Analysis of nuclear β^+ decay using longitudinal polarization measurements*

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Possible β^+ decay experiments are discussed with respect to a newly developed polarimeter.

[RADIOACTIVITY: Calculated longitudinal β^+ polarization for allowed decays.]

A recent article by Gerber *et al.*[1] suggests the feasiblity of measuring relative positron polarizations with an accuracy of one part in a thousand. It is the purpose of this note to examine the implications for possible β-decay studies, as suggested in their paper.

First assume the canonical V-A form for the weak interaction. Then

$$T=\frac{G}{\sqrt{2}}\cos\theta_c<\beta_{p_2}|V_\lambda+A_\lambda|\alpha_{p_1}>\bar{u}_\nu(k)\gamma^\lambda(1+\gamma_5)v_e(p)\,, \tag{1}$$

where p_1, p_2, p, and k represent the respective four-momenta of parent nucleus α, daughter nucleus β, positron, and neutrino, G ($\simeq 10^{-5}m_p^{-2}$) is the weak decay constant, and θ_C (~15°) is the Cabibbo angle. Letting M_1 and M_2 be parent and daughter masses, we define also

$$P=p_1+p_2\,,\quad q=p_1-p_2=p+k\,,$$

$$M=\tfrac{1}{2}(M_1+M_2)\,,\quad \Delta=M_1-M_2\,.$$

Then to first order in recoil the decay spectrum is[2]

$$d\Gamma=\frac{|T|^2}{(2\pi)^5}\left(1+\frac{3E-E_0-3\vec{p}\cdot\hat{k}}{M}\right)(E_0-E)^2pE\,dE\,d\Omega_e d\Omega_\nu\,, \tag{2}$$

where $E(\vec{p})$ is the electron energy (momentum), $\hat{k}$ is a unit vector in the direction of neutrino momentum, and E_0 is the maximum permissible positron energy

$$E_0=\Delta\frac{1+m_e^2/2M\Delta}{1+\Delta/2M}\,. \tag{3}$$

We write for an arbitrary allowed ($\Delta J=0,\pm1$; no) transition[3]

$$\langle\beta_{p_2}|V_\lambda+A_\lambda|\alpha_{p_1}\rangle l^\lambda=\frac{1}{2M}aP\cdot l\,\delta_{JJ'}\delta_{MM'}-\frac{i}{4M}C^{M'}_{J'}{}^{k}_{1;}{}^{M}_{J}\epsilon_{ijk}[2bl_iq_j+i\epsilon_{ij\lambda\eta}l^\lambda(cP^\eta+dq^\eta)]+\cdots\,, \tag{4}$$

where J and J' are the spins of parent and daughter nucleus, respectively, and M and M' represent the initial and final components of nuclear spin along some axis of quantization. Here a and c represent the usual Fermi and Gamow-Teller matrix elements, b is the so-called "weak magnetism" contribution, while d, often called the induced tensor, is uniquely correlated with the existence of a second class axial current if α and β are isotopic analogs.[4]

Each form factor—a, b, c, and d—is a function of the four-momentum transfer q^2. However, for present purposes it is sufficient to include this feature only for the leading a and c terms via

$$a(q^2)=a_1+a_2q^2+\cdots\,,\quad c(q^2)=c_1+c_2q^2+\cdots\,. \tag{5}$$

A straightforward, though tedious, calculation then yields the positron longitudinal polarization

$$P_L\cong\frac{p}{E}\,\frac{1}{1+[1/(a_1^2+c_1^2)](m_e^2/3ME)[c_1(-2c_1+d+2b)+4M(E_0-E)(a_1a_2-\frac{1}{3}c_1c_2)]}\,. \tag{6}$$

The preceding discussion has assumed the absence of Coulomb effects. These are included systematically in the Appendix and are shown not to modify Eq. (6) to any appreciable extent for $Z\alpha\ll 1$.

