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πNN vertex operator and πN scattering*

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An approximate πNN vertex operator is used to compute πN phase shifts.

[NUCLEAR REACTIONS Theory of πNN vertex and πN elastic scattering.]

I. INTRODUCTION AND RESULTS

Chew and Low[1] showed that the main features of low-energy pion-nucleon scattering can be understood as arising from a basic πNN vertex of the Yukawa type with the vertex operator for pseudoscalar meson absorption given by

$$\Gamma_{PS} = ig\frac{\vec{\sigma}\cdot\vec{k}}{2M}, \quad \vec{k} = \vec{p} - \vec{q}, \tag{1}$$

where $\vec{p}$ and $\vec{q}$ are the final and initial nucleon momenta and M is the nucleon mass. (For the present discussion, isospin factors are not relevant; they are to be understood as implicit.) However, actual calculations with the vertex operator Γ_{PS} would give divergent or nonsense results because Γ_{PS} is ill-behaved as $k\to\infty$. Therefore, in practice a form factor $v(k)$ is introduced so as to give sensible answers; the vertex operator used is Γ_v

$$\Gamma_v = igv(k)\frac{\vec{\sigma}\cdot\vec{k}}{2M}. \tag{2}$$

Of course, the results depend on $v(k)$, which can be chosen to give theoretical pion-nucleon p-wave phase shifts that agree with the experimental ones.[2] Naturally, the arbitrariness of $v(k)$ has led to discussion of the relative advantages of various forms of $v(k)$.[3] The importance of the form factor v lies in the desire to use a πNN vertex operator in calculations of pion-nucleus interactions.

In a recent article,[4] it was shown that a low-momentum-transfer approximation to the relativistic pseudoscalar Yukawa vertex gives a vertex operator

$$\begin{aligned}
\Gamma_{LMT} &= ig\sigma\cdot\vec{k}_{red}/2\epsilon(K), \\
\vec{k}_{red} &= \vec{k} - \frac{\vec{k}\cdot\vec{K}}{\epsilon(\vec{K})[\epsilon(\vec{K})+M]}\vec{K}, \\
\vec{K} &= \tfrac{1}{2}(\vec{p}+\vec{q}), \\
\epsilon(\vec{K}) &= (\vec{K}^2+M^2)^{1/2}.
\end{aligned} \tag{3}$$

In the nonrelativistic approximation $K/M\to 0$, $\Gamma_{LMT}\to\Gamma_{PS}$. As was noted in Ref. 4, the vertex operator Γ_{LMT} has good convergence properties; the no-pair quantum field theory based on Γ_{LMT} has no divergent integrals.

The vertex operator Γ_{LMT} can be used to compute pion-nucleon scattering phase shifts in the nucleon-ladder approximation with nucleon recoil. The extra convergence that comes from replacing $\vec{k}$ by $\vec{k}_{red}$ and M by $\epsilon(\vec{K})$ in Γ_{PS} suffices to produce sensible phase shifts in this approximation without a cutoff function $v(k)$. The results are shown in Fig. 1. Note that the only parameter is the

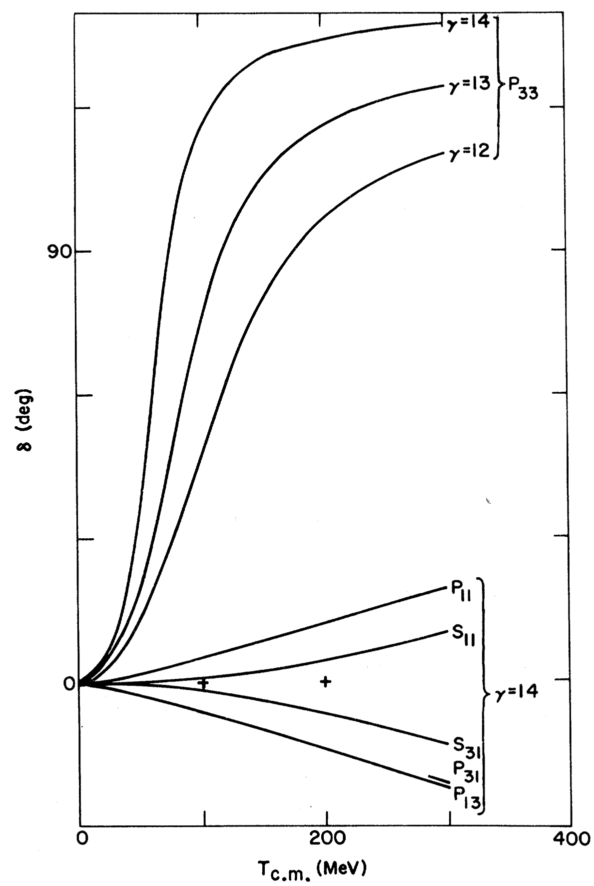

FIG. 1. Calculated πN phase shifts.

coupling constant g, with

$$\gamma = \frac{g^2}{4\pi}. \tag{4}$$

It is evident from Fig. 1 that the vertex operator Γ_{LMT} is able to reproduce the qualitative features of πN phase shifts. That is to say, the $P33$ phase is resonant between 100 and 200 MeV and the other phases have signs that agree with the data and are small. It also seems clear that other processes besides the nucleon-ladder graphs are contributing significantly to πN scattering.

In a recent letter,[5] Noble has compared the energy dependence of the forward (p,π) reaction with that to be expected from the vertex operator Γ_{PS} and the Galilean vertex operator

$$\Gamma_G = ig\frac{\vec{\sigma}}{2M}\cdot\left(\vec{k} - \frac{m}{M}\vec{K}\right), \tag{5}$$

where m is the pion mass. More generally, he shows that if the vertex operator is Γ_α,

$$\Gamma_\alpha = ig\frac{\vec{\sigma}}{2M}\cdot(\vec{k} - \alpha\vec{K}), \tag{6}$$

then the experimental data favor a value of α near zero and, in any case, much smaller than m/M.

