⁶⁹Ge and ⁷¹Ge using the ⁷⁰Ge(\vec{d} , t) and ⁷⁰Ge(\vec{d} , p) reactions*[†]

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Differential cross sections and vector analyzing powers were measured at an incident deuteron energy of 16 MeV for the ⁷⁰Ge(d, t)⁶⁹Ge reaction and at 12 MeV for the ⁷⁰Ge(d, p)⁷¹Ge reaction. The data are generally well reproduced by the distorted-wave Born approximation except for the forward-angle analyzing-power data for (d, t) transitions to $1/2^-$ states in ⁶⁹Ge. Several spin assignments in ⁶⁹Ge are made from the vector analyzing power of the Ge(\vec{d} , t) reaction. A high-resolution study of the ⁷⁰Ge(d , p)⁷¹Ge reaction identified over 50⁷¹Ge levels as well as a number of additional states corresponding to contaminants in the target. Assignments of I_n were made to 44 levels in ⁷¹Ge. On the basis of analyzing-power measurements at 16° and 20°, values of J^{π} could be assigned to 29 levels, of which 15 are new assignments. A number of tentative J^{π} assignments are also made. Some of the results of the present work are relevant to the weak-coupling interpretation of 69,71 Ge by Eberth *et al.* Some $l = 0$ and 2 transitions were observed in ⁷⁰Ge(d, t) but the $l_n = 0$ cross-section data are not well fitted by the distorted-wave Born approximation. A sum-rule analysis was made and results are compared with previous work and with the predictions of the simple pairing theory.

> NUCLEAR REACTIONS ⁷⁰Ge(\bar{d} , t)⁶⁹Ge. Measured $\sigma(\theta)$ and $A_{\mathbf{y}}(\theta)$, θ_{lat}
=15°-80°, E=16 MeV. ⁷⁰Ge(\bar{d} , p)⁷¹Ge. Measured $\sigma(\theta)$, θ_{lab} =15°-90°, $A_{\mathbf{y}}(\theta)$, θ_{lab} =16°, 20°. E =12 MeV. DWBA analyses. Extracted J^{π} and S_i .

I. INTRODUCTION

The mass region around germanium was formerly considered to consist of nuclei of sphericalvibrational character. Forssten et $al.$ ¹ have discussed high-spin positive-parity states in $^{69, 71, 73}$ Ge in terms of a weak-coupling vibrational model which accounts for a number of levels, but not all positive-parity high-spin states. Eberth et $al.^2$ have also used the weak-coupling model to explain positive-parity states in "Ge and states of both parities in 69 Ge. Their results indicate the presence of both particle and collective features in $69,71$ Ge. However, some experimental facts relating to the odd- A Ge isotopes are not well explained by the weak-coupling model. In particular the existence of low-lying $\frac{5}{2}^+$ and $\frac{7}{2}^+$ states, some lying below the lowest $\frac{9}{2}^*$ state, are unaccounte for. Kregar and $\rm Elbek^3$ encountered difficultie in the description of the even Ge isotopes in terms of a vibrational model. In particular, the presence of a low-lying 0^* state in ⁷⁰Ge and in ⁷²Ge is hard to understand in terms of this model.

Heller and Friedman' applied a deformed core model with an effective pairing-type residual interaction and rotational-particle coupling⁵⁻⁷ to odd N nuclei in the $1g_{9/2}$ shell. They have succeeded in reproducing some of the general features of the spectra of both positive- and negativeparity levels of these nuclei.

Experimentally, the odd-X Ge isotopes have been investigated by neutron-transfer studies, $8-16$

 γ -ray experiments, 1,2,17 -19 and β decay. 20,21 The recent (\bar{d}, p) measurements of Yoh, Darden, and $Sen¹¹$ in this laboratory permitted spin and parity assignments to be made to a number of states in $71.73.75.77$ Ge. While states up to excitation energies of \sim 2.6 MeV were studied, the limited energy resolution of these measurements prevented the assignments of spins to many of the states. One surprising result of this work was the assignment surprising result of this work was the assignme
of $J^{\pi} = \frac{7}{2}^*$ to a level in ⁷¹Ge at 1.698 MeV which is populated fairly strongly $(S \sim 0.2)$ in the (d, p) reaction. No $\frac{7}{2}$ states have previously been observed to be strongly populated in (d, p) reactions to any of the odd Ge isotopes.

Neutron-pickup studies on the Ge isotopes have Neutron-pickup studies on the Ge isotopes have been limited mainly to (p,d) reactions,¹³⁻¹⁵ and no pickup reactions initiated with polarized particles have been reported. At the time the present work was begun, the structure of 69 Ge was not as well known as that of the other odd-A Ge isotopes. Discrepancies exist among earlier investigations, and 69 Ge cannot be studied by the (d, p) reaction.

Only one high-resolution study of the ${}^{70}Ge(d, p)$ ⁷¹Ge reaction has been made, which is the work of Goldman,⁸ with a resolution of 25 keV . A number of discrepancies exist between Goldman's work and the very high-resolution (p,d) man's work and the very high-resolution (p,d)
work of Show *et al.*¹⁵ Since Goldman's work was published, high-resolution studies of was published, high-resolution studies of
 $T^{2,74,76}$ Ge(d, p)^{73,75,77}Ge have been published by
Hasselgren⁹ and by Kato.¹⁰ The availability Hasselgren⁹ and by Kato.¹⁰ The availability of these data facilitates the identification of contam-

inant groups in the ⁷⁰Ge(d, p) spectrum, and a reinvestigation of the $^{70}Ge(d, p)$ reaction with good resolution, including the measurement of vector analyzing power (VAP), should clarify considerably the structure of ${}^{71}Ge$. Since the distortedwave Born approximation (DWBA) was found by
Yoh *et al*,¹¹ to be quite reliable in predicting th Yoh et $al.^{11}$ to be quite reliable in predicting the sign of the (d, p) VAP at forward angles, it should be possible to make spin assignments to many states on the basis of VAP measurements at a few forward angles.

The DWBA has been somewhat less successful in reproducing (d, t) VAP data for energies above the Coulomb barrier than it has for the (d, p) reaction. In analyses of the 58,60,62 Ni(\tilde{d} , t) 22 and the ${}^{40}\text{Ar}(\bar{d}, t)$ ²³ reactions the DWBA was able to reproduce the general shape of the VAP for $l=1$ and $J^{\prime}=\frac{5}{2}$ transitions in Ni, but failed to predict the shape of the VAP data for transitions to $\frac{7}{2}$ states in the Ni isotopes. For ${}^{40}Ar(d, t){}^{39}Ar$, the DWBA predicts the shape of the VAP for $l=3$ states better than it does for $l=1$ transitions. It should be of interest to study the (d, t) reaction on Ge isotopes to test the ability of the DWBA to reproduce the data in this mass region, and to investigate the reliability of spin assignments based on the j dependence of (d, t) VAP data by studying transitions to states of known spin.

In summary, the aims of the present study are: (1) to study the structure of 69 Ge and 71 Ge by means of the ⁷⁰Ge(d, t) and ⁷⁰Ge(d, p) reactions, respectively, and (2) to see to what extent the (d, t) reaction data can be understood within the framework of the DWBA.

II. EXPERIMENTAL DETAILS

Targets used for this work consisted of isotopically enriched GeO₂ evaporated onto $10-\mu g/cm^2 C$ foil backings. Rutherford scattering experiments indicated that the target thicknesses were approximately 150 μ g/cm². The enrichment of the targets was 84.6% in 70 Ge.

Cross sections for the (d, t) transitions to the first four states in 69 Ge were measured using ΔE -E telescopes. The technique employed was similar to that employed in previous (d, t) measimilar to that employed in previous (d, t) mest surements.²³ The resolution obtained with the counter telescopes was typically 60 keV.

