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Three-body calculation of ^{6}Li and d-α elastic scattering

B. Charnomordic, C. Fayard, and G. H. Lamot
Institut de Physique Nucléaire, Université Claude Bernard Lyon-I and IN2P3, 43 Bd du 11 novembre 1918, 69621 Villeurbanne, France
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The Faddeev-Lovelace equations have been solved for the $A = 6$ system with the use of new N-α separable interactions. The ^{6}Li spectrum is given and the (d-α) elastic scattering observables (angular distributions, vector and tensor polarizations) are calculated and compared with recent experimental data. A noticeable improvement on Shanley's results is obtained.

[NUCLEAR REACTIONS ^{6}Li; d-α elastic scattering $E_d = 0$–25 MeV, $d\sigma/d\Omega$, vector and tensor polarizations, three-body calculation; separable interactions.]

I. INTRODUCTION

The Faddeev-Lovelace equations are mainly applied to the three-nucleon system and recently important work has been done concerning the n-d elastic scattering observables. Other physical systems like π-d or d-α can be described by these equations and some work on the π-d system is reported in the literature. For the $A = 6$ system, the three-body model is constructed by considering two nucleons and a structureless α particle. Most of these studies deal with ^{6}Li and ^{6}He bound states and we can mention a recent paper by Ghovanlou and Lehman[1] where an extensive study of the ^{6}He ground state is given.

The d-α system was studied by Shanley[2] in 1969 and we first point out the main features of his work. The isospin states $I = 0$ and $I = 1$ are determined and the d-α scattering is considered. Shanley uses Amado's quasiparticle formalism for the (N-N-α) system. Coulomb effects are neglected and the three particles interact by pairs through simple separable interactions of the Yamaguchi type. The N-N system is described by the $^1S_0$ and $^3S_1$ partial waves while some calculations include the D state of the deuteron. The parametrizations used are those of Phillips.[3] The N-α interaction proceeds through the $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves, and only the $S_{1/2}$ interaction is repulsive. Shanley's fits to the N-α phase shifts were correct only up to 2 MeV, and his calculations lead to a fair fit of d-α elastic scattering observables existing at that time in the 0–30 MeV energy range.

Recently, two experimental groups have extensively studied the d-α scattering observables.
(i) The Los Alamos group[4,5] gives the differential cross sections and the analyzing powers at 12, 14, and 17 MeV,[4] and the analyzing powers and polarization transfer coefficients between 4.8 and 9 MeV.[5]
(ii) The Zürich group[6-11] presents a phase shift analysis of the (d-α) scattering between 3 and 17 MeV starting from the measurement of differential cross sections and vector and tensor analyzing powers. This group also makes some predictions on the polarization transfer coefficients.[11]

Let us mention the recent work of the Berkeley group[12] on the vector analyzing power in ($\vec{d}$ ^{4}He) scattering between 15 and 45 MeV.

It thus appears interesting to test the theoretical calculation with these new experimental results. In some aspects our study is similar to Shanley's. However, we improve the description of the N-N and N-α separable interactions. On the other hand, the exact resolution of the three-body equations is done with the "Padé approximant technique" previously used in (n-d) elastic scattering.

The second part of the paper is devoted to the formalism. We give an explicit description of the coupled integral equation system including the partial antisymmetrization coming from the identity of two among the three particles. We present in Sec. III the N-N and N-α separable interactions which we have constructed. The results are presented and discussed in Sec. IV: (i) the ^{6}Li spectrum obtained with our interactions; (ii) The (d-α) elastic scattering observables. Primarily, we study the effect of the two-body parametrizations on (d-α) observables. For this purpose we compare our interaction results obtained at E_d(lab) = 12 MeV with those given by the simpler interactions of Shanley and Ghovanlou-Lehman. We then present an extensive study of the angular distribution and vector and tensor polarizations given by our new interactions in the 0–25 MeV energy range. The last part contains our conclusions while the three appendices define the Born term and the numerical method used in this paper.