For the particular case of forward (p,π) reactions, k_{red} can easily be computed; in the energy range up to 200 MeV considered in Ref. 5, k_{red} is nearly proportional to k, $k_{red} \approx 0.88k - 0.01K$, corresponding to $\alpha \approx 0.01$. Thus, the vertex operator Γ_{LMT} is also consistent with the data on the forward (p,π) reaction.

In summary, the πNN vertex operator Γ_{LMT} seems suited to theoretical considerations of pion-nucleon interactions involving low-momentum pions. It does not require the sort of form factor that is needed to make Γ_{PS} useful and thus avoids the extra parameters that characterize $v(k)$.

II. DETAILS OF THE PHASE-SHIFT CALCULATION

The potential $V_{\Lambda t}(\vec{k},\vec{q})$ that gives the same Born term for going from pion momentum $\vec{q}$ to pion momentum $\vec{k}$ in the center-of-momentum frame as the one that results from the crossed graph with the vertices given by Eq. (3) is

$$V_{\Lambda t}(k,q) = \frac{\gamma_t}{16\pi^2[\omega(k)\omega(q)]^{1/2}}\,\frac{\vec{\sigma}\cdot\vec{q}_{red}\vec{\sigma}\cdot\vec{k}_{red}}{\epsilon(\vec{k}+\frac{1}{2}\vec{q})\epsilon(q+\frac{1}{2}k)[\Lambda - \omega(\vec{k}) - \omega(\vec{q}) - \epsilon(\vec{k}+\vec{q}) + i0]}, \tag{7}$$

where t is the isospin ($\frac{1}{2}$ or $\frac{3}{2}$),

$$\gamma_t = (3t - \tfrac{5}{2})\gamma, \quad \omega(\vec{k}) = (\vec{k}^2 + m^2)^{1/2}, \tag{8}$$

and Λ is the total energy

$$\Lambda = \epsilon(\vec{p}) + \omega(\vec{p}), \tag{9}$$

where $\vec{p}$ is the incident pion momentum. The vectors $\vec{q}_{red}$ and $\vec{k}_{red}$ are given by

$$\vec{q}_{red} = \vec{q} - \frac{\vec{q}\cdot(\vec{k}+\frac{1}{2}\vec{q})}{\epsilon(\vec{k}+\frac{1}{2}\vec{q})[\epsilon(\vec{k}+\frac{1}{2}\vec{q})+M]}(\vec{k}+\tfrac{1}{2}\vec{q}) = \vec{q}(1-B) - 2\vec{k}B, \qquad \vec{k}_{red} = \vec{k}(1-B^T) - 2\vec{q}B^T,$$
$$B = B(k,q,x) = \frac{q(q+2kx)}{4f(k,q,x)[f(k,q,x)+M]}, \qquad B^T = B(q,k,x), \qquad f(k,q,x) = \epsilon(\vec{k}+\tfrac{1}{2}\vec{q}), \qquad x = \vec{k}\cdot\vec{q}/kq. \tag{10}$$

The potential in the tlj partial wave is

$$\tilde{V}_{\Lambda tlj}(k,q) = kq\int Y^{lj\dagger}(\hat{k})V_{\Lambda t}(\vec{k},\vec{q})Y^{lj}(\hat{q})d\Omega_k d\Omega_q, \tag{11}$$

where Y^{lj} is the vector spherical harmonic for the lj partial wave. Standard algebra gives

$$\tilde{V}_{\Lambda tlj}(k,q) = \frac{\gamma_t(kq)^2}{8\pi[\omega(k)\omega(q)]^{1/2}}\int_{-1}^{1}\frac{2F(k,q,x)P_l(x) - G(k,q,x)P_{\bar{l}}(x)}{f(k,q,x)f(q,k,x)[\Lambda - \omega(k) - \omega(q) - f_+(k,q,x) + i0]}dx,$$

$$f_+(k,q,x) = \epsilon(\vec{k}+\vec{q}), \qquad F(k,q,x) = x(1-B)(1-B^T) - \frac{k}{q}B(1-B^T) - \frac{q}{k}B^T(1-B), \tag{12}$$

$$G(k,q,x) = 1 - B - B^T - 3BB^T,$$

and $\tilde{l}$ is the orbital angular momentum "opposite" to l for the given j.

The corresponding potential for the vertex operator Γ_{PS} is obtained by setting $B = B^T = 0$ and replacing $f(k, q, x)$ [but not $f_+(k, q, x)$] by M; in that case, the behavior of the potential as $k \to \infty$ does not allow a well-behaved solution to the Lippmann-Schwinger equation. (A similar result holds when $B = B^T = 0$ with f left as is.)

The Lippmann-Schwinger equation

$$u_p^{tlj}(k) = \delta(p-k) + [E(p) - E(k) + i0]^{-1} \times \int_0^\infty \tilde{V}_{\Lambda tlj}(k,q) u_p^{tlj}(q) dq \qquad (13)$$

with

$$\Lambda = \epsilon(\vec{p}) + \omega(\vec{p}), \qquad E(k) = \omega(k) + \frac{k^2}{2M} \qquad (14)$$

was solved numerically by using the method of Ref. 6 to give the results shown in Fig. 1. Since Γ_{LMT} is a low-momentum-transfer vertex operator, the nonrelativistic nucleon kinetic energy was used in $E(k)$.

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