

$0^- \leftrightarrow 0^+$ beta decay and muon capture in the $A = 16$ nuclei*

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The muon-capture process $\mu^- + {}^{16}\text{O}(0^+) \rightarrow {}^{16}\text{N}^*(0^-) + \nu_\mu$ and the β decay ${}^{16}\text{N}^*(0^-) \rightarrow {}^{16}\text{O}(0^+) + e^- + \bar{\nu}_e$ are analyzed in the elementary particle treatment. Theoretical results are shown to be consistent with experiment and do not support any necessity of a significant upward renormalization of the nucleon form factor g_p in nuclei as suggested by Palfy *et al.* Slight downward renormalization of g_p is consistent with experimental data.

$$\left[\text{NUCLEAR REACTIONS } {}^{16}\text{O}(\mu, \nu_\mu){}^{16}\text{N}^*(0^-); \text{ calculated capture rate. } {}^{16}\text{N}^*(0^-), \right. \\ \left. \text{calculated } \beta\text{-decay rate.} \right]$$

I. INTRODUCTION

Application of the partially conserved axial-vector current (PCAC) hypothesis¹ to the nuclear matrix element of the axial-vector weak current leads to a relationship between the axial-vector form factor $F_A(q^2)$ [or $g_A(q^2)$ for nucleon transitions] and the induced pseudoscalar form factor $F_P(q^2)$ [or $g_P(q^2)$ for nucleon transitions]. This relation provides us with theoretical predictions on the values of $F_P(q^2)$, which are compared with observed values usually obtained from muon capture rates. This serves as a test of PCAC. The contribution of the induced pseudoscalar term in the muon capture rate, however, is typically of order of 10% for allowed transitions. Therefore, the test of PCAC is not a sensitive one.

On the other hand, in the case of the forbidden transitions, in particular, the $0^+ \rightarrow 0^+$ transitions, the capture rate is much more sensitive to $F_P(q^2)$ than in the allowed transitions. This is due to the suppression of usually dominant axial-vector terms. Earlier attempts² to obtain the value of the nucleon induced pseudoscalar form factor $g_P(q^2 \simeq m_\mu^2)$ from the observed rates of $\mu^- + {}^{16}\text{O}(0^+) \rightarrow {}^{16}\text{N}^*(0^-) + \nu_\mu$, Γ_μ , had to rely on various nuclear models because of the lack of the experimental rate, Γ_β , of the corresponding β decay ${}^{16}\text{N}^*(0^-) \rightarrow {}^{16}\text{O}(0^+) + e^- + \bar{\nu}_e$. Palfy *et al.*³ measured the rate of this β decay and obtained a value of $g_P(q^2 \simeq m_\mu^2)$ in more or less model independent way from the observed ratio Γ_μ/Γ_β . The range of the values of $g_P(q^2 \simeq m_\mu^2)$ obtained from this analysis was

$$13 \lesssim |g_P(q^2 \simeq m_\mu^2)| \lesssim 20. \quad (1)$$

These values are considerably larger than the value $|g_P(q^2 \simeq m_\mu^2)| = 9$ predicted by PCAC for the nucleon process, $\mu^- + p \rightarrow n + \nu_\mu$. The discrepancy is even more serious in light of the recent works⁴

which have suggested that $|g_P|$ as well as g_A in the nuclei have tendency to be renormalized *downward*. For example, in the $A = 16$ system, one expects, according to Ref. 4, the effective value of $|g_P|$ in the nuclei to be about $|g_P| \simeq 5 - 6$.

Contrary to the work by Palfy *et al.*, Donnelly and Walecka⁵ have shown that the observed values of Γ_μ and Γ_β can be reasonably well reproduced in the impulse approximation with the use of the unrenormalized values of g_A and g_P within uncertainties of experiment and theory.

