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Pion production and absorption in nuclear reactions. I. The vertex function*

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We have performed a model calculation of the pion-nucleon vertex function for the case in which one nucleon is allowed to go far off its mass shell. We discuss the relevance of this vertex function for the calculation of pion production and absorption in nuclear reactions, such as (π^+,p), (p,π^+), and for the pionic disintegration of the deuteron. The model used is based upon an approximation to an exact equation for the vertex function derived from a field-theoretic model with pseudoscalar coupling. Our calculations indicate a strong dependence of the vertex function on the invariant mass of the off-shell nucleon. The results are dominated by the presence of the 1470 MeV, P_{11} resonance.

[NUCLEAR REACTIONS Pion production and absorption, calculation of pion-nucleon vertex function.]

I. INTRODUCTION

In this work we direct our attention to the calculation of the pion-nucleon vertex function and indicate how this function may play an important role in the calculation of various nuclear reactions involving pion production and absorption.

As has been noted in the literature,[1,2] there are many ambiguities in a nonrelativistic analysis of such reactions and we believe that the use of a covariant theory will resolve many of the problems associated with the nonrelativistic reduction of the vertex function.

In the covariant analysis the need for the π-N vertex function, with various of the particles off their mass shells, is manifest. In Sec. II we comment on some features of such covariant calculations, indicating the various diagrams whose evaluation requires knowledge of the vertex function. In Sec. III we indicate how our model calculation is performed, relegating many of the details to the Appendixes. Section III also contains a discussion of the results of our calculation. Finally, in Sec. IV, we summarize our results and attempt to place our calculation in perspective. In that section we also present a simple parametrization of the vertex function which should be useful for future calculations.

II. COVARIANT CALCULATION OF THE PION PRODUCTION AND ABSORPTION REACTIONS

Recently there has been interest in understanding pion production and absorption experiments involving nuclear targets. The simplest of such targets is the deuteron, and the reaction $\pi^+ + d \to p+p$ has received much attention over the years.[3] In Fig. 1 we indicate two Feynman diagrams which are expected to be important for this reaction. The evaluation of such diagrams requires the knowledge of the deuteron wave func-

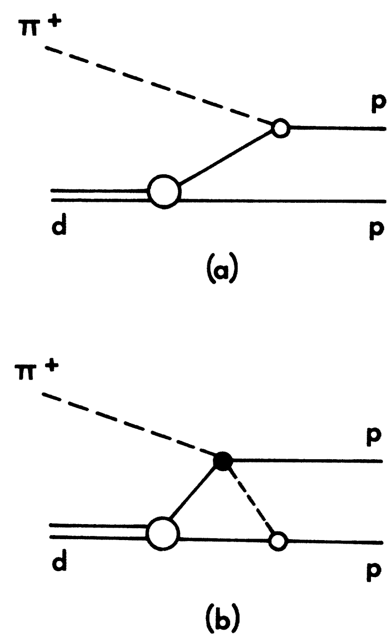

FIG. 1. (a) Diagrammatic representation of the simplest process in the pionic disintegration of the deuteron. The small circle denotes the π-N vertex function and the large circle is the deuteron vertex function. (b) Diagrammatic representation of a process leading to pionic disintegration of the deuteron where the filled circle denotes a π-N scattering amplitude.

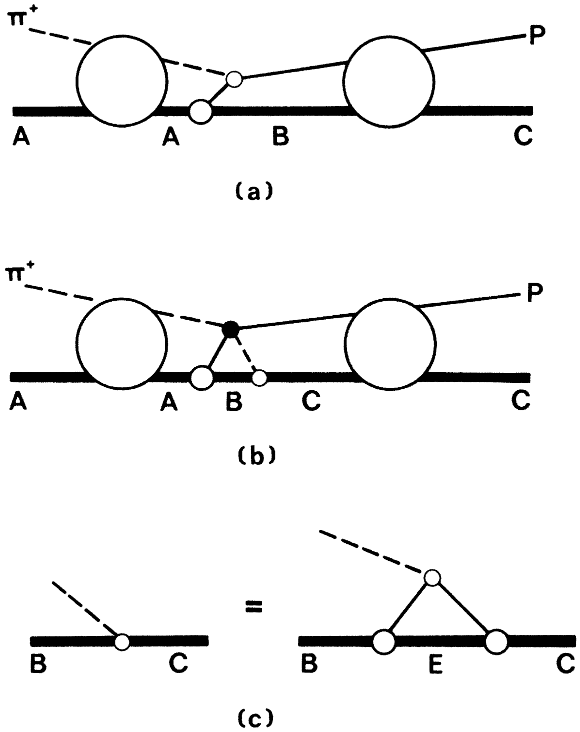

FIG. 2. (a) Diagrammatic representation of a process contributing to the reaction $A(\pi^+,p)C$. The large circles denote π-nucleus or proton-nucleus initial or final state interactions and the heavy lines denote various nuclei. (b) Another process contributing to the reaction $A(\pi^+,p)C$ where the filled circle denotes a π-N scattering amplitude. (c) Representation of a pion-nucleus vertex function and a model which may be used in the calculation of that quantity.

tion, (or alternatively the amplitude $d \to n+p$). Further, in both diagrams (a) and (b), one needs to know the π-N vertex function. The calculation of diagram (b) also requires the knowledge of the π-N scattering amplitude (shown as a black dot in the figure).

Similar information is needed for the analysis of the (π^+,p) reaction, however, in this case it is clearly necessary to include the initial state interaction of the pion and the final state interaction of the outgoing protons. A model for the (π^+,p) reaction is indicated in Fig. 2.[4,5] In Figs. 2(a) and 2(b) the large circles denote optical T matrices for the π-nucleus or N-nucleus interaction. The heavy lines denote various nuclei which are distinguished by capital letters, $A,B,C,\ldots,$ and the small open circles are vertex functions. Again, as in the case of the pion absorption on the deuteron, we need to know the π-N vertex function. The vertex function for $A \to N+B$ is readily expressed in terms of nuclear wave functions. For the evaluation of the process depicted in Fig. 2(b),[6] we need the π-N scattering amplitude (black dot), the vertex function for $A\to N+B$, and the amplitude for $\pi+B\to C$. The latter amplitude may be approximated as in Fig. 2(c); this approximation again requires the knowledge of the π-N vertex function.

In this work we will provide a model calculation of the π-N vertex function. We will consider the amplitude for an off-mass-shell nucleon of four-momentum p to become a nucleon of momentum p' and a pion of momentum $p-p'$. While it would be most desirable to have knowledge of the vertex function in the case that the final nucleon and pion are off-mass shell, we will direct our attention to the simpler vertex function which has $p'^2=-m_N{}^2$ and $(p-p')^2=-\mu^2$, m_N and μ being the nucleon and pion masses, respectively. This particular vertex function appears in calculation of the stripping mechanism Fig. 1(a). Even if initial and final state interactions are included, as in Fig. 2(a), the kinematic situation is such that the vertex function considered here is one needed for the evaluation of this diagram.

In the process represented in Fig. 1(b), however, one needs the vertex function for an off-mass-shell pion and (nearly) on-mass-shell nucleons. This function has been discussed in the literature[7] and we review some properties of this function in Sec. IV.

In the next section we discuss a model for calculating the vertex function in the case one nucleon is off its mass shell and the other nucleon and the pion are on their mass shells. As noted above, this vertex function is needed when one wishes to calculate the "stripping mechanism", Fig. 1(a). Further, we remark that this same vertex function will play an important role in calculations of (real or virtual) pion photoproduction from nucleons. Application of this vertex function to the description of pion photoproduction will be discussed elsewhere.

III. CALCULATION OF THE VERTEX FUNCTION

In Appendix A it is shown that the vertex function for the emission of a pion of momentum p-p' by a nucleon of momentum p, is given by

$$\Gamma(p,p')=1+\Gamma_1(p,p')+\Gamma_2(p,p'), \tag{3.1}$$

where

$$\Gamma_1(p,p')=-i\int\frac{d^4k}{(2\pi)^4}\mathcal{G}(k)G(p-k) \times M^{1/2}(p-k,k;p',p-p') \tag{3.2}$$

and

$$\Gamma_2(p,p') = 3ig_0{}^2 \int \frac{d^4k}{(2\pi)^4} \mathcal{G}(k)G(p-k)\gamma_5\Gamma(p-k,p) \times G(p)\gamma_5\Gamma(p,p'). \quad (3.3)$$

As remarked previously, the function $\Gamma(p,p')$ has been studied extensively in the case the nucleons are on their mass shells ($p'^2 = -m_N{}^2$ and $p^2 = -m_N{}^2$). Matrix elements of $\gamma_5\Gamma(p,p')$ taken between nucleon spinors yield a function which is usually called "the vertex function." This function depends upon the single variable $\tau = -(p-p')^2$ and is denoted as $f_2(\tau)$ in this work. [See Eqs. (3.4) and (3.5) for a precise definition.] A parametrization of $f_2(\tau)$ was obtained in Ref. 7—see Eq. (4.8).

