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# Spectator expansion in multiple scattering theory

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A finite expansion for the scattering of a structureless projectile from a complex target of $A$ particles is presented. This development is given as a spectator expansion, in the sense that the first term represents the scattering of the projectile from single target constituent particles, with all other target particles playing a passive role (i.e., acting as spectators). Similarly, the second term represents the scattering from pairs of target particles with $(A-2)$ spectators, and so on. It is demonstrated that such expansions, one of which has been obtained previously as a resummation of the multiple scattering series, are very general in nature and obtain under circumstances for which the standard multiple scattering treatment is not valid.

[NUCLEAR REACTIONS Scattering theory, nonperturbative finite expansion of the transition operator for an $A$-body target.]

## I. INTRODUCTION

Recently Ernst, Londergan, Miller, and Thaler[1] (ELMT) offered an alternative to the multiple scattering theory of Watson[2] or Kerman, McManus, and Thaler[3] (KMT). This treatment is referred to as a correlation expansion. This expansion yields a finite series of $A$ terms, where $A$ is the number of target particles. The $n$th term of this series represents the scattering from $n$ $(n \leq A)$ target particles. This series is presented as a resummation of the standard multiple scattering series.

In this paper, this result is obtained in a somewhat different manner which indicates more clearly the general nature of this type of formulation. Furthermore, this present approach makes it easy to obtain a variety of different developments, all within the spirit of the correlation expansion. Moreover, since the present treatment is nonperturbative, it is more readily adapted to problems where perturbative expansions can be shown not to exist. Thus the present discussion is not only more general than the original presentation, but it will be seen to be applicable to a wider class of problems. In particular, it will be shown that this expansion, unlike the multiple scattering treatment, can be applied without modification in the case where many-body forces play an important role.

We begin by presenting some of the main results of ELMT in Sec. II. We also discuss in that section the utility of the ELMT type expansion. In Sec. III, we show that an expansion of the full Green's operator allows one to work with the solved form of the many-body Lippmann-Schwinger equation and to obtain the ELMT expansion without any iteration. In Sec. IV, an expansion of the full Green's operator is given as a spectator expansion. That is to say, the $n$th term in this expansion is characterized by the treatment of $n$ target particles as active participants in the scattering process, while $(A-n)$ target particles are passive spectators. Such an expansion has advantages which will become apparent in the text. We demonstrate in Sec. V that the inclusion of many-body forces results in no changes in the formalism. In the Appendix we apply these techniques to the expansion of the optical potential. All the results obtained in the text for the $T$ matrix have their analogs in the discussion of the optical potential.

## II. CORRELATIVE EXPANSION OF ELMT

In the paper of ELMT the transition operator is taken to be the solution of the linear operator equation

$$T=\sum_i v_{0i}+\sum_i v_{0i}G_0(E)T = \sum_i v_{0i}+TG_0(E)\sum_i v_{0i}\,, \tag{2.1}$$

with $G_0(E)$ defined to be

$$G_0(E)=(E-h_0-H_A+i\zeta)^{-1}\,, \tag{2.2}$$

where all the symbols have the standard meanings, i.e., $E$ is the parametric energy, $h_0$ is the kinetic energy operator for the projectile, and $H_A$ is the target Hamiltonian. The definition of $t_{0i}$ is also given via a linear operator equation, viz.,

$$t_{0i}=v_{0i}+v_{0i}G_0t_{0i}=v_{0i}+t_{0i}G_0v_{0i}\,. \tag{2.3}$$

In the usual way, Eq. (2.3) may be used to eliminate $v_{0i}$ from Eq. (2.1) in favor of $t_{0i}$.

The standard result is

$$T_i=t_{0i}+t_{0i}G_0\sum_{j\neq i} T_j\,, \tag{2.4}$$

with

$$T \equiv \sum_i T_i . \tag{2.5}$$

Iteration of Eq. (2.4) leads to the Watson multiple scattering series, viz.,

$$T = \sum_i t_{0i} + \sum_i t_{0i} G_0 \sum_{j \neq i} t_{0j} + \sum_i t_{0i} G_0 \sum_{j \neq i} t_{0j} G_0 \sum_{k \neq j} t_{0k} + \cdots . \tag{2.6}$$

This series may be rearranged as follows:

$$T = \sum_i t_{0i} + \left\{ \sum_i t_{0i} G_0 \sum_{j \neq i} t_{0j} + \sum_i t_{0i} G_0 \sum_{j \neq i} t_{0j} G_0 t_{0i} + \sum_i t_{0i} G_0 \sum_{j \neq i} t_{0j} G_0 t_{0i} G_0 t_{0j} + \cdots \right\} + \left[ \sum t_{0i} G_0 \sum_{j \neq i} t_{0j} G_0 \sum_{k \neq i, j} t_{0k} + \cdots \right] + \cdots . \tag{2.7}$$

We then define the operator $t_{0,ij}$ by means of the linear operator equation

$$\begin{aligned} t_{0,ij} &= (v_{0i} + v_{0j}) + (v_{0i} + v_{0j}) G_0 t_{0,ij} \\ &= (v_{0i} + v_{0j}) + t_{0,ij} G_0 (v_{0i} + v_{0j}) , \end{aligned} \tag{2.8}$$

which may be solved, according to the steps leading to Eq. (2.6), to yield

$$t_{0,ij} = t_{0i} + t_{0i} G_0 t_{0j} + t_{0i} G_0 t_{0j} G_0 t_{0i} + \cdots + t_{0j} + t_{0j} G_0 t_{0i} + t_{0j} G_0 t_{0i} G_0 t_{0j} + \cdots . \tag{2.9}$$

From Eq. (2.9), we may identify the term in the curly brackets in Eq. (2.7) to be

$$\sum_{\substack{i \\ j \neq i}} (t_{0i} G_0 t_{0j} + t_{0i} G_0 t_{0j} G_0 t_{0i} + t_{0i} G_0 t_{0j} G_0 t_{0i} G_0 t_{0j} + \cdots) = \sum_{i<j} (t_{0,ij} - t_{0i} - t_{0j}) . \tag{2.10}$$

Similarly, the definition

$$\begin{aligned} t_{0,ijk} &= (v_{0i} + v_{0j} + v_{0k}) + (v_{0i} + v_{0j} + v_{0k}) G_0 t_{0,ijk} \\ &= (v_{0i} + v_{0j} + v_{0k}) + t_{0,ijk} G_0 (v_{0i} + v_{0j} + v_{0k}) \end{aligned} \tag{2.11}$$

permits us to identify the term in the square brackets in Eq. (2.7) as

$$\sum_{\substack{i \\ j \neq i \\ k \neq i, j}} t_{0i} G_0 t_{0j} G_0 t_{0k} + \cdots = \sum_{i<j<k} (t_{0,ijk} - t_{0,ij} - t_{0,ik} - t_{0,jk} + t_{0i} + t_{0j} + t_{0k}) . \tag{2.12}$$

Thus ELMT rearrange the infinite multiple scattering series of Watson as the finite series

$$T = \sum_i t_{0i} + \sum_{i<j} (t_{0,ij} - t_{0i} - t_{0j}) + \sum_{i<j<k} (t_{0,ijk} - t_{0,ij} - t_{0,ik} - t_{0,jk} + t_{0i} + t_{0j} + t_{0k}) + \cdots . \tag{2.13}$$

This finite series groups the $(A+1)$-body $T$ operator into $A$ terms: The first term, $\sum_i t_{0i}$, in some sense represents the projectile scattering from single target particles. The second term $\sum_{i<j}$ $(t_{0,ij} - t_{0i} - t_{0j})$ similarly represents the scattering from pairs, and so forth.

