# Spin assignments for the sub-Coulomb resonances of the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ reaction* 

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Excitation functions of the reaction ${ }^{12} \mathrm{C}\left({ }^{12} \mathrm{C}, \alpha\right){ }^{20} \mathrm{Ne}$ have been measured in $50-\mathrm{keV}$ energy steps from $E_{\text {c.m. }}=4.0-8.0$ MeV at 15 angles between $\boldsymbol{\theta}_{\text {c.m. }}=6^{\circ}$ and $90^{\circ}$. From the angle-integrated cross sections of the first six transitions to ${ }^{20} \mathrm{Ne}$ the following resonance energies have been deduced: $E_{\text {c.m. }}=4.25,4.46,4.62,4.88,5.0,5.64$, 5.92 , and 6.25 MeV . The spin-parity assignments for these resonances have been determined to be $0^{+}$, $4^{+}\left(2^{+}\right), 2^{+}, 2^{+}, 2^{+}, 2^{+}, 4^{+}$, and $2^{+}$, respectively.

$$
\left[\begin{array}{c}
\text { NUCLEAR REACTIONS } \quad{ }^{12} \mathrm{C}\left({ }^{12} \mathrm{C}, \alpha\right), E=4-8 \mathrm{MeV}, \text { measured } \sigma(\theta, E), \text { extracted } \\
\text { resonance energies and } J^{\pi} \text { values. }
\end{array}\right.
$$

The interpretation of the pronounced resonances found in the low-energy excitation functions of the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ reaction ${ }^{1-4}$ is a long-standing problem in heavy-ion physics. Theoretical investigations ${ }^{4-7}$ have been aggravated by the lack of or the poor knowledge of experimental facts. In particular, spin values for most of the resonances were not known experimentally. The exact knowledge of these spin values is, however, of great importance since the comparison of experimental and theoretical values can be used as a test of the assumed model. Thus the knowledge of the resonance spins can probably help to shed light upon the problems connected with the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ resonances.

In this paper we report the measurement of spin values for ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ resonances lying between $E_{\text {c.m. }}=4.25$ and 6.25 MeV . These values have been deduced from angular distributions of the $0^{+}$ground-state transitions $\left(\alpha_{0}\right)$ or the transition to the $4.97-\mathrm{MeV}\left(2^{-}\right)$state in ${ }^{20} \mathrm{Ne}\left(\alpha_{3}\right)$ depending on whether the resonance shows up in the respective excitation function. In cases where both excitation functions (for $\alpha_{0}$ and $\alpha_{3}$ ) exhibit the resonance, both the $0^{+}$and the $2^{-}$angular distributions have been used to determine the actual spin value. The $2^{-}$angular distributions have been chosen since they allow (besides the ground-state angular distribution) the most unambiguous spin determination. In the resonance case only two different $l$ values contribute in the exit channel. For a correct spin determination it is very important to use angular distributions measured at energies as close as possible to the true resonance energy, since it was found during the present investigations that the shape of the angular distributions often changes dramatically within small energy intervals. It is thus impossible to measure simply at those energies reported in the literature since these values are more or less affected by (i) the
energy calibration of the accelerator and (ii) the energy loss of the projectiles in the target (even the ${ }^{12} \mathrm{C}$ buildup on the target plays an important role). Thus, first of all excitation functions had to be measured from which the resonance energies could be deduced.

The measurements have been performed at the Erlangen EN tandem accelerator. Excitation functions have been measured for the $\alpha$-particle exit channels of the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ reaction in energy steps of typically 50 keV between $E_{\mathrm{c} . \mathrm{m} .}=4-8 \mathrm{MeV}$. Using a multidetector array, 15 excitation functions could be measured simultaneously in the angular range $\theta_{\text {c.m. }}=6^{\circ}-90^{\circ}$ (the angular distributions are symmetric about $90^{\circ}$ due to the identity of the nuclei in the entrance channel). Silicon detectors were used to detect the $\alpha$ particles. The detectors were covered with Al foils in order to stop all ${ }^{12} \mathrm{C}$ ions. Their depletion depth was chosen so that the protons from the reaction ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ gave only a small energy signal. For the relative and absolute normalization of the excitation functions both a Faraday cup and two monitor counters (fixed at $\theta_{1 a b}=15^{\circ}$ and $30^{\circ}$ ) were used. The ${ }^{12} \mathrm{C}$ targets had a thickness of $8-15 \mu \mathrm{~g} / \mathrm{cm}^{2}$, which was determined from the elastic scattering at $E_{\text {c.m. }}=4-6 \mathrm{MeV}$ and from the known energy loss of $\alpha$ particles. ${ }^{8}$ The thickness was determined both before and after the exposure to the beam in order to determine the increase of the target thickness due to ${ }^{12} \mathrm{C}$ buildup, too. In comparison with the monitor counters it was found that this buildup proceeds linearly with the beam current.

