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Alpha-transfer reactions and the pairing-vibration model*

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The pairing-vibration model with isospin is extended to include α-transfer reactions. Selection rules and expressions for transition strengths are derived and compared with experimental results for $A = 40$–66 nuclei. The selection rules are found to be followed quite well in the examples studied. The systematics of ground-state transition strengths are qualitatively quite well reproduced although the quantitative agreement is poor. When the changing nature of the pairing quanta is incorporated using two-particle transfer data the agreement becomes quantitatively good. Evidence is presented for clustering other than that due to pairing in ^{40}Ca and ^{44}Ti.

[NUCLEAR STRUCTURE Extension of pairing-vibration model to four-particle transfer reactions and comparison with experimental results for f-p shell nuclei.]

I. INTRODUCTION

In the past few years, there has been an increasing interest in α-particle transfer reactions leading to final states in $2s$-$1d$ and $1f$-$2p$ shell nuclei.[1-16] Such reactions are interesting in view of their sensitivity to correlations in the initial and final states that cannot be probed directly by the move extensively studied one- and two-particle transfer reactions. A microscopic analysis of the transfer strengths is, however, extremely complex and the traditional shell-model approach becomes unrealistic for all but the lightest nuclei. For $1p$ and $2s$-$1d$ shell nuclei, analyses in terms of SU(3) wave functions have proved particularly useful, especially as this basis is a natural one in which to calculate the overlap factors associated with the transfer of an α-particle.[17-19] Unfortunately, the decomposition of realistic wave functions in terms of SU(3) basis states rapidly loses its simplicity as the splitting between spin-orbit partners increases with increasing mass. Nevertheless, some attempts have been made[20,21] to use this approach for nuclei in the $1f$-$2p$ shell although the lack of large basis shell-model wave functions is a serious drawback. At the present time, however, it seems that other schematic models may provide more insight into the experimental results for nuclei in this mass region.

A simple model which has proved extremely useful in the Ca-Ni region is the pairing-vibration model with isospin,[22,23] This model has provided an understanding of many of the features observed in two-nucleon transfer reactions to $J^\pi = 0^+$ final states and in particular, the model provides a simple explanation of the existence or nonexistence of transitions to excited 0^+ states in (t,p), (p,t), $(^3\mathrm{He},n)$, etc. reactions. Although there is no *a priori* reason to assume that this model will be a useful one with which to describe α-transfer reactions, there is evidence[3,15,16] of the extreme sensitivity of α transfer to the pairing correlations in this mass region. Therefore, although the model (in its simplest form) specifically excludes the other types of correlations which can play a role in α transfer, it probably provides a useful yardstick by which to judge the data.

In this communication, the pairing-vibration scheme is used to treat α transfer. Selection and intensity rules are derived for transitions between model states. The extent to which the selection rules are followed is examined using two- and four-particle transfer data leading to final states in three nuclei in the $A = 40$–60 mass region. It is found that these selection rules are largely obeyed, indicating the validity of the coupling scheme assumed in the pairing-vibration model. The model predictions for the strengths of ground-state to ground-state transitions, are compared with the experimental values for target nuclei ranging from ^{40}Ca to ^{62}Ni. This comparison shows that, although the model fails to correctly predict these relative strengths, the discrepancies are similar to those found when the model is applied to two-nucleon transfer reactions. We may therefore conclude that, despite the potential complexity of α-transfer reactions in this mass region, the same types of correlations that influence two-particle transfer are governing the four-particle transfer between ground states. The only exception to this behavior is for the ^{40}Ca(^{6}Li, d)^{44}Ti reaction, the ground-state transition of which is considerably enhanced

over the value expected from the two-particle transfer systematics. This result may then provide the first quantitative evidence for α-particle clustering in this mass region.

II. THEORY

A. Pairing-vibration model with isospin

This model, first proposed by Bohr[22] and Nathan[23], has been reasonably successful in describing many of the features of two-nucleon transfer reactions in the Ca-Ni region.[23-25] The formulation of the model in this mass region has previously been discussed in some detail and will therefore only be summarized briefly here.

The ground state of the "doubly-magic" ($N=Z=28$) nucleus ^{56}Ni is taken as the vacuum state $|0\rangle$. Monopole pairing-vibration states are then formed by the successive action of the boson creation operators $a^\dagger$ and $r^\dagger$. The operator $a^\dagger(r^\dagger)$ creates a highly correlated pair of particles (holes) in orbits largely above (below) the vacuum. Each of these pairs carries spin 0 and isospin 1. The states in the model are thus characterized by the number of removal and addition quanta, and an isospin coupling scheme is chosen by coupling the isospins of the removal and addition quanta separately and then combining the two resulting isospins to the total isospin. Each state in the model is therefore labeled in the following manner:

$$\Psi = |n_a T_a n_r T_r T T_z\rangle , \tag{2.1}$$

where n_a equals the number of addition quanta, n_r equals the number of removal quanta, $T_a = n_a, n_a-2, \ldots,$ 0 or 1, $T_r = n_r, n_r-2, \ldots,$ 0 or 1, and $\vec{T} = \vec{T}_a + \vec{T}_r$. In the absence of seniority nonzero correlations in the vacuum, it is implicit in the monopole pairing model that all the model states have seniority zero, a fact which will result in a considerable simplification of the α-transfer amplitudes.

