excitation energies and that H_c is a rather strong function of excitation energy. The fact that there is no "typical" Coulomb matrix element for any given nucleus could be explained by the assumption that any given class 2 state mixes only with its antianalog[37] configurations. There is only the fixed number $2T_0$ of them available, which means that the average mixing matrix element has to decrease inversely proportional to the total number of levels in class 1.

VI. CONCLUSION

We have shown in the present investigation that the possibility of extracting mixing matrix elements for nuclear states at high excitation energy is hampered by external mixing which has so far been completely neglected in theoretical analyses. A formula for the determination of an upper limit of the Coulomb matrix element has been given in Eq. (5.11). Despite the fact that we were able to estimate the importance of external mixing, we should not overlook the inability of the present theory to predict reliably this quantity or alternatively the quantity x in Eq. (2.2). If $x \lesssim 1$ then external mixing is uniquely connected to the transmission coefficients [see Eqs. (2.1) and (2.3)].

The theory has been applied to experimental data. Two sets of experiments were analyzed, excitation functions of fluctuating cross sections for isospin-forbidden reactions and evaporation cross sections in mainly isospin-allowed nuclear processes. The former experiments are more sensitive to isospin mixing because it enters into the latter ones only as a correction. However, the comparison of an allowed and a forbidden reaction can only be done in selected cases.

The suppression factor f that is usually extracted from the comparison of an isospin-allowed and a forbidden compound nucleus reaction has been related to the fractional mixing parameter or the ratio of spreading to decay width [see Eqs. (3.8), (3.9), and (4.7)].

A new formula for the extraction of the fractional mixing parameter from evaporation spectra is given in Eq. (5.5). We note that by using the absolute values of all four cross sections $\sigma(\alpha,\alpha')$, $\sigma(p,p')$, $\sigma(\alpha,p')$, and $\sigma(p,\alpha')$ the three parameters of the theory can all be determined. These are the numbers N_1 and N_2 of open decay channels and the fractional mixing parameter μ. If only the cross section ratio R of Eq. (5.2) is used, then N_1/N_2 has to be calculated. This was implicitly done in Refs. 14–17 and 36. However, the formulas of these references have a structure basically different from the theory presented here.

The present formalism gives new results on the correlation functions of statistically fluctuating cross sections. They are characterized by two correlation lengths.

Two of us (H.L.H. and A.R.) thank D. Robson for extensive and enlightening discussions about the subject of isospin mixing in highly excited compound nuclei.

APPENDIX: EXTRACTION OF THE ISOSPIN MIXING PARAMETER FROM CROSS SECTION RATIOS

In this Appendix, Eq. (5.5) shall be derived.

We use the abbreviation (5.6), from which follows by virtue of Eq. (3.5)

$$\tilde{\Gamma}_1^{\downarrow}/\Gamma_1^{\uparrow} = \mu N_2/N_1 . \tag{A1}$$

Note that the transmission coefficients for the α-particle decay channel are

$$\tau_{\alpha 1} = \tau_\alpha , \tag{A2}$$

$$\tau_{\alpha 2} = 0 , \tag{A3}$$

and for the proton decay channel

$$\tau_{p1} = \tau_p 2T_0/(2T_0+1) , \tag{A4}$$

$$\tau_{p2} = \tau_p/(2T_0+1) . \tag{A5}$$

Using Eq. (4.12), the cross section ratio R reads

$$R = \tau_\alpha \frac{1}{N_1}\left(1-\mu\frac{N_2}{N_1}\right)\tau_{\alpha'}\left[4T_0^2\tau_p \frac{1}{N_1}\left(1-\mu\frac{N_2}{N_1}\right)\tau_{p'} + 4T_0\tau_p \frac{1}{N_1}\mu\tau_{p'} + \tau_p\frac{1}{N_2}(1-\mu)\tau_{p'}\right]$$
$$\times\left[\tau_\alpha\frac{1}{N_1}\left(1-\mu\frac{N_2}{N_1}\right)2T_0\tau_{p'} + \tau_\alpha\frac{1}{N_1}\mu\tau_{p'}\right]^{-1}\left[2T_0\tau_p\frac{1}{N_1}\left(1-\mu\frac{N_2}{N_1}\right)\tau_\alpha + \tau_p\frac{1}{N_1}\mu\tau_\alpha\right]^{-1} . \tag{A6}$$

If one assumes that N_1 and N_2 have the same dependence on the total spin, then

$$\left(\tau_\alpha\frac{1}{N_1}\tau_{\alpha'}\right)\left(\tau_p\frac{1}{N_1}\tau_{p'}\right)\left(\tau_\alpha\frac{1}{N_1}\tau_{p'}\right)^{-1}\left(\tau_p\frac{1}{N_1}\tau_{\alpha'}\right)^{-1}$$

is equal to R_{CM} defined in Eq. (5.3), and R may be rewritten in the form

$$R = R_{\text{CM}}[((1-\mu N_2/N_1)2T_0)^2 + 2(1-\mu N_2/N_1)2T_0\mu + (1-\mu)(1-\mu N_2/N_1)N_1/N_2]\,[(1-\mu N_2/N_1)2T_0+\mu]^{-2}$$
$$= R_{\text{CM}} + R_{\text{CM}}[(1-\mu)N_1/N_2 - \mu]\,[(1-\mu N_2/N_1)2T_0+\mu]^{-2} , \tag{A7}$$

which proves Eq. (5.5).

For $\mu \ll 1$ and $N_1/N_2 \gg 1$, Eq. (A7) can be approximated by

$$R/R_{\text{CM}} \approx 1 + (1-\mu)(2T_0+\mu)^{-2}N_1/N_2 . \tag{A8}$$

This formula resembles Eq. (6) of Ref. 15. However, the theory of Ref. 15 as well as its more sophisticated version in Ref. 36 is different from the results of the present paper even to first order in μ. This is owing to the fact that in Ref. 36 isospin mixing is introduced through a modification of the transmission coefficients so that all three factors of the Hauser-Feshbach formula, τ_α, N_m, and τ_β, carry the isospin impurity. In our formulation, which is based on a rigorous theory, all isospin mixing is contained in the matrix inverse appearing on the right-hand side of Eq. (2.4). This suggests that the procedure used in Ref. 36, although based upon intuitively appealing arguments, leads to some kind of double counting of isospin-breaking effects.

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[24] For the low-energy reactions considered below it is not necessary to introduce more than two isospin values. This would be necessary only at substantially larger excitation energies.

[25] The use of the isospin formalism should not obscure the fact that, in some channels, isospin conservation is badly violated. Most noteworthy here are the proton channels which are linear combinations of states with isospin T_1 and T_2 and thus mainly responsible for "external" isospin mixing. A similar statement applies to all channels in which one fragment has $T_z < T$.

[26] For simplicity of notation, we suppress here and in the sequel the case of elastic scattering and the associated elastic enhancement factor.

[27] Note that our way of expressing the cross section is rather formal since the channel indices α and β represent vector-coupled partial waves of definite total angular momentum J and parity π. For simplicity of presentation we suppress all angular momentum recoupling coefficients and the summations over them which arise in cross section formulas.

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[34] Part of this result was already obtained in Eqs. (2.15) and (2.16) of Ref. 19. There the situation of dual isospin is formulated but without isospin mixing. In this case no terms appear which contain two poles as in Eq. (3.6) of the present paper.

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