

Microscopic ion-ion spin-orbit potential*

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A microscopic, ion-ion spin-orbit potential is derived in a double-folding model approach starting from a nucleon-nucleon interaction. Its geometry is determined by the structure of the target and projectile. An estimate of its strength is presented.

[NUCLEAR REACTIONS Ion-ion spin-orbit potential, double-folding model,
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I. INTRODUCTION

The standard Thomas form for the nucleon-nucleus spin-orbit potential can be derived, for example, in an impulse approximation¹ or a Glauber model approach.² The shape of the spin-orbit potential is given by $(1/r)(d/dr)\rho(r)$, where $\rho(r)$ is the target density, and at high energies the strength of the potential is proportional to the forward spin-flip nucleon-nucleon scattering amplitude.

Attempts have also been made to derive an ion-ion spin-orbit potential. Rawitscher³ derived an α -nucleus spin-orbit potential from a phenomenological α -nucleon spin-orbit potential. Love⁴ also used this approach, but he included nucleon-nucleon exchange effects also. He found that the exchange effects provided the dominant contribution to the spin-orbit potential. Neither was able to obtain agreement with phenomenological α -nucleus spin-orbit potentials that were derived in fitting elastic α -nucleus scattering from non-zero spin targets.⁵⁻⁸

More recently, Thompson⁹ investigated the role of a spin-orbit potential in heavy-ion scattering. In his approach, a phenomenological nucleon-nucleus spin-orbit potential is folded over the valence (non-spin-saturated) nucleon's wave function. For example, for $^{13}\text{C} + ^{24}\text{Mg}$ a spin-orbit potential representing the nucleon- ^{24}Mg interaction is convoluted with the $1p_{1/2}$ neutron wave function in ^{13}C .

Finally, vector polarized ^6Li -nucleus elastic scattering and asymmetry measurements¹⁰ have been analyzed¹¹ by considering the ^6Li as an α -deuteron bound state and then folding the deuteron wave function with a phenomenological deuteron-nucleus spin-orbit potential. This analysis was also carried out for ^{14}N projectiles.

In this paper, a formalism is presented for deriving an ion-ion spin-orbit potential from only the structure of the target and projectile nuclei

and from the nucleon-nucleon interaction. This will be done in the context of a double-folding model that has proved quite successful in describing both elastic^{12, 13} and inelastic^{14, 15} heavy-ion scattering. In the folding-model approach of Refs. 12 and 14 it was found that the geometry of the ion-ion potential is well determined when accurate nuclear densities are used. Also, a smoothly energy-dependent complex strength emerges from the analysis of many different sets of elastic heavy-ion data.^{12, 14} This strength can, in principle, be related to a spin-isospin averaged, effective nucleon-nucleon forward laboratory scattering amplitude.¹² In a similar way, the geometry of the ion-ion spin-orbit potential can be fixed in the double-folding model approach and the strength can be determined from a comparison to phenomenological potentials (unfortunately, there are very few) that have been derived.

In the energy range we will be considering (~ 5 MeV/nucleon) the effective nucleon-nucleon amplitude derived from fitting the data bears little resemblance to the free amplitude.¹² The real part of the free laboratory amplitude goes through zero at about a lab energy of 6 MeV and stays negative below that energy. The effective amplitude is fairly constant in that energy region at a value of about 1.8 fm.¹² At 5 MeV, the imaginary part of the free amplitude is large (~ 6 fm) and growing with decreasing energy, whereas for the effective amplitude it is small and decreasing.¹² Hence, it will not be surprising to find that the spin-flip amplitude needed to fit phenomenological spin-orbit potentials differs substantially from the free amplitude.

Of course, the folding model is a static approach. Dynamical changes in both the target and projectile densities are not taken into account. When these changes arise from the virtual excitation of non-zero-spin states, then, presumably, the folded spin-orbit potential will be modified. For deuteron-nucleus scattering, some of these dy-

namical effects have been calculated by Rawitscher.¹⁶ We are guided here by the fact that there has been no need to alter the geometry of the central part of the folded potentials calculated for elastic and inelastic scattering. Some of the difference between the fitted strengths and the predicted ones may be attributed to the dynamical processes. However, Satchler and Love¹³ have been able to correctly predict both the strength and geometry of the central potential when they used an effective nucleon-nucleon interaction that was derived by fitting G matrix elements.

