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Properties of the $d_{3/2}$-hole states in the $1f_{7/2}$ nuclei*

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On the assumption that the most important configurations in the description of the $3/2^+$-hole states in ^{43}Sc and ^{47}Sc are $[(\pi d_{3/2})^{-1}\times {}^{A+1}\mathrm{Ti}(I)]_{(3/2)m}$ ($I=0^+$ and 2^+) and $[(\pi s_{1/2})^{-1}\times {}^{A+1}\mathrm{Ti}(2^+)]_{(3/2)m}$, empirical wave functions are deduced that fit both the magnetic moments and the severely inhibited $M2$ decays of the states. The data require a 10–15% admixture of the $\pi s_{1/2}$ hole. The fitted wave functions are compared with the shell-model eigenfunctions that are obtained when the model space consists of nucleons in the $1f_{7/2}$ shell and a single hole in either the $1d_{3/2}$ or $2s_{1/2}$ orbits. Although the shell-model wave functions are very similar to the empirical ones [the probability $|\langle\psi(\mathrm{fit})|\psi(sm)\rangle|^2$ is at least 0.94], the experimental data are not extremely well reproduced with these wave functions. The implications of this result are discussed in detail. Considering the fact that the values of the quantities to be described are anomalously small we conclude that this model space provides an adequate description of the $3/2^+$ states in question. The magnetic moment of the $d_{3/2}$-hole state in ^{45}Ti is calculated on the basis of the shell model and shown to be in agreement with a recent experiment.

[NUCLEAR STRUCTURE Shell-model calculation for magnetic moments and $M2$ decays of the $\frac{3}{2}^+$-hole states in ^{43}Sc, ^{47}Sc, and ^{45}Ti.]

I. INTRODUCTION

The magnetic moments of the $d_{3/2}$-hole states in two odd-A Sc isotopes are known. The value of the moment of the 760-keV $\frac{3}{2}^+$ state in ^{47}Sc was reported[1] several years ago to be $\mu=(0.35\pm0.05)\mu_N$ (where μ_N is the nuclear magneton) and recently it was found[2] that the 150-keV state in ^{43}Sc has $\mu=(0.348\pm0.006)\mu_N$. In both cases the value is close to that observed in the single-$d_{3/2}$-proton-hole nucleus ^{39}K which has[3] $\mu=0.39142\mu_N$. Considerations of the moment alone would therefore lead to the conclusion that these states correspond to a proton-$d_{3/2}$ hole coupled to the ground state of the neighboring even-even Ti nucleus. Such a description, however, is in conflict with the result[4] that the $M2$ γ decay of both of these states to their respective $\frac{7}{2}^-$ ground states is very severely inhibited. The transition rates can be understood[5,6] if the $\frac{3}{2}^+$ eigenfunctions contain an appreciable component ($\cong$40%) in which the hole is coupled to an excited state of the respective Ti core. Because of this, one would expect the moments to be different from the ^{39}K value. In this paper we show that the magnetic moments are extremely sensitive to small admixtures of proton excitation from the $s_{1/2}$ shell and that one can simultaneously understand the inhibition in the $M2$ decays and the value of the magnetic moments if one allows about a 15% $(\pi s_{1/2})^{-1}$ admixture in the $\frac{3}{2}^+$ eigenfunctions of ^{43}Sc and ^{47}Sc.

We first construct empirical $\frac{3}{2}^+$ wave functions for ^{43}Sc and ^{47}Sc that fit both the $M2$ decay and magnetic moment data. These wave functions are then compared with the shell-model eigenfunctions obtained when the model space is taken to be the $1f_{7/2}$ orbit for particles and either the $1d_{3/2}$ or $2s_{1/2}$ level for the hole. It is found that when the modified surface δ interaction (MDSI) of Glaudemans, Brussaard, and Wildenthal[7] is used to describe the $(1d_{3/2}$-$1f_{7/2})$ and $(2s_{1/2}$-$1f_{7/2})$ interactions the $\frac{3}{2}^+$ wave function required to fit the ^{43}Sc data is reproduced to a high degree of accuracy. On the other hand, neither the modified surface δ interaction nor a Yukawa-Rosenfeld potential leads to the empirically required ^{47}Sc $\frac{3}{2}^+$ eigenfunction. Since the ^{43}Sc energy matrix depends mainly on the sum of the $T=0$ plus $T=1$ $(d_{3/2}$-$f_{7/2})$ interaction energies it appears that the MSDI interaction reproduces this combination quite well. In ^{47}Sc the difference between the $T=0$ and $T=1$ energies comes in and apparently neither of the considered interactions adequately reproduces this combination.

Finally we discuss the MSDI shell-model calculation for the magnetic moment of the $d_{3/2}$-hole state in ^{45}Ti and show that reasonable agreement between theory and experiment is obtained.

