Inelastic interactions of pions with the complex nuclei of emulsions

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(Received 29 April 1976; revised manuscript received 2 June 1977)

Cross sections for inelastic interactions of pions with complex nuclei of emulsion have been measured at various pion energies in the (3,3) resonance energy region. The results have been interpreted in terms of the pion-nucleon interaction cross section through Fraenkel's isobar model using the optical model formalism. A detailed analysis of the stars produced by the positive pions in that energy range was also made and the energy and the opening angle distributions were fitted with those obtained from the Monte Carlo calculations based on the isobar model. Good agreement in each case supports the isobar model for the inelastic interaction of pions in complex nuclei.

[NUCLEAR REACTIONS (π^+, emulsion), $E = 10 - 300$ MeV, measured and calculated $\sigma(E)$, $N-E$ distribution, isobar model.]

I. INTRODUCTION

A study on the interaction of pions with complex nuclei plays an important role in understanding the nuclear structure. A pion can interact with a nucleus either elastically or inelastically. In an inelastic interaction the struck nucleus is excited and the striking pion is either absorbed, scattered with reduced energy, or suffers charge exchange. Whatever the case may be, the pion gets a chance to enter the nucleus and the interaction must be strongly dependent on the behavior of the nucleons inside the nucleus.

Since the discovery of pion a number of investigations have been made to study its interaction with complex nuclei from different view points. Although a vast amount of work has been carried out on various theoretical and experimental aspects of the problem, no systematic experimental data exist for the variation of inelastic interaction cross section of pions with complex nuclei in the low energy region. Bernardini[1] measured the mean free path for the interaction of both π^+ and π^- in emulsion. They obtained cross sections for star formation by π^+ mesons at mean energies between 35–50 and 70–80 MeV which were 380 and 670 mb, respectively. These results give the magnitude of the cross sections but are not very useful for quantitative analysis due to the large energy spread of the pions. Rankin and Bradner[2] also studied the interaction of 45.5 MeV π^+ with the emulsion nuclei. Blau, Oliver, and Smith[3] worked with a π^+ beam having an energy between 50 and 80 MeV. Using laminated emulsion they observed that 24 to 30% of the emulsion stars originated from the light nuclei of emulsion. Stork[4] investigated the interaction of π^+ of energies 33, 46, and 68 MeV with beryllium, carbon, aluminum, and copper. Using cloud chamber technique Tracy[5] obtained the interaction cross sections of π^+ with aluminum at average energies of 38, 57, and 82 MeV. Similar work was carried out by Saphir[6] with lead at 50 ± 15 MeV. Blinov *et al.*[7] studied the interaction of π^+ mesons with freon using a bubble chamber. More recently Kirillov-Ugryumov, Sergeev, and Fesenko[8] studied the stars formed by the absorption of 70 ± 8 MeV π^+ with the complex nuclei of emulsion. In most cases, however, the results cannot be used for rigorous quantitative analysis either due to insufficient statistics or large spread of the pion beam energy or because of the lack of exact knowledge or muon contamination. There is thus a general lack of systematic experimental study of inelastic interactions of pions with complex nuclei and the need of such a study was emphasized by Ericson[9] and by Jean.[10]

In the present work, therefore, we have studied the stars formed by the inelastic interaction of positive pions with the complex nuclei of emulsions. The star formation cross sections have been measured at various energies up to about 60 MeV and at 168, 225, and 254 MeV and the results have been interpreted using the optical model formalism through Fraenkel's isobar model. A detailed analysis of the stars was also made and the nature and kinematics of the outgoing particles have been investigated experimentally and compared statistically with the results of a Monte Carlo calculation based on the isobar model of interaction.

TABLE I. Characteristics of emulsion stacks.

Stack No.	No. of pellicles	Nominal energy (MeV)	Pion flux (cm^2)	Pellicle dimension (cm)	Angle of incidence with respect to the surface (nominal) (deg)
1	13	72	7×10^4	$2\times10\times0.066$	0
2	12	168	3×10^5	$10\times10\times0.060$	90
3	13	224	3×10^5	$10\times10\times0.060$	30
4	13	254	1.4×10^6	$10\times10\times0.060$	90

II. EXPERIMENTAL DETAILS

A. Irradiation of stacks

The experiments were conducted with four different emulsion stacks irradiated with highly monoenergetic pion beams. The first stack consisting of 13 pellicles was irradiated with 72 MeV positive pions incident parallel to the plate in the conventional manner and was used for measuring cross sections at various energies upto 60 MeV. Stacks 2, 3, and 4 were irradiated with pions of 168, 224, and 254 MeV, respectively, and were used for cross-sectional and other measurements at these energies. In stacks 2 and 4, the pion beams were incident at right angles to the emulsion surface, whereas in stack 3 the beam was incident at an angle of 30° with respect tothe emulsion surface (these stacks were also used for some other experiments involving inclined beam). Each plate was printed with a square reference grid graduated in millimeters to facilitate the measurements. Characteristics of the stacks are given in Table I.

