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Study of odd vanadium isotopes

A. K. Dhar and K. H. Bhatt
Physical Research Laboratory, Ahmedabad-380009, India
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Deformed configuration mixing shell model calculations of the spectra and electromagnetic properties of ^{47,49,51}V have been carried out. Agreement between the calculations and available observations is good. On the basis of the correspondence between the calculated and observed decay properties, we suggest the spin assignments $J = 1/2$ to the 2.21 MeV level in ^{47}V and $J = 1/2, 9/2, 7/2, 11/2$, and $13/2$ to the levels at 1.64, 2.35, 2.41, 2.67, and 2.73 MeV, respectively, in ^{49}V. An interesting possibility of the existence of an excited highly deformed $K = 3/2$ band of states in ^{51}V is also suggested.

NUCLEAR STRUCTURE: ^{47,49,51}V, calculated spectra, $B(E2)$, $B(M1)$, $(E2/M1)$ mixing ratios, branching intensities, lifetimes, deformed configuration mixing shell model calculations based on projected Hartree-Fock theory in $(fp)^n$ space, modified Kuo-Brown interaction, comparison with experiment, and rotation particle-coupling model calculation.

I. INTRODUCTION

The spectra of the isotopes of ^{47}V, ^{49}V, and ^{51}V show interesting similarities as well as differences. The isotope ^{51}V has often been considered to be "spherical" with $f_{7/2}$ neutron subshell closed. The spectrum of the low-lying states is then attributed to the excitations of the three $f_{7/2}$ protons. On the other hand, marked deformation effects have been observed in ^{47}V and ^{49}V.

We have studied these nuclei in the framework of the deformed configuration mixing[1] (DCM) shell model. Modified[2] Kuo-Brown effective interaction[3] matrix elements (labeled MWH2) in the full fp shell space were used together with the experimental single-particle energies relative to the ^{40}Ca core. In the DCM calculation the axially symmetric deformed Hartree-Fock (HF) calculations[4-7] were first carried out for these nuclei and prolate and oblate HF states were obtained. States of definite angular momenta were projected from these two HF states as well as from a number of low-lying one particle-one hole excited intrinsic states obtained from these HF states. The number of intrinsic states included in the calculations for ^{47,49,51}V are 9, 6, and 7, respectively. The energy spectra were obtained by diagonalizing the Hamiltonian in the space of the states of good J projected from these intrinsic states, taking care of their nonorthogonality. The resulting wave functions were used to study the electromagnetic properties. For the calculation of $E2$ transitions, effective charges $e_p = 1.25e$, $e_n = 0.47e$, and $e_p = 1.32e$, $e_n = 0.89e$ were used. The former charges were obtained recently by Kuo and Osnes[8] in a microscopic calculation for some of the fp shell nuclei. The latter ones were obtained[9] by a least-squares fit between the experimental and our calculated[10] $B(E2, 2 \rightarrow 0)$ values for transitions in even-even fp shell nuclei.

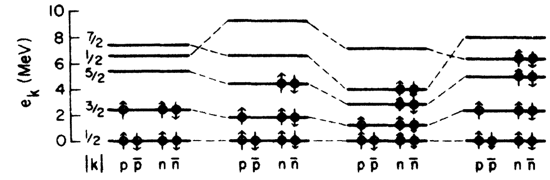
The basic similarity in the spectra of these isotopes, namely, a "collective" band of states with lowest angular momentum $J = \frac{3}{2}$, is easily understood from the point of view of this model to arise predominantly from the $K = \frac{3}{2}$ HF intrinsic states of these nuclei. The structure of these HF states changes with the neutron number. It is interesting to verify whether the observed changes in the deformation and their effects as one goes from ^{47}V to ^{51}V are reproduced by the calculations.

In Sec. II we discuss the structure of the HF states of these nuclei. In Secs. III, IV, and V we compare the spectra and electromagnetic properties of ^{47,49,51}V obtained in our calculation with the experimental ones and the ones obtained in previous calculations. A brief summary follows in Sec. VI.

II. HF STRUCTURE

Prolate and oblate HF states of ^{47}V, ^{49}V, and ^{51}V were obtained[11] in the full $(fp)^n$ space using a MWH 2 effective interaction. The energy difference between prolate and oblate HF states is 2.28 MeV in ^{47}V, 1.73 MeV in ^{49}V, and almost vanishes in ^{51}V. Significant admixture of the $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits occur in the HF states of ^{47}V and ^{49}V, but for ^{51}V the admixture is very small.

An important aspect of ^{51}V is that in addition to the normal prolate and oblate HF states with a dominant $(f_{7/2})^{11}$ structure, the interaction gives rise to an excited prolate HF state with $K = \frac{3}{2}$,



	^{47}V $(fp)^7$	^{47}V $(f_{7/2})^7$	^{49}V $(fp)^9$	^{49}V $(f_{7/2})^9$	^{51}V $(fp)^{11}$	^{51}V $(f_{7/2})^{11}$	^{51}V $(fp)^{11}$
Q_P	11.4	5.6	10.0	5.6	7.8	5.6	12.1
Q_N	13.5	6.9	11.7	6.0	3.2	0.0	19.4
Q	24.9	12.5	21.7	11.6	11.0	5.6	31.5
R	2.0		1.9		2.0		

FIG. 1. A sketch of the occupied and some of the unoccupied single particle orbits $|k\rangle$ of the prolate HF intrinsic states of ^{47,49,51}V. $\bar{p}$ and $\bar{n}$ represent protons and neutrons in $|-k\rangle$ orbits. The differences in the single-particle energies e_k of the protons and neutrons are ignored in drawing the sketch. For ^{51}V, in addition to the normal HF state the highly deformed excited prolate HF state is also sketched. Q_P and Q_N are the mass quadrupole moments (in $b^2=\hbar/m\omega$) for the protons and neutrons, respectively, obtained in $(fp)^n$ and $(f_{7/2})^n$ configuration spaces, $Q=Q_P+Q_N$ is the total quadrupole moment. The ratio $R=Q(fp)^n/Q(f_{7/2})^n$.

which has a large deformation. This suggests the possible existence of a highly deformed band of states in the spectrum of ^{51}V.

It is interesting to compare the deformations of the HF intrinsic states of these nuclei in the full fp shell space with the ones obtained in $(f_{7/2})^{11}$ space. In Fig. 1, we have sketched the occupied, and some unoccupied, orbits of the prolate HF states of these nuclei. We have also listed the mass quadrupole moments Q_P and Q_N of the protons and neutrons and the total quadrupole moment $Q=Q_P+Q_N$. The ratio $R=Q(fp)^n/Q(f_{7/2})^n$ provides some measure of the importance of the other configurations in producing deformations in these nuclei. The near constancy of this ratio for the three nuclei implies that deformation effects are as important for the nearly "spherical" ^{51}V as they are for the more deformed ^{47}V. Note that the neutrons contribute quite significantly to the quadrupole moments of ^{51}V in contrast to the assumption of the $(f_{7/2})^{11}$ model.

III. NUCLEUS ^{47}V

Spectrum

The properties of ^{47}V have been investigated by a variety of experiments.[12-27] Theoretical studies of this nucleus have been carried out in the framework of the $(f_{7/2})^n$ shell model[28,29] and in the rotation particle-coupling (RPC) model calculations.[30,31] The DCM calculated spectrum is compared with the experimental one in Fig. 2. The calculated yrast band of states, drawn in thick lines, is in good agreement with the observed band up to $J=\frac{15}{2}^-$ at 2.61 MeV. It is surprising that the calculated $J=\frac{13}{2}^-$ state at 2.22 MeV does not have an experimental counterpart although the $J=\frac{15}{2}^-$ member of the band is populated in the heavy-ion reaction of Blasi *et al.*[24] The nonobservation of this $J=\frac{13}{2}^-$ state perhaps implies that it might be occurring either very close to the $J=\frac{15}{2}^-$ state or above it. The $J=\frac{17}{2}$ and $\frac{19}{2}$ members of the band are predicted to occur at 4.02 and 3.76 MeV, respectively. This band of states belongs dominantly to the prolate $K=\frac{3}{2}$ HF intrinsic state. The average "band mixing" in these states is only about 6%. The other states have a complex structure.

It is interesting to point out that the structure of the $K=\frac{3}{2}$ prolate HF intrinsic state corresponds to a deformation of about $\beta=0.17$. In the RPC calculations of Haas, Taras, and Styczen[31] and Malik, and Scholz[30] (MS), the intrinsic states considered have deformations of $\beta\approx0.22$ and $\beta>0.5$, respectively.