The Michigan polarimeter, if teamed with a spectrometer, would provide a sensitive measurement of the relative longitudinal polarization as a function of energy. That is, one might undertake comparison of P_L for a pure Fermi transition ($b=c=d=0$)

$$P_L^{\rm Fermi} \cong \frac{p}{E}\,\frac{1}{1+[4m_e^2(E_0-E)/3E](a_2/a_1)}\,, \tag{7}$$

with a superallowed (SA) transition wherein all form factors are permitted. Then

$$P_L^{\rm SA}-P_L^{\rm Fermi} \approx -\frac{p}{E}\,\frac{m_e^2}{3ME}\,\frac{1}{a_1^2+c_1^2}\{c_1(-2c_1+d+2b)+4M(E_0-E)[-\tfrac{1}{3}c_1c_2-(c_1^2/a_1^2)a_1a_2]\}\,. \tag{8}$$

As a specific example, consider an analog transition, e.g.,

$$\mathrm{Ne}^{19}\to \mathrm{F}^{19}+e^+ +\nu_e\,.$$

Then the weak magnetism term b is predicted via conserved vector current (CVC) in terms of the measured parent and daughter magnetic moments[2]

$$b=\sqrt{3}\,A(\mu_{\rm Ne}-\mu_{\rm F})=-148.60\pm 0.03\,, \tag{9}$$

while d is required to vanish in the absence of second class axial currents. However, a recent experiment by Calaprice *et al.* suggests the value[5]

$$d^{\rm exp}=+250\pm 100 \tag{10}$$

if the CVC prediction for b is valid. The form factor dependence on q^2 is negligible for this case. Thus at $E=1$ MeV we expect an effect

$$P_L^{\rm Ne}-P_L^{\rm Fermi}\approx(1.1\times 10^{-3})p/E\,, \tag{11}$$

which may prove possible to measure in the not too distant future. This would provide a needed independent measurement of recoil order form factors in the mass-19 system and when combined with the Calaprice *et al.* measurement would yield values for b and d and thus CVC, second class currents separately rather than the present situation wherein only a linear combination is known.

Such a definitive measurement on a superallowed transition requires pushing the polarimeter somewhat beyond its present capabilities and thus may not prove feasible in the immediate future. Another interesting line of attack, however, could be a verification of the basic structure of the weak interaction by improving present limits on the absence of so-called Fierz interference terms,[6] which provide a measure of possible scalar and/or tensor interactions in terms of the factor[7]

$$B_{\rm Fierz}\cong 2\,\frac{a^2\rho(C_VC_S+C_{V'}C_{S'})+c^2\tau(C_AC_T+C_{A'}C_{T'})}{a^2(C_V^2+C_{V'}^2)+c^2(C_A^2+C_{A'}^2)}\,, \tag{12}$$

where a and c are the vector and axial form factors for the decay under consideration, while the definitions of C_V, $C_{V'}$, C_S, etc., are standard.[8] Here $\rho\approx 0.6$ ($\tau\approx 1.2$) are scalar (tensor) renormalization factors. Of course $B_{\rm Fierz}=0$ if the decay is strictly V-A. The presence of a Fierz term modifies the longitudinal polarization to become[9]

$$P_L\approx\frac{p}{E}\,\frac{1}{1-(m_e/E)B_{\rm Fierz}}\,. \tag{13}$$

For our purposes it is convenient to separate the Fierz term into two components:

$$B_{\rm Fierz}^{\rm F}=2\rho\,\frac{C_VC_S+C_{V'}C_{S'}}{C_V^2+C_{V'}^2}\,,\qquad B_{\rm Fierz}^{\rm GT}=2\tau\,\frac{C_AC_T+C_{A'}C_{T'}}{C_A^2+C_{A'}^2}\,, \tag{14}$$

which are probed in pure Fermi and Gamow-Teller (GT) transitions. Assuming $C_V\approx C_{V'}\approx C_A\approx C_{A'}\approx 1$ as in the conventional V-A interaction, we have then

$$B_{\rm Fierz}\approx\frac{\rho}{1+x^2}B_{\rm Fierz}^{\rm F}+\frac{\tau x^2}{1+x^2}B_{\rm Fierz}^{\rm GT}\,, \tag{15}$$

where $x=c/a$.