Excitation functions have been evaluated using a computer program. This program automatically applies corrections to projectile energies and absolute cross sections due to (i) the energy loss in the target and (ii) the gradually increasing target thickness. The code also determines angleintegrated cross sections using a Legendre-poly-


FIG. 1. Angle-integrated excitation functions of the $0^{+}$ ground-state transition, the $4.97-\mathrm{MeV}\left(2^{-}\right)$transition, and the sum of the transitions $0-5$ from the reaction ${ }^{12} \mathrm{C}\left({ }^{12} \mathrm{C}, \alpha\right){ }^{20} \mathrm{Ne}$. The solid lines mark the position of the resonances deduced from the nuclear structure factor $\tilde{S}(E)$ in Fig. 2.


FIG. 2. Nuclear structure factor $\tilde{S}(E)$ for the transitions $0-5$ of the reaction ${ }^{12} \mathrm{C}\left({ }^{12} \mathrm{C}, \alpha\right){ }^{20} \mathrm{Ne}$. On top of the resonances the $J^{\pi}$ values determined in the present work are shown.

TABLE I. Resonance energies and $J^{\pi}$ values determined in the present work compared with previously reported values.

| $E_{\text {c.m. }}(\mathrm{MeV})$ <br> Present <br> work | Ref. 4 | Present <br> work | Ref. 9 | Ref. 2 |
| :---: | :---: | :--- | :--- | :--- |
| 4.25 | 4.22 | $j^{j^{\pi} \text { values }}$ |  |  |
| 4.46 | 4.46 | $4^{+}\left(2^{+}\right)$ |  |  |
| 4.62 | 4.58 | $2^{+}$ |  |  |
| 4.88 | 4.85 | $2^{+}$ |  | $4^{+}$ |
| 5.00 | 5.00 | $2^{+}$ |  |  |
| 5.64 | 5.63 | $2^{+}$ | $2^{+}$ |  |
| 5.92 | 5.98 | $4^{+}$ | $4^{+}$ |  |
| 6.25 | 6.26 | $2^{+}$ |  |  |

nomial fit. From the errors involved in these procedures, the deduced resonance energies are accurate within $\pm 25 \mathrm{keV}$, the absolute value of the cross sections has a relative error of $\pm 15 \%$.

Figure 1 shows angle-integrated excitation functions for the transitions to the $0^{+}$ground state and the $4.97-\mathrm{MeV}\left(2^{-}\right)$state in ${ }^{20} \mathrm{Ne}$. The figure also contains the angle-integrated excitation functions for the first six transitions to ${ }^{20} \mathrm{Ne}$. This curve is just the sum of the angle-integrated excitation functions of transitions $0-5$. From this excitation function the nuclear structure factor $\tilde{S}(E)$ was determined according to Ref. 2:

$$
\begin{equation*}
\sigma(E)=\tilde{S}(E) E^{-1} \exp -(2 \pi \eta+g E) . \tag{1}
\end{equation*}
$$

Figure 2 shows the nuclear structure factor $\tilde{S}(E)$. It exhibits pronounced resonances at $E_{\text {c.m. }}$. $=4.25,4.46,4.62,4.88,5.0,5.64,5.92$, and 6.25 MeV . A structure at $E_{\text {c.m. }}=4.15 \mathrm{MeV}$ is probably not a resonance since among all measured excitation functions only the one for the transition to the $2^{-}$state exhibits a resonance behavior. The widths of the measured resonances in this $\tilde{S}(E)$ representation is typically $\Gamma=60-80 \mathrm{keV}$. Only the peak at $E_{\text {c. m. }}=5.64 \mathrm{MeV}$ has a width which is twice as large. This is probably due to the existence of two unresolved resonances. The measured resonance energies differ only slightly from the values reported in Ref. 4 (see Table I). These differences are, however, very important for the choice of the appropriate angular distribution which determines $J^{\pi}$.