In the experimental spectrum states with $J=\frac{5}{2}^-$ and $\frac{3}{2}^-$ occur[23] at 0.66 and 1.97 MeV. Our calculation does not reproduce these states. Also, the RPC calculation of Haas *et al.*[31] fails to reproduce them. On the basis of the single-particle energy

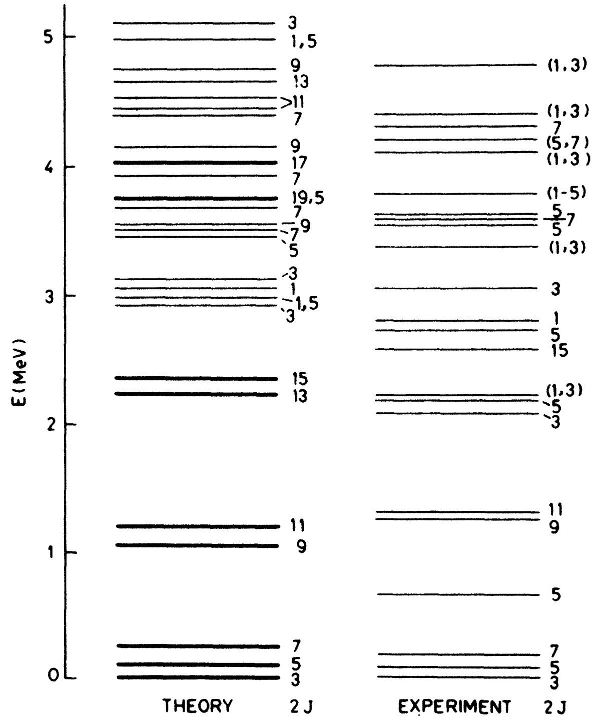


FIG. 2. Comparison of the theoretical and experimental spectrum of ^{47}V. The states drawn in thick lines are the members of the ground state band.

TABLE I. $B(E2,J_i \to J_f)$ values in $e^2\text{fm}^4$ for the transitions within the ground state band of ^{47}V.

J_i	J_f	Expt.[a]	DCM[b]	DCM[c]	RPC[d]
$\frac{5}{2}$	$\frac{3}{2}$	>25	342	198	253
$\frac{7}{2}$	$\frac{3}{2}$	312± 121[e]	147	86	110
	$\frac{5}{2}$		148	147	160
$\frac{9}{2}$	$\frac{5}{2}$	161± 91[f]	203	116	157
	$\frac{7}{2}$	141± 81[f] 907± 504[f]	124	70	96
$\frac{11}{2}$	$\frac{7}{2}$	200^{+100}_{-80}	254	147	188
	$\frac{9}{2}$		112	67	
$\frac{13}{2}$	$\frac{9}{2}$		270	156	198
	$\frac{11}{2}$		62	34	47
$\frac{15}{2}$	$\frac{11}{2}$	>110	278	160	210
	$\frac{13}{2}$		60	36	
$\frac{17}{2}$	$\frac{13}{2}$		275	161	
	$\frac{15}{2}$		35	19	
$\frac{19}{2}$	$\frac{15}{2}$		253	145	
	$\frac{17}{2}$		31	18	

[a] Reference 24.
[b] $e_p = 1.32e$, $e_n = 0.89e$.
[c] $e_p = 1.25e$, $e_n = 0.47e$.
[d] Reference 31.
[e] Reference 25.
[f] Reference 27.

TABLE II. $B(M1,J_i \to J_f)$ values in $(\mu_N)^2$ for the transitions within the ground state band of ^{47}V.

J_i	J_f	Expt.[a]	DCM	RPC[b]
$\frac{5}{2}$	$\frac{3}{2}$	>0.084 0.07± 0.01	0.22	0.07
$\frac{7}{2}$	$\frac{5}{2}$	>0.29[c] 0.45± 0.09	0.34	0.36
$\frac{9}{2}$	$\frac{7}{2}$	0.072± 0.039 0.0036± 0.002	0.26	0.09
$\frac{11}{2}$	$\frac{9}{2}$		0.66	
$\frac{13}{2}$	$\frac{11}{2}$		0.20	0.09
$\frac{15}{2}$	$\frac{13}{2}$		0.90	
$\frac{17}{2}$	$\frac{15}{2}$		0.14	
$\frac{19}{2}$	$\frac{17}{2}$		1.05	

[a] Reference 27.
[b] Reference 31.
[c] Reference 24.

gaps of the prolate and oblate HF states of ^{47}V we do not anticipate any intrinsic state that would give a $J=\frac{5}{2}^-$ state at 0.66 MeV. We feel that these states might be the "intruder" states arising by particle-hole excitations from ^{40}Ca core. This is quite likely in view of the existence of a $J=\frac{3}{2}^+$ state at only 260 keV excitation in ^{47}V.

Electromagnetic properties

The $B(E2)$ and $B(M1)$ values are presented respectively in Tables I and II for the decays between the members of the ground state band.

It is seen that the $B(E2)$ values as obtained in the RPC calculation are similar to our values calculated with Kuo-Osnes charges. However, significant differences between DCM and RPC results occur for the $B(M1)$ and $(E2/M1)$ mixing ratios, more particularly for the transitions between the high-spin states of the ground state band.

In Fig. 3 are presented the experimental and calculated mean lifetimes and branching intensities for the transitions between the low-lying states in ^{47}V. We have associated the calculated states with the corresponding observed states for the calculation of these decay properties. In this figure the vertical lines indicate the transitions from a state marked with a filled circle to the states where the arrows end. The calculated decay intensity is given on the top of the state from which the decay is considered while the corresponding observed value is given immediately below in the parentheses.

TABLE III. Static electric quadrupole and magnetic dipole moments of the ground state triplet of ^{47}V.

	E_J	Q ($e\,\text{fm}^2$)			μ (μ_N)			
J	keV	DCM[a]	RPC I[b]	RPC II[c]	DCM	RPC I[b]	RPC II[c]	$(f_{7/2})^7$[d]
$\frac{3}{2}$	0	16.75	17.4		2.12	2.96		
$\frac{5}{2}$	88	−7.77	−6.3		2.23	3.44		3.10
$\frac{7}{2}$	146	−13.47	−16.9	10	2.90	4.82	4.71	3.83

[a] $e_p = 1.25e$, $e_n = 0.47e$.
[b] Reference 31.
[c] Reference 30.
[d] Reference 29.

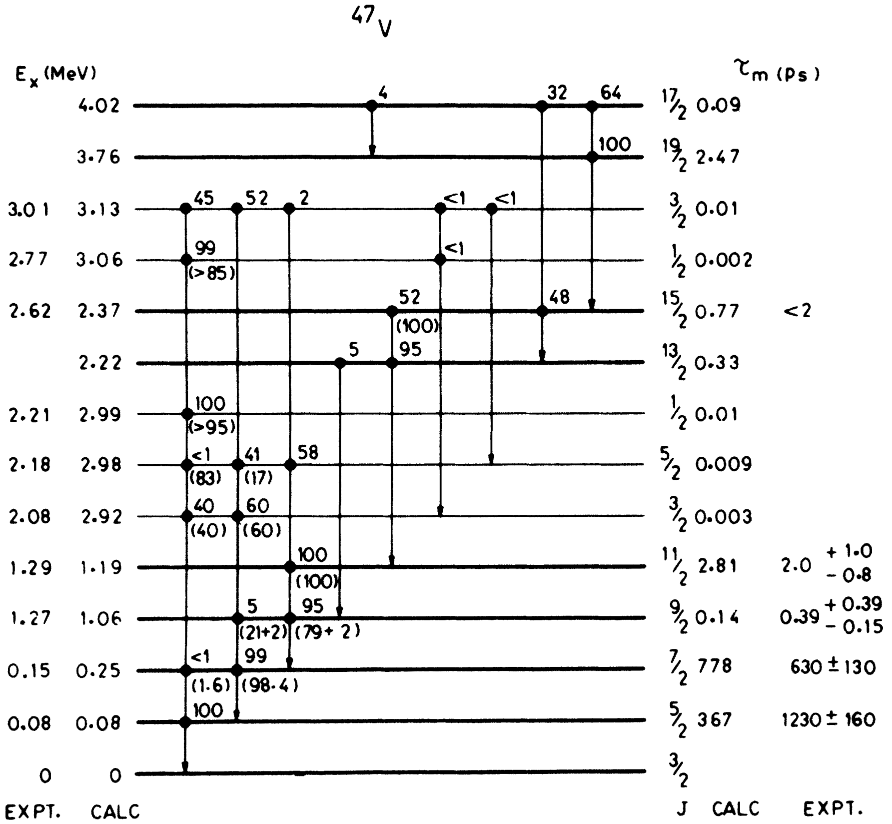


FIG. 3. Comparison of the calculated and observed excitation energies, mean lifetimes, and branching ratios in ^{47}V. The states drawn in thick lines are the members of the ground state band. The vertical line indicates a transition from a state marked with a filled circle to the state where the arrow ends. The calculated branching ratio is given on the top of that state while the observed one is given immediately below in parentheses.

The agreement between theory and experiment for the ground state band up to $J=\frac{11}{2}$ at 1.29 MeV is good except that the mean lifetime of the $J=\frac{5}{2}$ state observed at 88 keV is not well reproduced.