In the simplest harmonic pairing-vibration scheme the transfer of pair addition and removal modes may be thought of as transitions between the eigenstates of two uncoupled harmonic oscillators where the number of addition and removal pairs n_a and n_r play the role of the principal quantum numbers and isospin that of angular momentum. The pair creation and annihilation operators are then exactly equivalent to the raising and lowering operators of the normal harmonic oscillator, and have their usual matrix elements.

B. Cluster DWBA

For a cluster transfer reaction on a spin zero target the cross section can be written[26] in the form

$$\sigma(\text{exp}) \sim \left| \sum_N G_{NL} \beta_{NL} \right|^2 , \tag{2.2}$$

where the spectroscopic amplitude G_{NL} contains all the structure information and the factor β_{NL} is the DWBA amplitude calculated using a cluster form factor with N nodes, orbital angular momentum L and a falloff at large radius corresponding to the cluster separation energy.

For a $0s$ α particle and assuming the same size parameter for the projectile and target, N and L are determined by the expression

$$2N + L = Q = \sum_{i=1}^{4} q_i ,$$

where q_i equals the number of oscillator quanta transferred by the ith particle in the cluster. In the mass region under consideration, all transfers involve the same number of oscillator quanta ($Q=12$) (in other words, we restrict the discussion to one major shell which neglects the presence of the $1g_{9/2}$ orbit) and we may therefore rewrite Eq. (2.2) as

$$\sigma(\text{exp}) \sim |G_{NL}|^2 |\beta_{NL}|^2$$

or

$$\sigma(\text{exp}) = N |G_{NL}|^2 \sigma_{NL}(\text{DW}) , \tag{2.3}$$

where N is an overall normalization factor and $\sigma_{NL}(\text{DW})$ is the cluster DWBA cross section.

The quantity $|G_{NL}|^2$ may then be thought of as a cluster "spectroscopic factor" which provides a measure of the intrinsic strength of a given transition independent of the experimental conditions (bombarding energy, Q value, etc.).

The deficiencies of the above procedure are well known, but it is hoped (and tacitly assumed in most analyses of experimental data) that the *relative* spectroscopic factors for transitions with the same L value will be fairly reliable.

C. Structure amplitude G_N

Following Ref. 27, the structure amplitude G may be written

$$G = \left(\frac{B}{B-4}\right)^{Q/2} \sum_\Gamma \langle \Psi_B \| \chi^\dagger(\Gamma) \| \Psi_A \rangle g(\Gamma | \alpha) , \tag{2.4}$$

where B is the mass of the final nucleus and Q is the number of transferred oscillator quanta. The two terms in the sum over Γ in Eq. (2.4) are the coefficients of fractional parentage and the overlap of the four transferred particles with the α particle in the projectile, respectively. The label Γ serves to specify the exact nature of the transferred nucleons in the basis in which the

initial and final state wave functions are written. It is here that the simplicity of the pairing-vibration description of the initial and final states becomes clear. Within this basis, there are only three possible forms for the four-nucleon creation operator $\chi^\dagger(\Gamma)$. namely,

$$\chi^\dagger(\Gamma)=\begin{cases}[a^\dagger\otimes a^\dagger]^{0^+,T=0}, & \Gamma=aa\\ 2[a^\dagger\otimes r]^{0^+,T=0}, & \Gamma=ar\\ [r\otimes r]^{0^+,T=0}, & \Gamma=rr,\end{cases}\tag{2.5}$$

where the factor of 2 in the second of the three forms arises from the distinguishability of the addition and removal pairs.

The above simplicity in the forms of the four-nucleon creation operator when expressed in the pairing-vibration basis contrasts with the complexity of the shell-model approach. Although the latter is more general, the sum over Γ can contain 70 terms in the case of an $L=0$ transition where all possible couplings of two neutrons and two protons in $1f$-$2p$ shell orbitals are included.[27]

In general, at most one of the above three combinations of pairing operators has a nonzero matrix element between any two pairing-vibration states, which implies some strong selection rules the validity of which is discussed in Sec. III A. The evaluation of the matrix elements for allowed transitions leads to the following expressions for their transition strengths:

