Microscopic calculations for the ^{4}H and ^{4}Li continuum*

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A new model for the ^{4}H and ^{4}Li nuclei in which structure and reaction aspects are treated on an equal footing is introduced. The internal states are expanded on a basis of properly antisymmetrized translationally invariant harmonic oscillator eigenstates including all states up to $4\hbar\omega$ oscillator excitation. All two-body breakup channels, namely $n + {}^3$H or $p + {}^3$He, are explicitly included. The two-body interaction is based on the Sussex matrix elements. Model widths and positions of the excited states and elastic scattering cross sections are in qualitative agreement with the data. Three- and four-body breakup channels are ignored.

[NUCLEAR STRUCTURE Calculated ^{4}H and ^{4}Li level positions and widths.

NUCLEAR REACTIONS Calculated differential cross sections for ^{3}H(n,n) and ^{3}He(p,p) for $E_{c.m.} = 5.1$–8.1 MeV.]

I. INTRODUCTION

Continued experimental and theoretical interest in the ^{4}H and ^{4}Li systems is well documented by the recent data compilation of Fiarman and Meyerhof.[1] Despite the recent accumulation of new data, the detailed resonance structure of the low energy continuum of the ^{4}H system[2–4] and the ^{4}Li system[5–7] is still not well understood. From a theoretical viewpoint, the ^{4}H and ^{4}Li systems are of fundamental interest because the $A = 4$ system is the lightest nuclear system in which excited states have been identified as well as the lightest system in which analog states ($T = 1$) can be systematically studied.

The ^{4}H and ^{4}Li systems have been the subject of a considerable body of theoretical work. Previous calculations can be roughly grouped into two categories: those which emphasize the structure features of the system and those which emphasize the reaction aspects. In the former category are the ^{4}He $T = 1$ shell model calculations of Kramer and Moshinsky,[8] Szydlik,[9] Szydlik, Borysowicz, and Wagner,[10] and Bevelacqua and Philpott.[11] There have been no major structure efforts for either ^{4}H or ^{4}Li except for calculations involving the position of the 2^- ground state of the ^{4}H system[6] and the ^{4}Li system.[6,7] Resonance widths have been estimated by applying R-matrix techniques to the shell-model eigenstates.[7,12] In the latter category, are the various resonating group calculations for elastic $n + {}^3$H scattering[2,13–15] and $p + {}^3$He scattering.[14,16,17]

In order to complete the picture, what is clearly needed is a comprehensive theoretical treatment in which both structure and reaction aspects are treated on an equal footing.[11,18] Since only four nucleons are involved, the theoretical description can afford to be microscopically explicit.

Here we attempt to describe the ^{4}H and ^{4}Li nuclei and their interactions with all possible two-body breakup channels (namely, the $n + {}^3$H or $p + {}^3$He channels) by means of a dynamical model constructed within the framework of the Lane-Robson R-matrix method.[19] In this paper, we calculate the positions and widths of the various resonances predicted by our model for comparison with the corresponding experimental quantities. We also obtain $n + {}^3$H and $p + {}^3$He elastic scattering cross sections. The preliminary results are encouraging. In later work, we intend to use the same model, with certain adjustments, to perform a comprehensive study of elastic as well as other reaction processes such as ^{3}He$(n,np)^2$H, ^{3}He$(p,pp)^2$H, ^{3}H$(n,np)2n$, and ^{3}He$(p,np)2p$.

II. FORMALISM

The model for the bound and continuum states represents an application of the dynamical equations of the Lane-Robson[19] R-matrix methodology to the ^{4}H and ^{4}Li systems. The application, here, is very similar to that employed for the ^{4}He system.[11,18] The choice of basis states and evaluation of matrix elements was discussed in Ref. 11. The dynamical equations can be written in the form[20]

$$\sum_{\lambda'} \left[\langle \lambda | H - E | \lambda' \rangle + \sum_c \gamma_{\lambda c} (b_{\lambda' c} - b_c) \gamma_{\lambda' c} \right] A_{\lambda' c} = 0 , \qquad (1)$$

where H is the Hamiltonian describing the system of interest and $\gamma_{\lambda c}$ and $b_{\lambda c}$ are the reduced widths[21] and logarithmic derivatives associated with the expansion states $|\lambda\rangle$.