Cross-section data were taken in the 100-cm spectrograph for both the (d, t) and (d, p) reactions, using a position-sensitive proportional counter. Typical spectra for the (d, t) reaction are shown in Fig. 1. The upper part of Fig. 1 shows a spectrum obtained using a counter telescope, while the expanded spectrum shown in the lower part of the figure was taken in the spectrograph. The

FIG. 1. Spectrum of the ${}^{70}Ge(d, t) {}^{69}Ge$ reaction. The upper portion of the figure shows a spectrum obtained using a counter telescope. The lower spectrum was obtained in the spectrograph and covers the excitation energy region in 69 Ge between 0.70 and 2.2 MeV.

spectrograph spectra had a resolution of about 18 keV full width at half maximum (FWHM) for the (d, t) reaction and about 20 keV FWHM for the (d, p) reaction.

Absolute normalization of the cross-section data was made by a comparison to Rutherford scattering. The uncertainty in the absolute calibration is a result of a number of factors, including the statistical error, any error due to peak fitting, uncertainty in determining the angle, and any error in the assumption that the scattering is Rutherford at 40° in the lab with $E_a = 3$ MeV. Opticalmodel calculations indicate that in this case; the cross section differs from the Rutherford cross section by less than 1% . The estimated overall uncertainty in the absolute (d, t) cross sections is generally between 15 and 20%.

Fifty- μ m NTA Kodak nuclear emulsion plates covered with 0.25 mm of Mylar to stop inelastically scattered deuterons were used for the spectrograph measurements of the ⁷⁰Ge(d, p)⁷¹Ge VAP at 16' and 20'. Figure ² shows an example of these spectra for $\theta = 20^\circ$. The resolution is about 18 keV.

FIG. 2. Proton spectrum from the ⁷⁰Ge(d, p)⁷¹Ge reaction obtained using nuclear track plates. Group numbers correspond to those given in Table IV. Asterisks indicate the ends of plates where tracks were not recorded.

Since the target contains only 85% ⁷⁰Ge, additional spectra were taken to identify states due to isotopic contaminants in the "Ge target. These spectra were taken at 20' and 45' using thin targets, and excitation energies were determined for all states as excited states in "Ge as well as if they were excited states in the final nucleus of any contaminant present in the target. A resolution of approximately 12 keV was obtained in these spectra. Contaminants were identified by isotopic shifts as well as by comparison with the excitation energies of any states known to be excited strongly in the (d, p) reactions of the contaminants.

Three or four counter telescopes were used for the (d, t) VAP measurements. During the course of these measurements and the (d, p) VAP measurements, the beam polarization was calibrated by a measurement of asymmetry for a reaction of known analyzing power. Calibration standards used in this study were ${}^{12}C(d, d_o)$ at 12.1 MeV $[iT_{11}(\theta_{\rm c.m.} = 34.8^{\circ}) = 0.419 \pm 0.004]$,²⁴ and ¹²C(d, p₀) at
15 MeV $[A_{\rm o}(\theta_{\rm c.m.} = 59.9^{\circ}) = -0.54 \pm 0.05]$,²⁵ 15 MeV $[A_y(\theta_{\text{c.m.}} = 59.9^\circ) = -0.54 \pm 0.05]$.²⁵

III. RESULTS AND ANALYSIS

A. Choice of optical-model parameters

Results of the measurement of the deuteron elastic cross section and VAP and the opticalmodel calculation resulting from use of the search $program$ SNOOPY II arepresented in Fig. 3. Initial deuteron parameters used in the search were deuteron parameters used in the search were
those used by Yoh *et al*.¹¹ in fitting ⁷²Ge(*d*,*d*) data

at $E_d = 12$ MeV. Deuteron parameters used in the present work are given in Table I.

Triton optical-model parameters were taken from the Becchetti-Greenlees global triton param
eters.²⁶ Variations of well depths and geometries eters.²⁶ Variations of well depths and geometrie were undertaken in an attempt to achieve good DWBA fits for (d, t) transitions to states in 69 Ge having representative values of l_n and j_n , specifically the ground state, 0.374-, 0.813-, 0.995-, 1.468-, and 1.763-MeV states. No significant

FIQ. 3. Cross section and vector-analyzing power for 70 Ge(d, d)⁷⁰Ge measured at $E_d = 16$ MeV. The solid curves are optical model fits corresponding to the parameters given in the first row of Table I.

Particle	Reaction	Energy ^a	v	r_0	a_0	W	r_{w}	a_w	$V_{\rm so}$	$r_{\rm{so}}$	$a_{\rm so}$	r_{C}
d	70 Ge(d, d) 70 Ge	16.00	109.0	1.06	0.81	11.30	1.34	0.85	6.28	0.59	0.22	1.25
t	70 Ge $(d, t) {}^{69}$ Ge	10.73	162.6	1.20	0.72	8.26 ^b	1.40	0.84	2.50	1.20	0.72	1.30
\boldsymbol{d}	${}^{70}Ge(d,p){}^{71}Ge$	12.00	113.0	1.06	0.80	13.00	1.38	0.75	5.70	0.70	0.50	1.30
Þ	${}^{70}Ge(d,p){}^{71}Ge$	17.10	56.9	1.17	0.75	11.00	1.32	0.57	6.20	1.01	0.75	1.25
Þ	${}^{72}Ge(d,p) {}^{73}Ge$	16,45	55.8	1.17	0.75	10.14	1.32	0.61	6.20	1.01	0.75	1.25
Þ	${}^{74}Ge(d,p){}^{75}Ge$	16.18	55.8	1.17	0.75	9.81	1.32	0.61	6.20	1.01	0.75	1.25
Þ	${}^{76}Ge(d,p){}^{77}Ge$	15.58	59.86	1.17	0.75	10.83	1.32	0.63	6.20	1.01	0.75	1.25

TABLE I. Optical-model parameters. All energies are in MeV; radii and diffuseness parameters are in fm.

'Equivalent laboratory bombarding energy for the triton and proton parameters, the energy dependence given in Refs. 26 and 29 were used.

 $b W = W_v$.

improvement over the fits obtained with the Becchetti-Greenlees parameters was found. Calculations were also made using recent Los Alamos triton parameters 27 derived from elastic crosssection and VAP data on ${}^{52}Cr$ and ${}^{90}Zr$. However, the calculations using these parameters and variations thereof resulted in poorer fits to the (d, t) data than those obtained using the Becchetti-Greenlees parameters. The final set of optical-model parameters used is given in Table I.

Attempts to fit the (d, p) data by using deuteron

parameters determined from the folding-model prescription of Harvey and Johnson²⁸ were not as successful as fits using deuteron parameters obsuccessful as fits using deuteron parameters ob
tained by Yoh *et al.*11 in fitting ⁷²Ge(d,d)⁷²Ge data at $E_a = 12$ MeV. Proton parameters used in the DWBA calculations were essentially those used by these authors¹¹ in their analysis of $Ge(d, p)$. The energy dependence of the parameters suggested by Becchetti and Greenlees²⁹ was used. Proton optical-model parameters used in the (d, p) calculations are given in Table I.

B. The ⁷⁰Ge(d,t)⁶⁹Ge reaction

The ${}^{70}Ge(d, t) {}^{69}Ge$ cross-section and VAP data 0.087 Mev $\frac{1}{2}$ $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ E_x= 0.374 MeV $\frac{1}{3}$ are shown in Figs. 4-7. The excitation energies

FIG. 4. Results for the ${}^{70}Ge(d, t){}^{69}Ge$ reaction leading to the 0.087-, 0.233-, 0.374-, and 0.995-MeV states. The curves are DWBA predictions.

FIG. 5. Results for the ⁷⁰Ge(d, t)⁶⁹Ge reaction leading to the 0.0- 1.159-, 1.306-, 1.414-, and 1.611-MeV states. The curves give the DWBA predictions.