For $|n_a T_a n_r T_r T T_z\rangle \rightarrow |n_a+2T_a\, n_r T_r T T_z\rangle$,

$$\sigma(\exp)=N\left(\frac{B}{B-4}\right)^{12}(n_a+T_a+3)\times(n_a-T_a+2)g^2(aa|\alpha)\sigma(\mathrm{DW});\tag{2.6}$$

for $|n_a T_a n_r T_r T T_z\rangle \rightarrow |n_a+1T_a' n_r-1T_r' T T_z\rangle$,

$$\sigma(\exp)=4N\left(\frac{B}{B-4}\right)^{12}\begin{Bmatrix}T_a' & T_r' & T\\ T_r & T_a & 1\end{Bmatrix}^2[(n_a+T_a+3)(T_a+1)\delta(T_a',T_a+1)+(n_a-T_a+2)T_a\delta(T_a',T_a-1)]\times[(n_r-T_r)(T_r+1)\delta(T_r',T_r+1)+(n_r+T_r+1)T_r\delta(T_r',T_r-1)]g^2(ar|\alpha)\sigma(\mathrm{DW});\tag{2.7}$$

for $|n_a T_a n_r T_r T T_z\rangle \rightarrow |n_a T_a n_r-2T_r T T_z\rangle$,

$$\sigma(\exp)=N\left(\frac{B}{B-4}\right)^{12}(n_r+T_r+1)\times(n_r-T_r)g^2(rr|\alpha)\sigma(\mathrm{DW}),\tag{2.8}$$

where in the evaluation of the transition matrix elements we have used the fact that the pair operators transform as isovectors in the space defined by the model, and assumed a harmonic pairing-vibration scheme as discussed in Sec. II A.

The overlap factors $g^2(\Gamma|\alpha)$ reflect the microscopic structure of the pair addition and removal modes. In a first approximation the structure of these modes is assumed to be independent of the nucleus in which they exist—except to the extent that size and binding energy effects are included in the cluster form factor used in the distorted-wave calculations. In this case, then, all transfers involving the same combination of pairing quanta could now be compared on the same footing. (See Sec. III B.)

Although the above approach is appealing in its simplicity, it is known from (t,p) and $(^3\mathrm{He},n)$ studies[28-31] that the structure of the pairing modes does *not* remain constant in this mass region and that Pauli principle effects, etc. are important. It does seem, however, that the basic coupling scheme and its consequent selection rules are maintained.[25] This fact enables the changing nature of the pairing quanta to be incorporated into the present discussion of α transfer.

Again, following Ref. 27 we note that the four-particle overlap factor can be expressed as a product of two two-particle overlap factors,

$$g^2(\Gamma|\alpha)=g^2(\gamma_{nn}|0s)\,g^2(\gamma_{pp}|0s),\tag{2.9}$$

where γ_{nn} and γ_{pp} identify the pairing modes occupied by the neutron and proton pairs, respectively ($\gamma=a$ or r). [We could also write $g^2(\Gamma|\alpha)=g^2(\gamma_{np}|0s)g^2(\gamma_{np}|0s)$, but this is less convenient as will be clear.] The evaluation of the matrix elements of the operators given in Eq. (2.5) involved sums over complete sets of intermediate pairing-vibration states which contribute coherently to the final cross section. These intermediate states, together with the initial and final ground states are shown schematically for three typical cases in Fig. 1. The solid lines represent the two-neutron and the dashed lines two-proton transitions. All allowed transitions are shown, but only those which connect the initial and final ground states can contribute to the α transfer process. In principle, therefore, a knowledge of all of the relevant two-particle transfer routes could be utilized to calculate the four-particle transfer. As is evident from the figure, however, not all the required transitions are known from

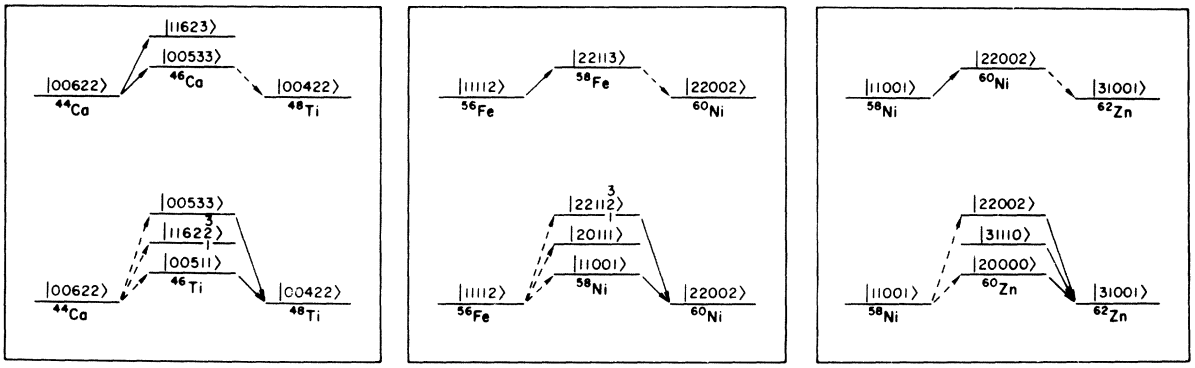

FIG. 1. Possible (t,p) and $(^3\mathrm{He},n)$ routes connecting target and final nucleus ground states. (t,p) transitions are shown as solid (——) and $(^3\mathrm{He},n)$ as dashed (---) lines. States are labeled by their pairing-vibration model quantum numbers $|n_a T_a n_\tau T_\tau T\rangle$.

two-particle transfer data and we must make the further simplifying assumption that so far as the overlap factors go, all routes are equivalent and use the values of $g^2(\gamma_{nn}|\,0s)$ and $g^2(\gamma_{pp}|\,0s)$ obtained from the ground-state transitions. Thus we are able to incorporate the changing nature of the pairing quanta from nucleus, but not within a given nucleus. The expressions for obtaining the values of $g^2(\gamma|\,0s)$ are[28,32] as follows:

For $|n_a T_a n_\tau T_\tau T\, T_z\rangle \rightarrow |n_a+1\, T_a' n_\tau T_\tau T' T_z'\rangle$,

$$\sigma(\mathrm{exp}) = Ng^2(a|0s)(2T+1)\begin{Bmatrix} T_\tau & T' & T_a' \\ 1 & T_a & T \end{Bmatrix}^2 \times |\langle TT_z 1(T_z'-T_z)|T'T_z'\rangle|^2 \times [(T_a+1)(n_a+T_a+3)\delta(T_a', T_a+1) + T_a(n_a-T_a+2)\delta(T_a', T_a-1)]\sigma(\mathrm{DW}); \tag{2.10}$$

for $|n_a T_a n_\tau T_\tau T T_z\rangle \rightarrow |n_a T_a n_\tau - 1\, T_\tau' T' T_z'\rangle$,

$$\sigma(\mathrm{exp}) = Ng^2(\tau|0s)(2T+1)\begin{Bmatrix} T_a & T & T_\tau \\ 1 & T_\tau' & T' \end{Bmatrix}^2 \times |\langle TT_z 1(T_z'-T_z)|T'T_z'\rangle|^2 \times [(T_\tau+1)(n_\tau-T_\tau)\delta(T_\tau', T_\tau+1) + T_\tau(n_\tau+T_\tau+1)\delta(T_\tau', T_\tau-1)]\sigma(\mathrm{DW}), \tag{2.11}$$

where N is an overall normalization factor for the (t,p) or $(^3\mathrm{He},n)$ reaction whichever is appropriate. The application of this procedure is discussed in the next section.

III. COMPARISON WITH EXPERIMENT

A. Selection rules

As discussed, if the pairing-vibration scheme does in fact provide a good description of 0^+ states in this mass region it follows that there are stringent requirements on the mechanism and selection rules for α transfer between these states. The origin of the selection rules lies primarily in the isospin coupling scheme used, and a comparison with experiment for both two- and four-particle transfer reactions should therefore provide a rigorous test of this scheme. In this section we present and discuss the population of 0^+ states in three nuclei from different regions of the f-p shell where relevant two- and four-particle transfer data are available, namely, ^{44}Ti, ^{58}Ni, and ^{62}Zn.

Figure 2 shows the spectrum of pairing-vibrational states with $T = T_z$ for ^{44}Ti; the vertical scale is of no significance. Allowed transitions predicted by the model for α stripping on ^{40}Ca, two-proton stripping on ^{42}Ca and two-neutron pick-

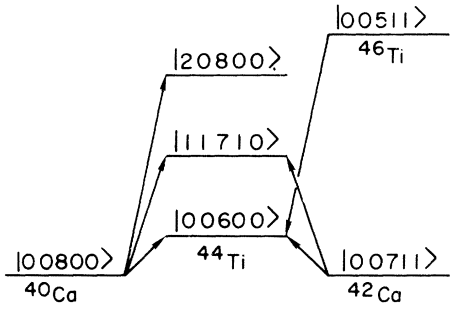

FIG. 2. Pairing-vibration states in ^{44}Ti and allowed transitions for $(^6\mathrm{Li},d)$, $(^3\mathrm{He},n)$, and (p,t) reactions on the appropriate targets.

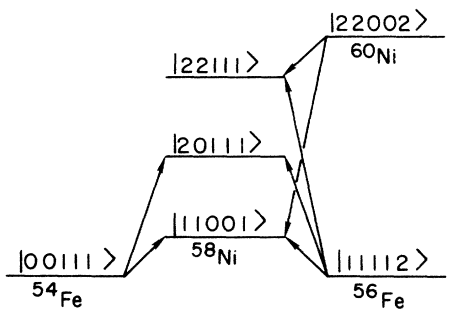

FIG. 3. Same as Fig. 2 but for ^{58}Ni.

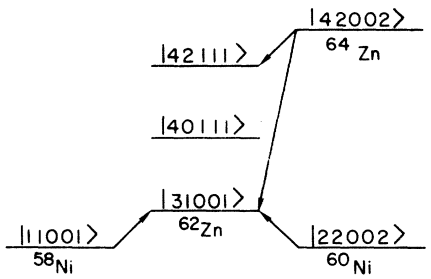

FIG. 4. Same as Fig. 2 but for ^{62}Zn.

up on ^{46}Ti are indicated. Experimentally, the situation is in good agreement with these expectations. The ^{40}Ca(^{6}Li, d)^{44}Ti data of Strohbusch *et al.*[8] shows three strong $L=0$ transitions to the ground-state and excited 0^+ levels at 4.84 and 8.55 MeV, respectively—there is also a somewhat weaker transition to the 1.91 MeV 0^+ state. The (^{3}He, n) study of Ref. 29 shows an $L=0$ transition to a state at 4.86 MeV as well as to the ground state; the 1.91 MeV state is evidently only very weakly excited and no $L=0$ strength is observed near an excitation energy of 8.5 MeV. Only the ground-state transition shows any appreciable strength in the ^{46}Ti(p, t)^{44}Ti reaction[33]—the cross section observed to the 4.86 MeV level is of the order of 0.5% of the ground-state cross section. These results suggest that the ground state, 4.86, and 8.55 MeV levels should be identified with the three pairing-vibration configurations shown in Fig. 2 and indicate that the expected selection rules are quite well obeyed. The exception to this is the existence of an additional 0^+ state at 1.90 MeV which is probably deformed in nature as are the first excited 0^+ states in the even Ca isotopes.

