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Spin-orbit coupling in a relativistic Hartree model for finite nuclei

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Single-particle motion and spin-orbit splitting in finite nuclei are described in a relativistic Dirac-Hartree model for nucleons interacting through exchange of scalar and vector bosons. Parameters are adjusted to reproduce properties of nuclear matter as described by Walecka. The model, applied to ^{16}O, successfully explains the sign and size of the spin-orbit interaction. The sensitivity of this result to variations in the parameters of the effective scalar boson is discussed.

[NUCLEAR STRUCTURE ^{16}O; calculated single-particle energies. Relativistic Hartree method, spin-orbit coupling.]

Spin-orbit coupling in nuclei is well known to be a relativistic effect usually ascribed to relativistic corrections in one-boson exchange models of the nucleon-nucleon force, in particular to the exchange of vector and scalar mesons.[1,2] Understanding of the spin-orbit splitting therefore requires a treatment of relativistic degrees of freedom in nuclei.

One way of proceeding is to solve a relativistic Hartree-Fock problem with fully relativistic one-boson exchange interactions as input. Such a procedure has been carried out by Miller,[3] who solved the nuclear Hartree problem using the Dirac equation. Miller has been able to reproduce the known single-particle energies and the spin-orbit splitting in a number of nuclei. In his investigation, the coupling constants and partly also the masses of the exchanged mesons where handled as free parameters. Now, a refined analysis of the Dirac-Hartree problem by Brockmann[5] shows that the single-particle energies and the spin-orbit splitting become extremely sensitive to the mass and coupling constant of the scalar boson, so that additional information is required to fix these input parameters.

In this note we would like to draw connections to the work of Walecka,[6] who has used a relativistic theory of nucleons interacting via exchange of (isoscalar) effective scalar and vector bosons in the mean field approximation to obtain a description of nuclear matter both at normal and high densities. The restriction to scalar and vector fields is motivated by the fact that, in a spin-saturated system with equal number of protons and neutrons, scalar and vector exchange with $T=0$ dominates the Hartree energy in lowest order since spin- and isospin-dependent boson exchange interactions average out.

The interaction Lagrangian in Walecka's model is then

$$\mathcal{L}_{\rm int}=g_s\bar{\psi}(x)\phi(x)\psi(x)-g_v\bar{\psi}(x)\gamma_\mu V^\mu(x)\psi(x)\,, \qquad (1)$$

where ψ are the nucleon fields, ϕ and V^μ the scalar and vector fields, respectively. In the Hartree approximation, the ground state energy of nuclear matter in this picture is determined by only two parameters,

$$c_s=g_sM/m_s \quad\text{and}\quad c_v=g_vM/m_v\,, \qquad (2)$$

where m_s and m_v are the scalar and vector boson masses, respectively. These parameters are adjusted to give a binding energy $E_B/A=15.75$ MeV, and an equilibrium Fermi momentum of $k_F=1.42$ fm^{-1}. The result of Ref. 6 is

$$c_s^2=266.9\,, \quad c_v^2=195.7\,. \qquad (3)$$

Although there is no particular reason to identify the vector boson with the ω meson and the scalar boson with the σ meson as it appears in one-boson exchange (OBE) interactions, it is interesting to note that if we choose $m_v=783$ MeV and $m_s=550$ MeV, as is frequently done in such potentials, Eq. (2) gives

$$g_s^2/4\pi=7.3 \quad\text{and}\quad g_v^2/4\pi=10.8\,. \qquad (4)$$

These coupling constants happen to be quite close to those usually used in OBE potentials.[2,4]

It is now interesting to investigate whether the parameters of Eq. (3) reproduce the single particle energies and spin-orbit splitting in a finite nucleus, like ^{16}O, in the same (i.e., relativistic Hartree) approximation. For that purpose, we solve the Hartree problem for $A=16$ as in Ref. 5 using the Dirac equation

$$[-i\vec{\gamma}\cdot\vec{\nabla}-\gamma_0E_\alpha+\mathcal{V}_\alpha(\vec{r})+M]\psi_\alpha(\vec{r})=0\,. \qquad (5)$$