FIG. 6. Results for the ⁷⁰Ge(d, t)⁶⁹Ge reaction leading to the 0.813-, 1.468-, 2.091-, 2.106-, and 2.145-MeV states. The curves show the DWBA predictions.

given for states in 69 Ge are those of Show et al.¹⁵ The cross-section and analyzing-power data for the doublet at 1.468 MeV (Fig. 6) as well as the combined analyzing power of the 2.091-, 2.106-, and 2.145-MeV triplet (Fig. 6) were fitted as multiplets. A composite cross section $\bar{\sigma}$ was calculated using the relation

$$
\overline{\sigma}(\theta) = a_1 \sigma_1(\theta) + a_2 \sigma_2(\theta), \qquad (3.1)
$$

where a_1 and a_2 were obtained from a least-squares fit of the data to the DWBA calculations for (J^{\prime}) , and (J^{π}) , transitions. In Eq. (3.1), $\sigma_1(\theta)$ and $\sigma_2(\theta)$ are the cross sections calculated by the DWBA. For the (d, t) reaction, the coefficients are related to the spectroscopic factors by $a = (3.33S_J)/(2J+1)$. For the (d, p) reactions discussed in the next section, $a = 1.53S_J$. The composite analyzing power A_v was generated from the expression

$$
\overline{A}_y(\theta) = [a_1 \sigma_1(\theta) A_{y1}(\theta) + a_2 \sigma_2(\theta) A_{y2}(\theta)] / \overline{\sigma}(\theta). \quad (3.2)
$$

The $\frac{1}{2}^{\bullet}$ states could not be separated from other states in the VAP spectra, hence no comparison between $\frac{1}{2}$ ⁺ VAP data and calculations could be made. A summary of the results for the ⁷⁰Ge(d, t)⁶⁹Ge reaction and a comparison to other work are presented in Table II.

The principal uncertainties in the spectroscopic factors listed in Table II are uncertainties in the absolute normalization, uncertainties in the nor-

FIG. 7. ${}^{70}Ge(d, t) {}^{69}Ge$ results for the 1.438-, 1.537-, 1.724-, and 1.763-MeV states. The curves show the DWBA predictions.

malization of DWBA calculations to the data, and the uncertainty associated with ambiguities in the optical-model parameters. Ambiguities associated with the DWBA calculations will include uncertainties in the radial wave function of the transferred neutron used in the calculations. This effect has been discussed by Pinkston and Satchler³⁰ and may have a substantial effect on the magnitude of a spectroscopic factor if the state of interest is far from its corresponding centroid. In the best cases, involving strong, isolated states which are well reproduced by the DWBA, the uncertainty in S is estimated to be about 25%. For states determined by peak fitting or by least-squares fitting of a multiplet, the uncertainty in the spectroscopic factor is estimated to be as high as 50% for the weaker members of a doublet.

C. ⁷⁰Ge(*d*, *p*)⁷¹Ge reaction

Cross-section data for ⁷¹Ge states up to approximately 4 MeV in excitation have been taken in the angular range from 16° to 90° in the 100-cm spectrograph. Figures 8-12 show the (d, p) crosssection and VAP data. Excitation energies were determined from the high-resolution thin-target spectra referred to above. States seen in only one of the two high-resolution spectra have their excitation energies enclosed in parentheses. The uncertainty in these energies is generally 3 keV for states seen in both measurements, and 5 keV for states seen in only one measurement. For those states falling on the plate edge, excitation energies could not be determined from the high-resolution spectra, and the energies of Show et $al.^{15}$ were used. A summary of results for ${}^{70}Ge(d,p)$ and a comparison to previous work are presented in Table IV.

The uncertainties shown for the cross-section and vector-analyzing-power data in Figs. 8-12 are for the most part statistical. For the VAP data, a 4% uncertainty associated with plate-scanning efficiency is also included.

E_x	Present work			Ge(p,d) Show et al. (Ref. 15)			Ge(p,d) Hsu et al. (Ref. 13)			70 Ge (τ, α) Fou et al. (Ref. 12)		
(MeV)	ι	\boldsymbol{J}	\boldsymbol{S}	\mathfrak{r}	\boldsymbol{J}	S	\mathcal{L}	J	S	ι	\bm{J}	S
0.0	3	$\frac{5}{2}$	3.24 ± 0.81	3	$\frac{5}{2}$	3.80	3	$\frac{5}{2}$	2.56	3	$rac{5}{2}$	3.4
0.087	$\mathbf{1}$	$\frac{1}{2}$	0.61 ± 0.15	$\mathbf{1}$		0.56	$\mathbf{1}$	$\frac{1}{2}$	0.50	(3)	$(\frac{5}{2})$	$\mathbf{0.6}$
0.233	$\mathbf{1}$	$(\frac{3}{2})$	0.07 ± 0.02	$\mathbf{1}$		0.10	$\mathbf{1}$	$\frac{3}{2}$	0.09	(1)	$(\frac{3}{2})$	0.2
0.374	$\mathbf{1}$	$rac{3}{2}$	1.67 ± 0.42	$\mathbf{1}$		1.67	$\mathbf{1}$	$\frac{3}{2}$	1.64	$\mathbf{1}$	$\frac{3}{2}$	3.9
0.398				$\overline{\mathbf{4}}$	$\frac{9}{2}$	0.95	$\overline{\mathbf{4}}$	$\frac{9}{2}$	$\boldsymbol{0.8}$			
0.813	$\boldsymbol{2}$	$\frac{5}{2}$	0.09 ± 0.02	$\boldsymbol{2}$	$\frac{5}{2}$	0.09	1,3 1,4		0.06,0.29 0.05,0.26			
0.995	1	$\frac{1}{2}$	0.25 ± 0.06	$\mathbf{1}$	$\frac{1}{2}$	0.32	1		0.26	$\mathbf{1}$	$\frac{3}{2}$	0.7
1.159	$\mathbf{1}$	$(\frac{3}{2})$	0.04 ± 0.01	$\mathbf{1}$	$rac{3}{2}(\frac{1}{2})$	0.06						
1.306	1	$rac{3}{2}$	0.05 ± 0.01	$\mathbf{1}$	$rac{3}{2}(\frac{1}{2})$	0.08	$\mathbf{1}$		0.10			
1.414	3	$(\frac{5}{2})$	0.17 ± 0.06	3	$\frac{5}{2}$	0.28	(3)		0.35			
1.438	(0)	$(\frac{1}{2})$	0.01 ± 0.01									
1.468	$\overline{\mathbf{4}}$	$\frac{9}{2}$	0.66 ± 0.23	$\overline{\mathbf{4}}$	$\frac{9}{2}$	0.09						
	3	$(\frac{5}{2})$	0.21 ± 0.07	3	$\frac{5}{2}$	0.96	(3)		1.12			
1.537	$\mathbf{1}$	$(\frac{3}{7})$	0.02 ± 0.01	$\mathbf{1}$	$\frac{3}{2}$	0.03						
1.611	(3)	$(\frac{5}{2})$	0.24 ± 0.08	3	$rac{5}{2}$	0.25						
1.724	$\mathbf{1}$	$(\frac{3}{2})$	0.07 ± 0.02	1	$\frac{3}{2}$	0.03						
1.763	(0)	$(\frac{1}{2})$	0.02 ± 0.01	$\bf{0}$	$\frac{1}{2}$	0.12		much smaller)	(Probably a misprint; the graphs show S to be			
2.091	$\bf{0}$	$\frac{1}{2}$	$\textbf{0.02} \pm \textbf{0.01}$	$\bf{0}$	$\frac{1}{2}$	0.02						
2.106	$\mathbf{1}$	$(\frac{3}{2})$	0.11 ± 0.04	1	$\frac{3}{2}$	0.16	(1)		0.17			
2.145	$\overline{\mathbf{4}}$	$(\frac{9}{7})$	0.29 ± 0.07	4	$\frac{9}{2}$	0.30						

TABLE II. Summary of results for the ⁷⁰Ge(\tilde{d} , t)⁶⁹Ge reaction.