The situation as it applies to ^{58}Ni is shown in Fig. 3. From the available ^{56}Fe(^{3}He, n)^{58}Ni and ^{60}Ni(p, t)^{58}Ni data[30,34] it appears that the excited 0^+ states at 3.54 and 5.96 MeV should be identified with the pairing-vibration configurations shown, although the data are far from unambiguous. The (^{6}Li, d) data of Stein, Sunier, and Woods[16] show strong $L=0$ transitions to the ground-state and 3.54 MeV level, with no further $L=0$ strength below $E_x=4.5$ MeV despite the existence of a previously known 0^+ state at 2.94 MeV. The model also predicts that if the 5.96 MeV level seen in the (p, t) reaction[34] indeed has the $(n_a T_a n_\tau T_\tau T)=(22111)$ pairing configuration, then it should not be excited in the (^{6}Li, d) reaction. It will be of interest to see if this expectation is fulfilled.

The model states in ^{62}Zn are shown in Fig. 4. For (^{3}He, n) and (^{6}Li, d) reactions, the situation is particularly simple—only the ground-state transitions are allowed. Experiment bears this out as no excited 0^+ strength is seen below 4 MeV in the ^{58}Ni(^{6}Li, d) reaction[35] and the ^{60}Ni(^{3}He, n) reaction shows only a weak $L=0$ transition to a state at 2.39 MeV.[29] The location of the highest configuration shown is not certain, although it may be near 4 MeV, where $L=0$ (p,t) strength has been seen.[36]

The above examples show that, in general, the selection rules provided by the model are useful ones and that comparisons of two- and four-particle transfer can provide some insight into the pairing configurations involved. Clearly, the data presented above are somewhat limited in scope and a more general survey of α transfer and its comparison with two-nucleon transfer is desirable.

B. Ground-state transition strengths

The expressions (2.6), (2.7), and (2.8) have been used to calculate the expected relative strengths of ground-state to ground-state transitions for target nuclei from ^{40}Ca to ^{62}Ni. The results of these calculations are present in comparison with experimental results[12,37] in Tables I–III, where each table corresponds to transitions involving

TABLE I. Comparison between calculated and experimental strengths for ground-state transitions involving transfer of two removal quanta. Initial and final states are labeled by their pairing-vibration quantum numbers $(n_a T_a N_\tau T_\tau T)$.

Target nucleus	Final nucleus	$\frac{\sigma(\text{exp})}{\sigma(\text{DW})}$ Expt.[a]	Theory[b]
^{40}Ca(00800)	^{44}Ti(00600)	8.9	9.0
^{42}Ca(00711)	^{46}Ti(00511)	3.0	6.4
^{44}Ca(00622)	^{48}Ti(00422)	1.8	4.1
^{50}Cr(00311)	^{54}Fe(00111)	1.0[c]	1.0[c]

[a] See Refs. 12 and 37.
[b] Assumes $g^2(\tau\tau|\alpha)$ constant throughout.
[c] Normalized.

TABLE II. Comparison between calculated and experimental strengths for ground-state transition involving two addition quanta. Initial and final states are labeled by their pairing-vibration quantum numbers $(n_a T_a N_r T_r T)$.

Target nucleus	Final nucleus	$\frac{\sigma(\text{exp})}{\sigma(\text{DW})}$ Expt.[a]	Theory[b]
^{58}Ni(11001)	^{62}Zn(31001)	1.0[c]	1.0[c]
^{60}Ni(22002)	^{64}Zn(42002)	1.1	1.4
^{62}Ni(33003)	^{66}Zn(53003)	1.2	1.7

[a] See Refs. 12 and 37.
[b] Assumes $g^2(aa|\alpha)$ constant throughout.
[c] Normalized.

TABLE III. Comparison between calculated and experimental strengths for ground-state transitions involving both addition and removal quanta. Initial and final states are labeled by their pairing-vibration quantum numbers $(n_a T_a n_r T_r T)$.

Target nucleus	Final nucleus	$\frac{\sigma(\text{exp})}{\sigma(\text{DW})}$ Expt.[a]	Theory[b]
^{48}Ca(00444)	^{52}Ti(11334)	1.0[c]	1.0[c]
^{50}Ti(00333)	^{54}Cr(11223)	1.0	0.7
^{54}Fe(00111)	^{58}Ni(11001)	0.9	0.2
^{56}Fe(11112)	^{60}Ni(22002)	1.2	0.4

[a] See Refs. 12 and 37.
[b] Assumes $g^2(ar|\alpha)$ constant throughout.
[c] Normalized.

one of the three different possible combinations of pairing quanta. Without specification of the microscopic nature of the pairing quanta, it is not possible to relate the three different types of transfer and we have therefore normalized each group separately.