The underlying physical motivation in the work of ELMT as represented by Eq. (2.13) is clear. That is, ELMT wish to rearrange the multiple scattering series into a finite series, the first term of which contains only scatterings from single target particles, the second term of which contains only the scattering from pairs of target particles, and so on. In this sense Eq. (2.13) resembles the familiar linked-cluster decomposition[4] of nuclear structure.

In the extreme closure approximation the ELMT series becomes particularly appealing. While the work of ELMT does not necessarily stand or fall on the validity of this approximation, we shall scrutinize the achievements and shortcomings of ELMT in the form based on the extreme closure approximation. The extreme closure approximation as referred to here means operationally that the operator $H_A$ in Eq. (2.2) is set equal to zero. As has been discussed at length in ELMT and elsewhere,[5] this approximation implies that all the excitations which contribute to the scattering lie sufficiently low in energy that they can all be considered to have the same energy as the ground state energy, which we conventionally set equal to zero. What is achieved in the extreme closure approximation is an extraordinary simplification of the multiple scattering series and likewise the finite series of ELMT. This follows because in this approximation the propagator of Eq. (2.2) becomes a one-body (the projectile) operator.

If we simplify the problem to the extent indicated above, that is, if we truncate the propagator such that it becomes a one-body operator, then Eq. (2.3) is equivalently simplified. The operator $t_{0i}$ of Eq. (2.3) is, in general, not a two-body operator, with the propagator taken as defined in Eq. (2.1). In the extreme closure approximation, $t_{0i}$ is obviously a two-body operator. We note in passing that other approximations to the propagator are possible which will transform the many-body operator $t_{0i}$, as defined in Eq. (2.3), into a two-body operator. All such approximations suffer from shortcomings similar to those under present discussion.

We note that *only* in the case where $t_{0i}$ is a two-body operator can we treat the lowest order term of the Watson series or ELMT series for elastic scattering as

$$\langle \vec{p}_0' \Phi_A(0) \,|\, T \,|\, \vec{p}_0 \Phi_A(0)\rangle^{(1)} \cong \sum_{i=1} \int d\vec{p}_1 \cdots d\vec{p}_A d\vec{p}_i' \langle \vec{p}_0' \vec{p}_i' \,|\, t_{0i} \,|\, \vec{p}_0 \vec{p}_i\rangle \langle \Phi_A(0) \,|\, \vec{p}_1 \cdots \vec{p}_i' \cdots \vec{p}_A\rangle \langle \vec{p}_1 \cdots \vec{p}_A \,|\, \Phi_A(0)\rangle$$

$$= \sum_{i=1}^{A} \int d\vec{p}' d\vec{p}_i \langle \vec{p}_0' \vec{p}_i' \,|\, t_{0i} \,|\, \vec{p}_0 \vec{p}_i\rangle \rho^{(1)}(\vec{p}_i'; \vec{p}_i) \,, \tag{2.14}$$

where $|\Phi_A(0)\rangle$ represents the $A$-particle target ground state and $\rho^{(1)}(\vec{p}_i', \vec{p}_i)$ is the usual density

$$\rho^{(1)}(\vec{p}_i'; \vec{p}_i) \equiv \int d\vec{p}_1 \cdots d\vec{p}_{i-1} d\vec{p}_{i+1} \cdots d\vec{p}_A \langle \Phi_A(0) \,|\, \vec{p}_1 \cdots \vec{p}_i' \cdots \vec{p}_A\rangle \langle \vec{p}_1 \cdots \vec{p}_{(A)} \,|\, \Phi_{(A)}(0)\rangle \,. \tag{2.15}$$

It cannot be emphasized too strongly that this standard "$t\rho$" result only obtains if $t_{0i}$ is a two-body operator.

Similarly, the second term of the ELMT expansion, in the extreme closure limit in which the propagator $G_0$ is treated as a one-body operator, becomes a true three-body operator and hence it follows immediately that the next term in the ELMT series for elastic scattering can be represented as

$$\langle p_0' \Phi_A(0) \,|\, T \,|\, p_0 \Phi_A(0)\rangle^{(2)} \cong \sum_{i<j} \int d\vec{p}_i' d\vec{p}_j' d\vec{p}_i d\vec{p}_j \langle \vec{p}_0' \vec{p}_i' \vec{p}_j' \,|\, \{t_{0,ij} - t_{0i} - t_{0j}\} \,|\, \vec{p}_0 \vec{p}_i \vec{p}_j\rangle \rho^{(2)}(\vec{p}_i', \vec{p}_j'; \vec{p}_i \vec{p}_j) \,. \tag{2.16}$$

Thus in the extreme closure limit the series of ELMT is a series in the one-body density, the two-body density, and so on.

Now if the finite series of ELMT in the extreme closure limit is rapidly "convergent," i.e., contributions from higher order terms decrease rapidly from order to order, then the contributions from the higher order terms may be less significant than corrections to closure. Obviously then, we must at some point consider corrections to the closure approximation. At that point we immediately encounter the difficulty engendered by the fact that the propagator of Eq. (2.1) is a many-body propagator, and for that reason even $t_{0i}$ of Eq. (2.3) is no easier to calculate than $T$. Thus we must seek some systematic way to treat corrections to closure.

In this paper, we seek an expansion of $T$ that inherently posseses all the advantages of the ELMT series in the closure limit, namely that the first term is a two-body operator, the second term a three-body operator, and so on. Further, we would like to have this series become identical with the ELMT series in the closure limit. Obviously such a treatment is possible, and can represent a better over-all systematic approach to the problem. In this paper we present such an approach.

## III. NONITERATIVE TREATMENT

In the preceeding section the Watson multiple scattering series was resummed according to the prescription of ELMT to obtain a finite series of $A$ terms. The treatment of ELMT represents a particular way of "counting" or "arranging" the scatterings, i.e., according to the number of target particles which play an active role. The physical motivation behind this treatment clearly transcends the perturbative derivation of the Watson series and the selective resummation of this series as presented by ELMT. In this section we shall show how the ELMT result may be obtained without recourse to iteration. Such a derivation will make it easy to grasp the essential elements in the ELMT arrangement and hence to improve and extend this result.