In the next section formalism needed to derive the ion-ion spin-orbit potential is presented in some detail. The features of this potential are discussed in Sec. III. In Sec. IV the results of our calculations are shown and finally, a section of concluding remarks appears.

II. FORMALISM

The derivation of the central part of the ion-ion potential in the double-folding model has been given elsewhere.¹² In order to derive the spin-orbit term we need a nucleon-nucleon spin-orbit potential in coordinate space. We write

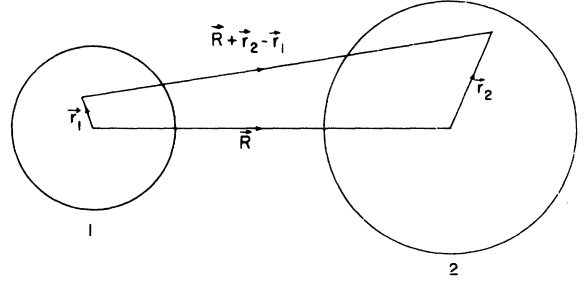


FIG. 1. The coordinate system used in deriving the spin-orbit potential.

$$v_{so}(\vec{R} + \vec{r}_2 - \vec{r}_1) = g(|\vec{R} + \vec{r}_2 - \vec{r}_1|) \times (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot (\vec{R} + \vec{r}_2 - \vec{r}_1) \times \hat{k}, \quad (1)$$

where g is a short-ranged radial function, the σ 's are the Pauli spin operators, \hat{k} is a unit vector in the direction of the nucleon-nucleon c.m. momentum, and the coordinates are defined in Fig. 1. We write the ion-ion spin-orbit potential as the sum of all the nucleon-nucleon spin-orbit interactions averaged over the bound-state nuclear wave functions, χ :

$$V_{so}(\vec{R}) = \sum_{i,j} \int \int \chi_{1,i}^2(r_{1,i}) \chi_{2,j}^2(r_{2,j}) g_{ij}(|\vec{R} + \vec{r}_{2,j} - \vec{r}_{1,i}|) (\vec{\sigma}_{1,i} + \vec{\sigma}_{2,j}) \cdot (\vec{R} + \vec{r}_{2,j} - \vec{r}_{1,i}) \times \hat{k} d^3r_{1,i} d^3r_{2,j}. \quad (2)$$

For simplicity, we will assume that nucleus 2 has total spin $I=0$, and that nucleus 1 has total spin I given by the total spin of a single valence nucleon. Then,

$$\begin{aligned} \sum_j \langle \chi_{2,j} | \vec{\sigma}_{2,j} | \chi_{2,j} \rangle &= 0, \\ \sum_i \langle \chi_{1,i} | \vec{\sigma}_{1,i} | \chi_{1,i} \rangle &= \langle \tilde{\chi}_1 | \vec{\sigma}_1 | \tilde{\chi}_1 \rangle, \end{aligned} \quad (3)$$

where $\tilde{\chi}_1$ is the wave function of the valence nucleon. If we assume that the radial function g is independent of i and j , Eq. (2) becomes

$$V_{so}(\vec{R}) = \int \int \rho_2(r_2) \tilde{\chi}_1^2(r_1) g(|\vec{R} + \vec{r}_2 - \vec{r}_1|) \vec{\sigma}_1 \cdot (\vec{R} + \vec{r}_2 - \vec{r}_1) \times \hat{k} d^3r_1 d^3r_2. \quad (4)$$

We will now introduce a change of coordinates and proceed in a manner similar to Glauber's² approach in deriving the nucleon-nucleus spin-orbit potential. Let us define

$$\begin{aligned} \vec{y} &\equiv \vec{R} + \vec{r}_2 - \vec{r}_1, \\ 2\vec{z} &\equiv \vec{R} + \vec{r}_2 + \vec{r}_1, \end{aligned} \quad (5)$$

then Eq. (4) becomes

$$V_{so}(\vec{R}) = \int \int \rho_2(|\vec{z} - \vec{R} + \vec{y}/2|) \tilde{\chi}_1^2(|\vec{z} - \vec{y}/2|) g(y) \vec{\sigma} \cdot \vec{y} \times \hat{k} d^3y d^3z.$$