II. EMPIRICAL EIGENFUNCTIONS

In this section we construct wave functions of the $\frac{3}{2}^+$ states of ^{43}Sc and ^{47}Sc that fit both the moment data and the $M2$ transition rates in these nuclei. These wave functions are assumed to have the form

$$\begin{aligned}\psi_{(3/2)^+m} = {} & \alpha[(\pi d_{3/2})^{-1} \times {}^{A+1}\mathrm{Ti}(0^+)]_{(3/2)^+m} \\ & + \beta[(\pi d_{3/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{(3/2)^+m} \\ & + \gamma[(\pi s_{1/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{(3/2)^+m}, \end{aligned} \tag{1}$$

where the notation $[\times]_{(3/2)^+m}$ stands for the vector coupling of j and I to angular momentum $\frac{3}{2}$ and $^{A+1}\mathrm{Ti}(I)$ denotes the first state of spin I in the neighboring Ti nucleus. We shall assume that one need only consider $f_{7/2}$ nucleons moving outside an inert $N=Z=20$ core to describe the states of the Ti nuclei.

It is easy to show that the magnetic moment associated with this state is

$$\begin{aligned}\mu = {} & (\alpha^2 + \tfrac{1}{5}\beta^2)\mu((\pi d_{3/2})^{-1}) + (\tfrac{3}{5}\beta^2 + \tfrac{9}{10}\gamma^2)\mu(2^+) \\ & + \tfrac{3}{5}\gamma^2\mu((\pi s_{1/2})^{-1}). \end{aligned} \tag{2}$$

For the moments on the right-hand side of this equation we make the following assumptions:

(i) The magnetic moment of the $(\pi d_{3/2})^{-1}$ state has the value observed in ^{39}K

$$\mu((\pi d_{3/2})^{-1}) = 0.391\,42\mu_N. \tag{3}$$

(ii) The moment associated with $f_{7/2}$ particles is that observed in ^{41}Ca and ^{41}Sc

$$\begin{aligned}&\mu_\nu(f_{7/2}) = -1.5946\mu_N; \\ &\mu_\pi(f_{7/2}) = 5.43\mu_N. \end{aligned} \tag{4}$$

Nuclei with $T=0$ or those that are particle-hole self-conjugate in the sense that they are describable by the configuration $(\pi j)^m(\nu j)^{-m}$ have moments that depend only on the isoscalar part of the magnetic moment operator.[8] Since ^{44}Ti and ^{48}Ti are examples of this it follows that the g factor for any level describable by the pure $f_{7/2}$ configuration will be

$$\begin{aligned} g &= \tfrac{1}{7}[\mu_\nu(f_{7/2}) + \mu_\pi(f_{7/2})] \\ &= 0.548\mu_N. \end{aligned} \tag{5a}$$

Consequently for both these nuclei

$$\mu(2^+) = 1.096\mu_N. \tag{5b}$$

(iii) There are no experimental data available concerning the moment of the $\pi s_{1/2}$-hole state. Therefore we take the Schmidt value for this quantity

$$\mu((\pi s_{1/2})^{-1}) = 2.79\mu_N. \tag{6}$$

Aside from Eq. (2) and the normalization condition

$$\alpha^2 + \beta^2 + \gamma^2 = 1 \tag{7}$$

one more relationship is needed to determine empirical values for α, β, and γ. This is provided by the $B(M2)$ values which have been measured to be

$$B(M2; \tfrac{3}{2}^+ \to \tfrac{7}{2}^-) = 1.4\mu_N^2\,\mathrm{fm}^2$$

for ^{43}Sc and

$$B(M2; \tfrac{3}{2}^+ \to \tfrac{7}{2}^-) = 0.7\mu_N^2\,\mathrm{fm}^2$$

for ^{47}Sc.

In order to calculate the $B(M2)$ values we must know the structure of the $\frac{7}{2}^-$ ground states in the Sc nuclei. Consistent with our description of the core-Ti states, we assume that these states can be described by the pure $f_{7/2}$ configuration. When use is made of the fact that an $s_{1/2}$ nucleon cannot be converted into an $f_{7/2}$ particle via the $M2$ operator, it follows that

$$\begin{aligned} B(M2; \tfrac{3}{2}^+ \to \tfrac{7}{2}^-) &= \frac{2I_f+1}{2I_i+1}|\langle {}^A\mathrm{Sc}(\tfrac{7}{2}^-)||M2||{}^A\mathrm{Sc}(\tfrac{3}{2}^+)\rangle|^2 \\ &= 2[\alpha B(0) + \beta B(2)]^2|\langle \tfrac{7}{2}^-||M2||\tfrac{3}{2}^+\rangle_\pi|^2, \end{aligned} \tag{8}$$

where $\langle \frac{7}{2}^-||M2||\frac{3}{2}^+\rangle_\pi$ is the proton single-particle reduced matrix element which satisfies the relationship

$$(-1)^{j_1-j_2}(2j_1+1)^{1/2}\langle j_1||M2||j_2\rangle = (2j_2+1)^{1/2}\langle j_2||M2||j_1\rangle.$$