It is interesting to consider the branching ratio for the $\frac{15}{2}$ state at 2.62 MeV. Blasi *et al.*[24] have observed a 100% decay of this state to the $J=\frac{11}{2}$ state at 1.29 MeV. As already mentioned they have not observed any $J=\frac{13}{2}^-$ state. Our calculation gives $J=\frac{13}{2}$ state 150 keV below the $\frac{15}{2}^-$ state. Using the theoretical energy of this $J=\frac{13}{2}^-$ state, the $J=\frac{15}{2}^-$ state is found to decay with a branching ratio of 52 : 48 to the $J=\frac{11}{2}$ and $\frac{13}{2}$ states at 1.29 and 2.22 MeV, respectively, with a mean lifetime consistent with the measurements of Blasi *et al.*[24] As already suggested, if in the experimental spectrum the $J=\frac{13}{2}^-$ state occurs above or very close to the *observed* $J=\frac{15}{2}^-$ state, then our calculated $J=\frac{15}{2}$ state would decay by a 100% branch to the $J=\frac{11}{2}$ state at 1.29 MeV, in concurrence with the measurements of Blasi *et al.* The mean lifetime (1.5 ps) of the $J=\frac{15}{2}^-$ state would still be consistent with the observed[24] lifetime of less than 2 ps. It would be interesting to look for this missing $J=\frac{13}{2}$ state in ^{47}V.

No experimental values are available either for the static electric quadrupole or the magnetic dipole moments in ^{47}V. In Table III are compared the DCM calculated moments of the ground state triplet of ^{47}V with those obtained in the RPC model by Haas *et al.*[31] and Malik and Scholz[30] and in the $(f_{7/2})^7$ configuration model.[29] An interesting discrepancy occurs for the sign of the quadrupole moment of the $J=\frac{7}{2}$ state as predicted by our calculation and that of Malik and Scholz. In general, our static electric quadrupole moments are similar to those obtained by Haas *et al.*, but the magnetic moments are slightly different.

IV. NUCLEUS ^{49}V

A lot of experimental data on ^{49}V have become available[14,15,32-54] very recently. Theoretically this nucleus has been studied within the $(\pi f_{7/2})^3 (\nu f_{7/2})^{-2}$ configuration[28] shell model and by the RPC model.[30,31,49,53,56]

Spectrum

The experimental and DCM calculated spectra are compared in Fig. 4. The calculated[11,55] yrast band of states with $J=\frac{3}{2}^-$ up to $J=\frac{15}{2}^-$ at 2.15 MeV is in good agreement with the observed spectrum. The $J=\frac{13}{2}$, $\frac{17}{2}$, and $\frac{19}{2}$ members of the band are not yet observed. This band of states belongs dominantly to that projected from the prolate $K=\frac{3}{2}$ intrinsic state. The average mixing in all these states is about 6%.

The RPC calculation of Tabor and Zurmühle[49] also reproduces well this band of states up to $J=\frac{11}{2}$, but the calculated first $J=\frac{15}{2}$ state is found to be about 700 keV higher than the observed state. The RPC calculation of Haas *et al.*[53] provides only a qualitative agreement.

It might be mentioned that our microscopic calculation gives rise to the prolate HF intrinsic state corresponding to $\delta \simeq 0.11$, while that needed by Tabor and Zurmühle correspond to $\delta \simeq 0.20$.

The triplet of states observed about 1.5–1.66 MeV is well reproduced by our DCM and RPC calculation of Tabor and Zurmühle.[49] These calculations favor a $J=\frac{1}{2}^-$ for the level observed at 1.64 MeV with probable $(\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-)$ spin assignment. The RPC calculation of Haas *et al.*[31,53] fails to reproduce the $\frac{1}{2}^-$ and $\frac{5}{2}^-$ members of this triplet.

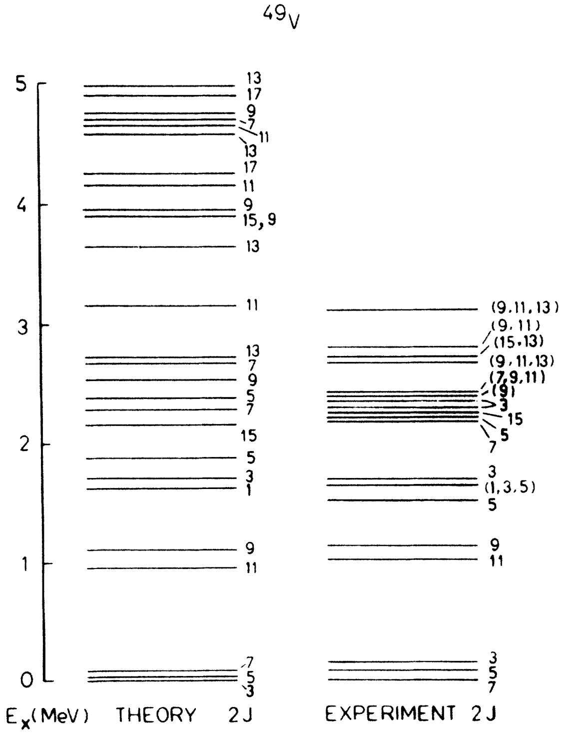


FIG. 4. Comparison of the theoretical and experimental spectrum of ^{49}V.

The $J=\frac{1}{2}$ and $\frac{3}{2}$ states of this calculated triplet and the second $J=\frac{7}{2}$ state at 2.27 MeV are found to be mainly the states projected from the one particle-one hole excited prolate $K=\frac{1}{2}$ intrinsic state. All other states are found to have a complex structure.

Electromagnetic properties

The information on $B(E2)$, $B(M1)$, and mean lifetimes of the low-lying states up to about 3 MeV is presented in Tables IV to VIII.

We have compared in the tables our results (labeled DCM) with the experimental results of Haas *et al.*[31,53] (Expt. I) and of Tabor and Zurmühle[49] (Expt. II), and with their corresponding RPC results labeled RPC I and RPC II, respectively.

The $B(E2)$ values for the transitions between the members of the ground state band of states are listed in Table IV and for the transitions from the states not belonging to the ground state band in Table V. The experimental values of Haas *et al.*[31,53] and Tabor and Zurmühle[49] are not similar. As seen from Table IV, there does not seem to be a unique preference for either of the sets of effective charges. The RPC values of Haas *et al.*[31,53] (RPC I) and their experimental values (Expt. I) are similar to the DCM values obtained with effective charges $e_p = 1.25e$ and $e_n = 0.47e$. In contrast, the calculated (RPC II) and experimental values (Expt. II) of Tabor and Zurmühle[49] appear to be in optimum agreement with the DCM results, using $e_p = 1.32e$ and $e_n = 0.89e$.

In general, however, the $B(E2)$ values for the transitions between the members of the ground state band in ^{49}V appear to be large. The $B(E2)$ values in Table V for the transitions from other states to the members of the ground state band are very small.

The $B(M1)$ values are given in Tables VI and VII for the transitions between the members of the ground state band and other states up to 2.7 MeV in ^{49}V.

The experimental $B(M1)$ values as given by Haas *et al.*[53] and Tabor and Zurmühle,[49] for the transitions between the members of the ground state band (Table VI) are similar. In contrast, the experimental values of Tabor and Zurmühle and Haas *et al.* given in Table VII for the transitions between the other low-lying states in ^{49}V are not similar and have large errors. In many cases only lower limits of $B(M1)$ values are known. The calculated results are in semiquantitative agreement with the experiment. A careful remeasurement of $B(M1)$ values would be desirable.

The DCM results for the $(E2/M1)$ mixing ratios

TABLE IV. Comparison of the theoretical and experimental $B(E2)$ values for transitions within the ground state band of ^{49}V.

		$B(E2, J_i \to J_f)$ (e^2fm^4)					
J_i	J_f	Expt. I[a]	Expt. II[b]	DCM[c]	DCM[d]	RPC I[a]	RPC II[b]
$\frac{5}{2}$	$\frac{7}{2}$			280	171	136	
$\frac{3}{2}$	$\frac{7}{2}$	197± 20		245	146	146	254
	$\frac{5}{2}$			354	206	220	
$\frac{11}{2}$	$\frac{7}{2}$	144 ± 28	172± 59	206	120	123	211
$\frac{9}{2}$	$\frac{7}{2}$	58 ± 33	106 ± 28	72	38	49	111
	$\frac{5}{2}$	83 ± 44	126 ± 17	140	77	96	173
	$\frac{11}{2}$			113	70	55	
$\frac{15}{2}$	$\frac{11}{2}$	279± 128	<71	291	127	137	
$\frac{13}{2}$	$\frac{11}{2}$			31	15		
	$\frac{9}{2}$	295^{+230}_{-125}		181	105	120	
	$\frac{15}{2}$			42	26	29	
$\frac{19}{2}$	$\frac{15}{2}$			161	87		
$\frac{17}{2}$	$\frac{13}{2}$			54	33		
	$\frac{15}{2}$			11	5		
	$\frac{19}{2}$			26	0.82		

[a] References 31 and 52.
[b] Reference 49.
[c] $e_p = 1.32e$, $e_n = 0.89e$.
[d] $e_p = 1.25e$, $e_n = 0.47e$.