To this end, we now present an identity which is central to our development. This identity, which is proved in Appendix B, is

$$\begin{aligned}\xi = &\sum_{i=1}^{A} \xi^{(i)} + \sum_{i<j} [\xi^{(ij)} - \xi^{(i)} - \xi^{(j)}] + \sum_{i<j<k} [\xi^{(ijk)} - \xi^{(ij)} - \xi^{(ik)} - \xi^{(jk)} + \xi^{(i)} + \xi^{(j)} + \xi^{(k)}] \\ &+ \sum_{i<j<k<l} [\xi^{(ijkl)} - \xi^{(ijk)} - \xi^{(ijl)} - \xi^{(ikl)} - \xi^{(jkl)} + \xi^{(ij)} + \xi^{(ik)} + \xi^{(il)} + \xi^{(jk)} + \xi^{(jl)} \\ &\qquad + \xi^{(kl)} - \xi^{(i)} - \xi^{(j)} - \xi^{(k)} - \xi^{(l)}] + \cdots \\ \equiv &\sum_{i} \Gamma^{(i)} + \sum_{i<j} \Gamma^{(ij)} + \sum_{i<j<k} \Gamma^{(ijk)} + \sum_{i<j<k<l} \Gamma^{(ijkl)} + \cdots \,,\end{aligned} \tag{3.1}$$

where the definitions of the Γ's are given by the term by term identification in Eq. (3.1). This series is a finite series of $A$ terms. It is an identity provided only that

$$\xi \equiv \xi^{(1,2,3,\cdots,A)} , \tag{3.2}$$

independent of the definitions of the other $\xi$'s. Although it is easy to prove Eq. (3.3) in general (cf. Appendix B), we shall content ourselves with illustrating how it works for the case in which $A=3$. In that case Eq. (3.3) becomes

$$\begin{aligned}\xi = &[\xi^{(1)}+\xi^{(2)}+\xi^{(3)}]\\ &+[\xi^{(12)}-\xi^{(1)}-\xi^{(2)}+\xi^{(13)}-\xi^{(1)}\\ &\quad-\xi^{(3)}+\xi^{(23)}-\xi^{(2)}-\xi^{(3)}]\\ &+[\xi^{(123)}-\xi^{(12)}-\xi^{(13)}-\xi^{(23)}\\ &\quad+\xi^{(1)}+\xi^{(2)}+\xi^{(3)}]=\xi^{(123)} ,\end{aligned} \tag{3.3}$$

as expected. Clearly, therefore, Eq. (3.1) is merely a combinatorial identity. However, we note that Eq. (2.13) is in the form of the identity given in Eq. (3.1).

From the above exercise we thus learn that the expansion given in Eq. (2.13) is by no means unique. Obviously, there are an infinitude of possible definitions of the operators $t_{0i}$, $t_{0,ij}$, $t_{0,ijk}$, ... for which Eq. (2.13) will hold. The definitions given in Eqs. (2.3), (2.8), (2.9), and (2.11) are those of ELMT and are physically motivated. From one point of view, therefore, Eq. (3.1) is empty. A better way of looking at this, however, is that Eq. (3.1) provides us with the flexibility to define the operators $\xi^{\{\nu\}}$ (where $\{\nu\}$ represents $i$, $ij$, $ijk$, or $ijkl$, and so on) as to put as much of the physical answer as possible into the lowest terms in a way consistent with computational reality.

With this prelude, we return to the problem under consideration. We begin with the solved form of Eq. (2.1), viz.,

$$T=\sum_i v_{0i}+\sum_i v_{0i}\,G(E)\sum_i v_{0i} , \tag{3.4}$$

where the full Green's operator for outgoing waves $G(E)$ is given by

$$G(E)=(E-H+i\eta)^{-1} , \tag{3.5}$$

with $H$ taken to be the Hamiltonian of the entire $(A+1)$-body system. In the case at hand of couse, $H=h_0+H_A+\sum_i v_{0i}$.

We now introduce the expansion of Eq. (3.1) for the propagator, viz.,

$$\begin{aligned}G(E)&=\sum_i G^{(i)}+\sum_{i<j}[G^{(ij)}-G^{(i)}-G^{(j)}]+\sum_{i<j<k}[G^{(ijk)}-G^{(ij)}-G^{(ik)}-G^{(jk)}+G^{(i)}+G^{(j)}+G^{(k)}]+\cdots\\ &=\sum_i\Gamma^{(i)}+\sum_{i<j}\Gamma^{(ij)}+\sum_{i<j<k}\Gamma^{(ijk)}+\cdots .\end{aligned} \tag{3.6}$$

At this point we shall treat Eq. (3.6) as a formal identity without defining the $G^{\{\nu\}}$ beyond the necessary identification of $G(E)$ with $G^{(1,2,\cdots,A)}$. Insertion of Eq. (3.6) into Eq. (3.4) yields

$$\begin{aligned}T&=\sum_i(v_{0i}+v_{0i}\Gamma^{(i)}v_{0i})+\sum_{i<j}\{(v_{0i}+v_{0j})(\Gamma^{(ij)}-\Gamma^{(i)}-\Gamma^{(j)})(v_{0i}+v_{0j})-v_{0i}\Gamma^{(i)}v_{0i}-v_{0j}\Gamma^{(j)}v_{0j}\}+\cdots\\ &=\sum_i[v_{0i}+v_{0i}G^{(i)}v_{0i}]+\sum_{i<j}\{[(v_{0i}+v_{0j})+(v_{0i}+v_{0j})G^{(ij)}(v_{0i}+v_{0j})]\\ &\qquad-[v_{0i}+v_{0i}G^{(i)}v_{0i}]-[v_{0j}+v_{0j}G^{(i)}v_{0j}]\}+\cdots .\end{aligned} \tag{3.7}$$

The definitions

$$\bar t_{0i}=v_{0i}+v_{0i}\,G^{(i)}v_{0i} , \tag{3.8}$$

$$\bar t_{0,ij}=(v_{0i}+v_{0j})+(v_{0i}+v_{0j})G^{(ij)}(v_{0i}+v_{0j}) , \tag{3.9}$$

$$\begin{aligned}\bar t_{0,ijk}=&(v_{0i}+v_{0j}+v_{0k})\\ &+(v_{0i}+v_{0j}+v_{0k})G^{(ijk)}(v_{0i}+v_{0j}+v_{0k}) ,\end{aligned} \tag{3.10}$$

and so forth, thus enable us to rewrite Eq. (3.7) as

$$\begin{aligned}T=&\sum_i\bar t_{0i}+\sum_{i<j}[\bar t_{0,ij}-\bar t_{0i}-\bar t_{0j}]\\ &+\sum_{i<j<k}[\bar t_{0,ijk}-\bar t_{0,ij}-\bar t_{0,ik}-\bar t_{0,jk}+\bar t_{0i}+\bar t_{0j}+\bar t_{0k}]\\ &+\cdots .\end{aligned} \tag{3.11}$$

If we now define $G^{(i)}$ to be

$$G^{(i)}=(E-h_0-H_A+v_{0i}+i\eta)^{-1} , \tag{3.12}$$

then $\bar t_{0i}$ as given by Eq. (3.8) is clearly the solved form of Eq. (2.3), and hence this choice of $G^{(i)}$ makes $\bar t_{0i}\equiv t_{0i}$. Similarly, the identification

$$G^{(ij)}=(E-h_0-H_A+v_{0i}+v_{0j}+i\eta)^{-1} \tag{3.13}$$

yields the equality of $\bar t_{0,ij}$ of Eq. (3.9) with $t_{0,ij}$ of Eq. (2.8). This process of identification can continue in this through all terms of the series to reproduce exactly the result of Eq. (2.13).

The exercise above indicates clearly that a particular expansion of the Green's operator is at the heart of the ELMT result given in Eq. (2.13). Once we recognize this, we are in a position to expand the Green's operator somewhat differently in order to retain the "connectedness" of the ELMT expansion in the extreme closure limit without making any approximations. In the next section we present a choice for the operators $G^{\{\nu\}}$ that generates the desired properties in the expansion for $T$.