Cluster wave functions for the three-nucleon

systems are needed in order to define the various breakup channels. These wave functions are obtained by diagonalizing the three-nucleon Hamiltonians within suitable sets of oscillator eigenfunctions. Thus, for example, an approximate triton cluster wave function formed from oscillator eigenfunctions in the coordinates $\vec{r}_{12}$ and $\vec{r}_B$ of Fig. 1 of Ref. 11 is employed in the calculation of quantities describing the connection to the $p+{}^3$H channel.

Solutions of the dynamical R-matrix equations are obtained by the procedures outlined in Ref. 22. After the transformations of Ref. 22 have been carried out, the R matrix takes on the standard form[21]

$$R_{cc'}=\sum_{\mu}\frac{\gamma_{\mu c}\gamma_{\mu c'}}{E_{\mu}-E}, \tag{2}$$

where the quantities E_μ and $\gamma_{\mu c}$ are calculated directly from the information which appears in Eq. (1). By this means, the resonance structure of the theory is made explicit. Scattering and reaction cross sections are obtained from the R matrix via the S matrix by means of standard formulas.[21]

Specific formulas for the positions and widths of R-matrix resonances are available in the literature.[21,23] In particular, the resonance energy E_R^μ corresponding to the level E_μ may be defined as the solution to the equation

$$E_R^\mu=\mathrm{Re}[E_\mu-\xi_\mu(E_R^\mu)]. \tag{3}$$

The total width of the resonance is then obtained from the equation

$$\Gamma_\mu=-2\,\mathrm{Im}[E_\mu-\xi_\mu(E_R^\mu)], \tag{4}$$

where ξ_μ is itself defined in terms of known R-matrix energies and reduced widths $E_\mu, \gamma_{\mu c}$, and standard Coulomb radial functions.[21,23]

III. CHOICE OF INTERACTION

In Ref. 11 an effective interaction for oscillator basis states was determined for the two, three, and four nucleon systems. This interaction was determined from the Sussex matrix elements[24] and is of the form

$$V^{\mathrm{eff}}=CV^{\mathrm{Sussex}}, \tag{5}$$

where C is a strength parameter of order unity. The parameter C and the oscillator size parameter b were varied independently. Triton and helion binding energies are both underpredicted by 5% with this interaction. The calculated rms radii are underpredicted by 8% for the helion and 2% for the triton. In addition, the ^{3}H-^{3}He Coulomb energy difference is in agreement with experiment. The changes from the original Sussex matrix elements implied by our choice of C are typically of the same order of magnitude as the expected uncertainties in the matrix elements themselves.[24]

TABLE I. Comparison of model resonances ($T=1$) and widths with experiment for the ^{4}H system.

E_{exp}[a] (MeV)	J^π	$E_R^{\mathrm{unshifted}}$ (MeV)	$\Gamma^{\mathrm{unshifted}}$ (MeV)	E_R^{shifted} (MeV)	$\Gamma^{\mathrm{shifted}}$ (MeV)	Γ^{exp}[b] (MeV)
3.4	2^-	6.2	1.2	3.4	0.7	~10.0
5.1	1^-	9.1	3.3	5.1	2.5	~10.0
~5.7	0^-	6.7	3.0	5.7	2.0	~10.0
7.5	1^-	10.0	2.6	7.5	2.5	~10.0

[a] Energy relative to the $n+{}^3$H threshold.
[b] Estimated from single particle widths of Ref. 12.