B. Emulsion characteristics

In stack 1, the pions were incident parallel to the emulsion surface and hence traversed a long distance in any given pellicle. The pion energy decreased as it penetrated the emulsion stack parallel to the emulsion surface and cross sections could be measured at various energies as determined from the residual range. The actual cross sections were measured at eight different points extending over strips of 3 mm on either side of the mean point at which the energy was calculated. In each case the energy was calculated from the residual range and suitable corrections were made for straggling as well as for the width of the strip. The incident energy was carefully measured from the π-μ decay point and was found to be 72 ± 2.5 MeV corresponding to a range of 64.8 ± 3 mm.

The pion flux was directly measured by actual counting of the number of meson tracks per unit area in the region corresponding to each of the experimental points. The tracks having normalized

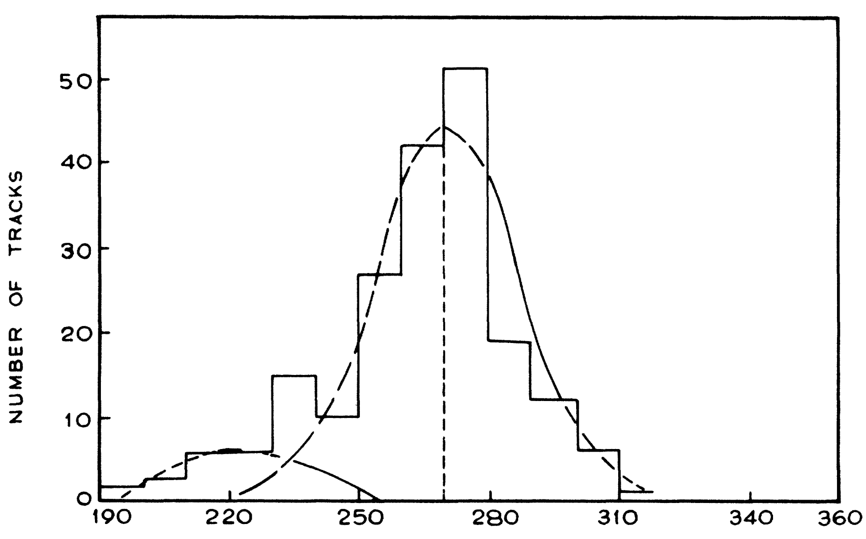

FIG. 1. Determination of muon contamination in stack 1.

grain densities between 1.0 and 1.5 were considered as meson tracks but also included muons. Corrections for muons were made on the basis that, at these energies pion grain densities are 20 to 30% higher than those of muons. Thus the grain density distribution histogram (220–360 grains/mm) could be superimposed by two Gaussian distribution curves and fitted by χ^2 minimizing technique. The curve with larger area represented the distribution of pions and the other with smaller area represented that due to muons. From the ratio of these areas, the percentage of muons was calculated and found to be $(12 \pm 1.5)\%$. Actual measurements were done at a number of places. The distribution curve corresponding to the residual range of 49.5 mm ($E_\pi \sim 58$ MeV) is shown in Fig. 1.

In stacks 2, 3, and 4 the pion beams were inclined (at right angles in stacks 2 and 4) and hence did not travel parallel to the emulsion surface. In penetrating the thickness of the stack the pions traversed only about a centimeter or so and the loss in energy was insignificant in the energy ranges concerned. Each stack could, therefore, be considered of a specific energy as determined by the irradiation characteristics. The measurements were, moreover, limited to only one pellicle (usually the second pellicle from the incident direction) in each stack to ensure minimum energy uncertainty.

The total flux was obtained by actual counting of the number of meson tracks passing through the pellicle concerned while the correction for muon contamination was made from the irradiation geometry and was taken to be 8%. Characteristics of the four stacks are given in Table I.

C. Measurement of cross section

To obtain the cross section at any energy, the stars were obtained by area scanning using a magnification of 100× 10. Each area was scanned twice to ensure high efficiency. The overall efficiency of star detection was determined by scanning a limited area repeatedly (extending over a few days to avoid any impressional optical effect) till no new star could be detected. The efficiencies were found to be $(85 \pm 5)\%$ and $(82 \pm 5)\%$ for lower energy pions (stack 1) and higher energy pions (stacks 2, 3, and 4), respectively. A selection criterion was imposed on the stars to eliminate the secondary stars. The stars with the incident tracks inclined at angles larger than 10° from the mean incident direction or having grain densities at least 10% higher than the average grain density of the pion tracks in that region were considered as secondary stars. Out of 783 stars scanned in the first stack 554 satisfied the selection criterion for primary stars. Similarly 551 stars satisfied the selection criterion out of 645 stars scanned in stacks 2, 3, and 4. Corrections were also made for the hydrogen stars which were produced due to the hydrogen present in the gelatin of the emulsion. From the corrected number of stars and the flux at that region, the interaction cross section σ was obtained from the following:

$$\sigma = \frac{N_s}{fAdN_e}, \qquad (1)$$

where A equals the area of the face on which the pion beam is incident, d equals the width of the strip, f equals the pion flux, N_s equals the number of stars in the volume Ad (corrected for efficiency, secondary stars, and hydrogen stars), and N_e equals the number of atoms of complex nuclei cm^3.