The resonance spins have been determined from angular distributions measured at those energies which are closest to the resonance energies found in the nuclear structure factor $\tilde{S}(E)$. These angular distributions are shown in Fig. 3. The figure also contains calculations which have been performed in order to establish the $J^{\pi}$ values in question. The $0^{+}$angular distributions have


FIG. 3. Angular distributions for the $0^{+}\left(\alpha_{0}\right)$, the $2^{-}$ $\left(\alpha_{3}\right)$, and the $\left(\alpha_{4}+\alpha_{5}\right)$ transitions measured at those energies which are closest to the determined resonance energies. The curves shown are calculations using Eqs. (2) and (4) or simply Legendre polynomials: $P_{l}{ }^{2}$ $=P_{l}{ }^{2}(\cos \theta), \quad \sum^{2,4}=\left|\sum_{l=0}^{2,4}(2 l+1) A_{l} P_{l}(\cos \theta)\right|^{2}$. The coefficients $a_{l}=\left|A_{l}\right|(2 l+1)^{1 / 2}$ and $B_{l}$ used in these calculations are given in Table II.
been fitted with a simple $P_{l}{ }^{2}(\cos \theta)$ distribution or using the expression

$$
\begin{equation*}
\sigma(\theta) \propto\left|\sum_{l=0}^{l_{\max }}(2 l+1) A_{l} P_{l}(\cos \theta)\right|^{2} \tag{2}
\end{equation*}
$$

with the $A_{l}$ being complex coefficients. In the following, the coefficients $a_{l}=\left|A_{l}\right|(2 l+1)^{1 / 2}$ are used as a measure for the mainly contributing partial wave. The $l$ values involved are only even due to the identity of the particles in the entrance channel; $l_{\text {max }}$ was gradually increased until no improvement of the fit could be achieved.
The analysis of the angular distributions for the $2^{-}$transition was performed along the lines worked out in Ref. 9. Assuming a pure resonance reaction the $2^{-}$angular distribution can be described using
the expression (notation of Ref. 10)

$$
\begin{align*}
\sigma\left(\theta ; 2^{-}\right) \propto \sum_{L=0}^{2 J} \sum_{l_{1}^{\prime}, l_{2}^{\prime}=J \pm 1} & \bar{Z}(J J J J ; 0 L) \bar{Z}\left(l_{1}^{\prime} J l_{2}^{\prime} J ; 2 L\right) \\
& \times g_{l_{1}} g_{l_{2}^{\prime}} \Phi\left(l_{1}^{\prime}, l_{2}^{\prime}\right) P_{L}(\cos \theta) . \tag{3}
\end{align*}
$$

Here $l_{1}^{\prime}$ and $l_{2}^{\prime}$ are the two possible $l$ values in the exit channel and $\Phi$ is the Coulomb phase angle. On the other hand, the $2^{-}$angular distribution can also be described using

$$
\begin{equation*}
\sigma\left(\theta ; 2^{-}\right) \propto \sum_{L \text { even }}^{L_{\max }} B_{L} P_{L}(\cos \theta) . \tag{4}
\end{equation*}
$$

Thus, only certain $B_{L}$ values are allowed for each ratio $g_{l_{1}^{\prime}} / g_{l^{\prime}}$ as outlined in Ref. 9. The coefficients $B_{L}$ and $a_{l}$ used to fit the measured angular distributions are displayed in Table II.
For the different resonances the following spin assignments have been made.
4.25 MeV. The angular distributions of all transitions (with the exception of the transition to the $2^{+}$state) show a more or less isotropic behavior (see Fig. 3). The fit of the $0^{+}$angular distribution using Eq. (1) gives a large $a_{0}$ coefficient. Thus $J^{\pi}=0^{+}$. Additional support for this assignment comes from the fact that the $2^{-}$excitation function exhibits a strong minimum at this energy (see Fig. 1).
4.46 MeV. A reasonable fit [Eq. (1)] of the $0^{+}$ angular distribution can be achieved only with

TABLE II. Coefficients $a_{l}=\left|A_{l}\right|(2 l+1)^{1 / 2}$ and $B_{L}$ used to reproduce the angular distributions shown in Fig. 3, using Eqs. (2) and (4).

| $E_{\text {res }}$ <br> $(\mathrm{MeV})$ | $\sum^{2,4}$ | $\sum_{B_{L} P_{L}}$ |
| :---: | :--- | :--- |
| 4.25 | $a_{0}=0.08$ |  |
|  | $a_{2}=0.02$ |  |
| 4.46 | $a_{0}=0.09$ | $B_{0}=1$ |
|  | $a_{2}=0.01$ | $B_{2}=-1$ |
|  | $a_{4}=0.03$ | $B_{4}=0$ |
| 4.62 | $a_{0}=0.09$ |  |
|  | $a_{2}=0.01$ |  |
| 4.88 |  | $B_{0}=1$ |
|  |  | $B_{2}=-1.23$ |
|  |  | $B_{4}=0.23$ |
|  |  | $B_{0}=1$ |
|  |  | $B_{2}=0.91$ |
|  |  | $B_{4}=0.09$ |
|  |  | $B_{0}=1$ |
|  |  | $B_{2}=0.6$ |
|  |  | $B_{4}=-1.6$ |

