Analysis of polarization and analyzing power in the reaction $^3H(p,n)^3He$†

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The difference between the polarization and analyzing power for the reaction $^3He(p,n)^3He$ has been calculated in a $4\hbar\omega$ model space within the framework of the generalized R-matrix method of Lane and Robson. The model calculations indicate neither the presence of f wave components nor a significant sensitivity of the $P-A$ difference to the position of the ^4He levels.

[NUCLEAR REACTIONS $P^y - A_y$ difference for the $^3H(p,n)^3He$ reaction.]

I. INTRODUCTION

In a recent letter, Donoghue *et al.*[1] measured A_y and P^y for $E_p < 4$ MeV in the reaction $^3H(p,n)^3He$. The A_y data of this letter confirmed previous data[2] but the measured P^y values were appreciably larger than earlier results.[3-6] In fact, the previously reported $A_y - P^y$ difference[2] was eliminated. A possible theoretical explanation of this difference was proposed by Arnold *et al.*[7] They suggested this difference can be explained in terms of the presence of $^3F_2 \rightarrow {}^3P_2$ transitions which are enhanced in the vicinity of the lowest 2^- state of ^4He. The $A_y - P^y$ difference is also significant because it implies a breaking of exact charge symmetry by the Coulomb interaction and hence provides a mechanism for investigating charge symmetry breaking terms in the nuclear interaction.[8] In view of the significance of this difference and because of conflicting experimental results, we have decided to analyze the $A_y - P^y$ difference in the reaction $^3H(p,n)^3He$.

II. THEORY AND FORMULATION

The model for the bound and continuum states of the ^4He system represents an application of the dynamical equations of the Lane-Robson[9] R-matrix methodology to the ^4He nucleus. The dynamical equations can be written in the form[10]

$$\sum_{\lambda'}\left[\langle\lambda|H-E|\lambda'\rangle+\sum_c \gamma_{\lambda c}(b_{\lambda' c}-b_c)\gamma_{\lambda' c}\right]A_{\lambda'}=0, \quad (1)$$

where H is the Hamiltonian describing the system of interest and $\gamma_{\lambda c}$ and $b_{\lambda c}$ are the reduced widths[11] and logarithmic derivatives associated with the expansion states $|\lambda\rangle$. The expansion states are introduced in order to describe the nuclear wave function within the interaction region $r_c \leq a_c$ in all channels. The quantities b_c are related to radial wave functions $U_c(r_c)$ in the physical channels by

$$b_c=\left(\frac{r_c}{U_c}\frac{dU_c}{dr_c}\right)_{r_c=a_c}. \quad (2)$$

They provide the needed connection between the interaction region and the various two-body breakup channels. The A_λ are expansion amplitudes which are to be determined, if necessary, by the solution of Eq. (1).

Within this framework, the model is defined by choosing a form for the Hamiltonian and a set of expansion states and cluster wave functions.[12] The calculations include the $p+{}^3H$, $n+{}^3He$, and $d+{}^2H$ breakup channels in addition to an explicit set of structure states, whose total oscillator energy does not exceed $4\hbar\omega$.

The nuclear Hamiltonian is expressed as

$$H=-\sum_K(\hbar^2/2\mu_K)\nabla_K{}^2+\sum_{i<j}V_{ij}, \quad (3)$$

where K runs over the α particle internal coordinates and i and j run over nucleon coordinates. Using standard techniques, the desired many-body matrix elements of the Hamiltonian can be expressed in terms of standard one- or two-body matrix elements evaluated over all space. The matrix elements of the Coulomb interaction and the kinetic energy require corrections to remove contributions arising from those parts of the oscillator eigenfunctions which extend beyond the interaction region. Similar corrections to the nuclear interaction matrix elements are taken to be negligible and are ignored.