In stack 1 the cross sections were measured at eight different mean energies extending from about 10 MeV up to 62 MeV. For this, 6 mm wide strips were chosen (identified by the graduations in the reference grid) at eight different places along the length of the emulsion plate (x axis) dividing the entire range in more or less equally spaced energy ranges. In each case the stars were scanned from the top to the bottom over a length of 10 mm (measured along the y axis of the reference grid).

To facilitate counting the stars, each millimeter square in the reference grid was considered as a block and scanning was carried out blockwise. More than one plate was scanned whenever a sufficient number of stars was not available in one plate. The stars, thus counted, were corrected for secondary stars and efficiency in a straightforward manner. An unambiguous identification of the hydrogen stars was, however, not always possible. For this reason the expected number of hydrogen stars was obtained in each case from the density of hydrogen atoms, N_H, and the $\pi^+ + p$ cross section using Eq. (1) and was subtracted from the corrected value to obtain N_s. For energies up to about 60 MeV $\sigma_{\pi^+ + p}$ is small and the contribution of the hydrogen stars to the total number of stars N_S was calculated to be less than 1%. At pion energies 168, 224, and 254 MeV, the contributions were 16.5, 13.5, and 12.0%, respectively. The mean energy at the center of each strip was calculated from the residual range and the corresponding spread was calculated from the width of the strip. The face area A was obtained by multiplying the length scanned in the y direction by the thickness of the emulsion plate and the number of plates scanned.

Measurements were carried out similarly in the stacks 2, 3, and 4 except that the face area A was the product of x and y. In stacks 2 and 4 where the particles were incident at an angle of 90°, the dis-

TABLE II. Experimental cross sections of interaction of positive pions with emulsion nuclei.

Pion energy E_π (MeV)	Face area A	Vol. scanned (mm^3)	d (mm)	N_S (corrected for efficiency and hydrogen stars)	Pion flux/cm^2 (corrected for efficiency)	Cross section (mb)
10.7 ± 7	19.8	120	6	17	5.51×10^4	55 ± 18
21.4 ± 6	19.8	120	6	34	5.67×10^4	107 ± 22
31.2 ± 4.3	19.8	120	6	73	5.77×10^4	224 ± 31
40.5 ± 3.5	13.2	80	6	83	5.93×10^4	377 ± 52
46.5 ± 3.5	13.2	80	6	98	6.06×10^4	435 ± 61
52 ± 3	13.2	80	6	112	6.17×10^4	492 ± 64
58 ± 3	13.2	80	6	106	6.25×10^4	457 ± 63
62 ± 2.5	13.2	80	6	129	6.35×10^4	548 ± 69
168 ± 2.5	30	18	0.6	188	3.0×10^5	730 ± 75
224 ± 1.1	15	9	1.2	162	2.9×10^5	652 ± 72
254 ± 0.5	10	6	0.6	215	1.35×10^6	565 ± 59

tance d was equal to the thickness of the plate, whereas in stack 3 d was larger by a factor of 2 due to the inclination of the beam at an angle of 30°. The details of the experimental measurements and the corresponding cross sections are given in Table II.

Besides using them for measurement of the cross sections, the stars were also analyzed to investigate their size dependence on the incident pion energy. For this, the prong distributions of the stars

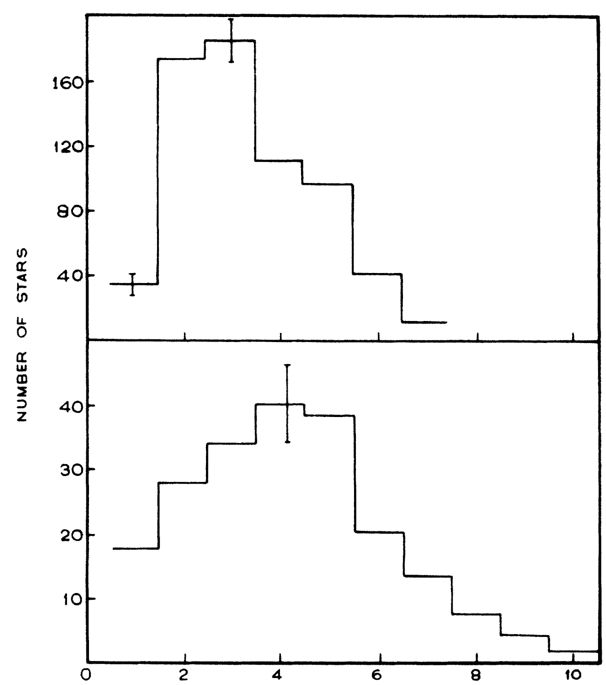

FIG. 2. Prong distribution of stars induced in emulsion nuclei by π^+ mesons of energies (a) 50 MeV, (b) 254 MeV.

at an average energy of 50 MeV and at 254 MeV were measured and are shown in Fig. 2. The average size of the stars was larger for high energy pions (~254 MeV) and had on an average four prongs against three prongs for the lower energy 50 MeV pions.