In this work we will consider the case in which $p'^2 = -m_N{}^2$ and $(p-p')^2 = -\mu^2$. Again, with two particles on their mass shells, one may consider certain spinor matrix elements of $\gamma_5\Gamma(p,p')$ which are proportional to a scalar function of a single variable. In this case it is convenient to use the variable $\sqrt{s}$, where $s = -p^2$. The scalar function defined in this manner could also be called "a vertex function." It is denoted as $F(\sqrt{s},\mu^2)$ and given a more precise definition in Eqs. (3.4) and (3.6).

In this work we will also discuss certain matrix elements of $\gamma_5\Gamma(p,p')$ with the restriction that only a single nucleon be on its mass shell, e.g., $p'^2 = -m_N{}^2$. In this case we define a "vertex function" that is a scalar function of *two* variables. These may be chosen as the variables $\sqrt{s}$ and τ defined above. The resulting function may be written as $F(\sqrt{s},\tau)$. [See Eqs. (3.4) and (3.7).] While we use the same terminology (vertex function) for $f_2(\tau)$, $F(\sqrt{s},\mu^2)$, $F(\sqrt{s},\tau)$, and for the matrix $\Gamma(p,p')$, this should not result in confusion as the reference will be clear from the context in which it appears.

We may make the foregoing discussion more precise by defining the general matrix product of the nucleon Green's functions $G(p)$ and $G(p')$ with the (renormalized) vertex function $\Gamma(p,p')$. We define a function $\mathfrak{F}$:

$$iG(p)\gamma_5\Gamma(p,p')G(p') = iG(p)\gamma_5 G(p')\mathfrak{F}(p^2,(p-p')^2,p'^2). \quad (3.4)$$

[We will in the course of this work ignore the negative frequency parts of nucleon Green's functions. The form taken by Eq. (3.4) is a consequence of this approximation.] The various functions described above may be obtained from the general function $\mathfrak{F}$:

$$f_2(\tau) = \mathfrak{F}(-m_N{}^2, -\tau, -m_N{}^2), \quad (3.5)$$

$$F(\sqrt{s},\mu^2) = \mathfrak{F}(-s, -\mu^2, -m_N{}^2), \quad (3.6)$$

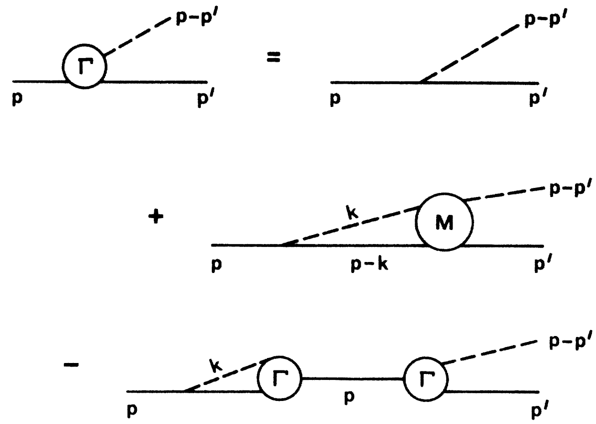

FIG. 3. Representation of the nonlinear equation determining the vertex function. [See Eqs. (3.1)–(3.3).] The quantity M is a pion-nucleon (invariant) scattering amplitude.

$$F(\sqrt{s},\tau) = \mathfrak{F}(-s, -\tau, -m_N{}^2). \quad (3.7)$$

We remark that the renormalization program leads to the constraint

$$\mathfrak{F}(-m_N{}^2, -\mu^2, -m_N{}^2) = 1. \quad (3.8)$$

Equation (3.1) is illustrated in Fig. 3. Here $\mathcal{G}(k)$ and $G(p-k)$ are meson and nucleon propagators and $M^{1/2}$ is a pion-nucleon scattering amplitude in the isospin $\frac{1}{2}$ channel. The term denoted as $\Gamma_2(p,p')$ serves to remove certain contributions contained in $\Gamma_1(p,p')$ which correspond to mass operator insertions on the nucleon line of momentum p. (See Appendixes B and C.) We find that the inclusion of $\Gamma_2(p,p')$ in the calculations leads to only small modifications of the results obtained when Γ_2 was neglected.

We now concentrate on the approximations necessary in order to calculate $\Gamma(p,p')$. We make the assumption that the main contribution to the k integral comes from the region where the nucleon of momentum $p-k$ is close to its mass shell. Thus in Eq. (3.2) we replace $G(p-k)$ by

$$\hat{G}(p-k) = 2i\pi \frac{m_N}{E_{\vec{p}-\vec{k}}} \delta(p^0 - k^0 - E_{\vec{p}-\vec{k}})\Lambda^+(\vec{p}-\vec{k}), \quad (3.9)$$

where

$$\Lambda^+(\vec{p}-\vec{k}) = \sum_s u^{(s)}(\vec{p}-\vec{k})\bar{u}^{(s)}(\vec{p}-\vec{k}) \quad (3.10)$$

is the projection operator for positive energy spinors. We also multiply $\Gamma_1(p,p')$ on the right by $\Lambda^+(\vec{p}')$ since we will ultimately be interested in taking matrix elements of Γ between positive energy spinors. At this stage we have

$$\Gamma_1(p,p')=\int\frac{d\vec{k}}{(2\pi)^3}\left(\frac{m_N}{E_{\vec{p}-\vec{k}}}\right)\frac{1}{\vec{k}^2+\mu^2-(p^0-E_{\vec{p}-\vec{k}})^2-i\epsilon}$$
$$\times\sum_{ss'}u^{(s)}(\vec{p}-\vec{k})\bar{u}^{(s)}(\vec{p}-\vec{k})M^{1/2}(p-k,k;p',p-p')u^{(s')}(\vec{p}')\bar{u}^{(s')}(\vec{p}'),\tag{3.11}$$

where $k^0=p^0-E_{\vec{p}-\vec{k}}$ for the evaluation of $M^{1/2}$.

We now recall that with the nucleons of momentum $p-k$ and p' on their mass shells, we can relate the invariant amplitude $M^{1/2}$ to a two-body T matrix[8]

$$[(2\pi)^6\,{}_{\rm NR}\langle\vec{k},\vec{p}-\vec{k},s\,|\,T_{1/2}(p^0)\,|\,\vec{p}-\vec{p}',\vec{p}',s'\rangle_{\rm NR}]$$
$$=(2\pi)^3[{}_{\rm NR}\langle\vec{k},s\,|\,T_{1/2}(p^0)\,|\,\vec{p}',s'\rangle_{\rm NR}]$$
$$=R^{1/2}(\vec{k},\vec{p}-\vec{k})[\bar{u}^{(s)}(\vec{p}-\vec{k})M^{1/2}(p-k,k;p',p-p')u^{(s')}(p')]R^{1/2}(\vec{p}-\vec{p}',\vec{p}'),\tag{3.12}$$

where

$$k^0=p^0-E_{\vec{p}-\vec{k}},\quad p'^0=E_{\vec{p}'},\ \text{etc., and } R^{1/2}(\vec{k},\vec{p}-\vec{k})=\left(\frac{m_N}{2\omega_{\vec{k}}E_{\vec{p}-\vec{k}}}\right)^{1/2}.\tag{3.13}$$

The notation NR refers to the normalization convention

$${}_{\rm NR}\langle\vec{k}\,|\,\vec{k}'\rangle_{\rm NR}=\delta(\vec{k}-\vec{k}').\tag{3.14}$$

The extra factor of $(2\pi)^3$ in Eq. (3.12) when compared with Eq. (2.11) of Ref. 8 is due to a different choice of the normalization of the invariant matrix made in this work.

As for the T matrix, we make use of a separable model introduced previously.[9] In the channel with $l=1$, $T=\frac{1}{2}, J=\frac{1}{2}$, we have

$$\langle\vec{k},s\,|\,T_{1/2}(E_s)\,|\,\vec{p}',s'\rangle=4\pi\sum_m C_{m-s\,s\,m}^{1\ \frac{1}{2}\ \frac{1}{2}}C_{m-s'\,s'\,m}^{1\ \frac{1}{2}\ \frac{1}{2}}Y_{1,m-s}(\hat{k})\,\lambda_{11}^1 v_{11}^1(|\vec{k}|)v_{11}^1(|\vec{p}'|)Y^*_{1,m-s'}(\hat{p}')/D_{11}^1(p^0),\tag{3.15}$$

where the $v_{11}^1(|\vec{k}|)$ and $D_{11}^1(p^0)$ have been determined from an inverse scattering problem.[9] We choose a model in which the $v_{11}^1(|\vec{k}|)$ is sufficiently small for large $|\vec{k}|$ such that we may further write

$$\frac{1}{\vec{k}^2+\mu^2-(p^0-E_{\vec{p}-\vec{k}})^2-i\epsilon}\simeq-\frac{1}{2\omega_{\vec{k}}}\left[\frac{1}{p^0-(\omega_{\vec{k}}-E_{\vec{p}-\vec{k}})+i\epsilon}\right].\tag{3.16}$$