Solutions of Eq. (1) are obtained by the methods outlined in Ref. 13. After utilizing the transformations of Ref. 13, the R matrix takes on the standard form[11]

$$R_{cc'}=\sum_\mu \frac{\gamma_{\mu c}\gamma_{\mu c'}}{E_\mu - E}, \quad (4)$$

where the quantities E_μ and $\gamma_{\mu c}$ are calculated

TABLE I. Properties of few-nucleon clusters using the modified Sussex interaction.

Cluster	Experimental binding energy (MeV)	% error in calculated value	Experimental rms radius (fm)	% error in calculated value
^{4}He	28.30	0	1.63	−6
^{3}He	7.72	−5	1.88	−8
^{3}H	8.49	−5	1.70	−2
^{2}H	2.21	−20	1.95	−13

directly from information appearing in Eq. (1). In this manner, the resonance structure of the theory is made more explicit. Scattering and reaction information are obtained from the R matrix via the S matrix by means of standard formulas.[11]

Specific formulas for the positions and widths of R-matrix resonances are available in the literature.[11,14] In particular, the resonance energy E_R^μ corresponding to the level E_μ may be defined as the solution to the equation

$$E_R^\mu = \mathrm{Re}[E_\mu - \xi_\mu(E_R^\mu)]. \tag{5}$$

The total width of the resonances is then obtained from the equation

$$\Gamma_R = -2\,\mathrm{Im}[E_\mu - \xi_\mu(E_R^\mu)], \tag{6}$$

where ξ_μ is itself defined in terms of known R-matrix energies and reduced widths E_μ, $\gamma_{\mu c}$ and standard Coulomb radial functions.[11,14]

III. CHOICE OF INTERACTION

In Ref. 12, an effective interaction for oscillator basis states was determined for the two, three, and four nucleon systems. This interaction was determined from the Sussex matrix elements[15] and is of the form

$$V^{\mathrm{eff}} = CV^{\mathrm{Sussex}}, \tag{7}$$

where C is a strength parameter of order unity. The parameter C and the oscillator size parameter b were varied independently. Good fits to the ground state properties, given in Table I, were obtained for $C = 1.168$ and $b = 1.60$ fm. Within our model space, $4\hbar\omega$, this effective interaction also predicts a 4% D-state probability in the deuteron ground state and yields a ^{3}H-^{3}He Coulomb energy difference in agreement with experiment. The changes from the original Sussex matrix elements implied by our choice of C are typically of the same order of magnitude as the expected uncertainties in the matrix elements themselves.[15]

IV. RESULTS AND DISCUSSION

Excitation spectra and level widths are summarized in Table II. Both $L = 1$ and $L = 3$ components are included in the negative parity states. This permits an evaluation of the contribution of both p and f waves to the $P-A$ difference. The energy spectrum $E^{\mathrm{unshifted}}$ is obtained from the modified Sussex interaction, Eq. (7). A second

TABLE II. Comparison of model resonances and widths with experiment.

E_{exp} (MeV)	J^π	$E_R^{\mathrm{unshifted}}$ (MeV)	$\Gamma^{\mathrm{unshifted}}$ (MeV)	E_R^{shifted} (MeV)	$\Gamma^{\mathrm{shifted}}$ (MeV)	Γ^{exp} (MeV)
20.1	0^+	30.9	11.9	20.1	2.8	0.27 [a]
21.1	0^-	22.4	4.6	21.1	2.1	0.80 [a]
22.1	2^-	27.8	1.8	22.1	3.2	1.80 [a]
25.5	$(0^+, 1^+)$	36.3	0.4	25.5	2.6	2.9–5.6 [a]
26.4	2^-	29.0	0.7	26.4	1.4	~10.0 [b]
27.4	1^-	31.7	2.5	27.4	4.4	~10.0 [b]
29.5	0^-	30.5	4.2	29.5	4.0	~10.0 [b]
30.5	1^-	33.0	5.7	30.5	5.1	~10.0 [b]
31.0	1^-	33.2	3.7	31.0	8.3	3.1–5.3 [a]
33.0	2^+	38.9	3.8	33.0	5.0	2.8–5.6 [a]

[a] Summarized in Ref. 19.
[b] Estimated from single particle widths of Ref. 18.