III. INTERACTION MECHANISM

A. Isobar model

In an inelastic interaction, the pion can either get absorbed inside the nucleus or come out of the nucleus with reduced energy. "Inelastic events" include all the incoherent processes, i.e., absorption, inelastic scattering, and single and double charge exchanges. All these events produce stars and have been included in our cross section calculations.

In absorption, the rest mass of the pion and its associated kinetic energy are transferred to the interacting nucleons. From a series of experiments on proton-proton (pp), proton-neutron (pn), and neutron-neutron (nn) pairs emitted in pion absorption,[11] it has been established that the absorption predominantly takes place on a pair of nucleons. This is also quite evident from the consideration of absorption kinematics. For, if all the available energy, which is the rest mass energy, 139 MeV or more, is given to one single nucleon, its momentum will be of the order of 500 MeV/c or more and this, by momentum conservation, must be of the order of magnitude of its initial momentum. It is, however, unlikely to find inside a nucleus a nucleon with momentum higher than the Fermi momentum, i.e., 225 MeV/c, and hence the emission of one nucleon must be strongly suppressed. On the other hand, if the available energy is shared about equally between a pair of nucleons, each nucleon carries with it only about 70 MeV kinetic energy and gets a momentum of about 375 MeV/c. The angle between the two outgoing nucleons can be such that the resultant final momentum is equal to the resultant of the momenta of the two participating nucleons. Thus it is more likely that the absorption occurs predominantly on a pair of nucleons rather than on a single nucleon. The absorption on a single nucleon or on more than two nucleons may, of course, be allowed but the cross sections of such absorptions will be negligible.

If the pion is not absorbed, it can interact with one of the nucleons and can transfer a part of the kinetic energy to the nucleon. The pion can further interact or may come out with the reduced energy. In any case, the energized nucleons can collide with other nucleons and start a cascade. During these collisions, the nucleus can become highly excited and evaporation may result emitting a few nucleons. If no outgoing pion is visible, the star can be either due to absorption, or charge exchange in case the pion is a charged one.

Assuming that the absorption of a pion takes place on a pair of nucleons, a model similar to Fraenkel's isobar model[12] is proposed in which the absorption takes place in two stages. According to this model, the incident pion interacts with one of the nucleons inside the nucleus to form an unstable state or an isobar:

(a) $\pi + N_1 \longrightarrow N^*$ (isobar),

where N_1 and N^* stand for a nucleon and an isobar, respectively. The isobar may then either (i) interact with another nucleon and the pion gets absorbed emitting two nucleons

(b) $N^* + N_2 \longrightarrow N_3 + N_4$ (absorption)

or (ii) the isobar may decay into a pion and a nucleon according to the reaction

(c) $N^* \longrightarrow \pi + N$ (charge exchange or scattering).

The resultant particles may further interact with the nuclear nucleons and thus produce a star. In this process, the pion can also exchange its charge resulting in a charge exchange reaction. This process may occur twice and the result will be a double charge exchange. Thus it can be tacitly assumed that any pion which gets a chance to interact with one of the nucleons inside the nucleus will result in an inelastic star through any one of the above mentioned channels. According to this model, therefore, the total inelastic cross section is simply determined by the interaction given in stage (a) which is known precisely over a wide range of energies. It is possible, therefore, to obtain the interaction cross section of pions with complex nuclei from the average π-nucleon interaction cross section $\bar{\sigma}$ inside the nucleus.