Finally choosing a Lorentz frame where $\vec{p}=0$, we have

$$\Gamma_1(p^0,\vec{p}')=\int\frac{k^2dk}{(2\pi)^3}\left(\frac{E_{\vec{p}'}\,\omega_{\vec{p}'}}{E_{\vec{k}}\omega_{\vec{k}}}\right)^{1/2}\frac{4\pi}{p^0-(\omega_{\vec{k}}+E_{\vec{k}})+i\epsilon}\lambda_{11}^1v_{11}^1(|\vec{k}|)v_{11}^1(|\vec{p}'|)$$
$$\times\sum_{ss'm}C_{m-s\,s\,m}^{1\ \frac{1}{2}\ \frac{1}{2}}C_{m-s'\,s'\,m}^{1\ \frac{1}{2}\ \frac{1}{2}}\frac{1}{D_{11}^1(p^0)}\int d\Omega_{\vec{k}}u^{(s)}(-\vec{k})Y_{1,m-s}(\hat{k})Y^*_{1,m-s'}(\hat{p}')\bar{u}^{(s')}(\hat{p}'),\tag{3.17}$$

from which we obtain

$$i\bar{u}^{(s'')}(\vec{0})\gamma_5\Gamma_1(p^0,\vec{p}')u^{(s''')}(\vec{p}')$$
$$=-\frac{1}{\pi^{3/2}}\int\frac{k^3dk}{[2m_N(E_{\vec{k}}+m_N)]^{1/2}}\frac{1}{p^0-(E_{\vec{k}}+\omega_{\vec{k}})+i\epsilon}\left(\frac{E_{\vec{p}'}\,\omega_{\vec{p}'}}{E_{\vec{k}}\omega_{\vec{k}}}\right)^{1/2}\frac{\lambda_{11}^1v_{11}^1(|\vec{k}|)v_{11}^1(|\vec{p}'|)}{D_{11}^1(p^0)}C_{s''-s'''\,s'''\,s''}^{1\ \frac{1}{2}\ \frac{1}{2}}Y_{1,s''-s'''}(\hat{p}')\tag{3.18}$$

and

$$i\bar{u}^{(s)}(\vec{0})\gamma_5[1+\Gamma_1(p_0,\vec{p}')]u^{(s')}(\vec{p}')=\frac{(2\pi)^{1/2}|\vec{p}'|}{[2m_N(E_{\vec{p}'}+m_N)]^{1/2}}C_{s-s'\,s'\,s}^{1\ \frac{1}{2}\ \frac{1}{2}}Y_{1,s-s'}(\hat{p}')$$
$$\times\left[1-\frac{\sqrt{2}}{2\pi^2}\int k^2dk\,\frac{|\vec{k}|}{|\vec{p}'|}\left(\frac{E_{\vec{p}'}+m_N}{E_{\vec{k}}+m_N}\right)^{1/2}\frac{1}{p^0-(E_{\vec{k}}+\omega_{\vec{k}})+i\epsilon}\right.$$
$$\left.\times\left(\frac{E_{\vec{p}'}\,\omega_{\vec{p}'}}{E_{\vec{k}}\omega_{\vec{k}}}\right)^{1/2}\frac{\lambda_{11}^1v_{11}^1(|\vec{k}|)v_{11}^1(|\vec{p}|)}{D_{11}^1(p^0)}\right]\tag{3.19}$$
$$\equiv i\bar{u}^{(s)}(\vec{0})\gamma_5u^{(s')}(\vec{p}')y^{(0)}(p^0,|\vec{p}'|).\tag{3.20}$$

We wish to introduce the usual renormalization procedure which requires evaluating $y^{(0)}$ at the unphysical point where all particles are on their mass shells. The value at that point cannot be obtained directly from the expression for $y^{(0)}$, Eq. (3.19), since $|\vec{p}'|$ would be imaginary, and we are assuming that we only know $v_{11}^{1}(|\vec{p}'|)$ for real $|\vec{p}'|$. Indeed the value of $|\vec{p}'|$, in the case of the final nucleon and meson are on their mass shells, is denoted as $|\vec{p}_c'|$ and is given by

$$|\vec{p}_c'| = \frac{1}{2p^0}[(p^0)^4 - 2(p^0)^2(m_N{}^2 + \mu^2) + (m_N{}^2 - \mu^2)^2]^{1/2}, \tag{3.21}$$

which is imaginary if $m_N - \mu < p^0 < m_N + \mu$.

We choose to continue $y^{(0)}(p^0, |\vec{p}_c'|)$ to the renormalization point via a dispersion relation in p^0. In the region of the cut, that is for $p^0 > m_N + \mu$, we define

$$\mathrm{disc}h(p^0) = \mathrm{disc}y^{(0)}(p^0, |\vec{p}_c'|) \tag{3.22}$$

and write a dispersion relation for $h(p^0)$,

$$h(p^0) = \frac{1}{2\pi}\int_{m_N+\mu}^{\infty} \frac{\mathrm{disc}h(p^{0\prime})dp^{0\prime}}{p^{0\prime} - p^0 - i\epsilon}, \tag{3.23}$$

$$= \frac{1}{2\pi}\int_{m_N+\mu}^{\infty} \frac{\mathrm{disc}y^{(0)}(p^{0\prime}, |\vec{p}_c'|)dp^{0\prime}}{p^{0\prime} - p^0 - i\epsilon}. \tag{3.24}$$

[The quantity $|\vec{p}_c'|$ defined by Eq. (3.21) is real along the cut.] We further define,

$$h(m_N) = 1/Z_1^{(0)} \tag{3.25}$$

and

$$s^{(0)}(p^0, |\vec{p}_c'|) = Z_1^{(0)} h(p^0), \tag{3.26}$$

where we have made use of the fact that $|\vec{p}_c'|$ is a function of p^0 [see Eq. (3.21)].

Now we note that from Eqs. (3.25)–(3.26),

$$s^{(0)}(p^0, |\vec{p}_c'|)|_{p^0 = m_N} = 1. \tag{3.27}$$

Thus, we see that $s^{(0)}(p^0, |\vec{p}_c'|)$ is the *renormalized* vertex function in the case that one nucleon is off its mass shell, i.e.,

$$i\bar{u}^{(s)}(\vec{0})\gamma_5[1 + \Gamma_1(p^0, |\vec{p}_c'|)]u^{(s')}(\vec{p}_c')$$
$$= \frac{i}{Z_1^{(0)}}\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}_c') s^{(0)}(p^0, |\vec{p}_c'|). \tag{3.28}$$

[The function $s^{(0)}(p^0, |\vec{p}_c'|)$, is the first approximation to the function defined in Eq. (3.4) for the case in which $\vec{p} = 0, p'^2 = -m_N{}^2$, and $(p - p')^2 = -\mu^2$.] To obtain this function we need to calculate $h(p^0)$. This is done by obtaining $\mathrm{disc}y^{(0)}(p^0, |\vec{p}_c'|)$ from Eqs. (3.19) and (3.20). [We have made use of the fact that

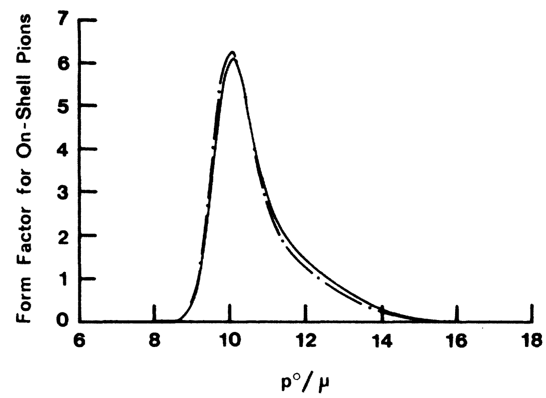

FIG. 4. The imaginary part of the quantity $s^{(0)}(p^0, |\vec{p}_c|)$ defined in Eq. (3.26) is denoted by the dashed-dotted line. The solid line denotes the quantity $\mathrm{Im}s(p^0, |\vec{p}|)$ obtained from the iteration procedure discussed in Appendixes B and C.

$$i\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')$$
$$= \frac{(2\pi)^{1/2}|\vec{p}'|}{[2m_N(E_{\vec{p}'} + m_N)]^{1/2}} C^{1}_{s-s'}{}^{\frac{1}{2}}_{s'}{}^{\frac{1}{2}}_{s} Y_{1,s-s'}(\hat{p}'), \tag{3.29}$$

so that $y^{(0)}(p^0, |\vec{p}'|)$ may be identified with the quantity in the square bracket in Eq. (3.19).] The evaluation of Eq. (3.24) yields $h(p^0)$.

For the calculations reported here we have used the form factors $v_{11}^{1}(|\vec{p}|)$ developed in a previous work.[9] These provide a good fit to the low-energy pion-nucleon phase shifts up to energies for which the pion-nucleon scattering has strong inelastic effects, $|\vec{p}| \simeq 400$ MeV/c.