TABLE V. Comparison of the theoretical and experimental $B(E2)$ values for transitions from the states J_i at energies E_i to the states J_f at energies E_f in ^{49}V.

				$B(E2, J_i \to J_f)$ (e^2fm^4)		
E_i (keV)	J_i	E_f (keV)	J_f	Expt. I[a]	DCM[b]	RPC I[a]
1514	$\frac{5}{2}$	0	$\frac{7}{2}$		3.6	0.65
		90	$\frac{5}{2}$	>0.3, <435	2.9	4.1
		153	$\frac{3}{2}$	570^{+595}_{-260}	2.4	1.9
1643	$\frac{1}{2}$	90	$\frac{5}{2}$		128.6	
		153	$\frac{3}{2}$		13.2	2.5
1661	$\frac{3}{2}$	90	$\frac{5}{2}$	<122	5.7	22
		153	$\frac{3}{2}$		0.40	8.9
		1643	$\frac{1}{2}$		84.2	
2182	$\frac{7}{2}$	0	$\frac{7}{2}$		0.12	0.06
		90	$\frac{5}{2}$	$4.2^{+8.8}_{-3.7}$	0.22	5.5
		1155	$\frac{9}{2}$		0.42	14.5
		1661	$\frac{3}{2}$		106.0	
2235	$\frac{5}{2}$	0	$\frac{7}{2}$		0.09	
		90	$\frac{5}{2}$		3.10	
		1155	$\frac{9}{2}$		32.4	
		1514	$\frac{5}{2}$		123.0	

[a] Reference 53.
[b] $e_p = 1.25e$, $e_n = 0.47e$.

TABLE VI. Comparison of the experimental and calculated $B(M1)$ values for transitions within the ground state band of ^{49}V.

J_i	J_f	$B(M1, J_i \rightarrow J_f)$ $(\mu_N)^2$ Expt. I[a]	Expt. II[b]	DCM	RPC[a]	RPC II[b]
$\frac{5}{2}$	$\frac{7}{2}$	0.12± 0.01	0.12± 0.01	0.21	0.37	0.57
$\frac{3}{2}$	$\frac{5}{2}$	0.004 ± 0.0005	0.0035± 0.0001	0.02	0.001	0.036
$\frac{9}{2}$	$\frac{7}{2}$	0.012 ± 0.005	0.016 ± 0.004	0.06	0.003	0.004
	$\frac{11}{2}$	<0.90	0.65± 0.16	0.50	0.67	0.87
$\frac{13}{2}$[c]	$\frac{11}{2}$		$(0.034^{+0.019}_{-0.010})$	0.0006		
	$\frac{15}{2}$	$0.39^{+0.41}_{-0.19}$	$2.4^{+1.3}_{-0.7}$	0.89	0.85	
$\frac{17}{2}$	$\frac{15}{2}$			0.15		
	$\frac{19}{2}$			0.70		

[a] Reference 53.
[b] Reference 49.
[c] 2727 keV level of Ref. 49.

TABLE VII. Comparison of the experimental and calculated $B(M1)$ values for transitions from the states J_i at energies E_i to the states J_f at energies E_f in ^{49}V.

E_i (keV)	J_i	E_f (keV)	J_f	$B(M1, J_i \rightarrow J_f)$ $(\mu_N)^2$ Expt. I[a]	Expt. II[b]	DCM	RPC I[a]	RPC II[b]
1514	$\frac{5}{2}$	0	$\frac{7}{2}$	$0.11^{+0.11}_{-0.05}$	>0.38	0.76	0.08	1.50
		91	$\frac{5}{2}$	>0.004, <0.10	>0.16	0.03	0.37	0.81
		153	$\frac{3}{2}$	$0.23^{+0.20}_{-0.09}$	>0.90	1.03	1.09	0.08
1643	$\frac{1}{2}$[c]	153	$\frac{3}{2}$	$0.34^{+0.14}_{-0.08}$	>0.55	1.59	0.70	2.34
1661	$\frac{3}{2}$	91	$\frac{5}{2}$	$0.59^{+0.68}_{-0.30}$	$0.41^{+0.31}_{-0.13}$	1.56	1.37	0.40
		153	$\frac{3}{2}$	$0.37^{+0.48}_{-0.19}$	$0.26^{+0.20}_{-0.08}$	0.84	0.72	0.12
2182	$\frac{7}{2}$	0	$\frac{7}{2}$	<0.01	$0.026^{+0.016}_{-0.010}$	0.08	0.03	
		91	$\frac{5}{2}$	$0.16^{+0.10}_{-0.05}$	$0.074^{+0.041}_{-0.02}$	0.67	0.71	
		1155	$\frac{9}{2}$	$0.36^{+0.26}_{-0.16}$	$0.22^{+0.14}_{-0.08}$	1.12	1.29	
2235	$\frac{5}{2}$[c]	0	$\frac{7}{2}$		$(0.064^{+0.1}_{-0.03})$	0.09		
		91	$\frac{5}{2}$		$0.25^{+0.39}_{-0.10}$	0.79		
2353	$\frac{9}{2}$[c]	0	$\frac{7}{2}$		$(0.05^{+0.03}_{-0.01})$	0.57		
		1021	$\frac{11}{2}$		$0.21^{+0.12}_{-0.06}$	0.61		
2408	$\frac{7}{2}$[c]	0	$\frac{7}{2}$		>0.30	0.81		
		1155	$\frac{9}{2}$		>0.34	0.06		
2671	$\frac{11}{2}$[c]	1021	$\frac{11}{2}$		>0.79	1.76		
2727	$\frac{13}{2}$[c]	1021	$\frac{11}{2}$		$(0.034^{+0.02}_{-0.01})$	0.001		
		2262	$\frac{15}{2}$		$2.4^{+1.3}_{-0.7}$	1.02		

[a] Reference 49.
[b] Reference 53.
[c] Spin assigned in this work.

TABLE VIII. Comparison of the calculated and experimental mean lifetimes of the low-lying states in ^{49}V.

Energy (keV)	J	DCM[a]	RPC[b]	τ_m (ps) Expt. I[c]	Expt. II[d]	Expt. III[e]	Expt. IV[f]
91	$\frac{5}{2}$	368	210	450 ± 30	430 ± 20[g]	330 ± 20[h]	
153	$\frac{3}{2}$	8631	50 000	$28\,700 \pm 500$[i]			
1021	$\frac{11}{2}$	6.2	5.9	$6.3^{+3.9}_{-2.5}$	$4.3^{+1.9}_{-1.1}$	>5	5.1 ± 1.0[j]
1155	$\frac{9}{2}$	0.53	2.6	$1.65^{+0.48}_{-0.34}$	1.06 ± 0.15	$4.3^{+5.6}_{-2.1}$	>0.4
1514	$\frac{5}{2}$	0.011	0.014	$0.045^{+0.021}_{-0.018}$	<0.014		$0.045^{+0.03}_{-0.02}$
1643	$\frac{1}{2}$	0.011	0.025	0.05 ± 0.02	<0.03		0.06 ± 0.03
1661	$\frac{3}{2}$	0.006	0.008	<0.04	0.023 ± 0.01	$0.016^{+0.012}_{-0.008}$	0.025 ± 0.005
2183	$\frac{7}{2}$	0.007	0.007	<0.08	0.048 ± 0.017	0.031 ± 0.010	
2235	$\frac{5}{2}$	0.005		<0.05	0.018 ± 0.011	0.021 ± 0.008	$0.03^{+0.030}_{-0.015}$
2263	$\frac{15}{2}$	2.2	2	$0.93^{+0.55}_{-0.28}$	>3.84	$0.064^{+0.025}_{-0.016}$	$0.045^{+0.030}_{-0.015}$, <3.5[j]
2363	$\frac{9}{2}$	0.006		<0.07	0.048 ± 0.017		
2408	$\frac{7}{2}$	0.003		<0.03	<0.011	<0.01	
2670	$\frac{11}{2}$	0.005		<0.03	<0.016		
2727	$\frac{13}{2}$	0.40		$0.35^{+0.15}_{-0.11}$	$0.138^{+0.058}_{-0.049}$		

[a] $e_p = 1.25e$, $e_n = 0.47e$.
[b] Reference 31.
[c] Reference 53.
[d] Reference 49.
[e] Reference 44.
[f] Reference 45.
[g] Reference 43.
[h] Reference 46.
[i] Reference 42.
[j] Reference 51.

are in agreement with both the experiment and RPC values. Significant discrepancies occur for the transitions $\frac{5}{2}^-$(1514 keV) → $\frac{3}{2}^-$(153 keV), and $\frac{7}{2}^-$(2183 keV) → $\frac{7}{2}^-$(g.s.), for which the opposite signs of ($E2/M1$) mixing ratios are obtained in RPC and DCM calculations.

The mean lifetimes are given in Table VIII. Different experiments have led to different values with large errors. Our DCM results appear to be in reasonable agreement with the experimental and RPC values, except for the $J = \frac{3}{2}$ state at 153 keV. For this state, the DCM calculated mean lifetime is a factor of about 3 smaller than the experiment.[42] The RPC value[31] is about twice the observed value. The discrepancy in the DCM value appears to be because of large $B(M1)$ value for the $\frac{3}{2}$-$\frac{5}{2}$ transition.