## IV. SPECTATOR EXPANSION

We now follow through on the suggestion of the previous section that the key to the proper expansion of the $T$ matrix is the appropriate treatment of the Green's operator. Thus we begin with the full Green's operator of Eq. (3.5), which we write as

$$G(E)=\left(E-h_0-H_A-\sum_i v_{0i}+i\eta\right)^{-1}, \tag{4.1}$$

with the target Hamiltonian $H_A$ given by

$$H_A=\sum_i h_i+\sum_{i<j} v_{ij}. \tag{4.2}$$

Now we may choose to concentrate our attention on a given target particle, the $i$th particle. To emphasize our interest in this particle we rewrite Eq. (4.2) as

$$\begin{aligned}H_A&=h_i+\sum_{j\neq i} v_{ij}+\sum_{j\neq i} h_j+\sum_{\substack{k<l\\ k\neq l\neq i}} v_{kl}\\ &=h_i+\sum_{j\neq i} v_{ij}+H_{(A-i)},\end{aligned} \tag{4.3}$$

where $H_{(A-i)}$ is the Hamiltonian governing the behavior of all the target particles in the absence of the $i$th particle. If we now insert Eq. (4.3) into Eq. (4.1) we have

$$\begin{aligned}G(E)&=\left\{E-(h_0+h_i+v_{0i})-H_{(A-i)}-\left[\sum_{j\neq i}(v_{0j}+v_{ij})\right]+i\eta\right\}^{-1}\\ &=\left\{E-H_{0i}-H_{(A-i)}-\left[\sum_{j\neq i}(v_{0j}+v_{ij})\right]+i\eta\right\}^{-1},\end{aligned} \tag{4.4}$$

where $H_{0i}$ is the first term in parentheses in Eq. (4.4) above and represents the Hamiltonian governing the projectile and the $i$th target particle. The term in the square brackets represents the coupling between particles 0 and $i$ and the remaining target particles.

This way of writing the full Green's operator conveniently simplifies in the situation where we wish to consider that only the $i$th target plays an active role in the scattering and the remaining $(A-1)$ target particles act as passive spectators. This simplification takes place in the case where the $(A-1)$ target particles are uncoupled from the projectile and the $i$th target particle, i.e.,

$$\sum_{l\neq i}(v_{0l}+v_{il})=0. \tag{4.5}$$

By setting the coupling term in the propagator to zero, we are dealing with two separate systems; the (A-1) target particles are one connected system and the projectile and $i$th target particle are another separate connected system. In this approximation the Hamiltonian is

$$H\simeq H^{(i)}\equiv H_{0i}+H_{(A-i)}. \tag{4.6}$$

We denote the eigenstates of the Hermitian Hamiltonian $H_{0i}$ through the relation

$$H_{0i}\,|\psi_{0i}^{(+)}(\beta)\rangle=\epsilon_{0i}(\beta)\,|\psi_{0i}^{(+)}(\beta)\rangle, \tag{4.7}$$

where the plus superscript indicates the outgoing wave boundary condition for two-body scattering states and is irrelevant for bound states, and $\beta$ is a parameter which selects a particular eigenstate among the complete orthonormal set of eigenstates of $H_{0i}$. Similarly, the eigenstates of $H_{(A-i)}$ are given by

$$H_{(A-i)}\,|\Phi_{(A-i)}(\alpha)\rangle=\epsilon_{(A-i)}(\alpha)\,|\Phi_{(A-i)}(\alpha)\rangle, \tag{4.8}$$

with the obvious definitions of the quantities which appear in Eq. (4.8). The eigenstates of $H^{(i)}$ are

$$H^{(i)}\,|\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)\rangle=[\epsilon_{0i}(\beta)+\epsilon_{(A-i)}(\alpha)]\,|\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)\rangle, \tag{4.9}$$

and in this approximation

$$G(E)\cong\sum_{\alpha,\beta}\frac{|\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)\rangle\langle\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)|}{E-\epsilon_{0i}(\beta)-\epsilon_{(A-i)}(\alpha)+i\eta}. \tag{4.10}$$

If we make the further approximation that $\epsilon_{(A-i)}(\alpha)$ is some average energy $\bar{\epsilon}_{(A-i)}$ independent of the state label $\alpha$, i.e., if we make the closure approximation for $H_{(A-i)}$, we have then simply assigned some average energy $\bar{\epsilon}_{(A-i)}$ to the $(A-1)$ passive (spectator) particles independent of their configuration. These approximations lead to

$$\begin{aligned}G(E)&\cong\sum_{\alpha,\beta}\frac{|\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)\rangle\langle\psi_{0i}^{(+)}(\beta)\Phi_{(A-i)}(\alpha)|}{e_i-\epsilon_{0i}(\beta)+i\eta}\\ &=(e_i-H_{0i}+i\eta)^{-1},\end{aligned} \tag{4.11}$$

with

$$e_i\equiv E-\bar{\epsilon}_{(A-i)}. \tag{4.12}$$

We define the approximate Green's operator of Eq.

(4.11) to be $\tilde{G}^{(i)}(E)$, i.e.,

$$\tilde{G}^{(i)}(E) \equiv (e_i - h_0 - h_i - v_{0i} + i\eta)^{-1} . \tag{4.13}$$

We observe that the operator $\tilde{G}^{(i)}(E)$ is a two-body operator, which is indeed a desirable property to be possessed by an operator which is supposed to represent the propagation of particle 0 interacting with particle $i$, in the presence of $(A-1)$ passive spectator particles. We note that the energy $e_i$ given by Eq. (4.12) is an appropriate energy for this two-particle system since it represents the energy of the entire $(A+1)$-body system less the average energy of the $(A-1)$ spectators.

In an analogous manner we may define the propagator $\tilde{G}^{(ij)}(E)$ through the treatment of $G(E)$ as

$$\begin{aligned} G(E) &= \Big\{ E - (h_0 + h_i + h_j + v_{ij} + v_{0i} + v_{0j}) \\ &\quad - H_{(A-i-j)} - \Big[ \sum_{l \neq i, j} (v_{0l} + v_{il} + v_{jl}) \Big] + i\eta \Big\}^{-1} \\ &= \Big\{ E - H_{0,ij} - H_{(A-i-j)} \\ &\quad - \Big[ \sum_{l \neq i, j} (v_{0l} + v_{il} + v_{jl}) \Big] + i\eta \Big\}^{-1} . \end{aligned} \tag{4.14}$$