To find the values of $B(0)$ and $B(2)$ in this equation we write the $\frac{7}{2}^-$ wave function in the form

$$\psi_{(7/2)m} = \sum_{J_N} \alpha_{J_N}[\pi f_{7/2} \times \phi_{J_N}]_{(7/2)m}, \tag{9}$$

where ϕ_{J_N} is the neutron wave function $(\nu f_{7/2})^2{}_{J_N}$ for ^{43}Sc or $(\nu f_{7/2})^{-2}{}_{J_N}$ for ^{47}Sc. In a similar manner the 0^+ and 2^+ eigenvectors in the Ti nuclei can be expressed as

$$\psi_{IM} = \sum_{J_P J_N} \beta_{J_P J_N}(I)[(\pi f_{7/2})^2{}_{J_P} \times \phi_{J_N}]_{IM}. \tag{10}$$

It then follows that

$$\begin{aligned} B(I) = {} & 8(2I+1)^{1/2} \sum_{J_P J_N} \alpha_{J_N}\beta_{J_P J_N}(I)(2J_P+1)^{1/2} \\ & \times W(\tfrac{7}{2}\tfrac{7}{2}IJ_N; J_P\tfrac{7}{2})W(\tfrac{7}{2}\tfrac{7}{2}\tfrac{3}{2}\tfrac{3}{2}; I2), \end{aligned} \tag{11}$$

where the W's are Racah coefficients.

We first show that for any isospin conserving two-body interaction $B(0)$ is independent of α_{J_N} and

$\beta_{J_PJ_N}(0)$ and always has the value $-\frac{1}{2}$. To see this we note that the Hamiltonian matrix from which the α_{J_N}'s are determined has the form

$$\langle[\pi f_{7/2}\times\phi_{J_N'}]_{(7/2)m}|H|[\pi f_{7/2}\times\phi_{J_N}]_{(7/2)m}\rangle$$

$$=E_{J_N}\delta_{J_NJ_N'}+n[(2J_N+1)(2J_N'+1)]^{1/2}$$

$$\times\sum_K(2K+1)W(\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2};J_NK)\,W(\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2}\tfrac{7}{2};J_N'K)E_K',\tag{12}$$

where $n=2$ or 6 is the number of $f_{7/2}$ neutrons in the state ϕ_{J_N}. E_{J_N} is the interaction energy of the two neutrons (note for $n=6$ that the particle-particle interaction is the same as the hole-hole interaction aside from an additive J-independent constant) and E_K' is the neutron-proton interaction. For the $I=0$ state in the Ti nucleus with n neutrons the only difference in the Hamiltonian matrix is that in Eq. (12)

$$E_{J_N}\delta_{J_NJ_N'}\to(E_{J_P}+E_{J_N})\delta_{J_PJ_N}\delta_{J_NJ_N'}=2E_{J_N}\delta_{J_NJ_N'},$$

provided isospin is conserved, and that

$$n\to 2n.$$

Thus the Hamiltonian matrix of the Ti 0^+ states, for the special case that $n=2$ or $=6$, has each matrix element a factor of 2 larger than the $\frac{7}{2}^-$ Sc energy matrix. Consequently the eigenvectors of the two Hamiltonians are identical and

$$\beta_{J_PJ_N}(0)=\alpha_{J_N}\delta_{J_PJ_N}.$$

When this result is combined with the properties of the Racah coefficients for $I=0$ it follows that

$$B(0)=-\tfrac{1}{2}.\tag{13}$$

On the other hand, $B(2)$ is not independent of the residual nucleon-nucleon interaction. In order to obtain a value for this quantity we proceed in a manner analogous to that of Bansal and French[6]:

(i) In ^{43}Sc α_{J_N} and $\beta_{J_PJ_N}(2)$ are determined by diagonalizing the Hamiltonian matrix using the $f_{7/2}$-interaction energies determined from the spectrum of ^{42}Sc. Equation (11) then leads to

$$B(2)=0.685;\tag{14a}$$

(ii) In ^{47}Sc the analogous matrix elements are taken from the spectrum of ^{48}Sc. The appropriate value of $B(2)$ calculated by use of Eq. (11) is

$$B(2)=0.592.\tag{14b}$$

To determine the value of the proton-reduced $M2$ matrix element to be used in Eq. (8) we make use of the $M2$ lifetime data on ^{39}K and ^{39}Ca. For the former, the mean γ-ray lifetime for the $\frac{7}{2}^-\to\frac{3}{2}^+$ transition is[10]

$$\tau_m(^{39}\mathrm{K};\tfrac{7}{2}^-\to\tfrac{3}{2}^+)=63\pm10\text{ psec}$$

and in ^{39}Ca the analogous transition has a lifetime[11]

$$\tau_m(^{39}\mathrm{Ca};\tfrac{7}{2}^-\to\tfrac{3}{2}^+)=90\pm24\text{ psec}.$$