Present limits on the size of $B_{\rm Fierz}^{\rm F}$ and $B_{\rm Fierz}^{\rm GT}$ come from:

(i) *e^-capture to β^+ ratios*. Unfortunately, most such measurements are for K capture only for which the results, although sensitive to Fierz interference, are also uncertain due to exchange and overlap corrections.[10] A case for which an accurate *total* capture rate (which is insensitive to exchange and overlap uncertainties) is measured is ^{22}Na. Two recent experiments give

$$B_{\rm Fierz}^{\rm GT}=-0.025\pm 0.006\ \ (\text{Ref. 11}),\qquad B_{\rm Fierz}^{\rm GT}=-0.024\pm 0.009\ \ (\text{Ref. 12}), \tag{16}$$

These results appear to indicate that $B_{\rm Fierz}^{\rm GT}\neq 0$. However, this is a strongly hindered transition ($\log ft=7.4$), so that second forbidden corrections may be able to resolve the discrepancy between theory and experiment without introduction of a Fierz term.[13]

(ii) *Analysis of β spectra*. Here also results are uncertain. There are a number of cases wherein measurements are inconsistent with $B_{\rm Fierz}=0$.[14] The most stringent limit presently quoted is from a shape factor measurement in ^{22}Na[15]:

$$B_{\rm Fierz}^{\rm GT}=0.0008\pm 0.0028. \tag{17}$$

However, this is *not* in agreement with the ϵ/β^+ data on ^{22}Na [Eq. (12)]. Also, the analysis does not include radiative corrections or the expected sizable second forbidden terms, which can mask the presence of $B^{\rm GT}_{\rm Fierz}$.[13] A result from a nonhindered transition is from a shape factor measurement on the analog decay $^{13}{\rm N} \to {}^{13}{\rm C} + e^+ + \nu_e$, which yielded[16]

$$B_{\rm Fierz} \approx 0.46 B^{\rm F}_{\rm Fierz} + 0.28 B^{\rm GT}_{\rm Fierz} = 0.0014 \pm 0.0237\,. \tag{18}$$

Finally, although not a spectral shape measurement, an analysis of 0^+-0^+ analog transitions by Hardy and Towner provides the best existing limit on $B^{\rm F}_{\rm Fierz}$[17]:

$$B^{\rm F}_{\rm Fierz} = -0.001 \pm 0.006\,. \tag{19}$$

(iii) e^- *helicity measurements*. Here the best result is from a recent measurement on ^{3}H[18]:

$$P_L = -(v/c)(1.005 \pm 0.026)\,,$$

implying

$$B_{\rm Fierz} \approx 0.25 B^{\rm F}_{\rm Fierz} + 0.71 B^{\rm GT}_{\rm Fierz} = -0.005 \pm 0.026\,. \tag{20}$$

Two older measurements,

$$P_L = -(1.00 \pm 0.02) v/c \quad \text{(Ref. 19)},$$

implying

$$\left.\begin{aligned} &B^{\rm GT}_{\rm Fierz} = 0.00 \pm 0.05\,,\\ &P_L = -(0.99 \pm 0.01) v/c \quad \text{(Ref. 20)}, \end{aligned}\right. \tag{21}$$

implying

$$B^{\rm GT}_{\rm Fierz} = 0.02 \pm 0.02\,,$$

are both for the transition

$$^{32}{\rm P} \to {}^{32}{\rm S} + e^- + \bar\nu_e\,,$$

which is strongly hindered ($\log ft = 7.9$) and is therefore subject to at least some of the same difficulties with respect to second forbidden contributions as is ^{22}Na. A summary of relevant experimental results is given by Pauli.[21] From these data, then, one can conclude that $|B^{\rm F}_{\rm Fierz}| \lesssim 0.006$, $|B^{\rm GT}_{\rm Fierz}| \lesssim 0.03$.

Use of the Michigan polarimeter could enable a substantial improvement on this limit. Suppose one compares the measured polarization at a given energy E for a pure Fermi and a pure Gamow-Teller decay. Then

$$P_L^{\rm Fermi} - P_L^{\rm GT} \approx -\frac{p}{E}\,\frac{m_e}{E}(B^{\rm GT}_{\rm Fierz} - B^{\rm F}_{\rm Fierz}). \tag{22}$$

A measurement on a pair of such nonhindered transitions which indicated that

$$|P_L^{\rm Fermi} - P_L^{\rm GT}| < 3 \times 10^{-3} p/E \ \text{at} \ E = 1 \ \text{MeV} \tag{23}$$

would allow a rather clean limit

$$|B^{\rm F}_{\rm Fierz} - B^{\rm GT}_{\rm Fierz}| < 6 \times 10^{-3} \tag{24}$$

to be set, which is a significant improvement on present results, as indicated above. Of course, should a nonzero effect be found at this level, that would be even more exciting in that one does not anticipate seeing V-A recoil effect corrections until the level of $\sim 1 \times 10^{-3}$.