In the approximation in which the $(A-2)$ target particles are spectators, i.e., uncoupled from the projectile and the two active particles ($i$ and $j$), this propagator becomes

$$G(E) \cong (E - H_{0,ij} - H_{(A-i-j)} + i\eta)^{-1} . \tag{4.15}$$

and in the closure approximation for the eigenstates of $H_{(A-i-j)}$,

$$\begin{aligned} H_{(A-i-j)} |\Phi_{(A-i-j)}(\alpha)\rangle &= \epsilon_{(A-i-j)}(\alpha) |\Phi_{(A-i-j)}(\alpha)\rangle \\ &\cong \bar{\epsilon}_{(A-i-j)} |\Phi_{(A-i-j)}(\alpha)\rangle , \end{aligned} \tag{4.16}$$

we obtain the definition for $\tilde{G}^{(ij)}(E)$ to be

$$\tilde{G}^{(ij)}(E) = (e_{ij} - H_{0,ij} + i\eta)^{-1} , \tag{4.17}$$

where

$$e_{ij} = E - \bar{\epsilon}_{(A-i-j)} \tag{4.18}$$

and

$$H_{0,ij} = h_0 + h_i + h_j + v_{0i} + v_{0j} + v_{ij} . \tag{4.19}$$

The propagator $\tilde{G}^{(ij)}(E)$ so defined is obviously a three-body operator. The continuation of this process to obtain $\tilde{G}^{(ijk)}$, etc., is obvious. We note that the procedure we have described leads to

$$\begin{aligned} \tilde{G}^{(1\cdots A)} &= (E - H_{0,1\cdots A} + i\eta)^{-1} \\ &= (E - H + i\eta)^{-1} = G(E) . \end{aligned} \tag{4.20}$$

That is, our procedure defining the operators $\tilde{G}^{\{\nu\}}$ automatically satisfies the required constraint in order that

$$\begin{aligned} G(E) &= \sum_i \tilde{G}^{(i)} + \sum_{i<j} [\tilde{G}^{(ij)} - \tilde{G}^{(i)} - \tilde{G}^{(j)}] + \cdots \\ &= \sum_i \Gamma^{(i)} + \sum_{i<j} \Gamma^{(ij)} + \cdots \end{aligned} \tag{4.21}$$

be an identity.

The finite series for the Green's operator of Eq. (4.21) is a spectator expansion. The lowest order term $\sum_i \tilde{G}^{(i)}$ is a two-body operator (one body in the same space of target particles), the next order term $\sum_{i<j} [\tilde{G}^{(ij)} - \tilde{G}^{(i)} - \tilde{G}^{(j)}]$ is a three-body operator, and so on. In the first term all but one of the target particles are truly spectators, i.e., they are uninvolved in the scattering process, in the second term all but two of the target particles are spectators, and so on.

We now follow the procedure outlined in the previous section, to obtain the result

$$\begin{aligned} T(E) &= \sum_i t_{0i}(e_i) \\ &\quad + \sum_{i<j} [t_{0,ij}(e_{ij}) - t_{0i}(e_i) - t_{0j}(e_j)] \\ &\quad + \sum_{i<j<k} [t_{0,ijk}(e_{ijk}) - t_{0,ij}) - t_{0,ik}(e_{ik}) - t_{0,jk}(e_{jk}) \\ &\quad + t_{0i}(e_i) + t_{0j}(e_j) + t_{0k}(e_k)] + \dots , \end{aligned} \tag{4.22}$$

where

$$t_{0i}(e_i) = v_{0i} + v_{0i}(e_i - H_{0i} + i\eta)^{-1} v_{0i} , \tag{4.23}$$

$$\begin{aligned} t_{0,ij}(e_{ij}) &= (v_{0i} + v_{0j}) \\ &\quad + (v_{0i} + v_{0j})(e_{ij} - H_{0,ij} + i\eta)^{-1}(v_{0i} + v_{0j}) , \end{aligned} \tag{4.24}$$

$$\begin{aligned} t_{0,ijk}(e_{ijk}) &= (v_{0i} + v_{0j} + v_{0k}) \\ &\quad + (v_{0i} + v_{0j} + v_{0k})(e_{ijk} - H_{0,ijk} + i\eta)^{-1} \\ &\quad \times (v_{0i} + v_{0j} + v_{0k}) , \end{aligned} \tag{4.25}$$

and so forth. Clearly the lower case $t$'s of Eqs. (4.23)–(4.25) are free $t$'s with energies shifted to account for the energy assigned to the passive particles.

We emphasize that the expansion given in Eqs. (4.22)–(4.25) has all the properties we proposed as desirable in such an expansion. The first term is a two-body operator, the second term is a three-body operator, etc. Of course, the simplicity of this expansion is somewhat illusory. To go beyond the first term in this expansion we need $t_{0,ij}(e_{ij})$. However, to obtain $t_{0,ij}(e_{ij})$ represents a formidable calculational task. Problems of connectedness,[6,7] which appear to have been dealt with here, reappear in the treatment of the $t_{0,\{\nu\}}(e_{\{\nu\}})$. However, these problems, although difficult, are not unsolved.

### V. MANY-BODY FORCES

Here we shall show that the treatment given above requires no modification in the case where many-body forces play a significant role. We assume, as before, that the behavior of the system of $(A+1)$ particles is governed by a Hamiltonian $H$, which is the sum of kinetic energy and potential energy operators. As usual, we may arrange this Hamiltonian as

$$H=\sum_{\lambda=0}^{A} h_\lambda+\sum_{\lambda<\mu} v_{\lambda\mu}+\sum_{\lambda<\mu<\nu} v_{\lambda\mu\nu}+\cdots , \tag{5.1}$$

where we have used Greek subscripts to call attention to the fact that in Eq. (5.1) the summations over the Green indices include the projectile (0) as well as the target particle $(1\ldots A)$ indices. We may then rearrange the Hamiltonian $H$ as

$$H=(h_0+h_i+v_{0i})+W_{0(A-i)}+W_{i(A-i)}+H_{(A-i)} , \tag{5.2}$$

where $W_{0(A-i)}$ represents the potential terms which couple the projectile to the remaining $(A-1)$ target particles, $W_{i(A-i)}$ represents the potential terms which couple particle $i$ to the remaining $(A-1)$ target terms, and $H_{(A-i)}$ is the Hamiltonian for the $(A-1)$ remaining target particles. As before we may then approximate $H$ by

$$H\cong H^{(i)}\equiv H_{0i}+H_{(A-i)} \tag{5.3}$$

by neglecting the coupling of particles 0 and $i$ to the $(A-1)$ other particles. This permits us to approximate $G(E)$ as

$$\begin{aligned}G(E)&\cong(e_i-H_{0i}+i\eta)^{-1}\\&=(e_i-h_0-h_i-v_{0i}-v_{0i}+i\eta)^{-1}\\&\equiv\tilde G^{(i)}(E) ,\end{aligned} \tag{5.4}$$

where all the definitions are just as defined in Sec. IV. Similarly, we define the propagator $\tilde G^{(ij)}(E)$ as

$$\begin{aligned}\tilde G^{(ij)}(E)&\equiv(e_{ij}-H_{0,ij}+i\eta)^{-1}\\&=(e_{ij}-h_0-h_i-h_j-v_{0i}\\&\quad-v_{0j}-v_{ij}-v_{0ij}+i\eta)^{-1} ,\end{aligned} \tag{5.5}$$

and

$$\begin{aligned}\tilde G^{(ijk)}&=(e_{ijk}-h_0-h_i-h_j-h_k-v_{0i}-v_{0j}-v_{0k}\\&\quad-v_{0ij}-v_{0ik}-v_{0jk}-v_{0ijk}-v_{ij}\\&\quad-v_{ik}-v_{jk}-v_{ijk}+i\eta)^{-1}\\&=(e_{ijk}-H_{0,ijk}+i\eta)^{-1} ,\end{aligned} \tag{5.6}$$