In this paper we present an analysis of the muon capture process, $\mu^- + {}^{16}\text{O}(0^+) \rightarrow {}^{16}\text{N}^*(0^-) + \nu_\mu$, and the corresponding β -decay process in the elementary particle treatment⁶ in order to minimize uncertainties due to the use of the impulse approximation. In our analysis, a recourse to the impulse approximation is made only in estimating the ratio of two unknown nuclear form factors. Our conclusion supports the work of Donnelly and Walecka.

II. CALCULATIONS

We start with the most general matrix elements of the vector and axial-vector currents, $V_\alpha^{(+)}(x)$ and $A_\alpha^{(+)}(x)$, for the $0^- \rightarrow 0^+$ β transition

$$\langle {}^{16}\text{O}(0^+) | V_\alpha^{(+)}(0) | {}^{16}\text{N}^*(0^-) \rangle = 0, \quad (2)$$

$$\langle {}^{16}\text{O}(0^+) | A_\alpha^{(+)}(0) | {}^{16}\text{N}^*(0^-) \rangle = F_A(q^2) Q_\alpha \\ + \frac{2M\Delta M}{m_\pi^2} F_P(q^2) q_\alpha;$$

$$q_\alpha = (p_f - p_i)_\alpha, \quad Q_\alpha = (p_i + p_f)_\alpha, \quad (3)$$

where M is the nuclear mass [$M = \frac{1}{2}(M_i + M_f)$] and $\Delta M = M_i - M_f$; p_i and p_f are, respectively, the momenta for the initial (${}^{16}\text{N}^*$) and final (${}^{16}\text{O}$) nuclei. From Eq. (2), we obtain the following β -decay rate

$$\Gamma_\beta \left(= \frac{\ln 2}{t} \right) = \frac{G^2 \cos^2 \theta_C}{2\pi^3} f(\Delta M, Z=8) [F_A(0)]^2 (1+C), \quad (4)$$

where G is the Fermi coupling constant and θ_C is the Cabibbo angle. In Eq. (4),

$$f(\Delta M, Z=8) = \int_{m_e}^{\Delta M} p E (\Delta M - E)^2 F(E, Z) dE \\ = 1.88 \times 10^5 m_e^5, \quad (5)$$

where the Fermi function $F(E, Z)$ contains the usual Coulomb corrections and the numerical value has been obtained in a standard way. The Coulomb correction through the induced term $F_P(q^2)$ which is not included in $F(E, Z)$ is given in the last factor in Eq. (4). It has been demonstrated⁷ that in the allowed transitions, the induced Coulomb correction is of order of $\alpha Z/m_p R$ which is roughly the same order of magnitude as the usual finite size Coulomb correction. However, in the $0^- \rightarrow 0^+$ transition, the contribution of the induced pseudoscalar term becomes, as mentioned already, comparable in magnitude to that of the leading axial-vector term, and hence, the induced Coulomb correction is dramatically enhanced.⁸ The quantity C in Eq. (4) is given by⁸

$$C \simeq \frac{\sqrt{10}\alpha Z}{m_e R} x, \quad x = \frac{\Delta M m_e F_P(0)}{m_\pi^2 F_A(0)}, \quad (6)$$

where R is the nuclear radius.

The matrix elements appropriate for the $0^+ \rightarrow 0^-$ muon-capture process are⁹

$$\langle {}^{16}\text{N}^*(0^-) | V_\alpha^{(-)}(0) | {}^{16}\text{O}(0^+) \rangle = 0, \\ \langle {}^{16}\text{N}^*(0^-) | A_\alpha^{(-)}(0) | {}^{16}\text{O}(0^+) \rangle \\ = \langle {}^{16}\text{O}(0^+) | A_\alpha^{(+)}(0) | {}^{16}\text{N}^*(0^-) \rangle^* (-1)^{\delta_{\alpha 4}} \\ = F_A(q^2) Q_\alpha + \frac{2M\Delta M}{m_\pi^2} F_P(q^2) q_\alpha, \quad (7)$$