We have also summarized in Fig. 5 the results of our lifetime calculations along with some of the observed values of Table IX with which they seem to be in optimum agreement.

The branching ratios are given in Fig. 5. The states drawn therein by thick lines belong to the ground state band. We have associated our calculated states with the corresponding observed states for calculations of these decay properties. The DCM calculated branching intensities are given on top of a state. The observed[40,49,53] values are given immediately below in parentheses. The agreement between the two is in general good. For the $J = \frac{3}{2}^-$ state observed at 0.15 MeV our calculated branches are in disagreement with the corresponding observed branches, mainly because of the disagreement in the $B(M1)$ values.

It may be pointed out that as seen from Fig. 4, there occur observed states with many probable spin assignments about 2.5 MeV. We have suggested possible spin assignments for these levels (marked with an open circle in Fig. 5) on the basis of overall agreement of the calculated and observed energies and decay properties of these states. Thus in Fig. 5 such states are labeled by definite spin as assigned by us.

In Table IX are given the static electric quadrupole and magnetic dipole moments of the ground state triplet of states in ^{49}V. The quadrupole moments are not measured so far. The RPC values of the quadrupole moments are in general smaller than ours but are of the same phase. Malik and Scholz[56] on the other hand predict the sign opposite

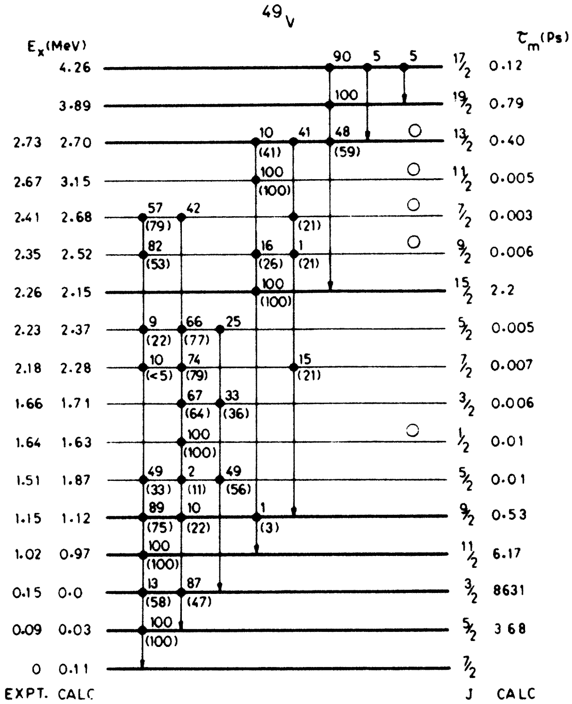


FIG. 5. Comparison of the calculated and observed excitation energies, mean lifetimes, and branching ratios in ^{49}V. The numbers on the top of the states are the calculated decay intensities while those below in brackets are the corresponding observed ones. The states drawn in thick lines are the members of the ground state band. For the states marked with open circles, the spin assignments have been made in this work.

to ours for the ground state quadrupole moment. Our magnetic moments are in better agreement with the observed values than the RPC values.

V. NUCLEUS ^{51}V

The recent experiments performed for the study of this nucleus are described in Refs. 58–64. Theoretically, this nucleus has been considered to have a relatively simple shell model structure with three protons outside the ^{48}Ca closed core. The $(f_{7/2})^3$ calculations[28,65] have been reasonably successful in describing the spectrum of the low-lying states of this nucleus. In this model the proton effective charges varying between $e_p = (1.72 \pm 0.04)e$ to $(1.94 \pm 0.07)e$ have been found[51] necessary to fit the observed $B(E2)$ values between the low-lying states. In the pure $(f_{7/2})^3$ configuration, the $M1$ strengths are strictly forbidden. However, $M1$ transitions though hindered have been observed[60,61] in ^{51}V. This indicates the necessity of other fp shell configurations for the description of low-lying states in ^{51}V. Auerbach[66] and Lips and McEllistrem[67] performed extended calculation for ^{51}V to include besides pure $(f_{7/2})^3$ configuration the $(f_{7/2}{}^2p_{3/2})$ configuration. The interaction matrix elements were determined by a least-squares fit of calculated-to-observed level energies. Lips and McEllistrem also included $(f_{7/2}{}^2f_{5/2})$ configuration by taking the matrix elements involving the $f_{5/2}$ orbit from a surface δ interaction and from the effective interaction of Kuo and Brown. Considerable improvement over the $(f_{7/2})^3$ configuration calculation was obtained.

Horoshko, Cline, and Lesser[60] performed $(fp)^3$ shell model calculation for ^{51}V. ^{48}Ca was regarded as the closed core and the corresponding Kuo-Brown two-body effective interaction[3] employed. The spectrum and transition rates were well accounted for. Proton effective charge $e_p = 1.57e$ was used in the calculation of $E2$ transition rates.

It must be pointed out that in all these shell model calculations, except the one based on pure $(f_{7/2})^3$ configuration, the calculated states do not have definite isospin. Osnes and Warke[68,69] constructed good isospin states in the $(f_{7/2})^{11}$ and $(f_{7/2})^{10}(p_{3/2})$ configuration and showed that although the $E2$ transition rates between the levels dominated by pure $(f_{7/2})^n$ configuration are only weakly affected, the $M1$ transitions are rather

TABLE IX. Comparison of the calculated and experimental static moments of the ground state triplet in ^{49}V.

J	E_J (keV)	Q (e fm^2) DCM[a]	RPC I[b]	MS[c]	μ (μ_N) Expt.	DCM	RPC I[b]	RPC II[d]	MS[c]	$(f_{7/2})$[e]
$\frac{7}{2}$	0	−10.6	−12.4	10	4.46 ± 0.05[f]	4.0	5.09	5.45	4.71	4.68
$\frac{5}{2}$	90	−9.7	−5.9			2.87	3.45			
$\frac{3}{2}$	153	−15.7	13.8		2.37 ± 0.12[g]	2.42	3.00			

[a] $e_p = 1.25e$, $e_n = 0.47e$.
[b] References 31 and 53.
[c] Reference 56.
[d] Reference 49.
[e] Reference 28.
[f] Reference 57.
[g] Reference 42.

strongly affected by this configuration mixing.

Scholz and Malik[70] performed the RPC calculation for ^{51}V. The calculated spectrum is found to be in qualitative agreement with the experiment for large deformation parameters $\beta = -0.32$ and for $\beta = 0.20$. The calculated $B(M1)$ values are in significant disagreement with the observed values.

It is seen that two opposing features about the structure of ^{51}V emerge from the observed[60,61] $B(E2)$ and $B(M1)$ values for the transitions between the low-lying states of this nucleus. The highly retarded $M1$ transitions in ^{51}V indicate the dominance of the $(f_{7/2})^{11}$ configuration. On the other hand, the large proton effective charges needed in the $(f_{7/2})^{11}$ configuration to fit the observed $B(E2)$ values suggest a substantial deformation. It is therefore of interest to see if the calculations using HF theory can explain these apparently contradictory features of ^{51}V structure.

The HF theory takes into account partially the effects of field-producing components of the effective interaction between the nucleons as well as the role of the single-particle energies in determining the dominant configurations. It is therefore likely that the deformed HF calculations, within $(fp)^{11}$ space, with a "suitable" effective interaction (like MWH2), might lead to the HF intrinsic state of smaller deformation with dominant $f_{7/2}$ component. It is, however, commonly felt that the projected HF theory is likely to be useful only for the nuclei with large deformations and hence well defined ground state shapes. It thus appears that for a "mildly" deformed nucleus like ^{51}V the projected HF calculations would not describe well its low-lying observed states. It is therefore of interest to examine the nucleus ^{51}V in the framework of DCM calculations and see how far the observed data can be explained.

In our HF calculations[11,71,72] the normal prolate and oblate HF states have a dominant $(f_{7/2})^{11}$ structure. In addition, as already mentioned in Sec. II, there occurs quite low in energy an excited HF state with $K = \frac{3}{2}$ (see Fig. 1) which has a much larger deformation than the normal HF states. The excited particle-hole intrinsic states with $T = \frac{5}{2}$ included in our calculation were formed from the normal HF states by limiting the p-h excitations to the single-particle orbits with a dominant $f_{7/2}$ component, so that all these intrinsic states have a dominant $(f_{7/2})^{11}$ component.

In the $(f_{7/2})^{11}$ configuration of ^{51}V there exists only one state of each angular momentum. It is, therefore, expected that the states projected from either of the above intrinsic states would provide an adequate description of the "spherical" states of ^{51}V. This is almost true except for the $J = \frac{9}{2}$ and $\frac{3}{2}$ states.