Within the $d_{3/2}$-$f_{7/2}$ configuration space the simplest assumption for the $\frac{3}{2}^+$ state in both nuclei is that it is a single-hole state and for the $\frac{7}{2}^-$ state that it has the form

$$\psi_{(7/2)m}=\sum_{JT}\gamma_{JT}[(d_{3/2}^{-2})_{JT}\times f_{7/2}]_{(7/2)m;(1/2)m_t}$$

there $(\frac{1}{2},m_t)$ is the isospin of the state with z component m_t. With this ansatz for the wave functions it follows[12] that the reduced matrix element governing the ^{39}K decay has the form

$$\langle\psi_{3/2^+}(^{39}\mathrm{K})||M2||\psi_{7/2^-}(^{39}\mathrm{K})\rangle$$

$$=-\frac{1}{2\sqrt{3}}[\gamma_{01}+(\tfrac{15}{7})^{1/2}\gamma_{21}](\langle\tfrac{3}{2}||M2||\tfrac{7}{2}\rangle_\nu$$

$$+2\langle\tfrac{3}{2}||M2||\tfrac{7}{2}\rangle_\pi)$$

$$-\frac{9}{2\sqrt{35}}[\gamma_{10}+(\tfrac{11}{27})^{1/2}\gamma_{30}]\langle\tfrac{3}{2}||M2||\tfrac{7}{2}\rangle_\nu.\tag{15}$$

In the corresponding expression for ^{39}Ca the role of the neutron and proton are interchanged. The Erné matrix elements,[13] which were chosen to fit the negative-parity states at the end of the (ds) shell lead to[12] $\gamma_{01}=0.957$, $\gamma_{21}=0.052$, $\gamma_{10}=-0.080$, and $\gamma_{30}=0.275$. Thus to fit the lifetime data one needs

$$\langle\tfrac{7}{2}||M2||\tfrac{3}{2}\rangle_\pi=10.8\mu_N\,\text{fm},\tag{16a}$$

$$\langle\tfrac{7}{2}||M2||\tfrac{3}{2}\rangle_\nu=-10.4\mu_N\,\text{fm}.\tag{16b}$$

The empirical proton reduced $M2$ matrix element, which is the only one that is needed with our assumed state vector, Eq. (1), has a value almost identical to the harmonic-oscillator single-particle estimate

$$\langle f_{7/2}||M2||d_{3/2}\rangle_\pi=3(2/\pi)^{1/2}(\mu_\pi-\tfrac{1}{3})b\,\mu_N,\tag{17a}$$

where

$$b=(\hbar/m\omega)^{1/2}$$

$$=1.006A^{1/6}\,\text{fm}.\tag{17b}$$

When the free-particle value, $\mu_\pi=2.79$, is used for the proton magnetic moment the single-particle matrix element has the value $10.9\mu_N$ fm for the $A=39$ system. Therefore we shall always use the harmonic-oscillator single-particle value given by Eqs. (17) for the single-particle $M2$ matrix element.

Equations (3), (5b), (6), (13), (14), and (17) provide all the values of the single-particle matrix elements needed to solve Eqs. (2), (7), and (8) for α, β, and γ. A unique solution is determined by the further condition that the state resembles the $\pi d_{3/2}$-hole state as closely as possible—that is α is as close to one as the experimental data allow. If α is arbitrarily chosen to be positive the foregoing condition implies that the experimental $M2$ matrix element, Eq. (8) must be taken to be negative. The results obtained with these assumptions are listed in Table I under the heading Expt.

For both nuclei a fit to the data within the very restricted model space of Eq. (1) demands slightly less than 15% admixture of the configuration $[(\pi s_{1/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{(3/2)^+m}$ into the $d_{3/2}$ hole state. A check on the consistency of these wave functions would be provided through measurement of the ${}^{A+1}\mathrm{Ti}(d, {}^3\mathrm{He}){}^A\mathrm{Sc}(\frac{3}{2}^+)$ spectroscopic factors. Some data do exist for the ^{48}Ti target[14] and if one assumes that all the $l=2$ strength seen below 5.75 MeV corresponds to $d_{3/2}$ pickup and moreover that this exhausts the sum rule one finds experimentally that

$$C^2\mathcal{S}(\tfrac{3}{2}^+, 760\text{ keV}) = 2.6,$$

whereas our empirical wave function gives

$$C^2\mathcal{S}(\tfrac{3}{2}^+, 760\text{ keV}) = 2.3.$$

In view of the difficulty in extracting reliable spectroscopic factors for hole states, there is no discrepancy between theory and experiment.

III. SHELL-MODEL CALCULATION

In this section we examine the form of the hole-state wave functions that emerge when a standard shell-model calculation is carried out. The model space employed restricts the particles to the $1f_{7/2}$ shell and the hole to either the $1d_{3/2}$ or $2s_{1/2}$ level.

In setting up the shell-model energy matrices we assume that for ^{43}Sc the diagonal $(f_{7/2})^2{}_I$ matrix elements can be taken from the ^{42}Sc spectrum[8] and those for ^{47}Sc from the data[9] on ^{48}Sc. The $d_{3/2}$-$f_{7/2}$ and $s_{1/2}$-$f_{7/2}$ interaction matrix elements are calculated in two different ways:

(i) By use of the modified surface δ interaction (MSDI)

$$V = -4\pi A_T \delta(\Omega_{12}) + B_T \tag{18a}$$

with the parameters

$$A_0 = 0.4\text{ MeV}, \quad B_0 = -1.8\text{ MeV},$$

$$A_1 = 1.2\text{ MeV}, \quad B_1 = 0.7\text{ MeV} \tag{18b}$$

determined by Glaudemans *et al.*[7]

(ii) By use of the Yukawa-Rosenfeld interaction (YURO)

$$V = V_0 \frac{e^{-r/r_0}}{(r/r_0)}(-0.13 + 0.93P_M + 0.47P_B - 0.27P_H), \tag{19a}$$

where P_M, P_B, and P_H have the usual meaning of space, spin, and space-spin exchange operators and where we choose

$$V_0 = -46\text{ MeV},$$

$$(\hbar/m\omega)^{1/2}/r_0 = 1.18. \tag{19b}$$

The excitation energy of the $s_{1/2}$-hole state relative to $d_{3/2}$ was taken to be 2.48 MeV, the value observed[3] in ^{39}Ca.