where we have used Eq. (2). We note that q_α in Eq. (7) is, as defined in Eq. (3), $[p({}^{16}\text{O}) - p({}^{16}\text{N}^*)]_\alpha$. If we redefine q_α for the muon-capture process as $[p({}^{16}\text{N}^*) - p({}^{16}\text{O})]_\alpha$, then Eq. (7) becomes

$$\langle {}^{16}\text{N}^*(0^-) | V_\alpha^{(-)}(0) | {}^{16}\text{O}(0^+) \rangle = 0, \\ \langle {}^{16}\text{N}^*(0^-) | A_\alpha^{(-)}(0) | {}^{16}\text{O}(0^+) \rangle = F_A(q^2) Q_\alpha \\ + \frac{2M\Delta M}{m_\pi^2} F_P(q^2) q_\alpha \\ q_\alpha = [p({}^{16}\text{N}^*) - p({}^{16}\text{O})]_\alpha \quad (8)$$

implying a sign change in the F_P term. We note that $F_A(q^2)$ and $F_P(q^2)$ in Eq. (8) are the same form factors that appear in the β -decay matrix elements in Eq. (2).

The muon-capture rate calculated from Eq. (8) is

$$\Gamma_\mu = \frac{G^2 \cos^2 \theta_C}{2\pi^2} \left(\frac{\alpha Z m_\mu}{1+m_\mu/M} \right)^3 \frac{E_\nu^2}{1+m_\mu/M} C_\mu [F_A(0)]^2 \\ \times [\mathcal{F}_A(q^2=0.8m_\mu^2)]^2 \left(1 - \frac{a m_\mu}{2M} \right)^2, \quad (9)$$

$$a = \frac{2M\Delta M}{m_\pi^2} \frac{F_P(q^2=0.8m_\mu^2)}{F_A(q^2=0.8m_\mu^2)}, \\ C_\mu = 0.84, \quad (10)$$

$$E_\nu = 94.9 \text{ MeV},$$

$$\mathcal{F}_A(q^2) \equiv \frac{F_A(q^2)}{F_A(0)}.$$

In Eq. (10), the factor C_μ is a correction factor⁶ in the muon wave function arising from the nonpoint charge distribution of the ${}^{16}\text{O}$ nucleus.

In the expressions (4) and (9), we have two unknown form factors $F_A(0)$ and $F_P(0)$, and their q^2 dependence. When the ratio Γ_μ/Γ_β is considered, the $F_A(0)$ is canceled, leaving $\mathcal{F}_A(q^2=0.8m_\mu^2)$ and F_P/F_A as unknown.

In the elementary particle treatment of muon-capture processes in nuclei, it is customary to obtain the q^2 dependence of $F_A(q^2)$, $\mathcal{F}_A(q^2)$, empirically from data on the corresponding inelastic electron scattering. This is done with the help of the impulse-approximation-based relation $\mathcal{F}_A(q^2) \simeq \mathcal{F}_M(q^2)$, where $\mathcal{F}_M(q^2)$ is the weak magnetism form factor normalized to unity at $q^2=0$. The $\mathcal{F}_M(q^2)$ is further related, by the conserved vector current hypothesis, to a measurable electromagnetic form factor. Unfortunately, this method fails to apply to the present case because the $0^- \rightarrow 0^+$ electron scattering process is forbidden, as the first equation in Eq. (2) suggests.

In order to obtain an estimate of the value of $\mathcal{F}_A(q^2=0.8m_\mu^2)$, we use the following result which can be verified by the impulse approximation: when $\mathcal{F}_A(q^2)$ is written as

$$\mathcal{F}_A(q^2) = \frac{1}{(1+q^2/M_A^2)^2} \quad \text{for } |q^2| \lesssim m_\pi^2 \quad (11)$$

the transition mean square radius, $\langle r^2 \rangle = 12/M_A^2$ depends mostly on the sizes of the nuclei involved and is insensitive to other nuclear properties such as spin and parity. This is also confirmed experimentally. For example, the experimental values of M_A^2 obtained, as described above, from the corresponding electron scattering data are^{6,10}

$$M_A^2 = \begin{cases} 2.6m_\pi^2 & \text{for } 1^+ \leftrightarrow 0^+ \text{ in } A=12 \text{ system, (12a)} \\ 2.2m_\pi^2 & \text{for } 2^- \leftrightarrow 0^+ \text{ in } A=16 \text{ system. (12b)} \end{cases}$$