IV. RESULTS AND DISCUSSION (STRUCTURE)

The calculated level positions and widths are summarized in Tables I and II for the ^{4}H and ^{4}Li systems. The energy spectrum $E_R^{\mathrm{unshifted}}$ is obtained from the modified Sussex interaction, Eq. (5). The level spectra of both ^{4}H and ^{4}Li are predicted reasonably well by our model and are qualitatively similar to the $T=1$ levels of the ^{4}He system.[11] The calculated levels lie above the corresponding experimental positions and have the order 2^-, 0^-, 1^-, and 1^-. This ordering is not in agreement with the 2^-, 1^-, 0^-, and 1^- scheme,[1] but resembles solution II of Werntz and Meyerhof.[12] Although no other states have been identified in experimental analyses, there is a host of positive parity states which lie above the four identified negative parity levels. Our calculated level position and order are similar to the prediction of Ref. 10.

The calculated total widths are typically found to agree with widths extracted from data to within a factor of 4. The model widths for ^{4}H and ^{4}Li are nearly equal. This equality is also suggested by reduced widths based on R-matrix parametrizations of existing ^{4}H and ^{4}Li data.[1] It is also important to note that the widths, summarized in Tables I and II, are based on single reaction channel calculations. The effect of omitting the three-

TABLE II. Comparison of model resonances ($T=1$) and widths with experiment for the ^{4}Li system.

E_{exp}[a] (MeV)	J^π	$E_R^{\mathrm{unshifted}}$ (MeV)	$\Gamma^{\mathrm{unshifted}}$ (MeV)	E_R^{shifted} (MeV)	$\Gamma^{\mathrm{shifted}}$ (MeV)	Γ^{exp}[b] (MeV)
4.7	2^-	7.3	1.2	4.7	0.8	~10.0
6.1	1^-	10.1	3.0	6.1	2.9	~10.0
7.9	0^-	8.9	2.9	7.9	2.4	~10.0
9.8	1^-	12.3	2.6	9.8	2.5	~10.0

[a] Energy relative to the $p+{}^3$He threshold.
[b] Estimated from single particle widths of Ref. 12.

TABLE III. Comparison of $T=1$ widths in the $A=4$ systems.

J^π	$\Gamma(^4\text{He})$ (MeV, $p+^3\text{H}$, $n+^3\text{He}$, and $d+^2\text{H}$ channels)	$\Gamma(^4\text{He})$ (MeV, $p+^3\text{H}$ channel)	$\Gamma(^4\text{Li})$ (MeV, $p+^3\text{He}$ channel)	$\Gamma(^4\text{H})$ (MeV, $n+^3\text{H}$ channel)
2^-	1.4	0.7	0.8	0.7
1^-	4.4	2.0	2.9	2.5
0^-	4.0	2.0	2.4	2.0
1^-	5.1	2.4	2.5	2.5

and four-body channels on the level width is difficult to determine without detailed analysis but a qualitative estimate can be gained from past ^{4}He calculations.[11,18]

The effect of including more than a single channel on the level widths of ^{4}He is illustrated in Table III. In general, the inclusion of the $p+^3$H, $n+^3$He, and $d+^2$H channels yields widths which are at least a factor of 2 larger than single channel $p+^3$H calculations. The widths for single channel calculations for the ^{4}H, ^{4}He, and ^{4}Li systems are very similar. These results suggest that the inclusion of the three- and four-body breakup channels in ^{4}H and ^{4}Li will have a sizable impact on the calculated widths.

These initial results are very encouraging, but there is much additional information which can be obtained by slight modifications to the existing solutions. For the moment, it will suffice to remark that the calculated cross sections are obtained as superpositions of the various angular momentum and parity states which themselves are each described as a sequence of resonances. If the actual resonances of the ^{4}H and ^{4}Li system are not accurately reproduced by our calculations, the resulting cross sections may be expected to suffer considerable distortion.[11]

This suggests the need for certain adjustments in our model parameters. In general, the search for an improved effective interaction is likely to be exceedingly tedious. However, some of the inadequacies of the present model can be at least partially compensated by the simple expedient of adjusting E_μ in a manner as to bring the predicted resonances closer to their observed positions. The approach of adjusting E_μ in Eq. (3) such that $E_R^\mu = E_{\text{exp}}$ produces the E_R^{shifted} level spectrum of Tables I and II. In this way, the resonance energies can be reproduced with eigenstates whose internal structure and corresponding partial widths have been determined by the model interaction. For these reasons, our scattering results will be based on the shifted levels.