and so on. In this way we quickly observe that

$$\begin{aligned}T(E)=&\sum_i t_{0i}(e_i)+\sum_{i<j}[t_{0,ij}(e_{ij})-t_{0i}(e_i)-t_{0j}(e_j)]\\&+\sum_{i<j<k}[t_{0,ijk}(e_{ijk})-t_{0,ij}(e_{ij})\\&\quad-t_{0,ik}(e_{ik})-t_{0,jk}(e_{jk})\\&\quad+t_{0i}(e_i)+t_{0j}(e_j)+t_{0k}(e_k)]+\cdots ,\end{aligned} \tag{5.7}$$

where

$$t_{0i}(e_i)=v_{0i}+v_{0i}(e_i-H_{0i}+i\eta)^{-1}v_{0i} , \tag{5.8}$$

$$\begin{aligned}t_{0,ij}(e_{ij})=&(v_{0i}+v_{0j}+v_{0ij})\\&+(v_{0i}+v_{0j}+v_{0ij})(e_{ij}-H_{0,ij}+i\eta)^{-1}\\&\times(v_{0i}+v_{0j}+v_{0ij}) ,\end{aligned} \tag{5.9}$$

$$\begin{aligned}t_{0,ijk}(e_{ijk})=&(v_{0i}+v_{0j}+v_{0k}+v_{0ij}+v_{0ik}+v_{0jk}+v_{0ijk})\\&+(v_{0i}+v_{0j}+v_{0k}+v_{0ij}+v_{0ik}\\&\quad+v_{0jk}+v_{0ijk})(e_{ijk}-H_{0,ijk}+i\eta)^{-1}\\&\quad\times(v_{0i}+v_{0j}+v_{0k}+v_{0ij}+v_{0ik}+v_{0jk}+v_{0ijk}) ,\end{aligned} \tag{5.10}$$

and so on. We observe that the $t$'s defined above are exactly what we would expect for the scattering of a projectile from a single particle ($i$), a pair of particles ($i$ and $j$), a triplet of particles ($i$, $j$, and $k$), and so forth.

Similarly, the results of Secs. II and III may be generalized for many-body forces in a completely straightforward manner. It thus becomes obvious that one of the unique advantages of the expansions under present discussion is that, unlike the multiple scattering expansions, these hold even if the potentials have significant many-body components.

### VI. CONCLUDING REMARKS

In summary, we remark on the advantages which accrue to the present treatment of the scattering of an elementary projectile from a composite target of $A$ particles.

We believe the "correlative" expansions for the optical potential given by ELMT to be more natural than those of Watson or KMT. Furthermore, these expansions show greater flexibility. It is much easier to build the transition operator or optical operator as a two-body operator plus a three-body operator, and so on, this way than any other way. It will be seen in another discussion that this treatment can be taken over almost verbatim in terms of creation and destruction operators. When this is done, these expansions become hole-line expansions.

We have seen in Sec. III that the present treatment is obviously independent of any perturbative considerations. We also see that the present expansions are completely contained within the expansion of the full Green's operator. Such a treatment is readily adapted to the demands of a field theoretic formulation of the many-body scattering problem.

Finally, in Sec. V we observed the present formulation to be sufficiently general to subsume many-body forces without alteration of the conclusion. This property, together with the nonperturbative nature of the treatment, suggest that such a framework might be more appropriate for the discussion of pion-nucleus scattering than is conventional multiple scattering theory. This is indeed the case. A formulation of the problem of the scattering of a pion from a nucleus which uses some of the ideas presented here is also in preparation.

## APPENDIX A

In this Appendix we shall demonstrate that the treatment presented in the text may be applied, without difficulty, to the optical potential. We also illustrate, in a simple case, how the target correlations enter the expansion of the optical potential.

Paralleling the text, we begin with a review of the ELMT treatment of the optical potential. The optical potential operator $U$ is conventionally defined by means of the linear operator equation

$$T = U + UG_0PT = U + TG_0PU , \tag{A1}$$

where $P$ is the projector onto the nuclear ground state. We may easily combine Eq. (2.1) and Eq. (A1) to obtain an explicit relation for $U$ in terms of the external potential $\sum_i v_{0i}$, viz.,

$$\begin{aligned} U &= \sum_i v_{0i} + \sum_i v_{0i}\, G_0 Q U \\ &= \sum_i v_{0i} + UG_0Q\sum_i v_{0i} , \end{aligned} \tag{A2}$$

where

$$Q \equiv 1 - P .$$

We note that Eq. (A2) for $U$ is identical to Eq. (2.1) for $T$ with the replacement of the propagator $G_0$ by the modified propagator $G_0Q$. Hence we may take over the results obtained above with a minimum of effort. We thus obtain in analogy with Eq. (2.13)

$$\begin{aligned} U = \sum_i \hat{t}_{0i} &+ \sum_{i<j} (\hat{t}_{0,ij} - \hat{t}_{0i} - \hat{t}_{0j}) \\ &+ \sum_{i<j<k} (\hat{t}_{0,ijk} - \hat{t}_{0,ij} - \hat{t}_{0,ik} - \hat{t}_{0,jk} \\ &\qquad + \hat{t}_{0i} + \hat{t}_{0j} + \hat{t}_{0k}) + \cdots , \end{aligned} \tag{A4}$$

where $t_{0i}$ is defined by means of

$$\begin{aligned} \hat{t}_{0i} &= v_{0i} + v_{0i}G_0Q\hat{t}_{0i} \\ &= v_{0i} + \hat{t}_{0i}G_0Qv_{0i} \end{aligned} \tag{A5}$$

and $\hat{t}_{0,ij}$ is defined through

$$\begin{aligned} \hat{t}_{0,ij} &= (v_{0i} + v_{0j}) + (v_{0i} + v_{0j})G_0Q\hat{t}_{0,ij} \\ &= (v_{0i} + v_{0j}) + \hat{t}_{0,ij}G_0Q(v_{0i} + v_{0j}) , \end{aligned} \tag{A6}$$

and so on. Comparison of the defining equation for $\hat{t}$ and $t$ indicates that $\hat{t}$ and $t$ are related as

$$\hat{t} = t - tG_0P\hat{t} = t - \hat{t}G_0Pt , \tag{A7}$$

where $t$ and $\hat{t}$ represent $t_{0i}$ and $\hat{t}_{0j}$, or $t_{0,ij}$ and $\hat{t}_{0,ij}$, or $t_{0,ijk}$ and $\hat{t}_{0,ijk}$, etc., respectively. We note that Eq. (A7) is simply a reflection of the relation between $U$ and $T$ as given in Eq. (2.16), i.e.,

$$U = T - TG_0PU = T - UG_0PT . \tag{A8}$$

A noniterative treatment for the optical operator $U$ is equally straightforward. The operator $U$ is defined through Eq. (A1). If we eliminate $T$ between Eq. (A1) and Eq. (3.4), we obtain after some manipulation

$$U = \sum_i v_{0i} + \sum_i v_{0i}\, Q\, \frac{1}{E - h_0 - H_A - \sum_j v_{0j}Q + i\eta} \sum_k v_{0k} . \tag{A9}$$