APPENDIX

A proper analysis of β^+ decay must take account of the electromagnetic interaction between the positron and nucleus. The elementary particle techniques for handling this problem were developed in a previous work[22] and will not be repeated here. The results can be described in terms of integrals A, B, C, and D of the weak charge density $\rho(r)$ and the lepton wave functions. Explicit definitions can be found in Ref. 22.

In terms of these we find

$$\begin{aligned} P_L = {} & +\beta\{a_1^2[|A|^2 - |B|^2 - |C|^2 + |D|^2 - 2\,{\rm Re}(A^*D - B^*C)] + c_1^2[|A|^2 - |B|^2 - |C|^2 + |D|^2 + \tfrac{2}{3}{\rm Re}(A^*D - B^*C)]\}\\ & \times [1 + (\alpha/2\pi) g_2(E)]/\{a_1^2[|A|^2 + |B|^2 + |C|^2 + |D|^2\\ & - 2\,{\rm Re}(A^*D + B^*C) + 2(m_e/E)\,{\rm Re}(A^*B + C^*D - A^*C - B^*D)] + c_1[c_1 + (m_e^2/3ME)(2b + d - 2c_1)]\\ & \times [|A|^2 + |B|^2 + |C|^2 + |D|^2 + \tfrac{2}{3}{\rm Re}(A^*D + B^*C) + 2(m_e/E)\,{\rm Re}(A^*B + C^*D) + \tfrac{1}{3}A^*C + \tfrac{1}{3}B^*D)]\}[1 + (\alpha/2\pi) g_1(E)]\,, \end{aligned} \tag{A1}$$

where the $g_i(E)$ are radiative correction factors given by

$$\left.\begin{aligned} g_1(E) &= \xi - 2(1-\beta^2)\frac{1}{\beta}\tanh^{-1}\beta + \frac{4}{3}\,\frac{E_0 - E}{E}\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right) + \frac{1}{6}\,\frac{(E_0 - E)^2}{E^2}\,\frac{1}{\beta}\tanh^{-1}\beta\,,\\ g_2(E) &= \xi + \frac{4}{3}\,\frac{E_0 - E}{\beta E}\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right) + \frac{1}{6}\,\frac{(E_0 - E)^2}{\beta E^2}\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right). \end{aligned}\right. \tag{A2}$$

Here ξ is the function

$$\xi = 3\ln m_p/m_e - \frac{3}{4} + 4\left(\frac{1}{\beta}\tanh^{-1}\beta - 1\right)\left[-\frac{3}{2} + \ln\frac{2(E_0-E)}{m_e}\right] + \frac{4}{\beta}L\left(\frac{2\beta}{1+\beta}\right) + \frac{4}{\beta}\tanh^{-1}\beta(1-\tanh^{-1}\beta)\,, \quad \text{(A3)}$$

where

$$L(x) = \int_0^x \frac{\ln(1-t)}{t}dt \quad \text{(A4)}$$

is the usual Spence function.

In the limit $Z\alpha \ll 1$ the integrals A–D can be evaluated analytically and we find that if one parametrizes the weak form factors as

$$F(q^2) \approx F(\vec{q}^{\,2}) \equiv F_1 + q^2F_2 + \cdots\,, \quad \text{(A5)}$$

then

$$A \approx [F_{BJ}(Z,E)]^{1/2}\left\{1 + \frac{F_2}{F_1}\left[2EE_0 - 2E^2 + m_e^2 + 3Z\alpha\frac{E}{R} - \frac{9}{4}\left(\frac{Z\alpha}{R}\right)^2 + \cdots\right]\right\},$$

$$B \approx C \approx 0\,,$$

$$D \approx -[F_{BJ}(Z,E)]^{1/2}\frac{F_2}{F_1}Z\alpha\frac{E_0-E}{R}\,, \quad \text{(A6)}$$

where R is the nuclear radius and $F_{BJ}(Z,E)$ is the definition of the Fermi function due to Behrens and Jänecke.[23]

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