Since $g(y)$ is a short-ranged function (as compared to the size of the nuclei) we can attempt to expand the density and wave function squared about $y=0$. The zeroth order term leads to an integral of the form

$$\int g(y) \vec{y} d^3y = 0,$$

so we must expand to first order in y . This leads to two terms involving derivatives, in one case of the density and in the other case of the wave function squared. Integrating by parts leads to the result

$$V_{so}(\vec{R}) \approx \vec{\sigma} \cdot \hat{k} \times \int d^3y \vec{y} g(y) \vec{y} \cdot \hat{R} \frac{d}{dR} \\ \times \int \rho_2(|\vec{z} - \vec{R}|) \bar{\chi}_1^2(z) d^3z. \quad (6)$$

Now, using the fact that $\int d\Omega_y y_i y_j = (4\pi/3) y^2 \delta_{ij}$, Eq. (6) becomes

$$V_{so}(\vec{R}) \approx \vec{\sigma} \cdot \hat{k} \times \frac{\vec{R}}{R} \frac{d}{dR} \left(\int \rho_2(|\vec{z} - \vec{R}|) \bar{\chi}_1^2(z) d^3z \right) \frac{4\pi}{3} \\ \times \int y^4 g(y) dy. \quad (7)$$

In an impulse approximation^{1,12} one could relate the radial factor $g(r)$ to the Fourier transform of the nucleon-nucleon isospin-averaged spin-flip laboratory amplitude $\bar{C}(q)$.^{1,12}

$$\frac{-2\pi\hbar^2}{m} \bar{C}(q) \vec{\sigma} \cdot \hat{n} = \int e^{-i\vec{q} \cdot \vec{r}} g(r) \vec{\sigma} \cdot \vec{r} \times \hat{k} d^3r, \quad (8)$$

where m is the nucleon mass and $\hat{n} = (\vec{k}_0 \times \vec{k}'_0) / |\vec{k}_0 \times \vec{k}'_0|$, where \vec{k}_0 and \vec{k}'_0 are the initial and final nucleon-nucleon c.m. momenta. Using Eq. (8), one can show that the integral in Eq. (7) becomes

$$\frac{4\pi}{3} \int y^4 g(y) dy = -i \frac{2\pi\hbar^2}{m} k \bar{C}(0), \quad (9)$$

where $\bar{C}(q) \equiv \bar{C}(q)/(k k_0 \sin\theta_0)$, \vec{k} is the ion-ion c.m. momentum, and θ_0 is the nucleon-nucleon scatter-angle. Note that

$$\bar{C}(0) \equiv \lim_{\theta_0 \rightarrow 0} \bar{C}(q)$$

is finite.

Finally, using $\vec{L} = \vec{R} \times \vec{k}$, where \vec{L} is the ion-ion c.m. angular momentum, Eq. (7) yields

$$V_{so}(\vec{R}) = i \vec{\sigma} \cdot \vec{L} \frac{2\pi\hbar^2}{m} \bar{C}(0) \frac{1}{R} \frac{d}{dR} \\ \times \int \rho_2(|\vec{r} - \vec{R}|) \bar{\chi}_1^2(r) d^3r. \quad (10)$$

Actually, the ion-ion spin-orbit potential should be written in terms of the total spin I of the nucleus. Using the Wigner-Eckhart Theorem, one can show that³:

$$\langle l_{\pm}^{\frac{1}{2}} I^{\pm} | \sigma | l_{\pm}^{\frac{1}{2}} I^{\pm} \rangle = \pm \frac{1}{l \pm \frac{1}{2}} \langle l_{\pm}^{\frac{1}{2}} I^{\pm} | I | l_{\pm}^{\frac{1}{2}} I^{\pm} \rangle,$$

where we define $I^{\pm} \equiv l \pm \frac{1}{2}$. Hence, the ion-ion spin-orbit potential becomes

$$V_{so}(\vec{R}) = \pm i \frac{2\pi\hbar^2}{m} \bar{C}(0) \vec{I} \cdot \vec{L} \frac{1}{l \pm \frac{1}{2}} \frac{1}{R} \frac{d}{dR} \\ \times \int \rho_2(|\vec{r} - \vec{R}|) \bar{\chi}_1^2(r) d^3r. \quad (11)$$

Since the nucleon-nucleon spin-flip amplitude is in general complex, the spin-orbit potential we

have derived will also be complex. However, at 5 MeV lab energy, the real part of the factor $\hat{C}(0) \equiv i\bar{C}(q)/\sin\theta_0|_{\theta_0=0}$ is about 4 times larger than the imaginary part. This would, at least partially, justify the use of real phenomenological spin-orbit potentials in this energy region.