TABLE I. The magnetic moment of the first excited $\frac{3}{2}^+$ state (in nuclear magnetons) and $B(M2; \frac{3}{2}^+ \to \frac{7}{2}^-)$ (in μ_N^2 fm^2) for ^{43}Sc and ^{47}Sc. The experimental values are listed in the columns headed Expt. The values of α, β, and γ correspond to the components in the wave function, Eq. (1). In the column headed MSDI we present the results of the shell-model calculation when the residual two-body interaction is the modified surface δ interaction of Eq. (18) and under the heading YURO the results when the potential is the Yukawa-Rosenfeld interaction of Eq. (19).

	^{43}Sc Expt.	^{43}Sc MSDI	^{43}Sc YURO	^{47}Sc Expt.	^{47}Sc MSDI	^{47}Sc YURO
α	0.813	0.836	0.802	0.753	0.867	0.836
β	0.483	0.451	0.500	0.547	0.422	0.483
γ	± 0.326	0.299	0.241	± 0.366	0.239	0.223
$\mu(\frac{3}{2}^+)$	0.348 ± 0.006 [a]	0.468	0.580	0.35 ± 0.05 [b]	0.477	0.376
$B(M2; \frac{3}{2}^+ \to \frac{7}{2}^-)$	1.4	5.1	2.0	0.7	7.6	7.5

[a] Reference 2.
[b] Reference 1.

A. ^{43}Sc

The dimensionality of the energy matrix to be diagonalized for the $\frac{3}{2}^+$ state in this nucleus is 57×57. Thirty-five of these states correspond to excitation of a proton out of the $(1d, 2s)$ shell and the remaining twenty-two to neutron excitation. For either residual interaction the probability of neutron excitation is small—0.49% for the MSDI and 3.28% for the YURO interaction.

In order to calculate the magnetic moment of the $\frac{3}{2}^+$ state we must—in addition to the moments given by Eq. (3), (5b) and (6)—know the value for a single-neutron hole. The recent Stanford measurement[15] of the moment of ^{39}Ca gives

$$\mu((\nu d_{3/2})^{-1}) = 1.0216\,\mu_N . \tag{20a}$$

There are no data on the $s_{1/2}$ hole and therefore we take the Schmidt value

$$\mu((\nu s_{1/2})^{-1}) = -1.91\,\mu_N \tag{20b}$$

for this moment. To obtain the shell-model prediction for the $M2$ decay we take the single-particle estimate, Eq. (17), for both the neutron and proton [to obtain the neutron matrix element from Eq. (17a) one merely replaces $(\mu_\pi - \frac{1}{3})$ by $\mu_\nu = -1.91$].

In the column headed MSDI we present the results obtained when the interaction of Eq. (18) is used. The overlap between the shell-model wave function and the "experimental" eigenfunction is in this case

$$\int \psi^*_{(3/2)m}(\text{fit})\psi_{(3/2)m}(\text{MSDI})d\tau = 0.995 .$$

Despite this large overlap, the magnetic moment predicted by the MSDI calculation is approximately 35% larger than the experimental value and the predicted $B(M2)$ is almost a factor of 4 larger than experiment. This, of course, illustrates the extreme sensitivity to slight changes in the wave function when the calculated quantity has a small value. For example, if one forgets about the small neutron contribution, the many-particle $M2$ matrix element has the value

$$\langle ^{43}\text{Sc}(\tfrac{7}{2}^-)\|M2\|^{43}\text{Sc}(\tfrac{3}{2}^+)\rangle = c\langle f_{7/2}\|M2\|d_{3/2}\rangle_\pi . \tag{21}$$

When Eq. (17) is used to calculate the proton, reduced-matrix element c must have the value

$$c = -0.076$$

to fit experiment. The value of c, which is -1 in the single-particle limit, becomes -0.142 for the MSDI calculation. Thus the coefficient is small, as required by experiment, and very minor changes in the wave function can bring the computed value into agreement with experiment.

Although the model space used in this shell-model calculation is much larger than that of Eq. (1), the neglected excitations from the $d_{5/2}$ state or to the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits will certainly give some contribution to the computed matrix elements. Because of this it must be concluded that the MSDI explains the currently known properties of the ^{43}Sc $\frac{3}{2}^+$ level to the accuracy one would expect with the model space used.

The results obtained with the Yukawa-Rosenfeld interaction, Eq. (19), are listed in Table I, column YURO. In this case an almost exact cancellation occurs in the $M2$ matrix element and so far as this quantity is concerned theory and experiment are in excellent agreement.