II. THEORY

A. Three-body formalism

We consider three pairwise interacting particles with masses m_i, m_j, and m_k, and we take the usual variables:

$$\vec{p}_i = -\rho_i \vec{q}_i - \vec{q}_j, \quad \rho_i = m_j/(m_j+m_k),$$
$$\vec{p}_j = \vec{q}_i + \rho_j \vec{q}_j, \quad \rho_j = m_i/(m_k+m_i), \tag{1}$$

where $\vec{p}_i$ is the relative momentum of the (jk) pair and $\vec{q}_i$ the momentum of particle i relative to the (jk) subsystem. We also define the corresponding reduced masses:

$$\mu_i^{-1} = m_j^{-1} + m_k^{-1}, \quad \mathfrak{M}_i^{-1} = m_i^{-1} + (m_j+m_k)^{-1}. \tag{2}$$

The Alt, Grassberger, and Sandhas (AGS)[13] equations for the three-body process $j+(ki) \to i+(jk)$ are written as

$$U_{ij}(s) = (1-\delta_{ij})(s-H_0) + \sum_{k=1}^{3} (1-\delta_{ik}) T_k(s) G_0(s) U_{kj}(s). \tag{3}$$

Here, s refers to total energy, U_{ij} are the transition operators, T_k the two-body scattering amplitude for particles i and j, and G_0 the resolvent of the free Hamiltonian:

$$G_0(s) = (s-H_0)^{-1}.$$

By choosing separable two-body interactions, the corresponding scattering amplitudes are made separable, and we get

$$V_i = \sum_n |in\rangle \lambda_{in} \langle in|, \tag{4a}$$

$$T_i(s) = \sum_n |in\rangle R_{in}(s) \langle in|, \tag{4b}$$

where n stands for the interacting pair quantum numbers. Using the separable form (4b) in Eq. (3), some classical algebra[13] leads to the equation for the three-body scattering amplitudes $X_{in,jm}$:

$$X_{in,jm}(s) = Z_{in,jm}(s) + \sum_{k,r} Z_{in,kr}(s) R_{kr}(s) X_{kr,jm}(s). \tag{5}$$

Here, R_{kr} is the propagator for the (ij) pair in the three-body Hilbert space and the Born term is given by

$$Z_{in,jm}(s) = (1-\delta_{ij}) \langle in | G_0(s) | jm \rangle. \tag{6}$$

We now move on to the d-α system and we follow the procedure of Afnan and Thomas[14] for the π-d system. Particle labels 1 and 2 will always refer to nucleons, while the α is particle 3. We must consider the three following processes:

$$\begin{aligned} 3+(12) &\to 3+(12) \\ &\to 1+(23) \\ &\to 2+(13). \end{aligned} \tag{7}$$

The first one corresponds to d-α elastic scattering, while the two others are settings for $d+\alpha \to N+(N\alpha)$. Taking into account Eq. (6), one can write the corresponding three-body equations:

$$\begin{aligned} X_{3n,3m} &= \sum_\alpha Z_{3n,1\alpha} R_{1\alpha} X_{1\alpha,3m} + \sum_\alpha Z_{3n,2\alpha} R_{2\alpha} X_{2\alpha,3m}, \\ X_{1\alpha,3m} &= Z_{1\alpha,3m} + \sum_\beta Z_{1\alpha,2\beta} R_{2\beta} X_{2\beta,3m} + \sum_n Z_{1\alpha,3n} R_{3n} X_{3n,3m}, \\ X_{2\alpha,3m} &= Z_{2\alpha,3m} + \sum_\beta Z_{2\alpha,1\beta} R_{1\beta} X_{1\beta,3m} + \sum_n Z_{2\alpha,3n} R_{3n} X_{3n,3m}. \end{aligned} \tag{8}$$