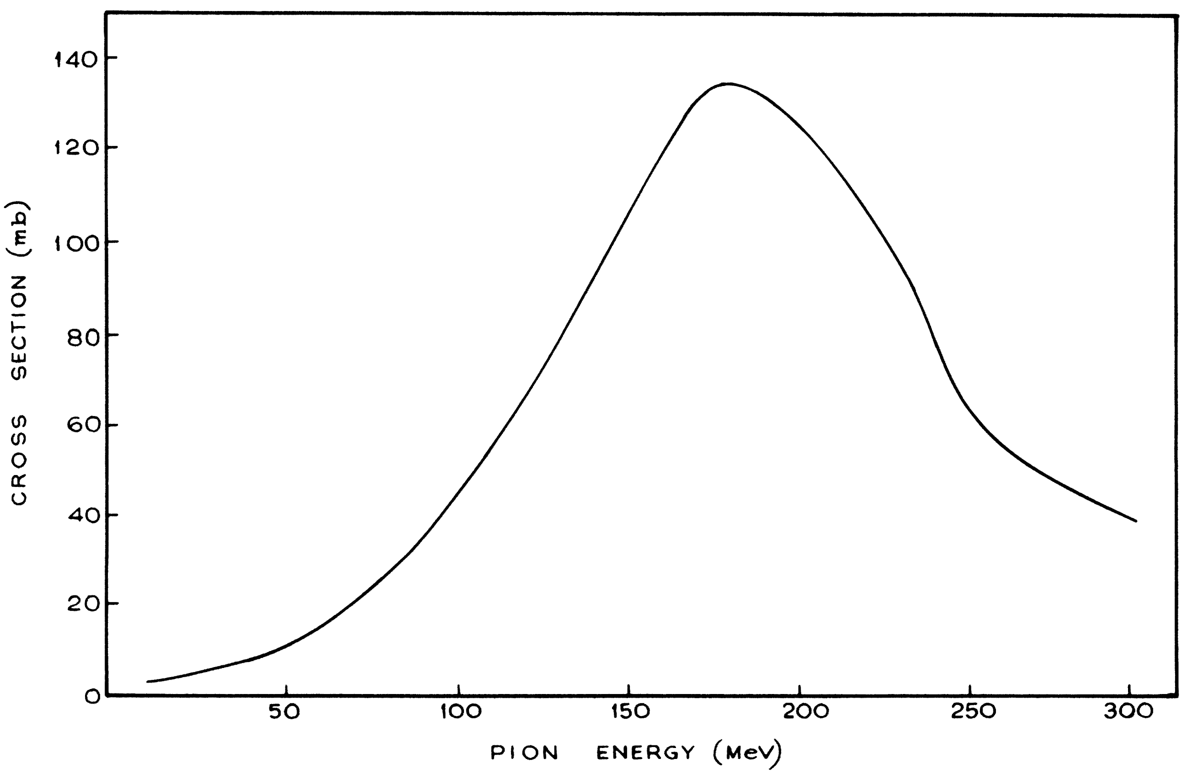

FIG. 3. Average total cross section for pions in the nuclear medium of emulsion.

B. Cross section calculation

Following Bethe,[13] Fernbach, Serber, and Tayler[14] the inelastic interaction cross section σ_i, in terms of $\bar{\sigma}$, can be obtained from the following:

$$\sigma_i = \pi R^2 \left(1 - \frac{V_c}{E}\right)\left[1 - \frac{1-(1+2KR)\,e^{-2KR}}{2K^2R^2}\right], \quad (2)$$

where V_c equals the Coulomb barrier of the nucleus, E equals the incident energy of the pion, R equals the nuclear radius, K equals the absorption coefficient $3\bar{\sigma}A/4\pi R^3$, $R = r_0 A^{1/3}$, and A equals the mass number of the nucleus. The subscript i denotes the type of nucleus. If the medium contains two or more different types of nuclei, as is the case with the emulsions, the cross section σ_i must be calculated for each type of atom and the results have to be suitably averaged over. In our calculation, the emulsion has been considered to be made up of two groups of nuclei with average mass numbers 14 and 94, corresponding to the light group (consisting of C, N, and O ~15.8% by weight) and the heavy group (Ag and Br ~84.2% by weight), respectively. With this averaging and using the standard values of $\sigma_{\pi^+ + p}$ and $\sigma_{\pi^- + p}$ (according to the isospin formalism $\sigma_{\pi^- + p} = \sigma_{\pi^+ + n}$) and considering the fact that pions can interact either with a proton or a neutron (according to their statistical weight in the nucleus concerned) in the emulsion nuclei, the average total cross section for the pion-nucleon interaction in the nuclear medium of emulsion, $\bar{\sigma}$, was calculated over the entire (3, 3) resonance energy region. These cross sections are shown in Fig. 3 and were used for calculating interaction cross sections of pions in emulsion (σ) using Eq. (2). In using Eq. (2), it should be noted that $\bar{\sigma}$ must be evaluated at the energy of the particle inside the nucleus. According to the optical model formalism, the energy of the incident particle is influenced by the real part of the optical model potential. Necessary energy correction was, therefore, introduced by using the required optical model potentials from the calculations of Frank, Gammel, and Watson[15] (1956) and Kimura and Nagashima[16] (1965) after correcting for the many body effect and Fermi motion of the nuclear nucleons. Cross sections were calculated for $r_0 = 1.3$, 1.4, and 1.5 fm. The results are plotted in Fig. 4, and compared with the measured cross sections. The general trend of the curves is similar in all the cases, but a good agreement is obtained for $r_0 = 1.4$ fm.

IV. ANALYSIS OF ABSORBTION STARS

In view of the very good agreement between the experimental and theoretical cross sections, it is worthwhile to study the stars formed in some detail. According to the isobar model an absorbed