$l_{\text {max }} \geq 4$ (see Fig. 3). Therefore $J^{\pi}=4^{+}$must be assumed for this resonance. On the other hand, the $2^{-}$angular distribution is consistent with an assumed resonance spin of $J=2$ (see Fig. 3). This discrepancy is not yet clear and thus the following assignment is made for this resonance: $4^{+}\left(2^{+}\right)$.
4.62 MeV. The two fits shown in Fig. 3 are consistent with the assignment $J^{\pi}=2^{+}$.
4.88 MeV. Since the $0^{+}$excitation function does not exhibit a clear resonance, the $2^{-}$angular distribution has been used for the spin determination. It can be fairly well reproduced with Eq. (4) using $B_{L}$ values, which are determined from Eq. (3) (see Fig. 3 and Table II). Therefore, the assignment $J^{\pi}=2^{+}$has been made. This is in contrast to the value $J^{\pi}=4^{+}$determined at $E_{\text {c. } \mathrm{m} .}=4.91 \mathrm{MeV}$ in Ref. 2. This discrepancy is probably due to the fact that the ground-state angular distribution has been used in Ref. 2 for a spin determination. This transition, however, does not exhibit any resonance behavior in the energy range in question (see Fig. 1). On the other hand, a spin determination using the angular distribution for the $2^{+}$transition (as tried in Ref. 2) is more ambiguous than a determination using the angular distribution for the $2^{-}$state.
5.00 MeV . The $0^{+}$angular distribution at 5.03 MeV clearly shows a $P_{2}{ }^{2}(\cos \theta)$ behavior (see Fig. 3). From this the spin-parity value $J^{\pi}=2^{+}$follows.
5.64 MeV. The $J^{\pi}$ value of this resonance has already been determined by Almqvist et al. ${ }^{9}$ They found $J^{\pi}=2^{+}$. This assignment is consistent with our result if we determine $J^{\pi}$ at $E_{\text {c.m. }}=5.62 \mathrm{MeV}$. At $E_{\text {c.m. }}=5.67 \mathrm{MeV}$, however, a large $l=0 \mathrm{com}-$ ponent can be found in the $0^{+}$angular distribution. This is probably a hint for the existence of a
double resonance, as mentioned before.
5.92 MeV. In contrast to Ref. 9, our data show that the ${ }^{20} \mathrm{Ne}$ ground state participates in the resonance. In order to prove that, the measurements have been repeated several times in the $6-\mathrm{MeV}$ energy range. They all gave the same result. In addition it should be noted that excitation functions for the transitions to the $2^{+}, 4^{+}$, and $3^{-} / 1^{-}$states in ${ }^{20} \mathrm{Ne}$ exhibit a pronounced resonance at exactly the same energy so that the resonance observed for the ground-state transition cannot be an unrelated Ericson fluctuation in this particular channel. We have used the ground-state angular distribution for the spin determination. It shows a $P_{4}{ }^{2}(\cos \theta)$ behavior. Thus, the $J^{\pi}$ value must be $J^{\pi}=4^{+}$in agreement with the result from Ref. 9. In addition, it should be noted that the total width of this resonance measured in this work is smaller than given in Ref. 9.
6.23 MeV. Imanishi ${ }^{5}$ has predicted a $J^{\pi}=0^{+}$ value for this resonance. This prediction is in contrast to the experimental results: (i) the $2^{-}$ excitation function exhibits a pronounced maximum at that energy (see Fig. 1); (ii) the $0^{+}$ angular distribution shows a $P_{2}{ }^{2}(\cos \theta)$ behavior (not shown in Fig. 3); and (iii) the $2^{-}$angular distribution at $E_{\text {c.m. }}=6.23 \mathrm{MeV}$ can be reproduced with Eq. (4) using $B_{L}$ values which are determined from Eq. (3) (see Fig. 3). Thus, the $J^{\pi}$ value must be $2^{+}$.
In conclusion, we have determined $J^{\pi}$ values for the sub-Coulomb resonances of the ${ }^{12} \mathrm{C}+{ }^{12} \mathrm{C}$ reaction. They are summarized in Table I. We hope that this new experimental information will be helpful for further theoretical investigations with the aim of explaining the nature of the ${ }^{12} \mathrm{C}$ $+{ }^{12} \mathrm{C}$ resonances.
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