In spite of quite different nature of the two transi-

tions, the values of M_A^2 are about the same. In fact, the slight decrease of M_A^2 from the $A=12$ to $A=16$ systems is consistent with the increase of the sizes of the nuclei. More specifically, the ratio $[M_A(A=16)/M_A(A=12)]=0.92$ obtained from Eq. (12) agrees with the ratio of the radii $[R(A=12)/R(A=16)]=(12/16)^{1/3}=0.91$ (since $R \sim 1/M_A$). Therefore, it is justified to use the experimental value of M_A^2 given in Eq. (12b) for the $0^- \rightarrow 0^+$ transition in the $A=16$ system to obtain

$$\mathcal{F}_A(q^2=0.8m_\mu^2)=0.69. \quad (13)$$

Taking the ratio of Eqs. (4) and (9) and using Eq. (13), we obtain

$$\frac{\Gamma_\mu}{\Gamma_\beta}=0.44 \times 10^3 \frac{1}{1+C} \left(1 - \frac{am_\mu}{2M}\right)^2. \quad (14)$$

The above ratio now has the two unknown parameters, C [Eq. (6)] and a [Eq. (10)] which depend on the values of the ratio $F_P(q^2)/F_A(q^2)$ at $|q^2| \simeq m_\sigma^2 \simeq 0$ (β decay) and at $q^2=0.8m_\mu^2$ (μ capture). The ratio of $F_P(q^2)/F_A(q^2)$ at $q^2 \simeq 0$ has been estimated in Ref. 11 to be

$$\begin{aligned} \frac{F_P(0)}{F_A(0)} &\simeq \left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{1 \pm \lambda} \text{ for } \beta^\mp \text{ decay,} \\ \lambda &= \frac{3}{2} \frac{\Lambda \alpha Z}{R \Delta M}, \end{aligned} \quad (15)$$

where $\Lambda=1$ to 2 depending on the detailed models used. When the energy difference between states differing by a neutron-proton substitution is dominated by the Coulomb energy difference (Pursey's estimate¹²), $\Lambda \simeq 2$ is expected. On the other hand, a partial cancellation of the electrostatic force by the nuclear force effects based on the semiempirical formula for the stable nuclear masses (Ahrens-Feenberg estimate¹³) yields $\Lambda \simeq 1$. Various examples¹⁴ tend to support the value of Λ close to 2.

In order to estimate the ratio $F_P(q^2)/F_A(q^2)$ at $q^2=0.8m_\mu^2$, we have to generalize the method of Ref. 11 to finite values of q^2 . We write down the impulse approximation expression for the matrix element of the axial-vector current

$$\begin{aligned} \langle {}^1\text{O}(0^+) | A_\alpha^{(+)}(0) | {}^1\text{N}^*(0^-) \rangle \\ = g_A(q^2) (-\langle \vec{\sigma} e^{i\vec{q}\cdot\vec{r}} \rangle, i\langle \gamma_5 e^{i\vec{q}\cdot\vec{r}} \rangle) \\ - \frac{g_P(q^2)}{m_\mu} \langle \gamma_4 \gamma_5 e^{i\vec{q}\cdot\vec{r}} \rangle (\vec{q}, iq_0), \end{aligned} \quad (16)$$

where we have used the definitions

$$\begin{aligned} A_\alpha &= (\vec{A}, iA_0), \\ \langle \mathcal{O} e^{i\vec{q}\cdot\vec{r}} \rangle &= \langle \psi_f | \sum_{\alpha=1}^A \mathcal{O}^{(a)} \tau_\alpha^{(a)} e^{i\vec{q}\cdot\vec{r}^{(a)}} | \psi_i \rangle. \end{aligned} \quad (17)$$