TABLE III. $P^y - A_y$ for various level schemes and L values for $\theta_{c.m.} = 45°$.

Proton energy $E_{c.m.}$ (MeV)	Unshifted $L=1$ only	Unshifted $L=1,3$	Shifted $L=1$ only	Shifted $L=1,3$
1.00	−0.000 23	−0.001 43	−0.001 21	−0.001 26
1.25	−0.000 31	−0.006 18	−0.004 20	−0.005 02
1.50	−0.001 30	−0.008 85	−0.005 84	−0.007 50
1.75	−0.001 77	−0.008 67	−0.006 99	−0.008 75
2.00	−0.001 95	−0.008 03	−0.008 22	−0.009 88
2.25	−0.001 90	−0.007 56	−0.009 22	−0.010 78
2.50	−0.001 76	−0.007 26	−0.009 88	−0.011 42
2.75	−0.001 61	−0.007 16	−0.010 31	−0.011 88
3.00	−0.001 48	−0.007 15	−0.010 63	−0.012 27
3.25	−0.001 36	−0.007 26	−0.010 96	−0.012 67
3.50	−0.001 26	−0.007 46	−0.011 36	−0.013 13
3.75	−0.001 17	−0.007 75	−0.011 86	−0.013 68
4.00	−0.002 23	−0.008 10	−0.012 50	−0.014 37

spectrum E^{shifted} is obtained by adjusting E_μ in Eq. (5) such that $E_R^\mu = E_{\text{exp}}$. Table II also summarizes the level widths for both shifted and unshifted levels. These two level schemes are very different and provide wide limits to test the importance of f waves in determining the $P-A$ difference. The shifted levels provide a reasonable description of all reaction processes between the $p+{}^3\text{H}$, $n+{}^3\text{He}$, and $d+{}^2\text{H}$ channels below a proton center of mass energy of 5 MeV.[16,17] The unshifted levels give a fair representation of scattering processes below the $d+{}^2\text{H}$ threshold, and a reasonable description of scattering near the $d+{}^2\text{H}$ threshold.[12]

Table III summarizes the model results for both shifted and unshifted levels for $\theta_{c.m.} = 45°$. In particular, the effects of f waves are considered. The results suggest that the $P-A$ differences are insensitive to the inclusion of f wave terms. The inclusion of f waves causes the $P-A$ difference to increase by only 0.002 for the shifted and 0.006 for the unshifted case. The magnitude and shape of both $P(E_p)$ and $A(E_p)$ are similar to that of Werntz and Meyerhof.[18] In addition, the singlet-triplet mixing parameter x for the two $T=1\,1^-$ levels has the value −0.5 which is in agreement with solution (I) of Ref. 18.

The f wave effect is very small for both shifted and unshifted level schemes, even though these levels differ by several MeV. This suggests that f wave effects would not be a sensitive test for proposed ${}^4\text{He}$ level schemes. Furthermore, penetrability considerations, which would apply to the reaction ${}^3\text{H}(p,n){}^3\text{He}$ for $E_p < 4$ MeV, seem to be an adequate argument for the small $P-A$ difference, because the width of the 2^- state at 22.1 MeV is 1.8 MeV.[19] This width is just broad enough to overlap the $d+{}^2\text{H}$ threshold at 23.8 MeV.

V. CONCLUSION

The results of this study suggest the $P-A$ difference is small and that the conclusions of Ref. 1 are justified. The results also suggest that f waves are not a sensitive level scheme test for $E_p < 4$ MeV and that these components do not significantly increase the $P-A$ difference in the reaction ${}^3\text{H}(p,n){}^3\text{He}$.

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