Of course, we recognize Eq. (A9) to be the solution to the integral equation Eq. (A2). We may now follow the course taken in Sec. III to obtain an expansion for the propagator in Eq. (A9) and from that an expansion for $U$. The steps are a completely straightforward extension of the argument in Sec. III. We begin by defining the propagator

$$\hat{G}(E) = \left(E - h_0 - H_A - \sum_i v_{0i}Q + i\eta\right)^{-1} . \tag{A10}$$

which we expand as

$$\begin{aligned} \hat{G}(E) &= \sum_i \hat{G}^{(i)} + \sum_{i<j} (\hat{G}^{(ij)} - \hat{G}^{(i)} - \hat{G}^{(j)}) + \cdots \\ &= \sum_i \hat{\Gamma}^{(i)} + \sum_{i<j} \hat{\Gamma}^{(ij)} + \cdots , \end{aligned} \tag{A11}$$

where

$$\hat{G}^{\{\nu\}} \equiv (E - h_0 - h_{\{\nu\}} - V_{\{\nu\}}\, Q + i\eta)^{-1} \tag{A12}$$

and

$$V_i = v_{0i}, \quad V_{ij} = (v_{0i} + v_{0j}) ,$$
$$V_{ijk} = (v_{0i} + v_{0j} + v_{0k}) , \text{ etc.}$$

We then insert Eq. (A11) into (A9) to obtain

$$U=\sum_i (v_{0i}+v_{0i}Q\hat{\Gamma}^{(i)}v_{0i}) + \sum_{i<j} [(v_{0i}+v_{0j})Q(\hat{\Gamma}^{(ij)}-\hat{\Gamma}^{(i)}-\hat{\Gamma}^{(j)})(v_{0i}+v_{0j}) - v_{0i}Q\hat{\Gamma}^{(i)}v_{0i} - v_{0i}Q\hat{\Gamma}^{(i)}v_{0j}]+\cdots , \quad \text{(A13)}$$

which we can rewrite as

$$U=\sum_i \hat{t}_{0i} + \sum_{i<j} (\hat{t}_{0,ij}-\hat{t}_{0i}-\hat{t}_{0j})+\cdots , \quad \text{(A14)}$$

where

$$\hat{t}_{0i}=v_{0i}+v_{0i}Q\hat{G}^{(i)}v_{0i} , \quad \text{(A15)}$$

$$\hat{t}_{0,ij}=(v_{0i}+v_{0j})+(v_{0i}+v_{0j})Q\hat{G}^{(ij)} \times (v_{0i}+v_{0j}) , \quad \text{(A16)}$$

and so on. With the choice of $G^{\{\nu\}}$ given in Eq. (A12), the operators $\hat{t}$ defined in Eqs. (A15)–(A16) are obviously identical to the operators $\hat{t}$ defined in Eqs. (A5)–(A6), from which it immediately follows that the series for $U$ given by Eq. (A14) is the same as that of Eq. (A4).

The spectator treatment of the optical potential may be obtained by expanding the propagator $\hat{G}$ of Eq. (A10) as

$$G(E)=\sum_i \hat{\mathcal{G}}^{(i)} + \sum_{i<j} (\hat{\mathcal{G}}^{(ij)}-\hat{\mathcal{G}}^{(i)}-\hat{\mathcal{G}}^{(j)})+\cdots, \quad \text{(A17)}$$

where the operators $\hat{\mathcal{G}}^{\{\nu\}}$ are the appropriate projections of the operators $\tilde{G}^{\{\nu\}}$ of Sec. IV. That is, for $\hat{\mathcal{G}}^{(i)}$ we have

$$\hat{\mathcal{G}}^{(i)}(E) \equiv (e_i - h_0 - h_i - v_{0i}Q + i\eta)^{-1} , \quad \text{(A8)}$$

which is analagous to the $\tilde{G}^{(i)}$ given in Eq. (4.13). For $\hat{\mathcal{G}}^{(ij)}$ we have

$$\hat{\mathcal{G}}^{(ij)}(E) \equiv [e_{ij} - h_0 - h_i - h_j - v_{ij} - (v_{0i}+v_{0j})Q + i\eta]^{-1} , \quad \text{(A19)}$$

which corresponds to the $\hat{G}^{(ij)}$ of Eq. (4.16), and so on. By inserting Eq. (A17) into (A9) we obtain the spectator expansion of the optical potential:

$$U=\sum_i \tau_{0i} + \sum_{i<j} (\tau_{0,ij}-\tau_{0i}-\tau_{0j})+\cdots , \quad \text{(A20)}$$

where

$$\tau_{0i}=v_{0i}+v_{0i}Q\hat{\mathcal{G}}^{(i)}v_{0i} , \quad \text{(A21)}$$

$$\tau_{0,ij}=(v_{0i}+v_{0j})+(v_{0i}+v_{0j})Q\,\hat{\mathcal{G}}^{(ij)}(v_{0i}+v_{0j}) , \quad \text{(A22)}$$

and so on.

Thus, the ideas and manipulations used in the text to obtain an expression of the $(A+1)$-body $T$ operator trivally go over to obtain an expansion of the optical operator. It is of interest to see how target particle correlations enter into the expansion for the optical potential.

We shall compare within the closure approximation the second-order $T$ matrix to the second-order optical potential. As mentioned in the text, the ELMT series and the spectator series become identical in this approximation. Hence we need not distinguish between them. Now, to obtain the first nonvanishing contribution of the second-order $T$ matrix, we insert the expansion of $t_{0,ij}$ given by Eq. (2.9) into Eq. (2.17) to obtain

$$\langle \vec{p}_0{}' \Phi_A(0) \,|\, T(E) \,|\, \vec{p}_0\Phi_A(0)\rangle^{(2)} \simeq \sum_{i\neq j} \int d\vec{p}_i{}' d\vec{p}_j{}' d\vec{p}_i d\vec{p}_j \times \langle \vec{p}_0{}'\vec{p}_i{}'\vec{p}_j{}' \,|\, t_i(E)G_0(E)t_j(E) \,|\, \vec{p}_0\vec{p}_i\vec{p}_j\rangle \times \rho^{(2)}(\vec{p}_i{}', \vec{p}_j{}'; \vec{p}_i, \vec{p}_j) , \quad \text{(A23)}$$

where the closure propagator is

$$G_0(E)=(E-h_0+i\eta)^{-1} . \quad \text{(A24)}$$

Using a complete set of plane waves we may write Eq. (A23) as

$$\langle \vec{p}_0 \Phi_A(0) \,|\, T(E) \,|\, \vec{p}_0\Phi_A(0)\rangle^{(2)} \simeq \sum_{i\neq j} \int d\vec{p}_i{}' d\vec{p}_j{}' d\vec{p}_i d\vec{p}_j \times \frac{\int d\vec{p}_0{}'' \langle \vec{p}_0{}'\vec{p}_i{}' \,|\, t_i(E) \,|\, \vec{p}_0{}''\vec{p}_i\rangle\langle \vec{p}_0{}''\vec{p}_j{}' \,|\, t_j(E) \,|\, \vec{p}_0\vec{p}_j\rangle \rho^{(2)}(\vec{p}_i{}', \vec{p}_j{}'; \vec{p}_i, \vec{p}_j)}{(E-(p_0{}'')^2/2m+i\eta)} . \quad \text{(A25)}$$