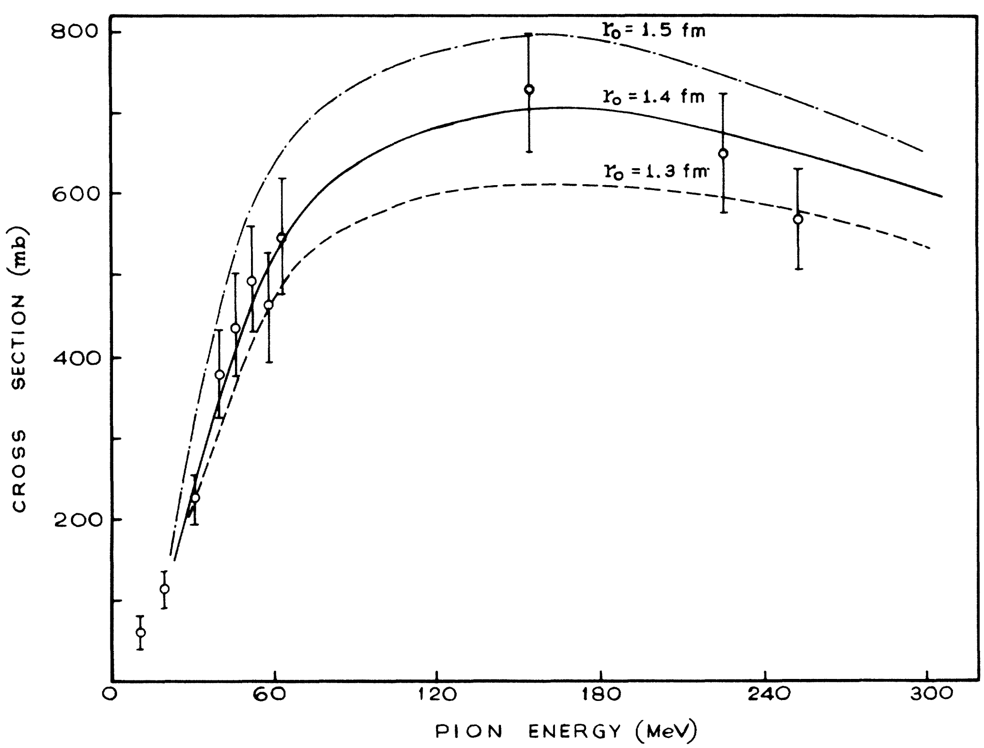

FIG. 4. Inelastic interaction cross section of π^+ mesons in emulsion.

pion interacts with two nucleons. The rest mass energy (~140 MeV) released is shared more or less equally by the two absorbing nucleons. Thus in low energy pion absorption both the nucleons will have energies of the order of 70 MeV. The study of these fast pairs can, therefore, yield information on the absorption process as well as on the behavior of the nucleons inside the nucleus. The resultant nucleons, however, interact with other nucleons in traversing through the nucleus, and hence change their initial energies and directions. The presence of other evaporated particles make it difficult to distinguish the primary nucleons from the secondaries. A selection criterion was, therefore, imposed on the energy of the protons under investigation. A simple calculation based on statistical thermodynamics shows that for incident pions of this energy range the average energy of the evaporated particles is of the order of 10 MeV and none of the evaporated particles have energy in excess of 30 MeV. A cutoff value of 30 MeV was, therefore, used to distinguish the primary protons from the evaporated ones; protons with energy greater than 30 MeV being taken as coming from the primary interactions.

882 pion induced stars in the energy range 30 MeV to about 60 MeV corresponding to an average energy of 45 ± 15 MeV were examined for fast protons with energy greater than 30 MeV. A total of 530 fast protons were found and 108 stars had fast proton pairs which presumably originated from the absorption of pions on a pair of nucleons. Energies of these fast protons as well as the opening angles between the two protons of the pairs were measured and the results are shown in Figs. 5–7 (continuous lines in histograms). Some of our results (Figs. 6 and 7) are similar to those of Kirillov-

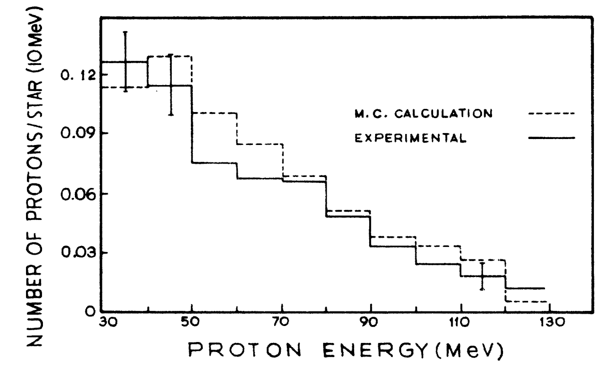

FIG. 5. Energy (laboratory) spectrum of the fast protons produced in positive pion absorption stars [$E_\pi \sim (45 \pm 15)$ MeV].

Ugryumov *et al.*[8] although the peaking observed at about 150° in Fig. 7 is not evident in their spectrum (Fig. 2, Ref. 8) due to wider angular grouping used for the abscissa. Our experimental results are finally compared with those obtained from the Monte Carlo calculations based on isobar model. The Monte Carlo calculations used are similar to those of Metropolis *et al.*,[17] Hébert and Mes,[18] and Werbrouck.[19] In the calculation, the nucleus has been considered as a spherical potential well in which neutrons and protons form two Fermi gases. The characteristics of the Fermi gas, such as the nuclear potential, the Fermi level, etc., were taken from Metropolis *et al.*[17] Calculations were done at pion energies between 30 and 60 MeV and the results were properly averaged for comparison with those of experiments in the energy range 45 ± 15 MeV.