IV. DISCUSSION

A. $^{70}Ge(d,t)$ reaction

1. $l_n = 1$ transitions

Data for $l_n = 1$ transitions are shown in Figs. 4 and 5. The strongest $l_n = 1$ transitions are to the 0.087- and 0.374-MeV states. The VAP data provide a clear distinction between the two possible j_n values and confirm the previous spin assignments to these states. However, at forward angles the VAP data are not well reproduced by the DWBA. This lack of agreement at forward angles between measured and calculated analyzing power.
has also been observed in the nickel isotopes.²² has also been observed in the nickel isotopes. Nevertheless, the overall qualitative agreement between measured and calculated VAP over most of the angular range permits unambiguous spin assignments to be made for these transitions. The cross-section data also appear to follow the i .

dependence around 70° which is predicted by the DWBA calculations. A similar j_n dependence in $l_n = 1$ cross-section data has been observed in Ge(p, d) reactions by Show *et al*,¹⁵ $Ge(p,d)$ reactions by Show et al.¹⁵

Cross-section and VAP data for the transition to the 0.233-MeV level are in fairly good agreement with the calculations and confirm the $\frac{3}{2}$ spinparity assignment of this state. It is interesting that the first minimum in the cross-section data for the 0.087-, 0.233-, and 0.374-MeV states is not as pronounced as in the calculations, yet this minimum is well described by the DWBA for $l = 1$ transitions to more highly excited states.

The 0.995-MeV state clearly has $J=\frac{1}{2}$. Except for the VAP data forward of 30', the data are well reproduced by the DWBA calculations.

The 1.306-MeV VAP data indicate a clear preference for the $J=\frac{3}{2}$ assignment. VAP data for the 1.159-MeV level exhibit the same behavior as those of the 1.306-MeV state in the angular region

FIG. 8. Cross-section and VAP data for $l_n = 1$ transitions in the ⁷⁰Ge(d, p)⁷¹Ge reaction. The curves show the predictions of the DWBA. Data for three contaminant transitions are also shown.

35°-60°, and indicate a tentative $j_n = \frac{3}{2}$ assignment for the former level.

2. $l_n = 3$ transitions

A strong transition to the ground state and weaker transitions to the 1.414- and 1.611-MeV states were observed to have $l_n = 3$ character and are shown in Fig. 5. The ground-state VAP data allow an unambiguous choice of spin and confirm the previous $\frac{5}{2}$ assignment to this state. The DWBA calculation reproduces the phase of the VAP data and also the magnitude except in the region near 50°. These data are very similar to $\frac{5}{2}$ analyzingpower data observed by Huttlin et al.²² in the Ni isotopes. Cross-section data for the 1.414- and 1.611-MeV states show disagreement with the calculations in the region of the first minimum. similar to that observed for the $l = 1$ transitions. Some of the disagreement around 50° in the data for the 1.611-MeV level is produced by a contaminant that could not be completely separated from the 1.611-MeV state. Although a firm $l = 3$

assignment can hardly be made to the 1.611-MeV state on the basis of these data, they do support the $\frac{5}{2}$ assignment to this level made by Show $et \ al.^{15}$

3. $l = 2$ transition

Cross-section data for the transition to the 0.813-MeV level are best fit with $l_n = 2$, in agreement with the results of Show et al.¹⁵ and with the $\frac{5}{2}$ assignment of Isoya et al.¹⁷ Hsu et al.¹³ expected only $l = 1$, 3, and 4 transitions to occur and therefore treated this level as one of two possible doublets, either an $l = 1, 3$ doublet or an $l = 1, 4$ doublet. The fit to the data with an $l_n = 2$ calculation is quite good for angles less than 30°, as was also found for the (p,d) data. The discrepancy between the calculated and measured cross sections in the region of the first minimum is similar to that observed in the $l = 1$ and $l = 3$ transitions. The VAP data appear to be shifted in phase from the predictions of the DWBA calculation but provide a clear indication of $j_n = \frac{5}{2}$.

FIG. 9. Cross-section and VAP data for $l_n = 2$ transitions in ⁷⁰Ge(d, p)⁷¹Ge. The curves show the predictions of the DWBA.

4. $I_n = 4$ transitions, $I_n = 0$ transitions, and multiplets

The (p,d) measurements of Refs. 13 and 15 showed an $l=4$ transition to a $\frac{9}{2}^*$ state at E_x = 0.398 MeV. In the present work, this state could not be resolved from the strong group corresponding to the $\frac{3}{2}$ level at 0.374 MeV. Using the spectroscopic factor of 0.95 quoted in Ref. 15 for the 0.398-MeV level, one would expect an enhancement in the cross section for the 0.374-MeV group in the angular range $30^{\circ} - 40^{\circ}$ of about 0.2 mb/sr. It is clear from the data shown for this group in Fig. 4 that an $l = 4$ contribution of this magnitude could easily be present and probably explains most of the discrepancy with the DWBA for angles near 30° .

States at 1.468 and 2. 145 MeV were also found to have $l_n = 4$ angular distributions and are shown in have $l_n = 4$ angular distributions and are shown in Fig. 6. The high-resolution work of Show *et al.*¹⁵ has revealed a doublet of $l = 4$ and $l = 3$ levels at 1468 and 1477 keV, respectively. An $l_n = 3$, $\frac{5}{2}$ admixture would explain the deviation of the crosssection data from the $l_n = 4$ calculation and would contribute a positive analyzing power (Fig. 5) to the negative $\frac{9}{2}$ ⁺ VAP, resulting in the composite analyzing power given by the solid curve in Fig. 6. The composite cross section and VAP given by the solid curves correspond to a mixture of 65% $1g_{9/2}$ and 35% $1f_{5/2}$. The (p,d) results of Show et al. indicate that the $l=3$ cross section is approximately four times the $l = 4$ cross section. proximately four times the $l = 4$ cross section.
Hsu *et al*.¹³ also assign the 1.477-MeV state $l = 3$. However, their cross-section data do not fall off as quickly with increasing angle as their $l = 3$ calculations predict, so it appears that their data include a contribution from a higher l_n transition. In the present work, the $l = 4$ transition is almost twice as strong as that for $l=3$. Since one expects comparable results from the (p, d) and (d, t) reactions, this rather large dependence of relative spectroscopic factor on reaction type is puzzling.

Transitions with $l_n = 0$ were seen at 1.438, 1.763, and 2.091 MeV. The 1.438-MeV state has not previously been assigned a value of l_n , but has a

FIG. 10. Cross-section and VAP data for $l_n = 0$ and $l_n = 2$ transitions in ⁷⁰Ge(d, p)⁷¹Ge. Data are also shown for a number of contaminant transitions. The curves show the predictions of the DWBA.

cross section similar to the 1.763- and 2.091-MeV states. For the latter two transitions, the l_n assignments are based on the results of Ref. 15.

The 2.145-MeV state could not be separated from the 2.091- and 2.106-MeV levels in the analyzing-power measurements. The cross section for excitation of the 2.091-MeV state is small compared to those of the 2.106- and 2.145-MeV states, especially for angles greater than 35°, where the calculated $l = 0$ VAP is appreciably different from zero. Consequently, an analysis of the VAP data in terms of an $l=1$, $l=4$ doublet is sufficient. Calculations were made using Eq. (3.2) for each of the four possible spin combinations.

The composite A_{ν} curve for the $\frac{1}{2}$, $\frac{7}{2}$ combination of spins is not shown in Fig. 13, but is a curve that is positive for all angles with values of A_v as high as 0.40. The statistical accuracy of the data do not permit a distinction between the

other three spin-parity combinations. However, on the basis of shell-model systematics, the spins and parities of the 2.106- and 2.145-MeV states can be tentatively assigned at $\frac{3}{2}$ and $\frac{9}{2}$, respectively.

Eberth et $al.^2$ have assumed the 1.306-MeV state to be the $\frac{1}{2}$ member of a multiplet arising from the coupling of $1f_{5/2}$ neutron to a quadrupole phonon. The present results show that neither the 1.306-MeV state nor probably the 1.159-MeV state can be identified with this level. However, the 0.995-MeV state could be considered as a candidate, since this $\frac{1}{2}$ level is in the excitation energy region of the expected weak-coupling state.