The imaginary part of the function $s^{(0)}(p^0, |\vec{p}_c'|)$ is shown in Fig. 4 and the real part in Fig. 5. The central feature of these figures is the importance of the P_{11} resonance at 1470 MeV. Since the calculation is resonance dominated and the scale is determined by the renormalization condition, Eq. (3.27), it is expected that the result is not partic-

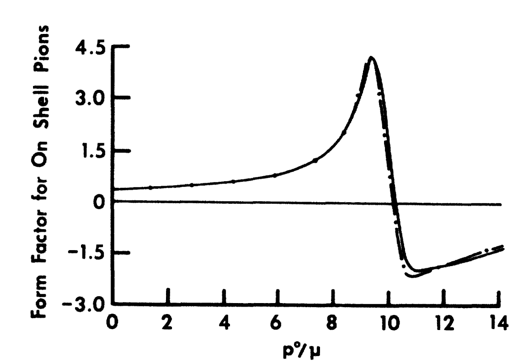

FIG. 5. The real part of the quantity $s^{(0)}(p^0, |\vec{p}_c|)$ and the real part of $s(p^0, |\vec{p}_c|)$. See caption of Fig. 4.

ularly sensitive to the various approximations used.

For future analytic work, we remark that it is possible to approximate the function $s(p^0, |\vec{p}_c|)$ by the following expression,

$$s(p^0, |\vec{p}_c'|) = \frac{m_N - E_R}{p^0 - E_R + i\frac{1}{2}\Gamma(p^0)}, \tag{3.30}$$

where

$E_R = 10.2\mu$,

$\Gamma(p^0)/2 = \theta(p^0 - E_0)\alpha \exp[-\beta/(p^0 - E_0)]$,

$E_0 = m_N + \mu$,

$\alpha = 1.88\mu$,

and

$\beta = 2.88\mu$.

The above discussion completes our treatment of the vertex function in the case that the pion and one nucleon are on their mass shells. We now turn to a more general situation in which we allow the pion to be off its mass shell and keep only a single nucleon (momentum p) on its mass shell. We remark that the value of $|\vec{p}'|$ is related to the off-shell pion mass τ. This relation is obtained by letting $|\vec{p}_c'| \to |\vec{p}'|$ and $\mu^2 \to \tau$ in Eq. (3.21).

We introduce the generalization of the function $s^{(0)}(p^0, |\vec{p}_c'|)$ and write

$$y^{(0)}(p^0, |\vec{p}'|) = s^{(0)}(p^0, |\vec{p}'|)h(m_N). \tag{3.31}$$

From Eq. (3.19) we see that the discontinuity in $y^{(0)}$ factorizes into a function of $|\vec{p}'|$ multiplied by a function of p^0;

$$\mathrm{disc}\, y^{(0)}(p^0, |\vec{p}'| = h)\ (m_N)\alpha(|\vec{p}'|)\mathrm{disc} f^{(0)}(p^0). \tag{3.32}$$

In Eq. (3.32)

$$\alpha(|\vec{p}'|) \equiv v_{11}^1(|\vec{p}'|)[(E_{\vec{p}'} + m_N)(E_{\vec{p}}\omega_{\vec{p}'})]^{1/2}/|\vec{p}'|, \tag{3.33}$$

and we have introduced a new function $f^{(0)}(p^0)$ for which we may write the following dispersion relation:

$$f^{(0)}(p^0) = \frac{1}{2\pi}\int_{m_N+\mu}^{\infty} \frac{\mathrm{disc} f^{(0)}(p^{0\prime})dp^{0\prime}}{p^{0\prime} - p^0 - i\epsilon}. \tag{3.34}$$

Again using Eqs. (3.19) and (3.20) we may obtain $\mathrm{disc}\, y^{(0)}(p^0, |\vec{p}'|)$ from which $\mathrm{disc} f^{(0)}(p^0)$ is found upon use of Eq. (3.32).

We see from Eqs. (3.31)–(3.34) that

$$s^{(0)}(p^0, |\vec{p}'|) = f^{(0)}(p^0)\alpha(|\vec{p}'|). \tag{3.35}$$

Equation (3.35) provides a natural generalization of the $s^{(0)}(p^0, |\vec{p}_c'|)$ which appears in Eq. (3.28). The function $s^{(0)}(p^0, |\vec{p}'|)$ is a first approximation to the function $\mathcal{F}$ defined in Eq. (3.4) in the special case in which $\vec{p} = 0$, and $p'^2 = -m_N^2$.

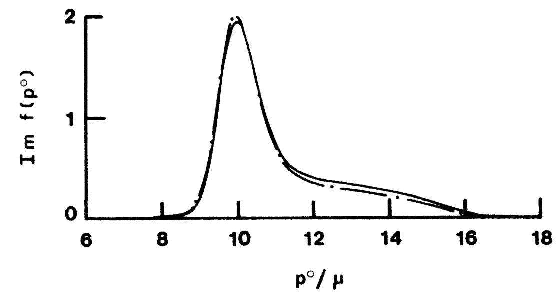

FIG. 6. The imaginary part of the quantity $f^{(0)}(p^0)$ defined in Eq. (3.36), is denoted by the dashed-dotted line. The solid line denotes the quantity $\mathrm{Im} f(p^0)$ obtained from the iteration procedure discussed in Appendixes B and C.

In practice one may avoid the evaluation of the dispersion integral of Eq. (3.34) by noting the simple relation

$$f(p^0) = \frac{s(p^0, |\vec{p}_c'|)}{\alpha(|\vec{p}_c'|)}, \tag{3.36}$$

which follows from Eq. (3.35). In Fig. 6 we present the real and imaginary parts of the function $f^{(0)}(p^0)$.

In Appendixes B and C we have discussed the modifications of the theory necessary if the term Γ_2 defined in Eq. (3.3) is included. There we have defined a function $s(p^0, |\vec{p}'|)$ which contains the effects of both Γ_1 and Γ_2 [see Eq. (B8)]. The details of the calculation of this quantity are discussed in the appendixes. In Figs. 4 and 5, the solid lines represent the imaginary and real parts of the function $s(p^0, |\vec{p}_c|)$. Correspondingly, in Figs. 6 and 7 we display the imaginary and real parts of the function $f(p^0)$ defined in Eq. (B13).

As may be seen from the figures, the inclusion of Γ_2 leads to relatively little change from the functions $s^{(0)}(p^0, |\vec{p}_c|)$ and $f^{(0)}(p^0)$. Again, we be-

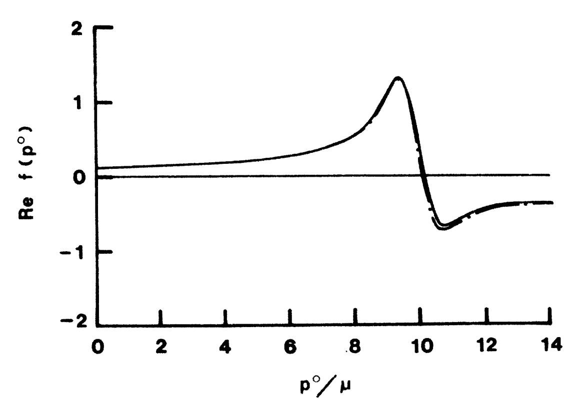

FIG. 7. The real part of the quantity $f^{(0)}(p^0)$ and $\mathrm{Re} f(p^0)$. See caption to Fig. 6.

lieve that this is a reflection of the dominant role of the resonance and the constraints imposed by renormalization procedure.

IV. SUMMARY AND CONCLUSIONS

We have presented a description of the vertex function in the case that one nucleon and the pion are on their mass shells. We have also provided a simple method for taking the *pion* off its mass shell. Using Eqs. (3.35) and (3.36) we have,

$$s(p^0, |\vec{p}\,'|) = s(p^0, |\vec{p}_c\,'|)\alpha(|\vec{p}\,'|)/\alpha(|\vec{p}_c\,'|). \tag{4.1}$$

Here, $\alpha(|\vec{p}\,'|)$ is given by Eq. (3.33) and $|\vec{p}_c\,'|$ [see Eq. (3.21)] is the value of $|\vec{p}\,'|$ when the pion is on its mass shell. Further, we have provided a useful parametrization of our results for the functions $s(p^0, |\vec{p}_c\,'|)$ in Eq. (3.30).