The scheme of the Monte Carlo calculation was as follows: The energy and the direction of the in-

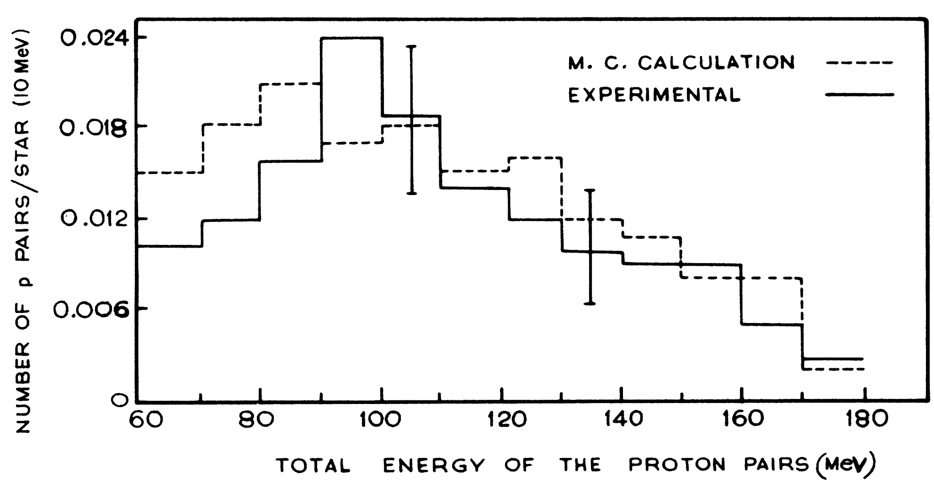

FIG. 6. Number of fast proton pairs produced in π^+ absorption stars as a function of the total kinetic energy (lab) of the pair [$E_\pi \sim (45 \pm 15)$ MeV].

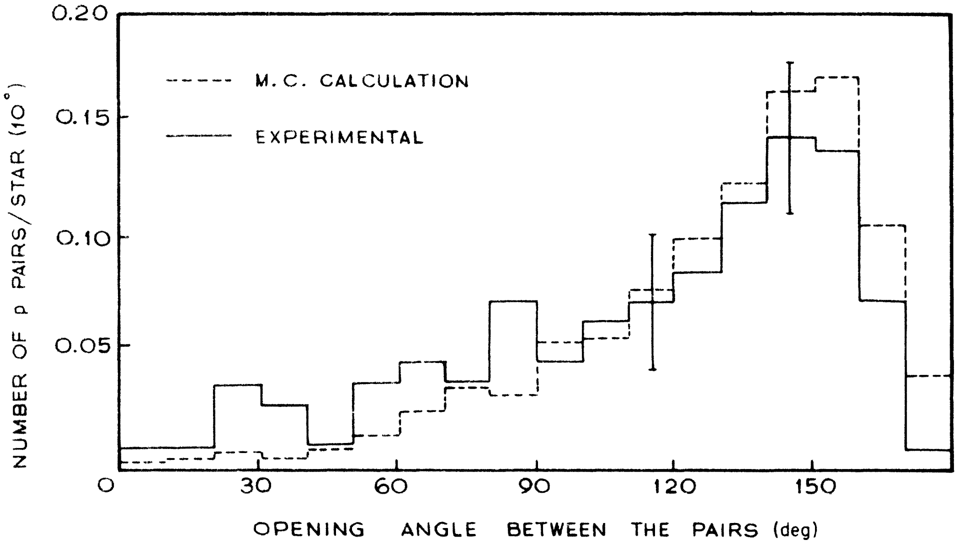

FIG. 7. Distribution of fast proton pairs produced in positive pion absorption stars ($E_\pi \sim 45$ MeV) as a function of their opening angles.

cident pions were known from the beam characteristics. From the known pion-nucleon interaction cross section, the mean free path of the pion in the nuclear matter was calculated and by using a weighted random number the distance of the collision was obtained. From the position of the entry point of the pion, the direction of motion, and the collision distance, the position of the expected collision was obtained. If the position in question fell inside the nucleus, the collision was considered to have taken place. By using other random numbers the charge of the target nucleon, its momentum and the type of interaction were determined. Thus the energies, the direction, and other characteristics of the interacting particles were completely known. In the laboratory frame both the particles move but not generally along the same line or along any of the axes. To accomplish an easy calculation a series of rotations and relativistic transformations were carried out. In the first rotation R_1, the target nucleon was made to move along the x axis. This was achieved by rotating the reference frame through an angle ϕ about the z axis and $\pi/2-\theta$ about the y axis where ϕ and θ are, respectively, the azimuth and colatitude of the momentum vector of the target nucleon in the spherical polar coordinate system. The transformation R_1 is, therefore, given by

$$R_1=\begin{pmatrix}\sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta\\ -\sin\phi & \cos\phi & 0\\ -\cos\theta\cos\phi & -\cos\theta\sin\phi & \sin\theta\end{pmatrix}. \quad (3)$$

This transformation is applied only to the incident momentum $\vec{P}_1$ since the target particle moves along the x axis. The quantities $\sin\theta$, $\cos\theta$, $\sin\phi$, and $\cos\phi$ were obtained with the help of random numbers.