Eberth et al. also interpreted several positiveparity levels in ⁶⁹Ge as members of a multiplet corresponding to the coupling of a $1g_{9/2}$ neutron to a quadrupole phonon and identified the $\frac{5}{2}^*$ member of this multiplet as the 0.813-MeV level. If

FIG. 11. Cross-section and VAP data for $l_n = 3, 4$ transitions and for unresolved doublets in ⁷⁰Ge(d, p)⁷¹Ge. Data are also included for some contaminant transitions. The curves give the DWBA predictions.

this state is a member of the proposed multiplet, the less-than-ideal fit to the data assuming a single-step reaction process may reflect a significant probability for forming the state in a two-step process involving pickup of a $g_{9/2}$ neutron from a process involving pickup of a $g_{9/2}$ neutron from
2^{*} excited ⁷⁰Ge core. Eberth *et al*, were unable
to identify the $\frac{7}{2}$ ^{*} and $\frac{9}{2}$ ^{*} members of the propos to identify the $\frac{7}{2}$ and $\frac{9}{2}$ members of the proposed $|2^{\star}\rangle \otimes |2^{\star}\rangle$ multiplet. The 1.468-MeV state might be considered a candidate for the $\frac{9}{2}^*$ member, since this level would be expected to have an excitation energy near 1.⁵ MeV. If this identification of the 1.458-MeV level as the $\frac{9}{5}$ ⁺ member of the weak-coupling multiplet is correct, it should decay preferentially to the lower-lying $\frac{9}{2}^+$ level. Forssten et al.¹ observe a reasonably intense γ transition of the right energy $(E_\gamma \sim 1070 \text{ keV})$ which they tentatively assign to a higher-spin level at 2475 keV. Their measured angular distribution for this γ ray also appears to be consistent with a $\frac{9}{2}$ ⁺ \div + $\frac{9}{2}$ ⁺ transition, so that the y-ray data lend some support to the idea that the 1.468-MeV level is the $\frac{9}{2}$ member of the $|2^{\ast}\rangle\otimes|\frac{9}{2}^{\ast}\rangle$ multiple

5. Sum-rule analysis

In order to study the gross structure of shellmodel states being filled in the Ge isotopes, a sum-rule analysis was carried out. Centroids for the distributions of spectroscopic strength for states of a given l and j were calculated along with the summed strengths, $\sum_i S_i(j)$. For a given J^{\dagger} , the centroid is given by $\sum_i E_i S_i / \sum_i S_i$. Values of centroids and fullnesses predicted by the simple pairing model³¹ were calculated using the equations

$$
E_j = [(\epsilon_j - \lambda)^2 + \Delta^2]^{1/2} - \text{constant},
$$
 (3.3)

$$
V_j^2 = \left(\frac{1}{2}\right) \left(1 - \frac{(\epsilon_j - \lambda)}{[(\epsilon_j - \lambda)^2 + \Delta^2]^{1/2}}\right),\tag{3.4}
$$

where ϵ , is the single quasiparticle energy, λ is the Fermi energy, Δ is the gap parameter. Values of ϵ , for the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals were interpolated from the values of Kisslinger were interpolated from the values of Kisslinger
and Sorenson.³¹ The value of $\epsilon_{1\epsilon_9/2}$ was taken to be 2.8 MeV, the value used in Ref. 11. The value

FIG. 12. Cross-section and VAP data for some unassigned transitions in ${}^{70}Ge(d, p)$ ⁷¹Ge. Data are also included for one contaminant transition. The curves show the results of DWBA calculations.

FIG. 13. Distributions of spectroscopic factors for the ⁷⁰Ge(*d*, *p*)⁷¹Ge reaction. The graphs include levels for which only tentative assignments have been made.

of the constant in Eq. (3.3) was set equal to Δ , following the method of Ref. 14. Δ was extrapolated from the Kisslinger and Sorenson values of Δ vs A for the Ge isotopes. λ was recalculated taking into account the change in $\epsilon_{s_{9/2}}$ and the extrapolated value of Δ . Results of the analysis are given in Table III along with the (p,d) results of Refs. 13 and 15.

The present results are in approximate agreement with the pairing model predictions, but it is clear that the simple theory underpredicts the fullness of the $p_{1/2}$ shell and appears to overpredict somewhat the fullness of the $p_{3/2}$ shell. It also is evident that a significant amount of $2p_{3/2}$ and $1f_{5/2}$ strength exists above $E_x = 2.1$ MeV and is missed in the present measurements. The

TABLE III. Comparison of ${}^{70}Ge(d,t){}^{69}Ge$ data with simple pairing theory.

		Σs_i			\boldsymbol{E} ,			
Shell-model orbital	Present work	Ref. 13	Ref. 15	Pairing theory	Present work	Pairing theory		
$2p_{3/2}$	2.03	$2.8 - 3.2$	2.0	3.40	0.56	0.56		
$1f_{5/2}$	3.86	5.16	5.10	5.22	0.24	0.67		
$2p_{1/2}$	0.86	$0.6 - 1.0$	0.73	0.30	0.35	0.57		
$1g_{9/2}$	1.90 ^a	1.0	1.40	1.40	1.04 ⁴	0.59		
$2d_{5/2}$	0.09	\cdots	0.12	\cdots	\cdots	\cdots		
$3s_{1/2}$	0.05	\cdots	0.02	\cdots	\cdots	\cdots		

^a Includes S for the 0.398-MeV level from Ref. 15.

 $g_{9/2}$ summed strength given in Table III includes the value of S for the 0.398-MeV level given in Ref. 15. The rather small amount of $(2d_{5/2})^2$ and $(3s_{1/2})^2$ configurations in ⁷⁰Ge indicated by the present results is quite consistent with the (p, d) results of Ref. 15.

B. $^{70}Ge(d,p)^{71}Ge$ reaction

The (d, p) cross sections of the present work are generally $20-25\%$ lower than those obtained by generally 20–25% lower than those obtained by
Yoh *et al*.¹¹ Better agreement is found for strong states, for which background subtraction is not very important, which implies that at least part of the discrepancy between the two measurements arises from differences in the amount of background subtracted in extracting yields. Absolute normalization of the cross section was obtained from the absolute normalization of the $^{70}Ge(d, t)$ reaction data. From Rutherford scattering on ' 70 Ge, the target thickness was determined using the isotopic abundance quoted by the isotope supplier, the known solid angle of the spectrograph, the measured charge, and the reaction yield. The (d, p) spectrum of 16° was remeasured periodically during the course of the $\sigma(\theta)$ measurements to check for target deterioration. No measurable target deterioration was observed. The estimated overall uncertainty in the absolute cross sections is about 15%.

In the discussion that follows, the results are grouped according to the l value of the transition.

I. $I_n = I$ transitions

In general, the DWBA calculations reproduce the cross-section data well, but tend to predict lower magnitudes of VAP than those observed (Fig. 8). However, the data provide a clear choice between j values, except for the 2.041-, 3.311-, and the 3.571-MeV states. Only the VAP data for the "Ge and "Ge ground states can be compared with the and ⁷⁵Ge ground states can be compared with th
data of Yoh *et al*.¹¹ The two measurements are in agreement, although a value of $A_v(\theta)$ almost twice as large as that obtained by Yoh et al. was measured at 16' in the present work.

The $l_n = 1$, 2 doublet at approximately 0.500 MeV was sufficiently well resolved to permit extraction of separate cross sections. In agreement with Goldman, ' the two states were found to have comparable cross sections. The data for the 0.501- MeV state indicate a $\frac{3}{2}$ assignment, confirmin
the tentative assignment of Yoh *et al*.¹¹ the tentative assignment of Yoh et $al.^{11}$.

VAP data for the 0.707-MeV state have been resolved from the contribution of the 73 Ge 0.066-MeV contaminant and are in fair agreement with the DWBA calculations. Yoh et al. were unable to resolve these states, which explains the discrepancy

between their measured and calculated VAP.

 $A^{\frac{1}{2}}$ state is found to exist near 1 MeV excitation in $^{69, 73, 75}$ Ge, so that a similar state might be expected at this excitation in "Ge. The VAP data for the 1.286-MeV level show this state to be $\frac{1}{2}$, consistent with the systematics of the other Ge isotopes.

The $l_n = 1$ assignments to the 2.742-, 2.896-, and 3.311-MeV states must be considered tentative. The data follow the trend of the calculation, but the deviation of the measurements from the predictions at several angles precludes a firm $l_n = 1$ assignment to these states. VAP data for the 3.311-MeV level would indicate a spin of $\frac{1}{2}$ for an l_n = 1 transition, hence this state is tentatively assigned $\frac{1}{2}$.