At this point it is useful to introduce the invariants $s=-p^2$ and $\tau=-(p-p')^2$ (see Fig. 3) and define a function of these invariants which is identical to $s(p^0, |\vec{p}\,'|)$ in the special frame where $\vec{p}=0$:

$$F(\sqrt{s}, \tau) \equiv s(\sqrt{s}, p'(\sqrt{s}, \tau)). \tag{4.2}$$

In the right hand side of Eq. (4.2) we have replaced p^0 by $\sqrt{s}$ and written $p'=|\vec{p}\,'|$ as a function of $\sqrt{s}$ and τ. Clearly $\sqrt{s}$ is the invariant generalization of p^0. (Note that $\tau=\mu^2$ for an on-mass-shell pion.) Further, the dependence of $p'=|\vec{p}\,'|$ on s and τ, when $|\sqrt{s}-m_N|>\sqrt{\tau}$, is given by,

$$|\vec{p}\,'| = \frac{1}{2\sqrt{s}}[s^2-2s(m_N{}^2+\tau)+(m_N{}^2-\tau)^2]^{1/2}. \tag{4.3}$$

In this case (where $|\sqrt{s}-m_N|>\sqrt{\tau}$) we may write Eq. (4.1) in an arbitrary frame as

$$F(\sqrt{s}, \tau) = F(\sqrt{s}, \mu^2) f_1(s, \tau), \tag{4.4}$$

where

$$f_1(s, \tau) \equiv \alpha(|\vec{p}\,'|)/\alpha(|\vec{p}_c\,'|). \tag{4.5}$$

It is useful to note that $f_1(s, \mu^2)=1$ and

$$F(\sqrt{s}, \mu^2) = s[\sqrt{s}, p'(\sqrt{s}, \mu^2)]. \tag{4.6}$$

{Note further that for $\vec{p}=0$ we have $s[\sqrt{s}, p'(\sqrt{s}, \mu^2)] = s(p^0, |\vec{p}_c\,'|)$.}

We now consider the case for which $|\sqrt{s}-m_N| < \sqrt{\tau}$. In this case Eq. (4.3) would give an imaginary value for $|\vec{p}\,'|$ and we cannot use Eq. (4.5). Therefore, for $|\sqrt{s}-m_N|<\sqrt{\tau}$ we make an alternative extension for $F(\sqrt{s}, \tau)$:

$$F(\sqrt{s}, \tau) \equiv F(\sqrt{s}, \mu^2) f_2(\tau). \tag{4.7}$$

In Eq. (4.7) we have an expression for the vertex function in the case that one nucleon is on its mass shell. We obtain the usual form factor if *both* nucleons are on their mass shells. This may be achieved by placing $\sqrt{s}=m_N$ in Eq. (4.7). We note that $F(m_N, \mu^2)=1$, so we see that $f_2(\tau)$ is the usual form factor for an off-mass-shell pion and on-mass-shell nucleons.

The factorized approximation given in Eq. (4.7) is expected to be good for small τ, since this expression is used in the domain $|\sqrt{s}-m_N|<\sqrt{\tau}$. Therefore, if τ is small we have $\sqrt{s}\simeq m_N$ and therefore $F(\sqrt{s}, \mu^2)\simeq 1$.

A reasonable parametrization for $f_2(\tau)$ was obtained in Ref. 7:

$$f_2(\tau) = \frac{\mu^2-\tau_R}{\tau-\tau_R+i\gamma(\tau)/2}, \tag{4.8}$$

where

$$\gamma(\tau) = 1.232(\tau-10\mu^2)\theta(\tau-10\mu^2) \tag{4.9}$$

and

$$\tau_R = 71\mu^2.$$

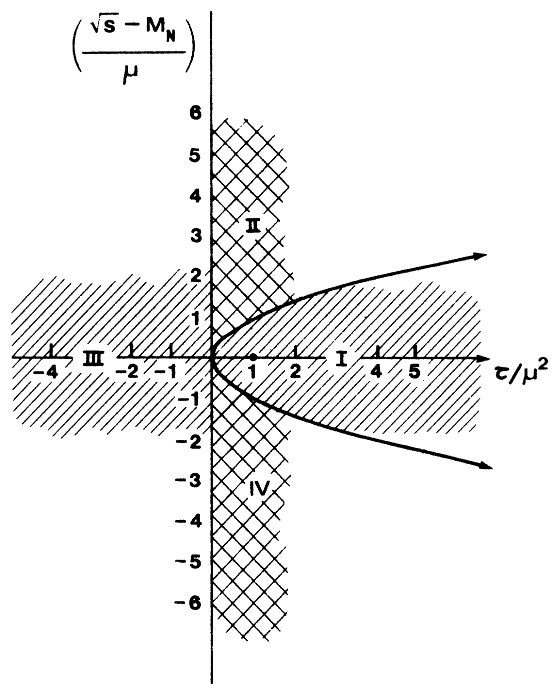

FIG. 8. Regions of applicability for our approximations to the form factor $F(\sqrt{s}, \tau)$. The parabola shown in the figure is given by the relation $|\sqrt{s}-m_N|=\sqrt{\tau}$. Region I is contained wholly within the parabola and extends up to $\tau\sim 30\mu^2$–$40\mu^2$. Regions II and IV (cross hatched) are the two regions of applicability of the product form $F(\sqrt{s}, \tau)=F(\sqrt{s}, \mu^2)f_1(s, \tau)$, Eq. (4.4). These regions are bounded on the left by $\tau=0$; however, the right hand boundary is uncertain ($\tau\sim 2\mu^2\rightarrow 10\mu^2$) and depends on the validity of the separable approximation. Region III is centered on $\sqrt{s}=m_N$ ($|\sqrt{s}-m_N|\lesssim 3\mu$). This region is bounded on the left by $\tau\sim -30\mu^2$ and on the right by $\tau=0$. In regions I and III one may use the approximation given in Eq. (4.11).

Thus using Eqs. (3.30), (4.2), (4.7), and (4.8) we have (when $|\sqrt{s}-m_N|<\tau$)

$$F(\sqrt{s},\tau)=\left(\frac{m_N-E_R}{\sqrt{s}-E_R+i\Gamma(\sqrt{s})/2}\right)\left(\frac{\mu^2-\tau_R}{\tau-\tau_R+i\gamma(\tau)/2}\right). \tag{4.11}$$

The regions of applicability of our two approximations, given in Eqs. (4.4) and (4.11), are shown in Fig. 8.

Since the π-N vertex function is important for the study of various processes in pion-nucleus, pion-nucleon, and nucleon-nucleon interactions we feel that the parametrizations given in Eqs. (4.4) and (4.11) will find extensive application. We will use these parametrizations in a future work to study pion production and absorption in nuclear reactions.

APPENDIX A

In this appendix we develop the equation satisfied by the pion-nucleon vertex function. We use the notation of Ref. 10 for simplicity and introduce an index η which refers to *both* the Dirac spin and isospin indices of the nucleons. When we wish to refer to the spin and isospin indices separately we will use the indices α,α', etc. for spin and β,β', etc., for isospin. The index j refers to the three (Hermitian) components of the meson field.

Following Ref. 10 we write the field equation for the nucleon field $\psi_\eta(x)$:

$$(\gamma\cdot p+m)_{\eta\eta'}\psi_{\eta'}(x)=-\Omega^j_{\eta\eta'}\psi_{\eta'}(x)\phi^j(x), \tag{A1}$$

where we have adopted a summation convention for repeated indices. In Eq. (A1), the quantity $\Omega^j_{\eta\eta'}$ is a numerical matrix which in the case of pseudoscalar coupling is equal to $g_0(\gamma_5)_{\alpha\alpha'}(\tau^j)_{\beta\beta'}$. We also introduce the nucleon Green's function

$$G_{\eta\eta'}(x,y)=i\langle 0|T\{\psi_\eta(x)\bar\psi_{\eta'}(y)\}|0\rangle \tag{A2}$$

and a Green's function for the pion,

$$\begin{aligned}\mathcal{G}^{jj'}(x,y)&=\delta_{jj'}\mathcal{G}(x,y)\\&=i\langle 0|T\{\phi^j(x)\phi^{j'}(y)\}|0\rangle.\end{aligned} \tag{A3}$$

We now make use of the field equation, Eq. (A1), to obtain,

$$\begin{aligned}&(\gamma\cdot p+m_0)_{\eta\eta''}G_{\eta''\eta'}(x,y)\\&\qquad=-\Omega^j_{\eta\eta''}G^j_{\eta''\eta'}(x,y;x)+\delta_{\eta\eta'}\delta^{(4)}(x-y).\end{aligned} \tag{A4}$$

In Eq. (A4) we find that the nucleon Green's function is coupled to a three-point function whose general definition is

$$G^j_{\eta\eta'}(x,y;z)=i\langle 0|T\{\psi_\eta(x)\bar\psi_{\eta'}(y)\phi^j(z)\}|0\rangle. \tag{A5}$$

We are now able to define a pion-nucleon vertex function through the following relation:

$$\begin{aligned}G^j_{\eta\eta'}(x,y;z)&=iG_{\eta\eta'}(x,x')\Omega^{j'}_{\eta_1\eta_2}\Gamma^{j'}_{\eta_2\eta_3}(x',y';z')\\&\quad\times G_{\eta_3\eta'}(y',y)\mathcal{G}^{j'j}(z',z).\end{aligned} \tag{A6}$$

We can show that

$$(\gamma\cdot p+m_0)_{\eta\eta''}G^k_{\eta''\eta'}(x,y;z)=i\Omega^j_{\eta\eta''}G^{jk}_{\eta''\eta'}(x,y;x,z), \tag{A7}$$

where

$$G^{jk}_{\eta\eta'}(x,y;x',z)\equiv i\langle 0|T\{\psi_\eta(x)\bar\psi_{\eta'}(y)\phi^j(x')\phi^k(z)\}|0\rangle. \tag{A8}$$

Now Eq. (A7) may be written

$$\begin{aligned}&(\gamma\cdot p+m_0)_{\eta\eta''}[G_{\eta''\gamma}(x,x')G_{\gamma\delta}^{-1}(x',x'')]G^k_{\delta\eta'}(x'',y;z)\\&\qquad=i\Omega^j_{\eta\eta''}G^{jk}_{\eta''\eta'}(x,y;x,z)\end{aligned} \tag{A9}$$

or

$$\begin{aligned}&[\delta_{\eta\gamma}\delta^{(4)}(x-x')-\Omega^j_{\eta\eta''}G^j_{\eta''\gamma}(x,x';x)]\\&\quad\times G_{\gamma\delta}^{-1}(x',x'')G^k_{\delta\eta'}(x'',y;z)\\&\qquad=i\Omega^j_{\eta\eta''}G^{jk}_{\eta''\eta'}(x,y;x,z),\end{aligned} \tag{A10}$$

where we have made use of Eq. (A4).