Similar considerations yield the first nonvanishing contribution of the second-order optical potential to be

$$\begin{aligned}\langle \vec{p}_0{}' \Phi_A(0) \,|\, U(E) \,|\, \vec{p}_0 \Phi_A(0)\rangle^{(2)} &\simeq \sum_{i\neq j} \langle \vec{p}_0{}' \Phi_A(0) \,|\, \hat{t}_i(E)QG_0(E)\hat{t}_j(E) \,|\, \vec{p}_0\Phi_A(0)\rangle \\ &= \sum_{i\neq j} \langle \vec{p}_0\Phi_A(0) \,|\, \{\hat{t}_i(E)G_0(E)\hat{t}_j(E) - \int d\vec{p}_0{}''\hat{t}_i(E) \,|\, \vec{p}_0{}''\Phi_A(0)\rangle\langle \vec{p}_0{}'' \Phi_A(0) \,|\, G_0(E)\hat{t}_j(E)\} \,|\, \vec{p}_0\Phi_A(0)\rangle \\ &= \sum_{i\neq j} \frac{d\vec{p}_i d\vec{p}_j d\vec{p}_i{}' d\vec{p}_j{}' d\vec{p}_0{}'' \langle \vec{p}_0{}'\vec{p}_i{}' \,|\, \hat{t}_i(E) \,|\, \vec{p}_0{}''\vec{p}_i\rangle\langle \vec{p}_0{}''\vec{p}_j{}' \,|\, \hat{t}_j(E) \,|\, \vec{p}_0\vec{p}_j\rangle}{(E-(p_0{}'')^2/2m+i\eta)} \\ &\quad\times [\rho^{(2)}(\vec{p}_i{}', \vec{p}_j{}'; \vec{p}_i, \vec{p}_j) - \rho^{(1)}(\vec{p}_i{}', \vec{p}_i)\rho^{(1)}(\vec{p}_j{}', \vec{p}_j)] , \end{aligned} \quad \text{(A26)}$$

where in going from the first equality to the second we have used the explicit representation of the projector $Q$ off the nuclear ground state as

$$Q=1-\int d\vec{p}_0'' \,|\vec{p}_0''\Phi_A(0)\rangle\langle\vec{p}_0''\Phi_A(0)| \,. \qquad (A27)$$

We observe that it is the optical potential that brings in the two-body correlation function[8]

$$\rho^{(2)}(\vec{p}_i',\vec{p}_j';\vec{p}_i,\vec{p}_j)\equiv\rho^{(2)}(\vec{p}_i',\vec{p}_j';\vec{p}_i,\vec{p}_j) - \rho^{(1)}(\vec{p}_i',\vec{p}_i)\,\rho^{(1)}(\vec{p}_j',\vec{p}_j)\,, \qquad (A28)$$

and not the $T$ matrix.

## APPENDIX B

In this Appendix we shall prove the lemma stated in Eqs. (3.1) and (3.2). For this purpose we write

$$\Lambda\equiv\sum_{i=1}^{A}\xi_i+\sum_{i<j}(\xi_{ij}-\xi_i-\xi_j)+\sum_{i<j<k}(\xi_{ijk}-\xi_{ij}-\xi_{ik}-\xi_{jk}+\xi_i+\xi_j+\xi_k)$$
$$+\sum_{i<j<k<l}(\xi_{ijkl}-\xi_{ijk}-\xi_{ijl}-\xi_{ikl}-\xi_{jkl}+\xi_{ij}+\xi_{ik}+\xi_{jk}+\xi_{il}+\xi_{jl}+\xi_{kl}-\xi_i-\xi_j-\xi_k-\xi_l)+\cdots\,. \qquad (B1)$$

We shall then prove that this finite series can be summed to be

$$\Lambda=\xi_{1\cdots A}\,. \qquad (B2)$$

To facilitate this demonstration we rewrite Eq. (B1) as

$$\Lambda=\lim_{x\to1}\Bigg\{\Bigg[\sum_{i=1}^{A}\xi_i\Bigg]+\Bigg[\sum_{i<j}(\xi_{ij}-x\xi_i-x\xi_j)\Bigg]+\Bigg[\sum_{i<j<k}(\xi_{ijk}-x\xi_{ij}-x\xi_{ik}-x\xi_{jk}+x^2\xi_i+x^2\xi_j+x^2\xi_k)\Bigg]$$
$$+\Bigg[\sum_{i<j<k<l}(\xi_{ijkl}-x\xi_{ijk}-x\xi_{ijl}-x\xi_{ikl}-x\xi_{jkl}+x^2\xi_{ij}+x^2\xi_{ik}+x^2\xi_{jk}$$
$$+x^2\xi_{il}+x^2\xi_{jl}+x^2\xi_{kl}+x^3\xi_i+x^3\xi_j+x^3\xi_k+x^3\xi_l)\Bigg]+\cdots\Bigg\}, \qquad (B3)$$

or

$$\Lambda=\lim_{x\to1}\Bigg\{\Bigg[\sum_{i=1}\xi_i\Bigg]+\Bigg[\sum_{i<j}\xi_{ij}-(A-1)x\sum_{i=1}^{A}\xi_i\Bigg]+\Bigg[\sum_{i<j<k}\xi_{ijk}-(A-2)x\sum_{i<j}\xi_{ij}+\frac{(A-1)(A-2)}{2!}x^2\sum_{i=1}^{A}\xi_i\Bigg]$$
$$+\Bigg[\sum_{i<j<k<l}\xi_{ijkl}-(A-3)x\sum_{i<j<k}\xi_{ijk}+\frac{(A-2)(A-3)}{2!}x^2\sum_{i<j}\xi_{ij}-\frac{(A-1)(A-2)(A-3)}{3!}x^3\sum_{i}\xi_i\Bigg]+\cdots\Bigg\}$$
$$=\lim_{x\to1}\Bigg\{\Bigg[1-(A-1)x+\frac{(A-1)(A-2)}{2!}x^2+\frac{(A-1)(A-2)(A-3)}{3!}x^3+\cdots\Bigg]\sum_{i=1}^{A}\xi_i$$
$$+\Bigg[1-(A-2)x+\frac{(A-2)(A-3)}{2!}x^2-\frac{(A-2)(A-3)}{3!}x^3+\cdots\Bigg]\sum_{i<j}\xi_{ij}$$
$$+\Bigg[1-(A-3)x+\frac{(A-3)(A-4)}{2!}x^2-\frac{(A-3)(A-4)}{3!}(A-5)x^3+\cdots\Bigg]$$
$$\times\sum_{i<j<k}\xi_{ijk}+[1-(A-4)x+\cdots]\sum_{i<j<k<l}\xi_{ijkl}+\cdots+\xi_{1\cdots A}\Bigg\}$$
$$=\lim_{x\to1}\Bigg\{(1-x)^{(A-1)}\sum_{i=1}^{A}\xi_i+(1-x)^{(A-2)}\sum_{i<j}\xi_{ij}+(1-x)^{(A-3)}\sum_{i<j<k}\xi_{ijk}+(1-x)^{A-4}\sum_{i<j<k<l}\xi_{ijkl}+\cdots+\xi_{1\cdots A}$$
$$=\xi_{1\cdots A}\,. \qquad (B4)$$

Thus we have proved that the series of Eq. (B1) sums to the result given in Eq. (B2) and the identity given in Eqs. (3.1) and (3.2) is proved.

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