III. DISCUSSION

It has been stated in the literature that the spin-orbit potential has a $1/A_1$ dependence (A_1 being the mass number of the nonzero spin nucleus) and hence is unimportant for heavy ions. For a fixed lab energy per nucleon the factor $C(q)/k_0 \sin\theta_0|_{\theta_0=0}$ is a constant, and there are only two possible sources of an A_1 dependence in Eq. (11). Firstly, there is the factor of k in the denominator [contained in $\bar{C}(0)$]:

$$\hbar ck = \{2mc^2[A_1 A_2 / (A_1 + A_2)][A_2 / (A_1 + A_2)] E_{lab}\}^{1/2}.$$

Writing E_{lab} as $A_1(E_{lab}/A_1) = \text{constant} \times A_1$, the A_1 dependence of k becomes $k \sim A_1 A_2 / (A_1 + A_2)$, which for $A_2 \gg A_1$ goes like A_1 , and for $A_1 \gg A_2$ goes like A_2 (independent of A_1). However, this A_1 dependence of the strength of the spin-orbit potential arising from the factor of k in the denominator is canceled by the factor of k in the angular momentum, L , which appears in the numerator. Thus, although the strength of the spin-orbit potential may be smaller for heavier ions, this is partly compensated for by the larger values of L that are important for larger A_1 .

Secondly, the nucleon-nucleus spin-orbit potential is usually written in terms of the derivative of the central potential. For a zero-range nucleon-nucleon potential, the central ion-ion potential is proportional to the convolution¹²:

$$\int \rho_2(\vec{r} - \vec{R}) \rho_1(\vec{r}) d^3r. \quad (12)$$

Hence, if one were to use the derivative of the central potential to describe the spin-orbit term, the strength would have to be reduced by what might naively appear to be a factor of A_1 . This is because $\rho_1(r)$ is normalized to A_1 , whereas $\bar{\chi}_1^2$ is normalized to 1. However, one must remember that heavy-ion reactions are sensitive to relatively large separations of the ions.¹⁷ [Satchler¹³ found that the radius $R_s = 1.5(A_1^{1/3} + A_2^{1/3})$ defined the important region for elastic scattering.] For these large R 's the density is dominated by the least-bound wave function which is just the valence wave function $\bar{\chi}$. Thus, for large enough R , the ratio of $[(1/R)(d/dR)]$ of the integral appearing in Eq. (12) and that appearing in Eq. (11) should approach 1. A plot of that ratio for ¹³C + ⁴⁰Ca is presented in Fig. 2. (At 14 fm the ratio is down to 1.5.)

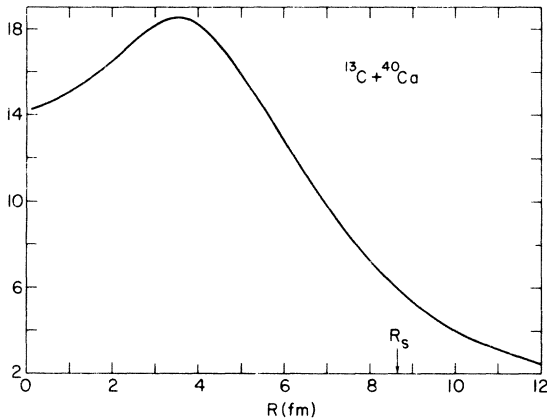


FIG. 2. The ratio

$$\frac{(1/R)(d/dR) \int \rho_2(|\vec{r}-\vec{R}|) \rho_1(r) d^3r}{(1/R)(d/dR) \int \rho_2(|\vec{r}-\vec{R}|) \chi_1^2(r) d^3r}$$

for $^{13}\text{C} + ^{40}\text{Ca}$. The radius $R_s = 1.5(A_1^{1/3} + A_2^{1/3})$ is indicated by an arrow.