This rotation was then followed by a relativistic transformation T_1 of both the particles to the rest frame of the nucleon. The transformation T_1 is given by

$$T_1=\begin{pmatrix}\gamma_1 & 0 & 0 & i\gamma_1\beta_1\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ -i\gamma_1\beta_1 & 0 & 0 & \gamma_1\end{pmatrix},$$

$$\gamma_1=\frac{1}{\sqrt{1-\beta_1^2}},$$

$$\beta_1=\frac{|\vec{P}_2|}{(\vec{P}_2^{\,2}+M_2^{\,2})^{1/2}}, \quad (4)$$

and M_2 and $\vec{P}_2$ are, respectively, the mass and momentum vector of the target nucleon. Under this condition the target nucleon is at rest while the incident pion approaches from some general direction. Another rotation similar to R_1 was, therefore, performed in the rest frame of the struck nucleon to bring the incident particle along the x axis. The particles were then transformed, using a transformation similar to T_1, to the center-of-mass system so that they approached each other with equal momenta and along the x axis. The center-of-mass momentum of each of the particles is then $|\vec{P}^*|=\beta_2\gamma_2 M_2$ where

$$\beta_2 = \frac{|\vec{P}_1|}{U_1' + M_2}, \quad \gamma_2 = \frac{1}{(1-\beta_2^2)^{1/2}}, \tag{5}$$

and U_1' is the total energy of the incident particle in the rest frame of the target particle. The total energy U^* in the center-of-mass system is, therefore, given by

$$U^* = [(\vec{P}^{*2} + M_1^2)]^{1/2} + [(\vec{P}^{*2} + M_2^2)]^{1/2}. \tag{6}$$

Since the total energy must be conserved and the masses of the scattered nucleons M_3 and M_4 are equal, the final momentum of each of the scattered nucleons P_f^* in the c.m. system can be obtained by equating the initial and final energies in the c.m. system and is given by

$$|\vec{P}_f^*| = [(\tfrac{1}{2}U^*)^2 - M_4^2]^{1/2}. \tag{7}$$

Subsequent to the collision they were allowed to move along the directions determined by random numbers and had equal but opposite momenta. Finally, the inverse transformations and rotations were applied to obtain the energies and directions of the outgoing particles in the laboratory system. The two outgoing particles were then free to collide with other nucleons as long as they were inside the nucleus. These two particles were, therefore, followed independently and for each of them the next point of collision was determined in the manner described earlier. The kinematical calculations for the collisions were continued as long as the point of the next collision was within the nucleus. If, however, the point of the next collision was found to be outside the nucleus it was considered as if the collision did not occur and the particles came out of the nucleus with the energies as determined in the previous collision. Thus each particle was followed until it came out of the nucleus or the energy of the particle was much lower than the potential barrier of the nucleus and remained inside the nucleus. Actual calculations were done for a large number of pions using an IBM 1620 electronic computer and in each case the energies and directions of the outgoing particles were printed out to give a complete information of the outgoing particles. Stars were simulated for $r_0 = 1.3$, 1.4, and 1.5 fm. From the data of these simulated stars, the energy and the angular distributions of the fast protons (>30 MeV) were obtained. The results with different r_0 do not statistically differ significantly from each other and hence the results corresponding to the value of $r_0 = 1.4$ fm ($r_0 = 1.4$ fm gave best fit in Fig. 4) are compared with the experimental data in Figs. 5–7. A good general fit is evident except that the calculated number of protons and proton pairs are somewhat higher than the data (Figs. 5 and 6) and the calculated spectrum shown in Fig. 7 is slightly more peaked than the data. These discrepancies are, however, due to inherent limitations of the Monte Carlo technique as well as to the simplicity of the nuclear model used in our calculations.

V. CONCLUSION

It is observed that inelastic interaction cross sections of π^+ with complex nuclei increases sharply with energy and follows a trend similar to that of π^+-nucleon interaction cross sections. Close agreement of the measured cross sections with those calculated from Eq. (2) indicates the success of the isobar model in explaining the interaction of pions with complex nuclei. A detailed analysis of the fast protons emitted in the pion absorbed stars tends to indicate that absorption takes place preferentially on two nucleons. The experimental distribution of the opening angles between the two fast protons produced in the π^+ absorption is well reproduced by the Monte Carlo calculations based on two-nucleon interaction mechanism. A general agreement is also observed between the experimental and calculated energy distribution of the fast proton pairs originating from the absorption of pions on a pair of protons.

ACKNOWLEDGMENTS

The authors would like to thank Professor E. Ekspong of the University of Stockholm for supplying the 72 MeV pion plates and Dr. N. Tanner and Dr. J. Domingo of CERN for allowing us to use their electronics for the irradiation of 168, 224, and 254 MeV stacks. Thanks are also due Dr. H. Mes for helping in the programming of Monte Carlo calculations and D. P. Dutta for going through the manuscript.

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