From Eq. (A10) we obtain

$$G^l_{\eta\eta'}(x,y;z)=iG_{\eta\gamma}(x,x')\Omega^j_{\gamma\delta}G^{jl}_{\delta\eta'}(x',y;x',z)+G_{\eta\gamma}(x,x')\Omega^j_{\gamma\delta}G^j_{\delta\rho}(x',y';x')G_{\rho\epsilon}^{-1}(y',x'')G^l_{\epsilon\eta'}(x'',y;z), \tag{A11}$$

and then using Eq. (A6),

$$\begin{aligned}\Gamma^j_{\eta\eta'}(x,x';z)&=\Omega^{-1j}_{\eta\eta_1}\Omega^j_{\eta_1\eta_2}G^{j'l}_{\eta_2\eta_3}(x,y;x,z')G_{\eta_3\eta'}^{-1}(y,x')\mathcal{G}^{-1lj}(z',z)+\Omega^{-1j}_{\eta\eta_1}\Omega^{j'}_{\eta_1\eta_2}G_{\eta_2\eta_3}(x,x'')\Omega^k_{\eta_3\eta_4}\Gamma^k_{\eta_4\eta_5}(x'',y'';z'')\\&\quad\times\mathcal{G}^{kj'}(z'',x)G^l_{\eta_5\eta_6}(y'',x'';z')G_{\eta_6\eta_7}^{-1}(x'',x')\mathcal{G}^{-1lj}(z',z).\end{aligned} \tag{A12}$$

We now use in Eq. (A6) again the form

$$G^l_{\eta_5\eta_6}(y'',x'';z')G_{\eta_6\eta_7}^{-1}(x'',x')\mathcal{G}^{-1lj}(z',z)=iG_{\eta_5\eta_6}(y'',y')\Omega^j_{\eta_6\eta_8}\Gamma^j_{\eta_8\eta_7}(y',x';z) \tag{A13}$$

to write Eq. (A12) as

$$\Gamma^j_{\eta\eta'}(x,x';z)=\Omega^{-1j}_{\eta\eta_1}\Omega^{j'}_{\eta_1\eta_2}G^{j'l}_{\eta_2\eta_3}(x,y;x,z')G_{\eta_3\eta'}{}^{-1}(y,x')\mathcal{G}^{-1lj}(z',z)+i\Omega^{-1j}_{\eta\eta_1}\Omega^{j'}_{\eta_1\eta_2}G_{\eta_2\eta_3}(x,x'')\Omega^k_{\eta_3\eta_4}\Gamma^k_{\eta_4\eta_5}(x'',y'';z'')$$
$$\times\mathcal{G}^{kj'}(z'',x)G_{\eta_5\eta_6}(y'',y')\Omega^j_{\eta_6\eta_7}\Gamma^j_{\eta_7\eta'}(y',x';z). \tag{A14}$$

Specializing to the case of pseudoscalar coupling we have upon suppressing the spin indices, and using Eq. (A3),

$$\Gamma^j(x,x';z)=\tau^j\tau^{j'}G^{j'j}(x,y;x,z')G^{-1}(y,x')\mathcal{G}^{-1}(z',z)+ig_0{}^2\tau^j\tau^{j'}G(x,x'')\tau^{j'}\gamma_5\Gamma^{j'}(x'',y'';z'')$$
$$\times\mathcal{G}(z'',x)G(y'',y')\tau^j\gamma_5\Gamma^j(y',x';z). \tag{A15}$$

[We remark that in Eqs. (A13) to (A15) there is no sum on the index j.]

At this point it is useful to define a scattering matrix M:

$$G^{j'j}(x,y;x',z')=\mathcal{G}(x',z')G(x,y)\delta_{j'j}-iG(x,y')\mathcal{G}(x',z'')M^{j'j}(y',z'';y''z)G(y'',y)\mathcal{G}(z,z'). \tag{A16}$$

Inserting the last expression into Eq. (A15) we find (with no sum on j)

$$\Gamma^j(x,x';z)=\delta^{(4)}(x-x')\delta^{(4)}(x'-z)-i\tau^j\tau^{j'}G(x,y')\mathcal{G}(x,z'')M^{j'j}(y',z'';x',z)$$
$$+ig_0{}^2\tau^j\tau^{j'}G(x,x'')\tau^{j'}\gamma_5\Gamma^{j'}(x'',y'';z'')\mathcal{G}(z'',x)G(y'',y')\tau^j\gamma_5\Gamma^j(y',x';z). \tag{A17}$$

It is useful to write

$$\Gamma^j(x,x';z)=\delta^{(4)}(x-x')\delta^{(4)}(x'-z)+\Gamma^j_1(x,x';z)+\Gamma^j_2(x,x';z), \tag{A18}$$

where Γ^j_1 is used to denote the second term of Eq. (A17) and $\Gamma^j_2(x,x';z)$ denotes the remaining term.

We first consider the equation for Γ in momentum space. We write

$$\Gamma(x,x';z)=\int\frac{e^{-ip\cdot x}}{(2\pi)^4}\frac{e^{ip'\cdot x'}}{(2\pi)^4}\Gamma(p,p';q)\frac{e^{iq\cdot z}}{(2\pi)^4}d^4p\,d^4p'\,d^4q \tag{A19}$$

and put

$$\Gamma(p,p';q)=(2\pi)^4\delta^{(4)}(p-p'-q)\Gamma(p,p') \tag{A20}$$

so that

$$\Gamma^j(x,x';z)=\int\frac{e^{-ip\cdot(x-z)}}{(2\pi)^4}\frac{e^{ip'\cdot(x'-z)}}{(2\pi)^4}\Gamma^j(p,p')d^4p\,d^4p'. \tag{A21}$$

With

$$G(x,y)=\int\frac{e^{ik\cdot(x-y)}}{(2\pi)^4}G(k)\,d^4k,\quad\text{etc.}, \tag{A22}$$

one finds, for example,

$$\Gamma^j_1(p,p')=-i\sum_{j'}\int\frac{d^4k}{(2\pi)^4}\mathcal{G}(k)G(p-k)\tau^j\tau^{j'}M^{j'j}(p-k,k;p',p-p'), \tag{A23}$$

where we have made the sum on j' explicit. Equation (A23) may be simplified further by writing $M^{j'j}$ in terms of an isospin symmetric and isospin antisymmetric part

$$M^{j'j}=\delta_{j'j}M_++\tfrac{1}{2}[\tau^{j'},\tau^j]M_-. \tag{A24}$$

Now,

$$\tau^j\sum_{j'}\tau^{j'}M^{j'j}=M_++2M_-\equiv M^{1/2}, \tag{A25}$$

where $M^{1/2}$ refers to the channels with isospin $\frac{1}{2}$ as expected. Thus, dropping the j superscript, we have

$$\Gamma_1(p,p')=-i\int\frac{d^4k}{(2\pi)^4}\mathcal{G}(k)G(p-k)M^{1/2}(p-k,k;p',p-p'). \tag{A26}$$

In an analogous fashion, we obtain

$$\Gamma_2(p,p')=3ig_0{}^2\int\frac{d^4k}{(2\pi)^4}\mathcal{G}(k)G(p-k)\gamma_5\Gamma(p-k,p)G(p)\gamma_5\Gamma(p,p'), \tag{A27}$$

which follows easily from Eq. (A17) if we note that $\Gamma^j(x,y;z)$ is actually independent of the index j.