The PCAC prediction for $g_P(q^2)$ is

$$\begin{aligned} g_P(q^2) &\simeq -g_A(q^2) \frac{1}{1+q^2/m_\pi^2} \frac{2m_\rho m_\mu}{m_\pi^2} \\ &\simeq -9 \text{ for } q^2=0.8m_\mu^2. \end{aligned} \quad (18)$$

Expanding the exponential factor and keeping the first nonvanishing terms in the expansions, we obtain

$$\begin{aligned} \langle \vec{\sigma} e^{i\vec{q}\cdot\vec{r}} \rangle &\simeq \frac{1}{3} \vec{q} \langle i\vec{\sigma} \cdot \vec{r} \rangle \mathcal{F}(q^2), \\ \langle \gamma_5 e^{i\vec{q}\cdot\vec{r}} \rangle &\simeq \langle \gamma_5 \rangle \mathcal{F}(q^2), \\ \langle \gamma_4 \gamma_5 e^{i\vec{q}\cdot\vec{r}} \rangle &\simeq \frac{1}{2m_\rho} \langle \vec{\sigma} e^{i\vec{q}\cdot\vec{r}} \rangle \cdot \vec{q} \\ &\simeq \frac{1}{3} \frac{1}{2m_\rho} \vec{q}^2 \langle i\vec{\sigma} \cdot \vec{r} \rangle \mathcal{F}(q^2), \end{aligned} \quad (19)$$

where we have assumed, as we have previously justified, that the q^2 dependence of the above three matrix elements are about the same and denoted them by $\mathcal{F}(q^2)$ with $\mathcal{F}(q^2=0)=1$. A comparison of Eqs. (16) and (19) with Eq. (8) gives

$$\begin{aligned} F_P(q^2) \frac{\Delta M}{m_\pi^2} &\simeq -\frac{1}{3} g_A(q^2) \langle i\vec{\sigma} \cdot \vec{r} \rangle \\ &\times \left[1 + \vec{q}^2 \frac{1}{2m_\rho m_\mu} \frac{g_P(q^2)}{g_A(q^2)} \right] \mathcal{F}(q^2), \\ F_A(q^2) - \left(\frac{\Delta M}{m_\pi}\right)^2 F_P(q^2) \\ &\simeq \langle \gamma_5 \rangle g_A(q^2) \left[1 + \frac{\Delta M}{3} \frac{\langle i\vec{\sigma} \cdot \vec{r} \rangle}{\langle \gamma_5 \rangle} \vec{q}^2 \frac{1}{2m_\rho m_\mu} \right. \\ &\quad \left. \times \frac{g_P(q^2)}{g_A(q^2)} \right] \mathcal{F}(q^2). \end{aligned} \quad (20)$$

Solving Eq. (20) for $F_P(q^2)/F_A(q^2)$, we obtain

$$\frac{F_P(q^2)}{F_A(q^2)} = \left(\frac{m_\pi}{\Delta M}\right)^2 \frac{1}{1 \pm \lambda} \left[1 + \vec{q}^2 \frac{1}{2m_\rho m_\mu} \frac{g_P(q^2)}{g_A(q^2)} \right], \quad (21)$$

where we have used the well-known relation¹⁴

$$\frac{\langle \gamma_5 \rangle}{\langle i\vec{\sigma} \cdot \vec{r} \rangle} = \mp \frac{\Lambda \alpha Z}{2R} \text{ for } \beta^\mp \text{ decay.} \quad (22)$$

Equation (21) is a generalization of Eq. (15) to the case of $|q^2| \lesssim m_\pi^2$.