It is interesting to observe that none of the prolate intrinsic states contains a $J = \frac{9}{2}$ state with a dominant $(\pi f_{7/2})^3(\nu f_{7/2})^8$ component. The $J = \frac{9}{2}$ state projected from the prolate $K = \frac{3}{2}$ HF intrinsic state has $(f_{7/2})^{10}(p_{3/2})$ dominant structure and occurs at 4.10 MeV. In a similar manner the oblate intrinsic states do not contain any dominantly $(f_{7/2})^{11}$ $J = \frac{3}{2}$ state. In contrast to these the $(f_{7/2})^{11}$ $J = \frac{9}{2}$ state is contained in the oblate states, while the $(f_{7/2})^{11}$ $J = \frac{3}{2}$ state is contained only in the prolate $K = \frac{3}{2}$ intrinsic state.

It is thus seen that for the overall description of the dominant $(f_{7/2})^{11}$ multiplet of states in ^{51}V, it is necessary to consider the mixing of both prolate and oblate projected states. Neither of them alone would provide agreement with the experimental spectrum of ^{51}V.

As a consequence of the $(f_{7/2})^{11}$ configuration in the intrinsic states, the states projected from these various intrinsic states have about 96% overlap with each other. This renders the basis space highly overcomplete. We find, however, that the inclusion of the p-h excited intrinsic states improves appreciably the energies of the $J = \frac{9}{2}$ and $\frac{15}{2}$ states and also slightly the agreement for $M1$ transitions. The Hamiltonian matrix was diagonalized in the basis of states projected from the six intrinsic states, taking care of their nonorthogonality.

Spectrum

The calculated spectrum is compared with the experiment[60-63] in Fig. 6(a). The agreement between the two up to $J = \frac{15}{2}$ at 2.70 MeV is good except that the $\frac{7}{2}$ and $\frac{5}{2}$ states have flipped in the calculated spectrum. Above 2.5 MeV there exist in the experimental spectrum the states that do not have dominant $(f_{7/2})^3$ structure.

It might be mentioned that in the band-mixing calculation[11,71,72] involving prolate and oblate HF states alone the $J = \frac{9}{2}$ and $\frac{15}{2}$ states are reproduced 300 keV higher than their observed energies.

The states projected from the highly deformed excited HF state do not mix with the other states included in our calculation. The spectrum of this highly deformed band of states in ^{51}V is plotted in Fig. 6(b). The experimental spectrum of ^{51}V does not contain such states so low in energy. It is likely that the chosen effective interaction does not reproduce correctly the position of the bandhead and that such a collective band may indeed exist at a somewhat higher energy. This expectation is based on the following observations.

It may be recalled that the *original* Kuo-Brown interaction[3] has a tendency[73,74] to produce HF ground states with large deformations. For ex-

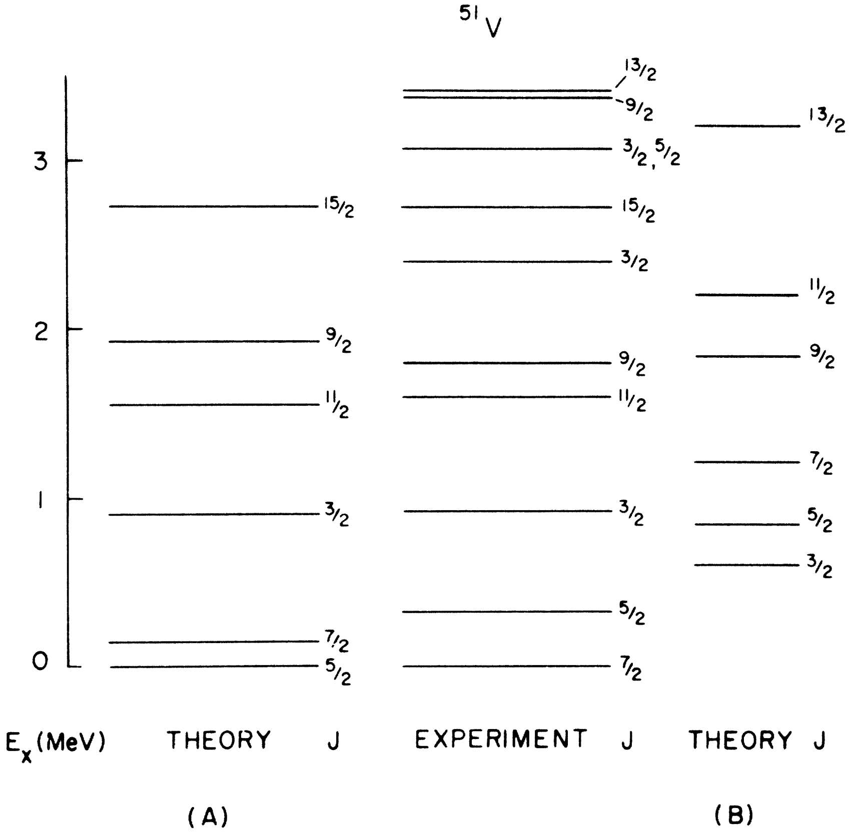


FIG. 6. Comparison of the theoretical and experimental spectrum of ^{51}V. At A are plotted the "spherical" states of ^{51}V. A highly deformed $K=\frac{3}{2}$ band of states that is predicted to exist in its spectrum is plotted at B.

ample, in ^{56}Ni the spherical $(f_{7/2})^{16}$ state is about 13.3 MeV above the highly deformed HF ground state. The *modified* Kuo-Brown interaction[2] has corrected[73,74] this undesirable aspect of the Kuo-Brown interaction. The HF ground state generated by it for the fp shell nuclei have a dominant $(f_{7/2})^n$ component. The more deformed HF states are obtained not as ground states but as excited states. However, it turns out that in general even the modified Kuo-Brown effective interaction employed in the present calculation gives rise[75] to excited deformed bands which are lower in energy than the observed ones, e.g., in ^{49}Cr the excited $K=\frac{1}{2}$ band is lower[76] than the observed one by about 500 keV. In view of this, the present calculations strongly suggest the possibility of the existence in ^{51}V of a highly deformed $K=\frac{3}{2}$ band of states. The spectrum and the deformation of this band, if observed, would be similar to the ones we have calculated. The limitations of the chosen effective interaction do not allow us to predict the exact location of this band. However, the occurrence[77,78] of what appear to be similarly excited bands in ^{51,53}Cr and ^{53}Fe suggests that the calculated $K=\frac{3}{2}$ deformed band in ^{51}V might indeed exist at *not too* high an energy.

Electromagnetic properties

We have compared our full DCM results with the results of only prolate-oblate (labeled PO in Tables X and XI) mixing calculations, and also with the $(f_{7/2})^3$ and $(fp)^3$ calculations and RPC calculation of Scholz and Malik.

The $B(E2)$ values are given in Table X. Our DCM calculated values in column 5, obtained with effective charges $e_p=1.32e$ and $e_n=0.89e$, are in remarkable agreement with the experiment.[60,61] Kuo-Osnes charges provide only a qualitative agreement. The $(fp)^3$ calculation of Horoshko *et al.*[60] uses $e_p=1.57e$, while in the $(f_{7/2})^{11}$ calculation[60] $e_p=1.77e$ is required. The $B(E2)$ value for $\frac{3}{2}\rightarrow\frac{5}{2}$ is poorly described by the pure $(f_{7/2})^{11}$ calculation. The PO values given in column 4 are similar to those obtained in the full DCM calculation. The $B(E2)$ values obtained by RPC model calculation[70] agree with the experimental ones for deformation parameter $\beta=-0.32$.

TABLE X. Comparison of the calculated and experimental $B(E2)$ values for transitions between the low-lying states in ^{51}V.

		$B(E2,J_i\rightarrow J_f)$ (e^2fm^4)					
J_i	J_f	Expt [a]	PO [b]	DCM [b]	DCM [c]	$(fp)^3$ [a]	$(f_{7/2})^3$ [a]
$\frac{5}{2}$	$\frac{7}{2}$	154 ± 7.6 [d]	160.6	164.4	125.2	169	174.6
$\frac{3}{2}$	$\frac{7}{2}$	72 ± 13 [d]	79.5	78.7	61.0	70.9	61.2
	$\frac{5}{2}$	107 ± 9	93.0	92.2	70.0	106.7	44.9
$\frac{11}{2}$	$\frac{7}{2}$	78 ± 14 [d]	88.0	90.0	68.3	80.4	79.3
$\frac{9}{2}$	$\frac{7}{2}$	27.5 ± 6.3 [d]	24.0	19.6	13.8	26.8	29.5
	$\frac{5}{2}$	27.5 ± 6.6 [d]	35.0	24.7	17.6	19.3	27.6
	$\frac{11}{2}$		59.3	47.3	36.0		
$\frac{15}{2}$	$\frac{11}{2}$	66 ± 5 [e]	78.0	63.6	47.5		

[a] References 60 and 79.

[b] $e_p=1.32e$, $e_n=0.89e$.

[c] $e_p=1.25e$, $e_n=0.47e$.

[d] Reference 61.

[e] Reference 51.

TABLE XI. Comparison of the calculated and experimental $B(M1)$ values for transitions between the low-lying states in ^{51}V.