APPENDIX B

In this appendix we present the approximations we have used to estimate the size of the contribution of the term Γ_2 to the vertex function. We recall Eq. (A27) and note that when written in terms of *renormalized* quantities, we have $[g_R = Z_3^{1/2} Z_2 Z_1^{-1} g_0]$,

$$\Gamma_2(p,p') = 3ig_R^2 \int \frac{d^4k}{(2\pi)^4} \mathcal{G}(k) G(p-k)\gamma_5 \Gamma(p-k,p) G(p)\gamma_5 \Gamma(p,p'). \tag{B1}$$

We are now interested in calculating the quantity

$$i\bar{u}^{(s)}(\vec{0})\gamma_5 [Z_1 \Gamma_2(\vec{p}=0, p^0, \vec{p}')] u^{(s')}(\vec{p}'), \tag{B2}$$

which may then be compared in importance with the expression $i\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')s^{(0)}(p^0, |\vec{p}'|)$. Indeed the complete vertex function is given by

$$i\bar{u}^{(s)}(\vec{0})[1+\Gamma_1(p^0,\vec{p}')+\Gamma_2(p^0,\vec{p}')]u^{(s')}(\vec{p}') = \frac{i}{Z_1}[u^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')s^{(0)}(p^0, |\vec{p}'|) + \bar{u}^{(s)}(\vec{0})\gamma_5 Z_1 \Gamma_2(p^0,\vec{p}')u^{(s')}(\vec{p}')]. \tag{B3}$$

We recall that in the calculation of Sec. III, where we neglected the quantity Γ_2, the right hand side of Eq. (B3) reduces to $[Z_1^{(0)}]^{-1}\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')s^{(0)}(p^0, |\vec{p}'|)$. The renormalization constant in the presence of the Γ_2 term is denoted as Z_1.

We will again use an approximation for the first two propagators appearing in Eq. (B1):

$$\hat{G}(p-k) = 2\pi i \delta(p^0 - k^0 - E_{\vec{k}}) \left(\frac{m_N}{E_{\vec{k}}}\right) \Lambda^+(-\vec{k}), \tag{B4}$$

with

$$\Lambda^+(-\vec{k}) = \sum_{s'} u^{(s')}(-\vec{k})\bar{u}^{(s')}(-\vec{k})$$

and

$$\mathcal{G}(k) = \frac{1}{k^2 + \mu^2 - i\epsilon} = \frac{1}{\omega_{\vec{k}}^2 - (p^0 - E_{\vec{k}})^2 - i\epsilon} \simeq -\frac{1}{2\omega_{\vec{k}}}\left(\frac{1}{p^0 - (E_{\vec{k}} + \omega_{\vec{k}}) + i\epsilon}\right). \tag{B5}$$

In addition we put

$$G(p) = G(p^0) \simeq G_+(p^0) \sum_{s''} u^{(s'')}(\vec{0})\bar{u}^{(s'')}(\vec{0}) \tag{B6}$$

to obtain

$$\begin{aligned}[\bar{u}^{(s)}(\vec{0})Z_1\gamma_5\Gamma_2(p,p')u^{(s')}(\vec{p}')] = 3ig_R^2 Z_1 \int\int \frac{dk^0 k^2 dk}{(2\pi)^4}\left[-\frac{1}{2\omega_{\vec{k}}}\left(\frac{1}{p^0 - (E_{\vec{k}}+\omega_{\vec{k}}) + i\epsilon}\right)\right] 2\pi i \delta(p^0 - k^0 - E_k)\left(\frac{m_N}{E_k}\right) \\ \times \int d\Omega_{\hat{k}} \sum_{s's''} [\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(-\vec{k})]\,[\bar{u}^{(s')}(-\vec{k})\gamma_5 u^{(s'')}(\vec{0})] s(p^0, |\vec{k}|) \\ \times [\bar{u}^{(s'')}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')] G_+(p^0) s(p^0, |\vec{p}'|).\end{aligned} \tag{B7}$$

In writing Eq. (B7) we have made the definitions for the *renormalized vertex functions*

$$\bar{u}^{(s')}(-\vec{k})\gamma_5\Gamma(p-k,p)u^{(s'')}(\vec{0}) = \bar{u}^{(s)}(-\vec{k})\gamma_5 u^{(s'')}(\vec{0})s(p^0, |\vec{k}|) \tag{B8}$$

and

$$\bar{u}^{(s'')}(\vec{0})\gamma_5\Gamma(p,p')u^{(s')}(\vec{p}') = \bar{u}^{(s'')}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')s(p^0, |\vec{p}'|). \tag{B9}$$

We now use the result

$$\sum_{s'} \int d\Omega_{\hat{k}} [\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(-\vec{k})][\bar{u}^{(s')}(-\vec{k})\gamma_5 u^{(s)}(\vec{0})] = 4\pi(E_{\vec{k}} - m_N)/2m_N \tag{B10}$$

to obtain

$$\begin{aligned}&\bar{u}^{(s)}(\vec{0})Z_1\gamma_5\Gamma_2(p^0,\vec{p}')u^{(s')}(\vec{p}') \\ &= \frac{3g_R^2 Z_1}{(2\pi)^2}\int_0^\infty \frac{k^2 dk (E_{\vec{k}} - m_N)}{2\omega_{\vec{k}} E_{\vec{k}}}\left[\frac{1}{p^0 - (E_{\vec{k}} + \omega_{\vec{k}}) + i\epsilon}\right] s(p^0, |\vec{k}|) G_+(p^0)[\bar{u}^{(s)}(\vec{0})\gamma_5 u^{(s')}(\vec{p}')] s(p^0, |\vec{p}'|),\end{aligned} \tag{B11}$$

so that Eq. (B3) becomes

$$i\bar{u}^{(s)}(\vec{0})[1+\Gamma_1(p^0,\vec{p}')+\Gamma_2(p^0,\vec{p}')]u^{(s')}(\vec{p}')$$

$$\equiv\frac{i}{Z_1}[\bar{u}^{(s)}(\vec{0})\gamma_5u^{(s')}(\vec{p}')]s(p^0,|\vec{p}'|)$$

$$=\frac{i}{Z_1}\left\{s^{(0)}(p^0,|\vec{p}'|)+\frac{3g_R{}^2Z_1}{(2\pi)^2}\int_0^\infty\frac{k^2dk(E_{\vec{k}}-m_N)}{2\omega_{\vec{k}}E_{\vec{k}}}\left(\frac{1}{p^0-(E_{\vec{k}}+\omega_{\vec{k}})+i\epsilon}\right)s(p^0,|\vec{k}|)G_+(p^0)s(p^0,|\vec{p}'|)\right\}$$

$$\times[\bar{u}^{(s)}(\vec{0})\gamma_5u^{(s')}(\vec{p}')] \tag{B12}$$

with the requirement that

$$s(p^0,|\vec{p}_c'|)=1 \quad \text{if } p^0=m_N.$$

Now let us recall that in our model the function $s(p^0,|\vec{p}'|)$ factorizes. We write

$$s(p^0,|\vec{p}'|)\equiv f(p^0)\alpha(|\vec{p}'|), \tag{B13}$$

where

$$\alpha(|\vec{p}'|)=v_{11}^1(|\vec{p}'|)[(E_{\vec{p}'}+m_N)(E_{\vec{p}'}\omega_{\vec{p}'})]^{1/2}/|\vec{p}'|. \tag{B14}$$

Thus we have from Eq. (B12),

$$f(p^0)=\left\{f^{(0)}(p^0)+\frac{3g_R{}^2Z_1}{2(2\pi)^2}\int_0^\infty\frac{k^2dk(E_{\vec{k}}-m_N)}{\omega_{\vec{k}}E_{\vec{k}}}\frac{\alpha(|\vec{k}|)f^2(p^0)G_+(p^0)}{p^0-(E_{\vec{k}}+\omega_{\vec{k}})+i\epsilon}\right\}. \tag{B15}$$

We may iterate this nonlinear equation by the following procedure. [We are reminded the $G_+(p^0)$ is a functional of $f(p^0)$ since the mass operator appearing in $G_+(p^0)$ depends on the vertex function.]

Let us denote an nth order approximation to $f(p^0)$ as $f^{(n)}(p^0)$, the corresponding value of Z_1 as $Z_1^{(n)}$, and the associated $G_+(p^0)$ as $G_+^{(n)}(p^0)$. We then write

$$f^{(n)}(p^0)=\left\{f^{(0)}(p^0)+\frac{3g_R{}^2Z_1^{(n-1)}}{2(2\pi)^2}\int_0^\infty\frac{k^2dk(E_{\vec{k}}-m_N)\alpha(|\vec{k}|)f^{(n-1)}(p^0)f^{(n)}(p^0)G_+^{(n-1)}(p^0)}{\omega_{\vec{k}}E_{\vec{k}}[p^0-(\omega_{\vec{k}}+E_{\vec{k}})+i\epsilon]}\right\} \tag{B16}$$

and obtain for an (unrenormalized solution)

$$f_U^{(n)}(p^0)=[f^{(0)}(p^0)]\bigg/\left(1-\frac{3g_R{}^2Z_1^{(n-1)}}{2(2\pi)^2}\int_0^\infty\frac{k^2dk(E_{\vec{k}}-m_N)\alpha(|\vec{k}|)f^{(n-1)}(p^0)G_+^{(n-1)}(p^0)}{\omega_{\vec{k}}E_{\vec{k}}[p^0-(\omega_{\vec{k}}+E_{\vec{k}})+i\epsilon]}\right) \tag{B17}$$

and

$$y^{(n)}(p^0,|\vec{p}'|)=f_U^{(n)}(p^0)\alpha(|\vec{p}'|). \tag{B18}$$

Again with

$$\rho^{(n)}(p^0)=\mathrm{Im}\,y^{(n)}(p^0,|\vec{p}_c'|)\equiv\mathrm{Im}\,h^{(n)}(p^0), \tag{B19}$$

we may write

$$h^{(n)}(p^0)=\frac{1}{Z_1^{(n)}}\left[\frac{1}{\pi}\int\frac{\rho^{(n)}(p^{0\prime})dp^{0\prime}}{p^{0\prime}-p^0+i\epsilon}\right]\bigg/\left[\frac{1}{\pi}\int\frac{\rho^{(n)}(p^{0\prime})}{p^{0\prime}-m_N}\right], \tag{B20}$$

where

$$\frac{1}{Z_1^{(n)}}\equiv h^{(n)}(m_N)=\frac{1}{\pi}\int\frac{\rho^{(n)}(p^{0\prime})dp^{0\prime}}{p^{0\prime}-m_N}. \tag{B21}$$