The n and m labels refer to $(N$-$N)$ pairs, and α and β to $(N$-$\alpha)$ pairs. The identity of nucleons 1 and 2 leads to symmetry properties for the Born terms:

$$\begin{aligned} Z_{1\alpha,3m} &= -Z_{2\alpha,3m}, \\ Z_{1\alpha,2\beta} &= Z_{2\alpha,1\beta}, \end{aligned} \tag{9}$$

and allows us to define the $(N$-$\alpha)$ propagator as

$$R_\alpha = R_{1\alpha} = R_{2\alpha}. \tag{10}$$

Using Eqs. (9) and (10), the three coupled equations (8) can be combined into the following new set:

$$\begin{aligned} X_{nm} &= \sum_\alpha Z_{n\alpha} R_\alpha X_{\alpha m}, \\ X_{\alpha m} &= Z_{\alpha m} + \sum_\beta Z_{\alpha\beta} R_\beta X_{\beta m} + \sum_n Z_{\alpha n} R_n X_{nm}, \end{aligned} \tag{11}$$

where we have introduced the antisymmetric amplitudes

$$\begin{aligned} X_{nm} &= \tfrac{1}{2} X_{3n,3m}, \\ X_{\alpha m} &= \tfrac{1}{2}(X_{1\alpha,3m} - X_{2\alpha,3m}). \end{aligned} \tag{12}$$

The Born terms and propagators in Eq. (11) are

defined as

$$\begin{aligned} Z_{\alpha\beta} &= -Z_{1\alpha,2\beta}\,, \\ Z_{n\alpha} &= Z_{3n,1\alpha}\,, \\ Z_{\alpha m} &= Z_{1\alpha,3m}\,, \\ R_n &= 2R_{3n}\,. \end{aligned} \tag{13}$$

We notice that it will be more convenient to write Eq. (11) in the compact form

$$X_{ij} = Z_{ij} + \sum_k Z_{ik} R_k X_{kj}\,. \tag{14}$$

This last equation will be solved to obtain the physical scattering amplitudes $X_{3n,3m} = 2X_{nm}$.

B. One-dimensional equations

On the next step, we have to write Eq. (14) in three-body Hilbert space. We can thus use the results obtained in the case of three interacting nucleons (i,j,k), where we assume a cyclic permutation of the (i,j,k) labels. We define the spin and isospin coupling schemes as follows:

$$\begin{aligned} &\vec{s}_j + \vec{s}_k = \vec{S}_i\,, \quad \vec{s}_i + \vec{J}_i = \vec{\Sigma}_i\,, \quad \vec{i}_j + \vec{i}_k = \vec{I}_i\,, \\ &\vec{L}_i + \vec{S}_i = \vec{J}_i\,, \quad \vec{l}_i + \vec{\Sigma}_i = \vec{J}\,, \quad \vec{i}_i + \vec{I}_i = \vec{I}\,. \end{aligned} \tag{15}$$

The particles i, j, and k have spin (isospin) s_i, s_j, and s_k (i_i, i_j, and i_k), and $J_i(I_i)$ is the total spin (isospin) of the (jk) subsystem, the orbital momentum of which is denoted by L_i. $J(I)$ is the total spin (isospin) of the three-body system, while Σ_i is the channel spin and l_i the orbital angular momentum of particle i relative to the (jk) pair. Using the usual procedure for angular momentum reduction,[15] we obtain the one-dimensional integral equation describing the process $j + (ki) \to i + (jk)$,

$$X^{JI}_{\tau_i\tau_j}(q_i,q_j,s) = Z^{JI}_{\tau_i\tau_j}(q_i,q_j,s) + \sum_k \sum_{\tau_k} \int_0^\infty dq_k q_k^2 Z^{JI}_{\tau_i\tau_k}(q_i,q_k,s) R_{c_k}(s - q_k^2/2\mathfrak{M}_k) X^{JI}_{\tau_k\tau_j}(q_k,q_j,s)\,; \tag{16}$$

s is the c.m. total energy, $c_i = \{J_i S_i I_i\}$ are the quantum numbers for the (jk) pair, and the $\tau_i = \{c_i l_i \Sigma_i\}$ specify the $i(jk)$ three-body channel.