		$B(M1,J_i\rightarrow J_f)\times 10^4$ $(\mu_N)^2$					
J_i	J_f	Expt [a]	PO	DCM	$(fp)^3$ [a]	RPC [b] ($\beta=-0.32$)	RPC [c] ($\beta=0.20$)
$\frac{5}{2}$	$\frac{7}{2}$	49 ± 4	14.7	42	12.7	0.36	2100
		36 ± 0.18 [c]					
$\frac{3}{2}$	$\frac{5}{2}$	29 ± 4	78	50	2.02	1200–4900	8900
$\frac{9}{2}$	$\frac{7}{2}$	4.7 ± 1.6	0.7	5.2	1.96	370	350
	$\frac{11}{2}$		38	6.4			

[a] Reference 60.

[b] Reference 70.

[c] References 81 and 82.

TABLE XII. Static moments of the first few states in ^{51}V.

	E_J	Q (e fm^2)				μ (μ_N)			
J	(MeV)	Expt [a]	DCM [b]	$(fp)^3$ [c]	$(f_{7/2})^3$ [c]	Expt.	DCM	$(fp)^3$ [c]	$(f_{7/2})^3$ [d]
$\frac{7}{2}$	0	−5.2 ± 1.0	−6.0	−6.4	−6.8	5.148	5.11	5.52	5.25
$\frac{5}{2}$	0.32		−14.8	−13.0	−19.0	(4.2 ± 0.7 [e] (3.86 ± 0.03 [f]	3.52	3.84	
$\frac{3}{2}$	0.93		13.7				1.88		

[a] Reference 84.

[b] $e_p=1.32e$, $e_n=0.89e$.

[c] Reference 60.

[d] Reference 28.

[e] Reference 81.

[f] Reference 85.

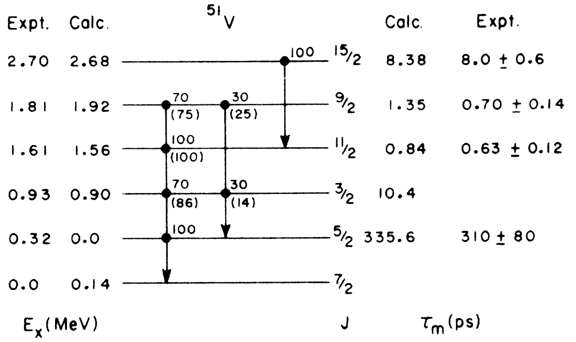


FIG. 7. Comparison of the calculated and experimental excitation energies, branching ratios, and mean lifetimes for the transitions between the "sperical" states of ^{51}V.

It should be noted that in comparison to the $(f_{7/2})^{11}$ calculation the additional deformation of the HF intrinsic state helps substantially in reproducing the $B(E2)$ values in ^{51}V with the reasonable effective charges $e_p = 1.32e$ and $e_n = 0.89e$. These charges were found to give good agreement[9] for the $E2$ transitions in other fp shell nuclei as well.

The $B(M1)$ values are given in Table XI. The experimental values are highly hindered and are strictly forbidden in $(f_{7/2})^3$ configuration. Our DCM calculation provides an improved agreement with the experiment[60,61,82] over the PO or $(fp)^3$ calculations.[60]

The RPC calculations[70] with large deformation give $M1$ values of about 2 to 3 orders of magnitude too large. This disagreement of the RPC results with the experiment is mainly due to the fact that the collective model is inherently incapable of taking into account the geometrical effects which forbid $M1$ transitions in $(f_{7/2})^n$ configuration. The microscopic DCM calculation reproduces the quadrupole collectively and also the $M1$ hindrances.

The branching ratios and mean lifetimes of the "spherical" states in ^{51}V are presented in Fig. 7. Good agreement between the calculated and experimental values[58,60,61,64,80,83] is obtained.

The DCM calculated values of the static moments of the first few states are compared with the experiments and other calculated ones in Table XII.

TABLE XIII. Calculated $B(E2)$ and $B(M1)$ values for transitions between the members of the highly deformed prolate $K = \frac{3}{2}$ excited band in ^{51}V.

J_i	J_f	$B(E2)$ [a] $(e^2\,\mathrm{fm}^4)$	$B(M1)$ $(\mu_N)^2$
$\frac{5}{2}$	$\frac{3}{2}$	614	0.27
$\frac{7}{2}$	$\frac{3}{2}$	258	
	$\frac{5}{2}$	399	0.41
$\frac{9}{2}$	$\frac{5}{2}$	372	
	$\frac{7}{2}$	240	0.37
$\frac{11}{2}$	$\frac{7}{2}$	444	
	$\frac{9}{2}$	186	0.55
$\frac{13}{2}$	$\frac{9}{2}$	460	
	$\frac{11}{2}$	115	0.37

[a] $e_p = 1.32e$, $e_n = 0.89e$.

In Table XIII are presented the $B(E2)$ and $B(M1)$ values for the *intraband* transitions between the members of the excited $K = \frac{3}{2}$ band. It is seen that these transitions are quite large compared to those for the "ground state band." Because of the near purity of these states, the crossover transitions from the members of this band to those with the dominant $(f_{7/2})^3$ multiplet are extremely hindered. Thus the $J = \frac{3}{2}$ member of this highly deformed band may be observed as an isomeric state.

VI. SUMMARY

The present DCM calculations are reasonably successful in describing the properties of the highly deformed ^{47}V as well as the almost "spherical" ^{51}V. The coherent effects of the field-producing part of the effective interaction are quite successful in substantially increasing the quadrupole moment of ^{51}V over that obtained in $(f_{7/2})^{11}$ space with very small admixture of the $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ single-particle states. It would be of interest to verify experimentally the spin assignments in ^{47}V and ^{49}V suggested by our calculations. Identification of the highly collective excited $K = \frac{3}{2}$ band in ^{51}V would be quite exciting.

We would like to thank S. B. Khadkikar and D. R. Kulkarni for extensive discussions, particularly on the ^{51}V nucleus.

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[1] A. K. Dhar, D. R. Kulkarni, and K. H. Bhatt, Nucl. Phys. A238, 340 (1975).

[2] J. B. McGrory, B. H. Wildenthal, and E. C. Halbert, Phys. Rev. C 2, 186 (1970).

[3] T. T. S. Kuo and G. E. Brown, Nucl. Phys. A114, 241 (1968).

[4] W. H. Bassichis, B. Giraud, and G. Ripka, Phys. Rev. Lett. 15, 980 (1965).

[5] M. R. Gunye, Phys. Lett. 27B, 136 (1968).

[6] G. Ripka, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum, New York, 1968), Vol. 1, p. 183.

[7] S. B. Khadkikar and M. R. Gunye, Nucl. Phys. A110, 472 (1968).

[8] T. T. S. Kuo and E. Osnes, Phys. Rev. C 12, 309 (1975).

[9] A. K. Dhar, Ph.D. thesis, 1976 (unpublished).

[10] A. K. Dhar, D. R. Kulkarni, S. B. Khadkikar, and K. H. Bhatt, in Proceedings of the International Conference on Gamma-ray Transition Probabilities, Delhi, 1974, edited by S. C. Pancholi and S. L. Gupta (Delhi U.P., to be published).

[11] A. K. Dhar, S. B. Khadkikar, D. R. Kulkarni, and K. H. Bhatt, in Proceedings of the International Conference on Gamma-ray Transition Probabilities, Delhi, 1974 (see Ref. 10).

[12] G. J. McCallum, A. T. F. Ferguson, and G. S. Mani, Nucl. Phys. 17, 116 (1960).

[13] H. Albinsson and J. Dubois, Phys. Lett. 15, 260 (1965); Ark. Fys. 34, 1 (1967).

[14] C. St. Pierre, P. N. Maheshwari, D. Doutriaux, and L. Lamarche, Nucl. Phys. A102, 433 (1967).

[15] W. E. Dorenbusch, J. Rapaport, and T. A. Belote, Nucl. Phys. A102, 681 (1967).

[16] B. Rosner and D. J. Pullen, Phys. Rev. Lett. 18, 13 (1967); Phys. Rev. 162, 1048 (1967).

[17] O. Redi and M. A. Graber, Bull. Amer. Phys. Soc. 12, 474 (1967).

[18] B. Cujec and I. M. Szoghy, Phys. Rev. 179, 1060 (1969).

[19] M. B. Lewis, Nucl. Data B4, 313 (1970).

[20] G. J. McCullum and K. P. Pohl, Nucl. Phys. A157, 552 (1970).

[21] W. Willmes, Phys. Rev. C 1, 1972 (1970).

[22] V. V. Okorokov, V. M. Serezhin, V. A. Smotryaev, D. L. Tolchenkov, I. S. Trostin, Yu. N. Cheblukov, V. S. Zolotarev, and V. S. Romanov, Yad. Fiz. 14, 490 (1971) [Sov. J. Nucl. Phys. 14, 275 (1972)].