V. RESULTS AND DISCUSSION (REACTIONS)

There exists a large amount of $p+^3$He as well as $n+^3$H data. The $p+^3$He data are generally expected to be somewhat more accurate than the $n+^3$H data and, for this reason, we will concentrate our efforts on the ^{4}Li system.

Elastic $p+^3$He scattering results are compared with the measurements of Ref. 25 in Fig. 1. The calculations agree with the experimental fact that the backward peaking of the angular distributions decreases with increasing energy. The 5.1 MeV results involve scattering processes which include

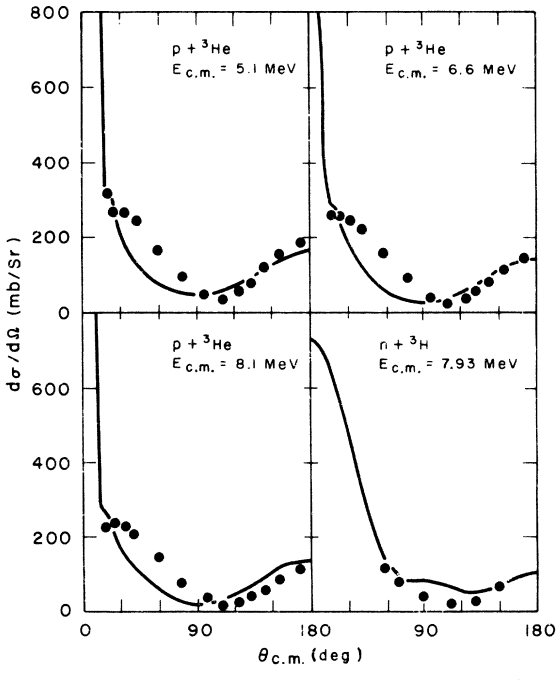

FIG. 1. Differential cross sections for elastic $p+^3$He and $n+^3$H scattering. The data are taken from Ref. 25 for ^{4}Li and from Ref. 26 for ^{4}H. $E_{\text{c.m.}}$ is measured with respect to the entrance channel threshold and is expressed in MeV.

only the $p+{}^3$He channel. The data are qualitatively reproduced by the model and the deviation from experiment are probably caused by inadequacies in our helion wave function and uncertainties in the position of the experimental levels. The 6.6 MeV data lie above the $2p+{}^2$H channel and the 8.1 MeV measurements are above the $n+3p$ threshold. The angular distributions for energies above the three- and four-body channels are also in qualitative agreement with the data except at forward angles. Although this effect is most likely caused by the omission of various reaction channels, the magnitude of this effect can be determined only after a more complete understanding of the ^{4}Li level scheme is obtained.

The discrepancies between theory and experiment described above have also been reported in single channel resonating group calculations.[14] Reichstein, Thompson, and Tang[14] report the discrepancy between theory and experiment around the Coulomb interference minimum at about 15° and the deep diffraction minimum at about 120° is a consequence of omitting reaction channels and using a purely central nucleon-nucleon interaction. Although our interaction includes central as well as other force components, we observe similar effects.

Elastic $n+{}^3$H scattering results, shown in Fig. 1, are qualitatively similar to the $p+{}^3$He results described above. The agreement between theory and data[26] is good at forward and back angles, but discrepancies occur near the minimum at about 120°. These results are similar to those from the ^{4}Li system and further suggest a more detailed analysis of the effects of level position and reaction channels is needed before a good understanding of the ^{4}H and ^{4}Li systems can be obtained.

VI. CONCLUSIONS

Despite the model character of our method, the results are in qualitative agreement with experiment. The ^{4}H and ^{4}Li states are found to be broad and overlap neighboring levels. Three- and four-body breakup channels appear to be important not only in cross section calculations but also in calculations of level widths and positions.

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