For the transitions involving two removal quanta shown in Table I, the data show a sharp decrease with increasing neutron number which is quite well accounted for by the model. The transitions involving two addition quanta are in similarly reasonable agreement, the data showing a somewhat smaller rise with increasing neutron number than do the expected values. The values for addition-removal transitions are experimentally quite constant whereas the model values fluctuate considerably, particularly in the case of ^{58}Ni. Overall, the qualitative agreement is reasonably good. Disagreements of the order of factor of 2 are not particularly surprising in light of experience with two-particle transfer reactions and in fact the α-transfer results are in somewhat better agreement than would be expected on this basis alone.

If we assume that the discrepancies arise as a result of the changing nature of the pairing quanta from nucleus to nucleus rather than any fundamental deficiency in the model we may incorporate this effect as discussed in Sec. II C. The two-neutron and two-proton overlap factors $g^2(\gamma_{nn}|0s)$ and $g^2(\gamma_{pp}|0s)$ are obtained from ground-state to

TABLE IV. Results of the inclusion of the values of the overlap factor from (t,p) and $(^3\text{He},n)$ data.

Final nucleus	S_{thy}[a]	$g^2(\gamma_{nn}\|0s)$[b]	$g^2(\gamma_{pp}\|0s)$[b]	S_{thy}[c]	S_{exp}[d]
^{44}Ti	$5.4g^2(rr\|\alpha)$	0.19	2.0	2.1	6.7
^{46}Ti	$3.9g^2(rr\|\alpha)$	0.36	2.2	3.0	2.8
^{48}Ti	$2.4g^2(rr\|\alpha)$	0.32	2.2	1.7	2.0
^{50}Ti	$1.2g^2(rr\|\alpha)$	0.63	1.0	0.74	...
^{52}Ti	$1.0g^2(ar\|\alpha)$[e]	1.0[e]	1.0[e]	1.0[e]	1.0[e]
^{50}Cr	$1.8g^2(rr\|\alpha)$	0.32	3.2	1.9	...
^{52}Cr	$0.87g^2(rr\|\alpha)$	0.57 0.40	2.1 3.2	1.1 1.1	...
^{54}Cr	$0.72g^2(ar\|\alpha)$	0.67	2.1	1.0	1.1
^{54}Fe	$0.60g^2(rr\|\alpha)$	0.40	3.8	0.9	0.67
^{56}Fe	$0.47g^2(ar\|\alpha)$	0.67 0.60	3.0 3.8	0.9 1.1	...
^{58}Fe	$0.90g^2(ar\|\alpha)$	0.47	3.0	1.3	...
^{58}Ni	$0.23g^2(ar\|\alpha)$	0.60	5.2	0.7	1.1
^{60}Ni	$0.44g^2(ar\|\alpha)$	0.47	4.2	0.9	1.5

[a] Defined as $\sigma_{\text{exp}}/N\sigma_{\text{DW}}$ from expressions 2.6, 2.7, and 2.8.
[b] Taken from experiment (Refs. 28, 29) and calculated using expressions 2.10 and 2.11.
[c] Calculated using expressions 2.6, 2.7, and 2.8 together with the experimental values of $g^2(\gamma_{nn}|0s)$ and $g^2(\gamma_{pp}|0s)$ and expression 2.9.
[d] See Refs. 12 and 37.
[e] Normalized.

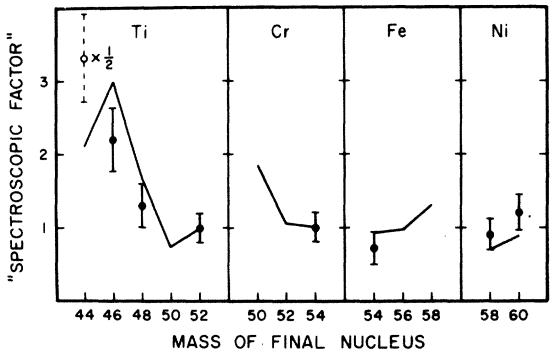

FIG. 5. Comparison of experimental α transfer strengths with the model predictions after inclusion of the two-particle transfer overlap factors. Experiment and theory are arbitrarily normalized to 1.0 for ^{48}Ca $\rightarrow$ ^{52}Ti.

ground-state (t,p) and $(^3\text{He},n)$ transitions utilizing the pairing-vibration expressions for the cross sections given in Eq. (2.10) and (2.11). With the exception of ^{40}Ca the (t,p) data were taken from the work of Casten *et al.*[28] and incorporate the increase in cross section at the $N = 28$ shell closure given in the text of that paper. The ^{40}Ca$(t,p)^{42}$Ca data used were taken from the work of Bjerregaard *et al.*[38] as quoted by Casten *et al.*[39] No (t,p) data with absolute cross sections are available for Ni targets. The $(^3\text{He},n)$ data are taken from the study of Evers *et al.*[29]