In proton-nucleus scattering, where one is sensitive to smaller separations of the target and projectile, our results would give that the spin-orbit potential arising from any target spin would be approximately $1/A_1$ of the spin-orbit term due to the proton's spin. This is in agreement with a remark made by Feshbach¹⁸ and with experiment, where there has been no need to introduce a spin-orbit potential due to the target's spin in proton-nucleus scattering.

In contrast, for the strongly absorbed ^3He (Ref. 19) and ^4He (Refs. 3-8) projectiles, target spin dependence has been introduced. In the ^3He studies, the strength of the $\vec{L} \cdot \vec{\sigma}$ term in the optical potential was compared for pairs of nuclei differing by one nucleon. It was found¹⁹ that the strength of the spin-orbit term increased by about 1 MeV for the nonzero spin member of the pair, independent of its spin. This is consistent with the result contained in Eq. (11), since the l dependence of the potential is essentially canceled by the $1/(l + \frac{1}{2})$ factor. The differences between the spin-orbit potential for strongly and weakly absorbed particles and its weak l dependence in ^3He scattering have been discussed in Ref. 19.

IV. RESULTS

The phenomenological spin-orbit potential derived in the analysis⁵ of α - ^9Be scattering at $E_{\text{lab}} = 20$ MeV is displayed in Fig. 3. Also shown is the calculated potential of Eq. (11) normalized to the peak at $R = 3.6$ fm. This normalization gives a value of $\text{Re}\hat{C}(0) = 0.39$ fm as opposed to the free space value of 0.023 fm.²⁰ As anticipated, these numbers differ dramatically.

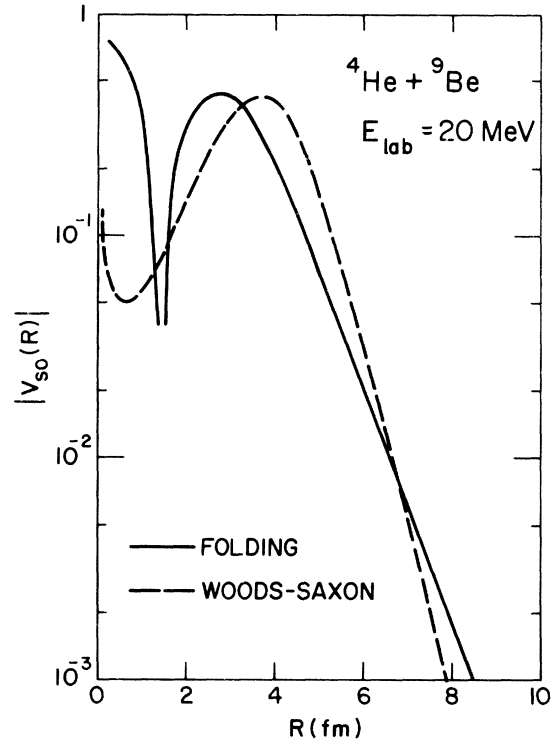


FIG. 3. The magnitude of the spin-orbit potential for $^4\text{He} + ^9\text{Be}$ at $E_{\text{lab}} = 20$ MeV. The solid line is the folding model result of Eq. (11). The dashed line is the Woods-Saxon derivative form of Eq. (13) using the parameters of Ref. 5.

The α density used is a three parameter Fermi distribution derived from electron scattering results.²¹ The ^9Be density and wave functions were obtained by using the Malaguti-Hodgson²² binding potentials with slight parameter adjustments to achieve better single-particle binding energies.

There is some doubt about the accuracy of the spin-orbit potentials derived for the ^3He and ^4He scattering. Firstly, there have been many careful analyses (e.g., Ref. 23) of ^4He -nucleus scattering with nonzero spin targets which have not needed to introduce a spin-orbit potential. Secondly, the ^3He -nucleus scattering has been successfully analyzed²⁴ without recourse to a target-spin-dependent spin-orbit potential. This has been done by introducing a term in the cross section corresponding to the interaction of the projectile with the quadrupole moment of the nonzero-spin target.