[23] M. Schrader, K. Bucholz, and H. V. Klapdor, Nucl. Phys. A213, 173 (1973).

[24] P. Blasi, T. Fazzini, A. Giannatiempo, R. B. Huber, and C. Signorini, Nuovo Cimento 15A, 521 (1973).

[25] N. Schulz and M. Toulemode, Nucl. Phys. A230, 401 (1974).

[26] M. Toulemonde, N. Schulz, J. C. Merdinger, and P. Engelstein, in Proceedings of the International Conference on Gamma-ray Transition Probabilities, Delhi, 1974 (see Ref. 10).

[27] J. V. Thompson, R. A. I. Bell, L. E. Carlson, and M. R. Najam, Aust. J. Phys. 28, 251 (1975).

[28] J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

[29] J. N. Ginocchio, Phys. Rev. 144, 952 (1966).

[30] F. B. Malik and N. Scholz in *Nuclear Structure*, edited by A. Hossain, H. A. Rashid, and M. Islam (North-Holland, Amsterdam, 1967); Phys. Rev. 147, 836 (1966).

[31] B. Haas, P. Taras, and J. Styczen, Nucl. Phys. A246, 141 (1975).

[32] W. M. Currie, Nucl. Phys. 47, 551 (1963).

[33] G. Brown, A. MacGregor, and R. Middleton, Nucl. Phys. 77, 385 (1966).

[34] P. Blasi, P. R. Maurenzig, R. A. Ricci, N. Taccetti, R. Giacomich, M. Lagonegro, and G. Poiani, Nuovo Cimento 51B, 241 (1967).

[35] D. Bachner, R. Santo, H. H. Duhm, R. Bock, and S. Hinds, Nucl. Phys. A106, 577 (1968).

[36] D. J. Pullen, B. Rosner, and O. Hansen, Phys. Rev. 177, 1568 (1969).

[37] J. C. Legg, D. G. Megli, D. R. Abraham, L. D. Ellsworth, and S. Hechtl, Phys. Rev. 186, 1138 (1969).

[38] J. N. Mo, B. Cujec, R. Dayras, I. M. Szoghy, and M. Toulemonde, Nucl. Phys. A147, 129 (1970).

[39] S. Raman, Nucl. Data B4, 397 (1970).

[40] P. Blasi, M. Mando, P. R. Maurenzig, and N. Taccetti, Nuovo Cimento 4A, 61 (1971).

[41] J. G. Malan, E. Barnard, J. A. M. de Villiers, J. W. Tepel, and P. Van der Merwe, Nucl. Phys. A195, 596 (1972).

[42] G. B. Vingiani, C. Rossi-Alvarez, A. Buscemi, F. Brandolini, and F. Cervellera, Phys. Lett. 40B, 638 (1972).

[43] H. C. Cheung and S. K. Mark, Nucl. Phys. A176, 219 (1971).

[44] C. M. Rosza, R. G. Arnes, B. J. Bruner, S. E. Caldwell, and J. W. Smith, Bull. Am. Phys. Soc. 17, 536 (1972).

[45] A. Kiuru, Z. Phys. 251, 93 (1972).

[46] O. B. Okon, H. Bakhru, M. K. Dewanjee, and I. L. Preiss, Phys. Rev. C 7, 239 (1973).

[47] Z. P. Sawa, J. Blomqvist, and W. Gullholmer, Nucl. Phys. A205, 257 (1973).

[48] J. Britz, J. Chevalier, B. Haas, and J. Styczen, in *Proceedings of the International Conference on Nuclear Physics, Munich, 1973*, edited by J. de Boer and H. J. Mang (North-Holland, Amsterdam/American Elsevier, New York, 1973), Vol. I, p. 210.

[49] S. L. Tabor and R. W. Zurmühle, Phys. Rev. C 10, 35 (1974).

[50] S. L. Tabor, L. K. Fifield, K. G. Young, Jr., and R. W. Zurmühle, Phys. Rev. C 10, 1484 (1974).

[51] B. A. Brown, D. B. Fossan, J. M. McDonald, and K. A. Snover, Phys. Rev. C 9, 1033 (1974).

[52] B. Haas, J. Chevallier, N. Schulz, J. Styczen, and M. Toulemonde, Phys. Rev. C 11, 280 (1975).

[53] B. Haas, J. Chevallier, J. Britz, and J. Styczen, Phys. Rev. C 11, 1179 (1975).

[54] S. V. Jackson, E. A. Henry, and R. A. Meyer, Phys. Rev. C 12, 2094 (1975).

[55] A. K. Dhar, D. R. Kulkarni, and K. H. Bhatt, in *Proceedings of the International Conference on Nuclear Structure and Spectroscopy, 1974*, edited by H. P. Blok and A. E. L. Dieperink (Scholar's Press, Amsterdam, 1974), p. 59.

[56] F. B. Malik and W. Scholz, Phys. Rev. 150, 919 (1966); 153, 1071 (1967); 176, 1355 (1968).

[57] V. S. Shirley, in *Hyperfine Interactions in Excited Nuclei*, Proceedings of a conference in Rehovot, 1970, edited by G. Goldring and R. Kalish (Gordon and Breach, Paris, New York, 1971), Vol. 4, p. 1262.

[58] E. N. Shipley, R. E. Holland, and F. J. Lynch, Phys. Rev. 182, 1165 (1969), and references therein.

[59] M. N. Rao and J. Rapaport, Nucl. Data B3, 37 (1970). (All references to previous experiments are given herein.)

[60] R. N. Horoshko, D. Cline, and P. M. S. Lesser, Nucl. Phys. A149, 562 (1970).

[61] A. S. Goodman and D. J. Donahue, Phys. Rev. C 5, 875 (1972).

[62] H. Gogelein, R. Huber, and C. Signorini, in *Proceedings of the International Conference on Nuclear Physics, Munich, 1973* (see Ref. 48), Vol. I, p. 176.

[63] D. Harrach, in *Proceedings of the International Conference on Nuclear Physics, Munich, 1973* (see Ref. 48, Vol. I, p. 175.

[64] A. R. Poletti, B. A. Brown, D. B. Fossan, P. Gorodetzky, J. J. Kolata, J. W. Olness, and E. K. Warburton, Phys. Rev. C 10, 997 (1974).

[65] R. D. Lawson and J. L. Uretsky, Phys. Rev. 106, 1369 (1957).

[66] N. Auerbach, Phys. Lett. 24B, 260 (1967).

[67] K. Lips and M. T. McEllistrem, Bull. Am. Phys. Soc. 14, 1205 (1969).

[68] E. Osnes and C. S. Warke, Nucl. Phys. A154, 331 (1970).

[69] E. Osnes, in *Proceedings of the Topical Conference on the Structure of $1f_{7/2}$ Nuclei, Padua, Italy, 1971*, edited by R. A. Ricci (Editrice Compositori, Bologna, 1971), p. 79.

[70] W. Scholz and F. B. Malik, Phys. Rev. 147, 836 (1966).

[71] A. K. Dhar, S. B. Khadkikar, D. R. Kulkarni, and K. H. Bhatt, Nucl. Phys. Solid State Phys. (India) 17B, 246 (1974).

[72] A. K. Dhar, S. B. Khadkikar, D. R. Kulkarni, and K. H. Bhatt, in *Proceedings of the International Conference on Nuclear Self-Consistent Fields, Trieste, 1975*, edited by G. Ripka and M. Porneuf (North-Holland, Amsterdam, 1975), p. 83.

[73] A. K. Dhar, D. R. Kulkarni, and K. H. Bhatt, Phys. Lett. 50B, 323 (1974).

[74] S. K. Sharma and K. H. Bhatt, Phys. Rev. Lett. 30, 620 (1973).

[75] A. K. Dhar, D. R. Kulkarni, and K. H. Bhatt, Nucl. Phys. (to be published); A. K. Dhar (unpublished).

[76] A. Dhar and K. Bhatt, Phys. Rev. C 14, 1630 (1976).

[77] W. Gullholmer and Z. P. Sawa, Nucl. Phys. A204, 561 (1973).

[78] Z. P. Sawa, Phys. Scri. 7, 5 (1973).

[79] P. G. Bizzeti, in *Proceedings of the Topical Conference on the Structure of $1f_{7/2}$ Nuclei, Padua, Italy, 1971* (see Ref. 69), p. 393.

[80] R. C. Ritter, P. H. Stelson, F. K. McGowan, and R. L. Robinson, Phys. Rev. 128, 2320 (1962).

[81] I. Y. Krause, Phys. Rev. 129, 1330 (1963).

[82] R. L. Robinson, N. R. Johnson, and G. D. O'Kelley, Nucl. Phys. 47, 506 (1963).

[83] *Nuclear Level Schemes $A = 45$ through $A = 257$* (Academic Press, New York and London, 1973), $A = 51$, Sheet 2.

[84] W. J. Childs, Phys. Rev. 156, 71 (1971).

[85] L. Keszthelyi, I. Demeter, Z. Szoskeflvi-Nagy, L. Varga, and Z. Zamori, Nucl. Phys. A120, 540 (1968).