If we ignore possible renormalizations of the nucleon form factors $g_P(q^2)$ and $g_A(q^2)$ in nuclei, the ratio g_P/g_A is given by Eq. (18) and Eq. (21) reduces to

$$\frac{F_P(q^2)}{F_A(q^2)} \simeq \frac{F_P(0)}{F_A(0)} \frac{1}{1+q^2/m_\pi^2} \text{ for } |q^2| \lesssim m_\pi^2, \quad (23)$$

where we have used $q^2 \simeq \vec{q}^2$ which is valid for muon-capture processes. In the muon capture, the positive sign in Eq. (15) and (21) should be used.

It should be remarked that although the ratio

$F_P(q^2)/F_A(q^2)$ has been calculated using the impulse approximation, our estimate of the ratio should be valid better than the limit of the impulse approximation indicates. Possible meson-exchange contributions are canceled in the ratios to a large extent.

The results of numerical calculations of the ratio Γ_μ/Γ_β are given in Table I. In view of the fact that the numerical values of F_P/F_A , and therefore the ratio Γ_μ/Γ_β , are sensitive to the value of Λ , theoretical values are given for three values of Λ . Experimental values of $(\Gamma_\mu/\Gamma_\beta)_{\text{exp}}$ are also listed in the last column of Table I.

III. DISCUSSION

The four values of $(\Gamma_\mu/\Gamma_\beta)_{\text{exp}}$ listed in Table I are based on the following measurements:

$$(\Gamma_\beta)_{\text{exp}} = 0.43 \pm 0.10 \text{ sec}^{-1}, \text{ Ref. 3}$$

$$(\Gamma_\mu)_{\text{exp}} = \begin{cases} 850_{-60}^{+140} \text{ sec}^{-1}, & \text{Ref. 15,} \\ 1100 \pm 200 \text{ sec}^{-1}, & \text{Ref. 16,} \\ 1560 \pm 170 \text{ sec}^{-1}, & \text{Ref. 17,} \\ 1600 \pm 200 \text{ sec}^{-1}, & \text{Ref. 18.} \end{cases}$$

First, our result $(\Gamma_\mu/\Gamma_\beta)_{\text{theor}} = 1.9 \times 10^3$ for $\Lambda=1.5$ is in agreement with the result obtained by Donnelly and Walecka,⁵ $(\Gamma_\mu/\Gamma_\beta)_{\text{theor}} = (1.9 \sim 2.0) \times 10^3$. Due to a wide range of the ob-

TABLE I. Theoretical and experimental values of $(\Gamma_\mu/\Gamma_\beta)$.

Λ	C	$a(m_\mu/M)$	$(\Gamma_\mu/\Gamma_\beta)_{\text{theor}} \times 10^{-3}$	$(\Gamma_\mu/\Gamma_\beta)_{\text{exp}} \times 10^{-3}$
2	0.51	6.5	1.4	1.98
1.5	0.60	7.4	1.9	2.56
				3.63
1	0.70	8.7	2.6	3.72

served values of Γ_μ , it is difficult at the present time to draw a definite conclusion. However, our theoretical predictions are consistent with experiment. This implies that the present analysis as well as that of Donnelly and Walecka do not indicate any necessity of an upward renormalization of $|g_P|$ as reported in Ref. 3. The downward renormalization of the g_A and g_P as suggested by the recent works⁴ tends to increase the ratio $(\Gamma_\mu/\Gamma_\beta)_{\text{theor}}$. For example, if we use the results⁴

$$\left[\frac{g_P(q^2 = 0.8m_\mu^2)}{g_A(q^2 = 0.8m_\mu^2)} \right]_{\text{eff}} = 0.8 \left[\frac{g_P(q^2 = 0.8m_\mu^2)}{g_A(q^2 = 0.8m_\mu^2)} \right],$$

the values of $(\Gamma_\mu/\Gamma_\beta)_{\text{theor}}$ would increase by 20%, further improving the agreement between theory and experiment. If we take the choice $\Lambda=2$ seriously, our analysis favors the lower experimental value of Ref. 15 over the higher values such as in Refs. 17 and 18.

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