In any case, using the effective spin-flip amplitude found for the α - ^9Be scattering, an estimate of the ion-ion spin-orbit potential was calculated for $^{13}\text{C} + ^{40}\text{Ca}$. It is shown in Fig. 4 along with a Woods-Saxon derivative-type potential that was fixed to agree with the calculated potential in the important peripheral region. This fit potential has

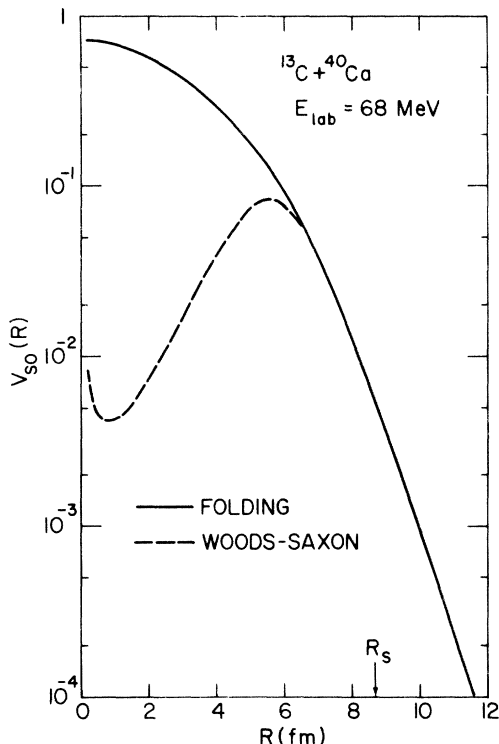


FIG. 4. The calculated spin-orbit potential for $^{13}\text{C} + ^{40}\text{Ca}$ at $E_{\text{lab}} = 68$ MeV (solid line) and the Woods-Saxon derivative potential fit to it (dashed line). Again, R_s is indicated by an arrow.

the standard form

$$V_{\text{so}}(r) = \left(\frac{\hbar}{m_\pi c}\right)^2 V_{\text{so}} \frac{1}{r} \frac{d}{dr} [1 + \exp(r-R)/a]^{-1} \vec{L} \cdot \vec{I}, \quad (13)$$

where m_π is the pion mass. The value of the parameters found are as follows: $V_{\text{so}} = -0.73$ MeV, $R = 5.78$ fm, and $a = 0.79$ fm. Note that since the valence neutron in ^{13}C is in a $1p_{1/2}$ orbital, we have $I = l - \frac{1}{2}$ and the potential of Eq. (11) has a sign opposite that of the normal nucleon-nucleus spin-orbit potential. The changing of sign of the ion-ion spin-orbit potential may be a helpful signature in identifying its effects in heavy-ion reactions.

The geometrical parameters given above should be accurate, since the folding integrals were performed with carefully determined densities. However, the strength of the potential was determined by a comparison to only one questionable pheno-

menological potential. It would be best to consider this number as only a very rough estimate for this reason. Above about 50 MeV/nucleon, there is hope that the impulse approximation, using the free nucleon-nucleon spin-flip amplitude, would give reasonably accurate results. Below that energy, the effective spin-flip amplitude should be taken as a free parameter.

The ^{13}C density and wave functions were also derived from modified Malaguti-Hodgson²² potentials. The ^{40}Ca density was produced from the results of a Hartree-Fock calculation.²⁵

Although the spin-flip probability measured in the inelastic excitation by ^{13}C of the first 2^+ state in ^{56}Ni is quite small,²⁶ there has been some recent evidence for the need of a spin-orbit potential in heavy-ion transfer reactions. Fits to the transfer reaction $^{28}\text{Si}(^{19}\text{F}, ^{16}\text{O})^{31}\text{P}$ have been greatly improved by the inclusion of a spin-orbit potential.²⁷ The reaction $^{40}\text{Ca}(^{13}\text{C}, ^{14}\text{N})^{39}\text{K}(\text{g.s.})$ at 68 MeV cannot be fitted in the distorted wave Born approximation using reasonable parameters.²⁸ There are indications that a spin-orbit potential will improve the fit there also.²⁹ The strength of the spin-orbit Woods-Saxon derivative potential estimated here was found to be too small to appreciably change the transfer cross section.³⁰ Invoking an helicity-flip mechanism has been shown to provide a fit to the ^{40}Ca transfer data.³¹

V. CONCLUSIONS

A microscopic, complex, ion-ion spin-orbit potential has been derived in the framework of a double-folding model. It is believed that it will provide a consistent geometry for the spin-orbit potential that can be used in semiphenomenological analyses of heavy-ion reactions. Of course, polarization and/or asymmetry measurements in heavy-ion scattering would allow one to determine the spin-orbit potential in a more straightforward way.

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