Here ψ_α is the spinor of a single nucleon,

$$\psi_\alpha(\vec{r}) \equiv \psi_{nljm\tau}(\vec{r}) = \begin{pmatrix} \dfrac{iG_{nlj}(r)}{r} \\ \dfrac{F_{nlj}(r)}{r^2}\vec{\sigma}\cdot\vec{r} \end{pmatrix} \varphi_{ljm}(\hat{r})\xi_{1/2\,\tau}, \tag{6}$$

and $\mathcal{V}_\alpha$ is the Hartree potential,

$$\mathcal{V}_\alpha(\vec{r}) = \sum_{\beta\neq\alpha}\int d^3r'\bar{\psi}_\beta(\vec{r}\,')[V_s(r_{12})+V_v(r_{12})]\psi_\beta(\vec{r}\,'), \tag{7}$$

$$r_{12}=|\vec{r}-\vec{r}\,'|,$$

where

$$V_s(r_{12}) = -\frac{g_s^2}{4\pi}\frac{e^{-m_s r_{12}}}{r_{12}} \tag{8a}$$

and

$$V_v(r_{12}) = \frac{g_v^2}{4\pi}\frac{e^{-m_v r_{12}}}{r_{12}}\gamma_\mu(1)\gamma^\mu(2) \tag{8b}$$

are the scalar and vector boson exchange interactions generated from $\mathcal{L}_{int}$ of Eq. (1). The coupled radial equation for the large and small spinor components G and F, respectively, reduce to

$$\frac{dF_a(r)}{dr} = [M-E_a+W_a^s(r)+W_a^v(r)]G_a(r) + \frac{\kappa}{r}F_a(r), \tag{9a}$$

$$\frac{dG_a(r)}{dr} = [M+E_a+W_a^s(r)-W_a^v(r)]F_a(r) - \frac{\kappa}{r}G_a(r), \tag{9b}$$

where $a = nlj$ denotes the set of radial quantum numbers. Here the reduced interactions $W_a^{s,v}$ follow from Eqs. (7), and (8) after performing angular integrations. For a closed-shell nucleus, one obtains

$$W_a^{s,v}(r) = \pm m_{s,v}\frac{g_{s,v}^2}{4\pi} \times\left\{\sum_b 2(2j_b+1)I_b^{s,v}(r) - I_a^{s,v}(r)\right\}, \tag{10a}$$

where

$$I_i^{s,v}(r) = \int_0^\infty dr' j_0(im_{s,v}r_<)h_0^{(1)}(im_{s,v}r_>) \times[G_i^2(r')\mp F_i^2(r')]. \tag{10b}$$

The different signs in Eqs. (10) refer to scalar or vector exchange, in that order. [Here j_0 and $h_0^{(1)}$ are the usual spherical Bessel and Hankel functions and $r_<$ ($r_>$) is the smaller (larger) value of either r or r'.] Furthermore,

$$\kappa = \pm(j+\tfrac{1}{2}) \quad \text{for} \quad j = l \mp \tfrac{1}{2}. \tag{11}$$

Subsequent reduction of Eqs. (9) to a second order equation, i.e., elimination of the small component F_a, reveals that the strongly attractive potential

$$U_a^{(-)} = W_a^s - W_a^v \tag{11}$$

appears in conjunction with the spin-orbit interaction, which takes the form

$$U_{s.o.}(r) = -\frac{(d/dr)U_a^{(-)}(r)}{2M[M+E_a+U_a^{(-)}(r)]}\frac{\vec{l}\cdot\vec{\sigma}}{r}. \tag{12}$$

Note that scalar and vector exchange contribute with equal sign to the spin-orbit interaction, whereas they enter with opposite sign into the central Hartree potential. The resulting $U_{s.o.}$ is sizeable even though the nucleon mass enters twice in the denominator. Note also that the spin-orbit force is confined to the nuclear surface, as it should be, and that in the limit $E_a \to M$, $U_a^{(-)} \ll M$, Eq. (12) reduces to the usual interaction of the Thomas type.