The value of the magnetic moment on the other hand comes out too large. Two effects contribute to this result: First, the YURO interaction gives almost a factor of 2 less $(\pi s_{1/2})^{-1}$ excitation than exists in the experimental wave function. As a consequence, this contribution reduces the magnetic moment by only about half the amount found by use of Eq. (1). Second, as pointed out at the beginning of this section, the Yukawa-Rosenfeld interaction leads to a comparatively large neutron excitation in the $\frac{3}{2}^+$ wave function (3.28%). Since neutron excitation corresponds to $T\geq 1$ states of the $(f_{7/2})^4$ configuration one can get an additional $f_{7/2}$ contribution to the magnetic moment

$$\mu_{\text{add}} = \langle[(\pi d_{3/2})^{-1}\times {}^{44}\text{Ti}(I=2,T=0)]_{(3/2)(3/2)}|\,\mu\,|[(\pi d_{3/2})^{-1}_\tau\times {}^{44}\text{Ti}(I=2,T=1)]_{(3/2)(3/2)}\rangle .$$

Because the two Ti states have different isospin, μ_{add} depends on the isovector part of the magnetic moment operator

$$\mu_{1\nu} = \tfrac{1}{7}[\mu(\pi f_{7/2}) - \mu(\nu f_{7/2})](\vec{J}_\pi - \vec{J}_\nu), \tag{22}$$

where $\vec{J}_\pi$ and $\vec{J}_\nu$ are the total angular momentum operators of the proton and neutron $f_{7/2}$ particles. Because $\mu_{1\nu}$ depends on the difference of the ^{41}Sc and ^{41}Ca moments, this contribution can have a substantial effect. Indeed, the contribution of the $f_{7/2}$ nucleons to the moment of the $\frac{3}{2}^+$ state is

$$\mu(f_{7/2}) = 0.386\,\mu_N$$

as compared to the value

$$\mu_{\text{expt}}(f_{7/2}) = 0.265\,\mu_N$$

obtained with the empirical wave function.

B. ^{47}Sc

In all calculations for this nucleus the $(f_{7/2})^2$ interaction energies were taken from the ^{48}Sc spectrum. The $\frac{3}{2}^+$ energy matrix to be diagonalized is 41×41—thirty five of the states correspond to proton excitation and for the MSDI these exhaust 99.94% of the wave function while for YURO they correspond to 99.91% of the eigenfunction. In calculating the properties of the $\frac{3}{2}^+$ state, listed in Table I, the magnetic moments of Eqs. (3), (5b), (6), and (20) were used and the single-particle value, Eq. (17), was taken for the $M2$ operator.

In the case of MSDI the overlap of the empirical and shell-model wave functions is

$$\int \psi^*_{(3/2)m}(\text{fit})\psi_{(3/2)m}(\text{MSDI})d\tau = 0.971.$$

The amount of $(\pi s_{1/2})^{-1}$ mixed into the wave function is considerably less than required by experiment (5.7% compared to 13.4%). However, because the wave function is more than 75% $[(\pi d_{3/2})^{-1} \times {}^{48}\text{Ti}(0^+)]_{(3/2)m}$ the predicted moment exceeds the experimental value by only about 36%—almost identical to the findings in ^{43}Sc. However, because α is so large the requisite cancellation in the $M2$ matrix element is not obtained. In order to fit experiment, c of Eq. (21) must be -0.053, whereas theoretically it has the value -0.171. Because of this the theoretical $B(M2)$ is more than a factor of 10 too large.

Qualitatively the same results are obtained for the Yukawa-Rosenfeld interaction. Although the probability of $\pi s_{1/2}$ excitation is somewhat smaller (5%) than with MSDI the fit to the magnetic moment is somewhat better. This comes about because the contribution of the $(f_{7/2})^8$ configuration to the moment is only $\mu(f_{7/2}) = 0.173\mu_N$, whereas the empirical wave function gives $\mu(f_{7/2}) = 0.329\mu_N$. This polarization effect on the ^{48}Ti(2$^+$) wave functions, which helps in the case of the magnetic moment, acts on the wrong direction for $B(M2)$ and leads to a value almost identical to that obtained with the MSDI.

Thus we see once more that the magnetic moment is fitted quite reasonably with either interaction. However, although a severe inhibition in the $M2$ transition rate is predicted, theory and experiment are at greater variance in this nucleus than in ^{43}Sc. In ^{43}Sc the hole state is dominated by the coupling of the $\pi d_{3/2}$ hole to the $T=0$ ^{44}Ti core. Because of this, the shell-model analog of the coefficients α and β of Eq. (1) are sensitive only to the sum

$$E_J(d_{3/2}f_{7/2}) = \tfrac{1}{2}[E_{JT=0}(d_{3/2}f_{7/2}) + E_{JT=1}(d_{3/2}f_{7/2})]$$

of the $(d_{3/2}\text{-}f_{7/2})$ interaction energies. On the other hand, in ^{47}Sc the $\pi d_{3/2}$ hole is coupled mainly to the $T=2$ states of the ^{48}Ti core. Consequently α and β depend not only on the sum of the $T=0$ and $T=1$ matrix elements, but also on their difference. Because the quality of the $M2$ fit is better in ^{43}Sc than in ^{47}Sc it would seem that both the MSDI and YURO interactions come closer to reality for the sum of these interactions than for their difference.