The results of the inclusion of the (t,p) and $(^3\text{He},n)$ data in the pairing-vibration expressions are listed in Table IV and are shown in comparison with the experimental $(^6\text{Li},d)$ results in Fig. 5. An arbitrarily determined error of $\pm 20\%$ has been associated with the experimental α-transfer results and the calculated and experimental values have been normalized for the ^{52}Ti transition. The overall agreement is remarkably good. Care should, however, be used in drawing conclusions from this agreement as the experimental $(^6\text{Li},d)$ strengths seem to be rather sensitive to the details of the distorted-wave analysis and different optical model prescriptions[14] could alter this good agreement. Additionally, the (t,p) and $(^3\text{He},n)$ results used are, to a lesser extent, subject to similar uncertainties, which would then affect the calculated values. Of particular interest are the two cases, ^{52}Cr and ^{56}Fe, for which (t,p) and $(^3\text{He},n)$ data exist for both possible ground-state routes. It is gratifying that in these cases the two possibilities give very similar values for S_{thy}. If for the moment, however, we discount possible problems as mentioned above, there are several conclusions that can be drawn from this analysis.

Firstly, the good overall quantitative agreement between the experimental results and the pairing-vibration analysis demonstrates the general validity of the coupling scheme implied by the model and further shows that the same types of correlations which govern the two-particle transfer are determining the four-particle transfer. Secondly, and perhaps more interesting, the large deviation for the ^{40}Ca$(^6\text{Li},d)^{44}$Ti(g.s.) is considerably outside anything that can be accounted for by distorted-wave ambiguities, etc. It is strongly suggestive that, of all the nuclei studied only ^{40}Ca and ^{44}Ti have $N = Z$ and $A = 4n$. That is, they are "α-particle nuclei" and as such might be expected to show clustering features not present in nuclei with $N > Z$.[40] We believe, therefore, that the present analysis provides quantitative evidence for such a clustering in ^{44}Ti which is apparently suppressed in other nuclei in this mass region. A similar and perhaps related effect has also been observed in the elastic scattering of α particles from various Ca isotopes.[41,42] Both ^{40}Ca and ^{41}Ca targets show an anomalous rise in the elastic scattering cross section at backward angles, whereas ^{42}Ca and ^{44}Ca do not. This observation has been interpreted as the result of α cluster exchange processes which are strong for ^{40}Ca but decrease as neutrons are added. The present results tend to support this conclusion.

IV. SUMMARY AND CONCLUSIONS

In this paper we have extended the pairing-vibration model with isospin to include four-particle transfer reactions. The model is used to provide selection rules and expressions for the strengths of α-transfer reactions between the various model states. The model is then applied to 0^+ states in $A = 40$–66 nuclei, first in terms of an analysis of the selection rules and subsequently in the comparison of experimental and theoretical values for ground-state transition strengths.

In the three cases examined, the selection rules expected from the model are quite well followed indicating the validity of the coupling scheme used in the model. The comparison of ground-state transition strengths with the calculated values assuming an unchanging microstructure of the various pairing quanta shows some deviations although these are no worse than those expected on the basis of experience with two-particle transfer reactions in this mass region. The changing nature of the pairing quanta from nucleus to nucleus may be incorporated in an approximate way using the factorization property of the overlap factor, and this is done using experimental values for the two-particle overlap factors taken from (t,p) and $(^3\text{He},n)$ studies in this mass region. When this is

done, the data are found to be in quantitative agreement over the range of nuclei studied. The exception to this good agreement is for the ^{40}Ca(^{6}Li, d)^{44}Ti(g.s.) transition, the strength of which is far in excess of the model predictions. This result suggests the importance of higher order correlations for these two nuclei and perhaps provides the first quantitative evidence for clustering other than that due to pairing in this mass region.

It seems therefore that, at least, the model provides a useful classification scheme for collating four- and two-particle transfer data in this mass region and that further comparisons, both qualitative and quantitative, will give insight into the importance of higher order correlations in these nuclei. Of particular interest will be the search for high-lying $L=0$ strength similar to that seen in ^{44}Ti near 8.5 MeV excitation and the mapping out of strength closer to the ground states.