We now give results for the (self-consistently determined) single-particle energies $\epsilon_a = E_a - M$, using $m_s = 550$ MeV, $m_v = 783$ MeV for the meson masses, together with the coupling constants of Eq. (4). Results are (for ^{16}O):

$$\begin{aligned}\epsilon(1s_{1/2}) &= -42.6\text{ MeV},\\ \epsilon(1p_{3/2}) &= -21.3\text{ MeV},\\ \epsilon(1p_{1/2}) &= -13.5\text{ MeV}.\end{aligned} \tag{13}$$

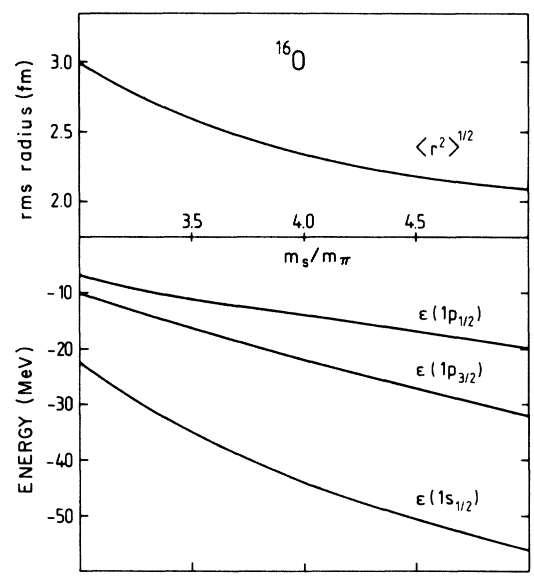

FIG. 1. Single-particle energies (lower section) and root-mean-square radius (upper section) of ^{16}O calculated in a self-consistent Hartree-Dirac model as described in the text, as a function of the mass m_s of the effective scalar boson. Scalar and vector boson exchange parameters have been fixed according to Walecka (Ref. 6) as in Eqs. (2 and 3). For the vector boson mass, $m_v = 783$ MeV has been used.

These numbers compare reasonably with measured separation energies.[7] In particular, the magnitude of the spin-orbit splitting is almost quantitatively correct. The binding is only slightly overestimated, so that as a consequence the rms radius comes out somewhat too small, $\langle r^2\rangle^{1/2} = 2.4$ fm.

We have noted already that there is no justification to identify, in particular, the scalar boson of Walecka's model with the σ meson appearing in one-boson exchange potentials. The scalar mass m_s is anyway not well localized, so that it might be interesting to study the sensitivity of our results to changes in m_s, keeping c_s of Eq. (2) at the constant value of Eq. (3). The results are summarized in Fig. 1 and show a relatively strong dependence of the single particle energies on the scalar mass. However, the qualitative conclusions about the nature of the spin-orbit interaction remain basically unchanged.

The reason for the larger binding with increasing scalar mass at fixed g_s^2/m_s^2 is that the attractive part of the two-body force becomes stronger at shorter internucleon distance, so that at the same time the rms radius decreases. Finally, we would like to mention that increasing the vector boson mass from 800 MeV to 1 GeV, while keeping a constant ratio g_v/m_v, raises the single-particle energies by only about 10%, with almost no change of the spin-orbit splitting.

The results obtained here are similar to those of Miller,[3] except that the freedom of choice in the scalar and vector boson parameters has now been reduced considerably by the fact that the ratios g/m are determined by properties of infinite nuclear matter.

[1] G. E. Brown and A. D. Jackson, *The Nucleon-Nucleon Interaction* (North-Holland, Amsterdam, 1976).
[2] K. Erkelenz, Phys. Rep. 13C, 191 (1974).
[3] L. D. Miller, Phys. Rev. Lett. 28, 1281 (1972); L. D. Miller and A. E. S. Green, Phys. Rev. C 5, 241 (1972); L. D. Miller, Ann. Phys. (N. Y.) 91, 40 (1975).
[4] A. D. Jackson, D. O. Riska, and B. Verwest, Nucl. Phys. A249, 397 (1975).
[5] R. Brockmann, Ph.D. thesis, 1976 (unpublished).
[6] J. D. Walecka, Ann. Phys. (N. Y.) 83, 491 (1974).
[7] G. Jacob and A. J. Maris, Rev. Mod. Phys. 45, 6 (1973).