C. ^{45}Ti

For cores that are neither $T=0$ nor particle-hole conjugates the isovector part of the magnetic moment operator, Eq. (22), will contribute and hence the moment of the $\frac{3}{2}^+$ state might be expected to be considerably different than the single-hole value. A nucleus where this effect comes in is ^{45}Ti and recently the magnetic moment of the 330-keV $\frac{3}{2}^+$ state has been measured.[16] Its value was found to be $\mu = (0.98 \pm 0.24)\mu_N$—almost identical to the ^{39}Ca moment, Eq. (20a). We have calculated the magnetic moment of this state using the MSDI, Eq. (18), and the ^{42}Sc $f_{7/2}$ matrix elements and have found that indeed the $f_{7/2}$ contribution to the moment is approximately twice that for the Sc nuclei. However, the shell-model calculation also predicts that the state is about $\frac{2}{3}$ neutron and $\frac{1}{3}$ proton excitation. Since the magnetic moment of $d_{3/2}$ proton is about a factor of 3 smaller than that of a neutron, it follows that the extra contribution of the $f_{7/2}$ nucleons is needed to bring the calculated moment up to the ^{39}Ca value. An analysis of the various contributions to the theoretical moment shows that $\mu(s_{1/2}) = 0.020\mu_N$, $\mu(f_{7/2}) = 0.650\mu_N$, and $\mu(d_{3/2}) = 0.584\mu_N$ giving a predicted moment for the $\frac{3}{2}^+$ state of $\mu(\frac{3}{2}^+) = 1.254\mu_N$. Thus the calculated value is again slightly higher than experiment but not appreciably outside the experimental uncertainty and certainly as close as one should expect to come when one neglects $(d_{5/2})^{-1}$ and $f_{5/2}$ admixtures in the wave function.

An analysis similar to that of Sec. II cannot be carried out since the $M2$ decay of the $\frac{3}{2}^+$ state in ^{45}Ti is not known. The lifetime of this 330-keV state for decay to the 37-keV $\frac{3}{2}^-$ level has been measured and has the value[17]

$$\tau_m = 1.72 \pm 0.10 \text{ nsec}.$$

If this value is taken in conjunction with our theoretical $M2$ lifetime for the crossover transition

$$\tau_m(M2; \tfrac{3}{2}^+ \to \tfrac{7}{2}^-) = 3.2 \times 10^{-6} \text{ sec}$$

we would predict that the branching ratio for decay to the ground state is 5×10^{-4}. This small value is consistent with the experimental fact that no crossover transition has been observed.

IV. SUMMARY AND CONCLUSIONS

Empirical wave functions for the $\frac{3}{2}^+$-hole states in ^{43}Sc and ^{47}Sc have been constructed to fit both the severely inhibited $M2$ decay rates of these levels and the single-hole nature of their magnetic moments (for both nuclei the observed moment is close to that of ^{39}K). If one assumes that the $\frac{7}{2}^-$ ground states of these nuclei are described by nucleons in the $f_{7/2}$ shell and that the $\frac{3}{2}^+$-hole states have the structure given by Eq. (1), the magnetic moment data require a 10–15% admixture of the configuration $[(\pi s_{1/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{I=(3/2)m}$ into the $\frac{3}{2}^+$ states of both nuclei. Because $g_\pi(s_{1/2}) = 5.58\,\mu_N$ is approximately 10 times larger than $g(2^+)$, Eq. (5a), the contribution from this configuration

$$\langle [(\pi s_{1/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{II} \,|\, \mu \,|\, [(\pi s_{1/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{II} \rangle$$

$$= \frac{g_\pi(s_{1/2})[I(I+1) - \frac{21}{4}] + g(2^+)[I(I+1) + \frac{21}{4}]}{2(I+1)}$$

is negative for $I = \frac{3}{2}$. Thus the moment due to this term has the opposite sign to that obtained for either $[(\pi d_{3/2})^{-1} \times {}^{A+1}\mathrm{Ti}(0^+)]_{(3/2)m}$ or $[(\pi d_{3/2})^{-1} \times {}^{A+1}\mathrm{Ti}(2^+)]_{(3/2)m}$ and independently of the sign of γ, Eq. (1), will reduce the theoretical moment as required to fit experiment.