We define

$$s^{(n)}(p^0,|\vec{p}'|)=\frac{y^{(n)}(p^0,|\vec{p}'|)}{h^{(n)}(m_N)} \tag{B22}$$

and note that

$$s^{(n)}(p^0,|\vec{p}'|)|_{p^0=m_N}=1. \tag{B23}$$

From these results we may define a renormalized $f^{(n)}(p^0)$ for use in iteration:

$$f^{(n)}(p^0) \equiv \frac{s^{(n)}(p^0, |\vec{p}'|)}{\alpha(|\vec{p}'|)}. \tag{B24}$$

At this point we may obtain $f_U^{(n+1)}(p^0)$ from Eq. (B17) using the result for $f^{(n)}(p^0)$ in Eq. (B24) if we also provide a scheme for calculating $G_+^{(n)}(p^0)$. This scheme is discussed in Appendix C.

APPENDIX C

In Appendix B we obtained an equation for the function $f(p^0)$ and discussed the renormalization procedure. [See Eqs. (B17)–(B24).] In this appendix we give some further details necessary for implementing our calculational scheme. In particular we need to calculate the nucleon Green's function $G_+(p^0)$. To this end we review some well known equations relating the Green's function to the mass operator. In this section we use a subscript U to denote unrenormalized quantities. Thus we may write

$$G_U = G_0 + G_0 M_U G_U, \tag{C1}$$

where we have suppressed isospin and spin indices. [The structure of M_U, however, may be obtained by combining Eqs. (A4) and (A6).] The relation between the renormalized Green's function G and G_U is $G_U = Z_2 G$ and we also have $Z_2 M_U = Z_1 M_f$ where M_f is the finite part of the mass operator. (See the discussion of "overlapping" divergences in Ref. 11.) Thus we may write

$$G^{-1} = Z_2 G_0^{-1} - Z_1 M_f, \tag{C2}$$

where $G_0^{-1} = \gamma \cdot p + m_0$. Equation (C2) may be decomposed to give relations for the positive and negative frequency parts of G^{-1}, denoted as G_+^{-1} and G_-^{-1}. Note that

$$G^{-1}(p) = G_+^{-1}(p)\Lambda_+(p) + G_-^{-1}\Lambda_-(p), \tag{C3}$$

where

$$\Lambda_\pm(p) = \frac{1}{2}\left[1 \mp \frac{\gamma \cdot p}{[(p^0)^2 - \vec{p}^2]^{1/2}}\right]. \tag{C4}$$

The projection operators in Eq. (C4) become the more familiar projection operators if the nucleon is on its mass shell, that is, if $(p^0)^2 = m^2 + \vec{p}^2$.

Our approximation requires that we neglect the coupling of the positive and negative frequency parts of G through the mass operator. Therefore we put

$$G_+^{-1} = Z_2 (G_0^{-1})_+ - Z_1 (M_f)_+, \tag{C5}$$

where from Eq. (B17) we can see that in this approximation (and with $\vec{p} = 0$)

$$[M_f^{(n)}(p^0)]_+ = -\frac{3 g_R^2}{2(2\pi)^2} \times \int_0^\infty \frac{k^2 dk (E_{\vec{k}} - m_N)\alpha(|\vec{k}|) f^{(n)}(p^0)}{\omega_{\vec{k}} E_{\vec{k}}[p^0 - (\omega_{\vec{k}} + E_{\vec{k}}) + i\epsilon]}. \tag{C6}$$

Using Eq. (C5) we may write [suppressing the subscript (+) for convenience]

$$Z_2 G_0^{-1} G = 1 + Z_1 M_f G. \tag{C7}$$

The right hand side of Eq. (C7) is seen to be the denominator of Eq. (B17) which may therefore be written as

$$f_U^{(n)}(p^0) = \frac{f^{(0)}(p^0)}{Z_2^{(n-1)} (G_0^{-1})^{(n-1)} G^{(n-1)}(p^0)} \tag{C8}$$

$$= \frac{f^{(0)}(p^0)}{Z_2^{(n-1)} [m_0^{(n-1)} - p^0] G^{(n-1)}(p^0)}. \tag{C9}$$

We will now obtain expressions to be used to calculate the quantities $Z_2^{(n-1)}$, $m_0^{(n-1)}$, and $G^{(n-1)}(p^0)$ which are to be inserted in Eq. (C9). We proceed by by defining

$$\sigma(p^0) = \frac{1}{\pi} \mathrm{Im}[Z_1 M_f(p^0)]. \tag{C10}$$

Then from Eq. (C5), we have

$$G^{-1}(p^0) = Z_2 [m_0 - p^0] - \int \frac{\sigma(p^{0\prime}) dp^{0\prime}}{p^{0\prime} - p^0 - i\epsilon}. \tag{C11}$$

We remark that $G^{-1}(p^0 = m_N) = 0$ so that

$$m_0 = m_N + \frac{1}{Z_2} \int \frac{\sigma(p^{0\prime})}{p^{0\prime} - m_N} dp^{0\prime}, \tag{C12}$$

which for the purposes of the iteration scheme may be written as

$$m_0^{(n)} = m_N + \frac{1}{Z_2^{(n)}} \int \frac{\sigma^{(n)}(p^{0\prime}) dp^{0\prime}}{p^{0\prime} - m_N}, \tag{C13}$$

with

$$\sigma^{(n)}(p^0) \equiv \frac{1}{\pi} \mathrm{Im}[Z_1^{(n)} M_f^{(n)}(p^0)]. \tag{C14}$$

The expression for m_0 given in Eq. (C12) may be inserted into Eq. (C11) to obtain

$$G^{-1}(p^0) = (m_N - p^0) \left[Z_2 + \int \frac{\sigma(p^{0\prime}) dp^{0\prime}}{(p^{0\prime} - p^0)(p^{0\prime} - m_N)}\right]. \tag{C15}$$

Since $G^{-1}(p^0)$ is the *renormalized* Green's function, it follows that

$$G^{-1}(p^0 \to m_N) \to (m_N - p^0)$$

or

$$Z_2 = 1 - \int \frac{\sigma(p^{0\prime})dp^{0\prime}}{(p^{0\prime} - m_N)^2}. \tag{C16}$$

Of course, as above, Eq. (C16) may be rewritten with superscripts (n) for purposes of iteration. This expression may be inserted into Eq. (C15) to yield

$$G^{-1}(p^0) = (m_N - p^0) \times \left[1 - (m_N - p^0) \int \frac{\sigma(p^{0\prime})dp^{0\prime}}{(p^{0\prime} - p^0)(p^{0\prime} - m_N)^2}\right] \tag{C17}$$

and a similar superscripted expression.

It would be desirable to carry through the above program in its entirety. However, in evaluating M_f we have only used the pole term of the nucleon Green's function. (Indeed, we have placed the nucleon on its mass shell in evaluating M_f.) Therefore it would be somewhat inconsistent with our treatment of the nucleon Green's function throughout the rest of this work to use the full spectral representation given by Eq. (C17). Therefore, we have used the approximation

$$G^{-1}(p^0) \simeq (m_N - p^0) \tag{C18}$$

in Eqs. (C8) and (C9).

Thus we have,

$$f^{(n)}(p^0) = \frac{f^{(0)}(p^0)(m_N - p^0)}{Z_2^{(n-1)}(m_0^{(n-1)} - p^0)}. \tag{C19}$$

The renormalization constant Z_1 may be obtained from Eq. (B21). Then $\sigma(p^0)$ may be calculated from Eq. (C10), and Z_2 is given by Eq. (C16). Finally, m_0 is calculated using Eq. (C13).

Even with the approximation of Eq. (C18), this is a nontrivial iteration scheme. However, it is convergent with the value of m_0 converging to 4.65μ. In the converged solution one finds $Z_1 \to \infty$ and $Z_2 \to \infty$. However, as the magnitude of Z_1 and Z_2 increase, m_0 and the various form factors reach stable values.

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