2. $l_n = 2$ transitions

Thirty transitions for which $l_n = 2$ were seen and the data are shown in Figs. 9 and 10. Cross-section and VAP data for the transitions are generally tion and VAP data for the transitions are genera
well represented by the DWBA. Yoh *et al*.¹¹ have seen five of these transitions in their investigation, and were able to make $\frac{5}{2}$ assignments to the 0,526-, 1.203-, 1.556-, 2.277-, and 2.644-MeV states in "Ge.

It is somewhat surprising that the 0.526-MeV state, which is the strongest $l_n = 2$ state observed in ⁷⁰Ge(d, p), is considered to be the $\frac{5}{2}$ ⁺ member of a $|2^{\star}\rangle \otimes | \frac{9}{2} \rangle$ multiplet by Forssten et al.¹ and Eberth et $al.^2$ One might expect a state of this nature to be weakly populated by a one-step mecha. nism and to display most of its strength in a twostep process, in which case a poor fit to the data by the DWBA might be expected. However, the cross-section and VAP data are well fit by the DWBA. Of course, the $|{\bf 2}^\star\rangle\otimes|{\bf 1}{\bf g}_{9/2}\rangle$ configura tion can still be dominant in the wave function of this state, since the (d, p) spectroscopic factor is only 0.1.

aly 0.1.
Show *et al*.¹⁵ report an $l_n = 3$ cross section for the 1.203-MeV state. They observe a weakly populated doublet that can be fitted either by an $l_n = 3$
state or an $l_n = 2, 3$ mixture. Malan *et al*.,¹⁸ using state or an $l_n = 2, 3$ mixture. Malan *et al*.,¹⁸ using the ${}^{71}Ga(p,n){}^{71}Ge$ reaction have seen a 7-keV doublet at this energy and report that the lowerenergy member can have $J = \frac{5}{2}, \frac{3}{2}, (\frac{7}{2})$, while the higher-energy member has $J^{\pi} = \frac{5}{2}$. The presential work shows the 1.203-MeV level to be definitely $\frac{5}{2}$

Goldman' tentatively assigns the 1.474-MeV state to be $l_n = 3$. Although one would expect a pickup reaction to populate an $l_n = 3$ level, Show et al. do not see this state. While the agreement with the DWBA calculations is not as good as for other states, the data are most consistent with a

 $\frac{5}{2}^+$ assignment for this state. Examination of Goldman's data reveals that the data for the 1.47-MeV state have a larger cross section at formard angles than those of the other $l_n = 3$ states. Perhaps what Goldman observed was the combined cross section for the $l_n = 2$ state at 1.475 MeV and the $l_n = 0$ state at 1.453 MeV, since the secondary maximum of an $l_n = 0$ cross section somewhat resembles an l_n = 3 shape.

The 2.345-MeV state is assigned $l_n = 2$ by this udy and $l_n = 1$ by Show *et al*.¹⁵ who see a stron study and $l_n = 1$ by Show *et al*.¹⁵ who see a strong transition to a state at this energy. This level is not strongly populated in the (d, p) reaction, but the forward-angle data are reasonably mell fitted by a calculation for $l_n = 2$ and are out of phase with the predictions for $l_n = 1$. Some of the strength of this state can be attributed to the 1.006-MeV state of ⁷⁶Ge(d, p) which is also an $l_n = 2$ transition, so that the spectroscopic factor for this state must be considered an upper limit. However, the 70 Ge target contains only 2% ⁷⁶Ge, so that the ⁷⁷Ge 1.006-MeV level can only account for about onethird of the observed strength.

The 3.286-MeV state has been assigned $l_n = 1$ by Show et al., although this assignment is made on the basis of rather limited data. The forwardangle data shown in Fig. 9 are mell fitted by an $l_n = 2$ cross section. However, an $l_n = 1$ calculation also fits the cross-section data if the 30' and 35'points are excluded. This state is therefore tentatively assigned $l_n = 2$ or 1. The VAP data indicate a $\frac{5}{2}^+$ assignment (l_n = 2) or a $\frac{1}{2}^-$ assignmen $(l_n = 1)$.

Since there are only eight data points for the '3.767-MeV state, only a tentative $l_n = 2$, $J^{\dagger} = \frac{5}{2}^{\dagger}$ assignment is made to this state. The 3.899- MeV state is tentatively assigned a J^{π} of $\frac{3}{2}^{\ast}$ on the basis of the VAP at 16° . Goldman⁸ was unable to resolve the states at this excitation energy, and his 3.90-MeV state probably contains contributions his 3.90 -MeV state probably contains contribution several states. Show $et \ al.^{15}$ observe an $l_n = 1$ state at 3.900 MeV. Examination of the 3.899- and 3.910-MeV cross sections shows that the data disagree with the $l = 2$ predictions at far forward angles and also near 40' in the case of the 3.910-MeV transition. These are the angular regions in which the maxima of an $l_n = 1$ transition occur. Hence, an $l_n = 1$ transition to the level seen by Show et a/. may be contributing to the 3.899 and 3.910-MeV cross sections. If the 3.899- and 3.910-MeV states are both $\frac{3}{2}$ ^{*}, it is not surprising that Show et al. did not observe them since the ground state of "Ge is not expected to contain any significant amount of $(d_{3/2})^2$ configuration.

Examining all the $l_n = 2$ transitions, one sees the energy dependence of the cross section and the analyzing power. As the energy of the outgoing proton decreases, the magnitude of the crossproton decreases, the magnitude of the cross-
section oscillations decrease and the $\frac{5}{2}^\ast$ analyzin power near 20' becomes more negative. As seen in the calculations of Ref. 11, the predicted VAP tends to become more negative and less oscillatory in structure as the excitation energy of the state increases.

A weak j dependence in the cross-section data between $\frac{3}{2}$ and $\frac{5}{2}$ states appears to be present for the 2.881-MeV states. A similar j dependence has been seen in a ⁷⁶Se(d, p) study by Macefield, Mid-
dleton, and Pullen.³² dleton, and Pullen.

3. $I_n = 0$ transitions

Thirteen $l_n = 0$ transitions were seen in this work, and eleven of these are shown in Fig. 10. In general, most of the cross sections are well reproduced by the DWBA calculations. Some disagreement between measured and calculated VAP exists but is not surprising in viem of the sensitivity of the $l_n = 0$ VAP to spin-dependent distortions.

Part of the strength of the transition to the 3.087-MeV group is apparently produced by the 2.459- MeV state in ⁷³Ge which is also an $l_n = 0$ transition. If all the strength of this composite group were attributed to the 2.459 -MeV state in 73 Ge, the cross section for this transition mould have to be several times larger than that measured by to be several times larger than that measured
Yoh *et al*.¹¹ Hence it is concluded that an $l_n = 0$ ⁷¹Ge state as well as the 73 Ge 2.459-MeV state contribute to the observed $l_n = 0$ distribution. The spectroscopic factor for the ⁷¹Ge level should be considered as an upper limit for the strength of this state.

A state at 3.232 MeV has been assigned $l_n = 0$, but is not shown in Fig. 10. A 3.23-MeV level has been seen by Goldman⁸ and a 3.232 -MeV level by been seen by Goldman⁸ and a 3.232-MeV level by
Show *et al*.,¹⁵ but neither have been given l_n assign ments.

Goldman assigns $l_n = 0$ to a state at 3.62 MeV. The present cross-section data for the 3.633-MeV level (Fig. 12) are not characteristic of a particular l_n . The VAP data are more positive than those of other $l_n = 0$ states in the same excitation energy region.

4. $I_n = 3$, 4 transitions

In general, the cross sections for $l_n = 3, 4$ transitions (Fig. 11) are reasonably well reproduced by the DWBA calculations. Unfortunately, the 0.747- MeV state fell upon a plate gap in both the 16' and 20 VAP measurements, so no VAP data were obtained.

tained.
The spectroscopic factor quoted by Yoh *et al*.,¹¹ for the 199-keV level is at least 20% too large. since the contribution from the cross section to

the 0.176-MeV state was not taken into account in their analysis. A $J^* = \frac{5}{2}^-$ assignment can be made to the latter level, although the magnitude of the VAP is much greater than that predicted by the DWBA.