The results of shell-model calculations for these quantities are also reported. For both nuclei the $\frac{7}{2}^-$ ground state is assumed to arise from the $(f_{7/2})^n$ configuration and the $\frac{3}{2}^+$ eigenfunction is found by diagonalizing the shell-model Hamiltonian using all basis states of the configurations $[(d_{3/2})^{-1} \times (f_{7/2})^{n+1}]_{(3/2)m}$ and $[(s_{1/2})^{-1} \times (f_{7/2})^{n+1}]_{(3/2)m}$. For ^{43}Sc ($n = 3$) the $(f_{7/2})^2$ interaction energies were taken from the spectrum of ^{42}Sc and for ^{47}Sc ($n = 7$) they were taken from ^{48}Sc. For both nuclei two forms of the ($d_{3/2}$-$f_{7/2}$) and ($s_{1/2}$-$f_{7/2}$) interactions were taken—the MSDI, Eq. (18), and the Yukawa-Rosenfield interaction of Eq. (19). The predicted magnetic moments are somewhat larger than observed experimentally but consistent with what one would expect from a model in which excitations to the $f_{5/2}$ level and out of the $d_{5/2}$ orbit are excluded—the difference between theory and experiment being $(0.1\text{–}0.2)\mu_N$. This discrepancy is similar to that which arises when the pure $f_{7/2}$ model is used to calculate the $\frac{7}{2}^-$ ground state magnetic moments of the Sc nuclei. For example, with the assumed $f_{7/2}$ interactions and the single-particle moments of Eq. (4) one predicts

$$\mu(^{43}\mathrm{Sc}, \tfrac{7}{2}^-) = 4.65\,\mu_N,$$

$$\mu(^{47}\mathrm{Sc}, \tfrac{7}{2}^-) = 5.18\,\mu_N$$

and these values are to be compared with the experimental results[3,18]

$$\mu_{\mathrm{expt}}(^{43}\mathrm{Sc}, \tfrac{7}{2}^-) = 4.62\,\mu_N,$$

$$\mu_{\mathrm{expt}}(^{47}\mathrm{Sc}, \tfrac{7}{2}^-) = 5.34\,\mu_N.$$

Part of the reason the magnetic moments of the hole states in ^{43}Sc and ^{47}Sc are close to the $^{39}\mathrm{K}(\pi d_{3/2})^{-1}$ value is because of the special structure of the core nuclei ^{44}Ti and ^{48}Ti. The former has $T = 0$ and consequently for the dominant configurations in the $\frac{3}{2}^+$ eigenfunction, Eq. (1), only the isoscalar part of the magnetic moment operator, Eq. (5a), is important. The latter nucleus can, to reasonable approximation, be expected to have the structure $(\pi f_{7/2})^2(\nu f_{7/2})^{-2}$ and for nuclei describable by the particle-hole configuration $(\pi j)^n(\nu j)^{-n}$ again only the isoscalar part of the $M1$ operator contributes. Therefore, since the major contribution to the magnetic moments from both the ^{44}Ti and ^{48}Ti cores should be proportional to the isoscalar part of the operator their effects should be small. On the other hand, for nuclei in which the $d_{3/2}$ hole is coupled to a core state which does not satisfy the above criteria the magnetic moment contribution from the core might be expected to be substantial. An example of this is provided by the $\frac{3}{2}^+$ state in ^{45}Ti. In this case the $(d_{3/2})^{-1}$ part of the wave function is predominantly

$$\psi_{(3/2)m;(1/2)(1/2)} = (\tfrac{1}{2}1 - \tfrac{1}{2}1 \,|\, \tfrac{1}{2}\tfrac{1}{2})[(\nu d_{3/2})^{-1} \times {}^{46}\mathrm{Ti}(I^+)]_{(3/2)m} + (\tfrac{1}{2}1\tfrac{1}{2}0 \,|\, \tfrac{1}{2}\tfrac{1}{2})[(\pi d_{3/2})^{-1} \times {}^{46}\mathrm{Ti}(I^+)]_{(3/2)m},$$

where $(\frac{1}{2}1\mu\nu \,|\, \frac{1}{2}\frac{1}{2})$ is the Clebsch-Gordan coefficient that ensures coupling to isospin $T = \frac{1}{2}$, $m_t = \frac{1}{2}$. Thus in the extreme-weak-coupling limit ($I^+ = 0$) the magnetic moment associated with the hole state would be

$$\mu = \frac{2\mu(\nu d_{3/2})^{-1}) + \mu((\pi d_{3/2})^{-1})}{3}$$

$$= 0.812\,\mu_N,$$

where the values of the proton and neutron moments have been taken from Eqs. (3) and (20a) in making the numerical estimate. The fact that the observed moment[16] of this state has a value larger than this is a clear indication that some contribution to the moment comes from an $I^+ \neq 0$ state of the ^{46}Ti core. A detailed shell-model calculation in which particles are restricted to the $f_{7/2}$ shell and the hole is in either the $d_{3/2}$ or $s_{1/2}$ orbit gives a larger value for the moment in agreement with experiment.

As far as the $B(M2)$ values are concerned, those predicted by the shell-model calculations for ^{43}Sc are in satisfactory agreement with experiment once one recognizes that very minor changes in the wave function can easily change the predictions by a factor of 2. On the other hand, the theoretical $B(M2)$ in ^{47}Sc is a factor of 10 greater than experiment. As in ^{43}Sc, the matrix element is small and consequently quite sensitive to small changes in the wave function. However the matrix element deviates substantially more from experiment than in ^{43}Sc where only the sum of the $T=0$ and $T=1$ interaction energies comes in because we deal with an isospin zero core. Thus one might conclude that the difference in the $T=0$ and $T=1$ $(d_{3/2}\text{-}f_{7/2})$ interaction energies is less well reproduced by the MSDI and YURO interactions than are the sum of these energies.

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