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[1]M. C. Lemaire, Phys. Rep. 7, 281 (1973).
[2]U. Strohbusch, in Proceedings of the XII International Winter Meeting on Nuclear Physics, Villars, Switzerland, 1973 (unpublished).
[3]R. R. Betts, in Proceedings of the XII International Winter Meeting on Nuclear Physics, Villars, Switzerland, 1973 (see Ref. 2).
[4]H. T. Fortune, R. R. Betts, J. D. Garrett, and R. Middleton, Phys. Lett. 44B, 481 (1973).
[5]R. A. Lindgren, J. P. Trentelman, N. Anantaraman, H. E. Gove, and F. C. Jundt, Phys. Lett. 49B, 263 (1974).
[6]N. Anantaraman, J. P. Draayer, H. E. Gove, and J. P. Trentelman, Phys. Rev. Lett. 33, 846 (1974).
[7]J. P. Draayer, H. E. Gove, J. P. Trentelman, N. Anantaraman, and R. M. DeVries, Phys. Lett. 53B, 250 (1974).
[8]U. Strohbusch, C. L. Fink, B. Zeidman, R. G. Markham, H. W. Fulbright, and R. N. Horoshko, Phys. Rev. C 9, 965 (1974).
[9]H. W. Fulbright, U. Strohbusch, R. G. Markham, R. A. Lindgren, G. C. Morrison, S. C. McGuire, and C. L. Bennett, Phys. Lett. 53B, 449 (1975).
[10]H. T. Fortune, R. R. Betts, J. N. Bishop, M. N. I. Al-Jadir, and R. Middleton, Phys. Lett. 55B, 439 (1975).
[11]R. M. DeVries, H. W. Fulbright, R. G. Markham, and U. Strohbusch, Phys. Lett. 55B, 33 (1975).
[12]U. Strohbusch, G. Bauer, and H. W. Fulbright, Phys. Rev. Lett. 34, 968 (1975).
[13]F. D. Becchetti, L. T. Chua, J. Janecke, and A. M. Van der Molen, Phys. Rev. Lett. 34, 225 (1975).
[14]N. Anantaraman, C. L. Bennett, J. P. Draayer H. W. Fulbright, H. E. Gove, and J. Toke, Phys. Rev. Lett. 35, 1131 (1975).
[15]R. R. Betts, H. T. Fortune, J. N. Bishop, M. N. I. Al-Jadir, and R. Middleton, Phys. Lett. 62B, 37 (1976).
[16]N. Stein, J. W. Sunier, and C. W. Woods, Phys. Rev. Lett. 38, 587 (1977).
[17]M. Ichimura, A. Arima, E. C. Halbert, and T. Terasawa, Nucl. Phys. A204, 225 (1973).
[18]J. P. Draayer, Nucl. Phys. A237, 157 (1975).
[19]K. T. Hecht and D. Braunschweig, Nucl. Phys. A244, 365 (1975).
[20]J. B. McGrory, Phys. Lett. 47B, 481 (1973).
[21]L. R. Grisham and D. Strottman, Nucl. Phys. A276, 130 (1977).
[22]A. Bohr, in *Nuclear Structure* (International Atomic Energy Agency, Vienna, 1968), p. 179.
[23]O. Nathan, *Nuclear Structure* (see Ref. 22), p. 191.
[24]R. A. Broglia, Ole Hansen, and C. Riedel, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1973), Vol. 6, p. 287.
[25]D. Bès, R. A. Broglia, Ole Hansen, and O. Nathan (unpublished).
[26]N. K. Glendenning, Phys. Rev. 137, B102 (1965).
[27]D. Kurath and I. S. Towner, Nucl. Phys. A222, 1 (1974).
[28]R. F. Casten, E. R. Flynn, Ole Hansen, and T. J. Mulligan, Phys. Rev. C 4, 130 (1971).
[29]D. Evers, W. Assmann, K. Rudolph, S. J. Skorka, and P. Sperr, Nucl. Phys. A230, 109 (1974).
[30]W. P. Alford, R. A. Lindgren, D. Elmore, and R. N. Boyd, Nucl. Phys. A243, 269 (1975).
[31]W. Bohne, H. Fuchs, K. Grabisch, D. Hilscher, U. Jahnke, H. Kluge, T. G. Masterson, and H. Morgenstern, Nucl. Phys. A245, 107 (1975).
[32]B. F. Bayman, D. R. Bès, and R. A. Broglia, Phys. Rev. Lett. 23, 1299 (1969).
[33]H. W. Baer, J. J. Kraushaar, C. E. Moss, N. S. P. King, R. E. L. Green, P. D. Kunz, and E. Rost, Ann. Phys. (N.Y.) 76, 437 (1973).
[34]D. H. Kong-A-Siou, A. J. Cole, A. Giorni, and J. P. Longequeue, Nucl. Phys. A221, 45 (1974).
[35]R. R. Betts, N. Stein, J. W. Sunier, and C. W. Woods (unpublished).
[36]R. A. Hinrichs and D. M. Patterson, Phys. Rev. C 10, 1381 (1974).
[37]U. Strohbusch (private communication).
[38]J. Bjerregaard, O. Hansen, O. Nathan, R. Chapman, S. Hinds, and R. Middleton, Nucl. Phys. A103, 33 (1967).
[39]R. F. Casten, E. R. Flynn, J. D. Garrett, S. Orbesen,

and O. Hansen, Phys. Lett. 43B, 473 (1973).
[40]A. Arima, V. Gillet, and J. Ginocchio, Phys. Rev. Lett. 25, 1043 (1970).
[41]G. Gaul, H. Lüdecke, R. Santo, H. Schmeing, and R. Stock, Nucl. Phys. A137, 177 (1969).
[42]H.-J. Appel, W. Gemeinhardt, R. Stock, R. R. Betts, Ole Hansen, A. Sperduto, R. Santo, and H. Fuchs, Nucl. Phys. A246, 477 (1975).