The 1.375-MeV state is tentatively assigned $l_n = 3$, although a value of $l_n = 2$ cannot be ruled out. An $l_n = 2$ assignment fits the first few points of the cross-section distribution, but the data as a whole agree better with $l_n = 3$. Since the 1.375-MeV state and the 1.346-MeV states were observed as a doublet, it is possible that in fitting the most forward angles some of the $l_n = 0$ strength was assigned to the 1.375-MeV state, giving too large a cross section at the most forward angles. The preferred assignment of $J^{\prime} = \frac{7}{2}$ to this state on the basis of the VAP data is somewhat surprising, since the $1f_{7h}$ subshell is expected to be essentially full in this mass region. However, the spectroscopic factor is only about 0.01.

The cross section for the 1.695-MeV level is well fit by an $l_n = 4$ calculation, but the VAP data The cross section for the 1,030-mey fever is
well fit by an $l_n = 4$ calculation, but the VAP data
indicate $J^{\dagger} = \frac{9}{2}^{\dagger}$, in conflict with the $\frac{7}{2}^{\dagger}$ assignment indicate $J^{\mathbf{r}} = \frac{9}{2}^{\mathbf{t}}$, in conflict with the $\frac{7}{2}^{\mathbf{t}}$ assignn
of Yoh *et al*.¹¹ A reexamination of the data of Yoh et al. revealed that the published uncertainties for the two most forward-angle VAP measurements are too small by about a factor of 2. The errors should overlap zero and hence no definitive j_n value is indicated by their forward-angle data. Moreover, Yoh et al. found it necessary to use an unusually deep proton potential well $(V = 65 \text{ MeV})$ to reproduce their VAP data for this state. On the basis of the present work, this state must be assigned $\frac{9}{5}$.

Data for the 1.941-MeV level arebest fitted by an l_n =4 distribution and the VAP data indicate a J^{\dagger} of Hasselgren has reported l_n =4 states betwee:
and 2.1 MeV in $^{\rm 73, 75, 77}$ Ge with comparable 1.5 and 2.1 MeV in 73,75,77 Ge with comparabl spectroscopic factors.

5. Transitions to contaminants

Data for contaminant transitions corresponding to various l_n values are given in the lower portions of Figs. 8-12. DWBA curves were generated using the parameters of Ref. 11 for the 72,74,76 Ge(d, p) reactions. These contaminant transitions help to clarify some results of previous ${}^{70}Ge(d, p)$ studies. Several states reported by Goldman were at excitation energies corresponding to the expected positions of the ground-state groups of other $Ge(d, p)$ reactions, suggesting that these groups are contaminants.

Using the isotopic abundances of the target material provided by the supplier, the absolute cross sections for the contaminant transitions are only about half as large as the cross sections given by

Yoh *et al*.¹¹ \hat{Q} values used to calculate the position of contaminant groups were taken from Hasselgren.⁹ Excitation energies quoted in Table IV for contaminant groups are also taken from Refs. 9 and 10. Contaminant states were identified by: (a) their excitation energy corresponding to that of a strong state in an isotope other than 71 Ge, (b) having an l_n value corresponding to the l_n of the suspected contaminant state, (c) having a spectroscopic factor of the expected order of magnitude, and (d) by having the correct J value if VAP data are available for this state. A result of this identification is that the 0.890-MeV state reported by Goldman is clearly the ground state of 75 Ge. This conclusion is supported by the fact that no evidence for a state in ⁷¹Ge near 890 keV
appears in the (p,d) measurements.^{14,15} appears in the (p,d) measurements.^{14,15}

6. Doublets

C ross-section angular distributions which correspond to states that were not resolved in this work are shown in the left-hand portion of Fig. 11. In addition to known unresolved states, other groups were determined to be doublets if data for a state of known l_n , e.g., a well-known contaminant transition, were not well fitted by the assigned l_n or if a cross section was found to have a significantly higher spectroscopic factor than a significantly higher spectroscopic factor the
that measured by Yoh et $al.^{11}$ or Hasselgren.⁹

The 2.363 -MeV group in ⁷¹Ge is clearly seen to correspond to an $l_n = 0$ transition from the forward-angle behavior. A contribution from another state may cause the disc repancy between the measured and calculated cross section in the angular region around 60'.

Data for the 73 Ge ground state and 0.013-MeV level are fitted with a mixture of 90% $1g_{9/2}$ and 10% $2d_{5/2}$, which is the mixture used by Yoh
et al.¹¹ The VAP data have the same sign a et $al.^{11}$ The VAP data have the same sign as the data of Yoh et $al.$, although the value at 16° has a substantially larger magnitude.

The 2.088 -MeV state in ⁷³Ge was not reported by Yoh et al. but was observed by Hasselgren. 9 The maximum cross section observed in the present work is approximately twice that observed by Hasselgren. Some of the additional cross section probably arises from the "Ge 1.386-MeV state. The cross section for the level in $75Ge$ at 0.884 MeV exhibits mainly an $l_x = 3$ or 4 contribution. An $l_n = 3$ component would agree with the $l_n = 3$ assignment of Show et al. to the 1.780-MeV state. The VAP data agree with a DWBA calculation for $J^{\prime}=\frac{1}{2}$, which is the assignment made to the ⁷⁵Ge 0.884-MeV state by Yoh et al.

The cross section for the transition to the ⁷⁵Ge 1.854-MeV level appears to contain a contribution

 $\frac{16}{1}$

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 $\frac{16}{1}$

 $\frac{16}{1}$

TABLE IV. (Continued)

1350

 $\underline{16}$

⁶⁹Ge AND ⁷¹Ge USING THE ⁷⁰Ge(d,t) AND ⁷⁰Ge(d,n)

TABLE IV. (Continued)

1352

 $\underline{16}$

 $\frac{16}{1}$

	${}^{70}Ge(d,p)$							70 Ge			
	\sum (2J + 1)S _i Present work	Present work Ref. 11	$\sum s_i$	Theory	Present work	E_i Ref. 11		$\sum (2J+1)S(d,p)+S(d,t)$ Theory ^a Present work	Limit		
$2p_{3/2}$	0.88	0.22	0.24	0.15	1.56	0.77	0.56	2.91	$\overline{4}$		
$1f_{5/2}$	1.61	0.27	0.19	0.13	0.22	0.92	0.67	5.47	6		
$2p_{1/2}$	0.74	0.37	0.40	0.85	0.30	0.32	0.57	1.60	$\overline{2}$		
$1g_{9/2}$	5.34	0.53	0.87	0.86	0.54	0.20	0.59	7.24	10		
$2d_{5/2}$	2.34	0.39	0.59		1.89	1.51		2.43	6		
$3s_{1/2}$	0.74	0.37	0.82		2.37	2.32		0.79	$\mathbf{2}$		
$2d_{3/2}$	0.20	0.05	0.06		3.39	3.50		0.20	4		

TABLE V. Results of pairing calculation and sum-rule analysis for $^{70}Ge(d, p)^{71}Ge$.

^a The quantity given is $E_i - \Delta$.

from another level, but the origin of this contribution could not be identified.

The ⁷⁷Ge 0.159-MeV state appears to involve an $l_n = 1$ transition with some $l_n = 2$ contribution. This cross section is approximately three times the cross section is approximately three times the $l_n^{}$ = 1 cross section obtained by Yoh *et al*.,¹¹ but the origin of this discrepancy is not clear.

Data for the 0.495- and 1.783-MeV states of ⁷⁷Ge are reasonably well reporoduced by $l_n = 2$ DWBA calculations. The 0.495-MeV level falls within 15 keV of the 71 Ge 1.841-MeV state. Since the proportional-counter resolution was approximately 20 keV, the latter state is expected to contribute to the measured cross section for the former. A similar situation is expected for the "Ge 1.783-MeV state, since it lies within 13 keV of the "Ge 3.102-MeV level in the spectrum.

7. Unassigned transitions

Data for a number of unassigned transitions are shown in Fig. 12. The cross sections for the 1.154- and 1.595-MeV levels are relatively featureless. Levels at 1.160 and 1.600 MeV have
been listed in the compilation,³³ but no additic been listed in the compilation, $^{\rm 33}$ but no additiona information is available.

The 2.210- and 3.380-MeV cross sections are equally well reproduced by either an $l_n = 0$ or an $l_n = 3$ calculation, since no forward-angle data were obtained for these states. The VAP measurement for the 3.380-MeV level agrees better with a $\frac{5}{2}$ assignment, but the poor statistics prevent a definite assignment from being made.

A. large number of groups seen in the thin target spectra taken with nuclear plates in the spectrograph were too weak or not sufficiently well resolved in the proportional-counter spectra to permit angular distribution measurements to be obtained. These states are listed in Table IV by excitation ene rgy.

8. $^{70}Ge(d, p)^{71}Ge$ Sum-Rule Analysis

A pairing-model calculation and sum-rule analysis similar to that carried out for the (d, t) reaction was performed for the (d,p) reaction. The same parameters as those used in the (d, t) calculation were used for the pairing-model calculation. Results of this analysis are given in Table V. As with the (d, t) results, there is only qualitative agreement between theory and the results of the present work. The measured centroid of the $2p_{3/2}$ subshell is substantially higher than the pairing model predicts, and the $2p_{3/2}$ and $1f_{5/2}$ subshells appear to be emptier than predicted, while the $2p_{1/2}$ and $1g_{9/2}$ subshells appear fuller. Apparently the neutron strength is spread even more uniformly over the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals than the pairing model predicts.

Comparison of the results of this work to that Yoh $et al.¹¹$ shows substantial differences in of Yoh et al.¹¹ shows substantial differences in the summed spectroscopic factors for the $1f_{5/2}$ and $1g_{9/2}$ subshells. For the $1g_{9/2}$ subshell, the difference in strength is a consequence of different optical model parameters used in the DWBA calculations, as well as the fact that the cross section for the 0.199-MeV state reported by Yoh *et al.* is approximately 40% larger than the cross section of the present work. The spectroscopic factor obtained for this state in the present work is only half that reported by Yoh et al. Moreover, the combined summed spectroscopic strengths $(2J+1)S_j$ for the $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$, and $1g_{9/2}$ subshells amount to only 8.57, which is far short of the expected value of 12.

The same problem is evident from a comparison of the combined summed strengths for (d, p) and (d, t) reactions given in Table V with the sumrule limits given in the last column of the table. It seems unlikely that the difference can be attributed entirely to levels not included in the analy-

sis. All of these discrepancies suggest that the (d, p) spectroscopic factors obtained in the present analysis may be systematically low by about 20- 30%. The measured cross sections are lower 30%. The measured cross sections are lower than those of Yoh *et al*.¹¹ by about 25%, but it is also possible that part of the difficulty resides in the optical model parameters used in the analysis. The discrepancy does not appear to be caused by the deuteron parameters taken from the results of Ref. 11, although the assumption of these authors that the same set of parameters can be used for all four Ge isotopes may be questionable. The proton parameters are basically the global parameters of Becchetti and Green-
lees.²⁹ lees.²⁹

Another possible source of error in the spectroscopic factors is the form factor customarily used in the DWBA calculations. This has been
discussed by Pinkston and Satchler,³⁰ who pro discussed by Pinkston and Satchler, $^{\rm 30}$ who proposed a method of correction for this effect. The correction was estimated for the spectroscopic factors obtained in Ref. 11 and found to increase the measured S_j by 10-30% for $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ levels. However, no significant correction was indicated for the $g_{9/2}$ and $d_{5/2}$ levels. Application of this correlation would improve somewhat the agreement between the present values of S_i and the sum-rule limit, but has no significant effect on the $g_{9/2}$ strength, which is where most of the discrepancy resides.

Distributions of spectroscopic strengths are shown in Fig. 13. The $2d_{5/2}$ and $3s_{1/2}$ strengths are seen to be highly fragmented. This is all the more evident when it is considered that some of the strength for these states must lie above 4 MeV of excitation. The amount of $p_{3/2}$ strength above $E_x = 2.5$ MeV is surprising. With the exception of $2d_{5/2}$, most of the available strength for a given orbital is concentrated in two or three states. This feature has already been noted for the other odd- A Ge isotopes by Hasselgren.⁹

V. SUMMARY AND CONCLUSIONS

Differential cross sections and vector-analyzing powers were measured at an incident deuteron energy of 16 MeV for the ${}^{70}Ge(d, t) {}^{69}Ge$ reaction and at 12 MeV for the ⁷⁰Ge(d , p)⁷¹Ge reaction. The data were compared with the predictions of the DWBA. Generally, the results are fairly well reproduced by the calculations except for the for-'ward-angle VAP data for $(d, t$) transitions to $\frac{1}{2}$ states in 69 Ge.

Several new spin assignments in ⁶⁹Ge were made from the VAP of the $Ge(d, t)$ reaction. For states which already had definite spin assignments, the VAP data confirm these assignments. Approximately 150 levels were observed in high-resolution ${}^{70}Ge(d, p)$ spectra in the region of excitation of "Ge up to ⁴ MeV. At least one-third of these levels are definitely "Ge states, while at least 22 states can be identified as belonging to Ge isotopes other than ⁷¹Ge. Values of l_n were assigned to 44 transitions to states in ⁷¹Ge, and j_n values were determined for 29 levels, of which 15 represent new assignments. A number of tentative J assignments were also made.

Some of the results in the present work are relevant to the weak-coupling interpretation of ^{69,71}Ge of Eberth et al. A $\frac{9}{2}$ state at 1.468 MeV in ⁶⁹Ge is a candidate for the missing $\frac{9}{2}$ ⁺ member of a $| \, 2^{\star} \rangle \otimes \, | \, 1_{\mathcal{S}_9/\mathfrak{2}} \rangle$ multiplet proposed by these auth ors. The missing $\frac{1}{2}$ member of a $|2^*\rangle \otimes |1f_{5/2}\rangle$ multiplet cannot be the 1.306-MeV level as proposed in Ref. 2, but other candidates for this level are proposed. The strength of the $J_n^{\pi} = \frac{5}{2}^+$ transition to the 0.526-MeV level in 7^{1} Ge does not preclude the interpretation of this state as a member of an excited corepar ticle multiplet.

A number of $l_n = 0$ and 2 transitions are observed in ⁷⁰Ge(*d*,*t*) but the $l_n = 0$ cross sections are poorly reproduced by the DWBA. The level at 0.890 MeV listed in the $A = 71$ compilation³³ is probably not a state in 71 Ge but rather a level in 75 Ge. The present work indicates that the spin of the ⁷¹Ge 1.695-MeV level is $\frac{9}{2}^+$ rather than $\frac{7}{2}^+$ as assigned by Yoh et al.¹¹

The summed spectroscopic strength for the ${}^{70}Ge(d, t)$ reaction indicates that most of the available strength has been observed. In comparing the summed (d, p) and (d, t) strengths to $2j+1$ for active orbitals in these reactions the measured strengths tend to be low, particularly for $1g_{9/2}$. It appears that the absolute (d, p) spectroscopic factors obtained in the present work are systematically low by around 25%. It appears that this discrepancy arises at least in part from optical model parameters used in the analysis.

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- ^TSee AIP document No. PAPS PRVCA-16-1333-36 for 36 pages of ${}^{70}Ge(d, t){}^{69}Ge$ and ${}^{70}Ge(d, p){}^{71}Ge$ cross-section and vector-analyzing-power data obtained in this experiment. Order by PAPS number and journal reference from American Institute of Physics, Physics Auxiliary Publication Service, 335 East 45th Street, New York, N. Y. 10017. The price is \$1.50 for microfiche or \$5.90 for photocopy. Airmail additional. Make checks payable to the American Institute of Physics. This material also appears in Current Physics Microfilm, the monthly microfilm edition of the complete set of journals published by AIP, on the frames immediately following this journal article.
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