The partial wave Born amplitudes have the explicit form

$$\begin{aligned} Z^{JI}_{\tau_i\tau_j}(q_i,q_j,s) = (1-\delta_{ij})^{\frac{1}{2}} (-)^{2s_k+s_j+S_j+J} \begin{bmatrix} J_j & S_j & \Sigma_j & l_j \\ J_i & S_i & \Sigma_i & l_i \end{bmatrix} C(I,I_iI_j) \\ \times \sum_{L_iL_j} [L_iL_j](-)^{L_i} \int_{-1}^{+1} du \frac{C_{c_iL_i} g_{c_iL_i}(p_i) g_{c_jL_j}(p_j) C_{c_jL_j}}{p_i^{L_i} i(s - q_i^2/2\mu_j - q_j^2/2\mu_i - q_iq_ju/m_k) p_j^{L_j}} G(q_i,q_j,u)\,, \end{aligned} \tag{17}$$

where $u = \cos(\hat{q}_i\hat{q}_j)$, $[x] = (2x+1)^{1/2}$, and p_i and p_j are expressed in terms of q_i, q_j, and u by Eq. (1). The isospin coefficient $C(I,I_iI_j)$ and the function $G(q_i,q_j,u)$ can be found in Appendix A.

In Eq. (17), $C_{c_iL_i}$ and $g_{c_iL_i}(p_i)$ represent the strength and form factor of the separable potential describing the (jk) pair in the c_i partial wave. In fact, Eq. (17) stands for rank-1 separable interactions. However, a matrix notation allows a straightforward extension for the case of higher rank interactions.

In the d-α system, the α particle (label 3) must be distinguished from the two nucleons, so that the coupling order must be the same in the two $(N$-$\alpha)$ pairs as written in Eq. (7). We have therefore to deal with noncyclic Born amplitudes which are evaluated in Appendix B.

C. Bound-state and scattering calculations

For practical calculation, we limit the number of two-body partial waves. In the N-α system we include only the $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves, and in the N-N system we take into account the $^1S_0$ and coupled $^3S_1$-$^3D_1$ partial waves. The parametrizations used will be described in Sec. III. We are now in a position to specify the quantum numbers of the various channels occurring in Eq. (16). In Table I, we set the possible values for $\tau = \{L'S'J'I'l\Sigma\}$ corresponding to a given value of the total three-body angular momentum J. Set A refers to positive (negative) parity if J is even (odd), and reciprocally for Set B. Parity conservation leads to two independent sets of equations. We can note here that, due to conservation of isospin, the $^3S_1$-$^3D_1$ partial waves of the N-N interaction contribute only if the total isospin I of the system is 0 (e.g., for d-α elastic scattering), and the $^1S_0$ wave only if $I = 1$.

The binding energies of the $(1^+, 0)$ ground state and $(0^+, 1)$ excited state of ^{6}Li are obtained by searching the zero value of the Fredholm determinant of Eq. (16) written in matrix form for

given J^π and I values. The continuum states (above the d-α elastic threshold) with total isospin 0 are studied by performing the phase shift analysis of d-α elastic scattering.

Evaluation of the d-α elastic scattering observables requires the knowledge of the physical scattering amplitudes which are the on-shell solutions of Eq. (16) with $I=0$ and $\tau_i, \tau_j = \tau_1, \tau_2$ (cf. Table I) for each J^π value. We have to solve a system of at most 10 coupled channels, and the classical method of matrix inversion cannot be used. As in the n-d case, there are two possibilities: we can solve exactly a part of the system, the remaining part being treated in a perturbative scheme[16]; we can also use the Padé approximant technique which allows us to take into account all the channels exactly.[17] We shall employ this last method, and some details are given in Appendix C. According to Table I, we get five physical amplitudes (four of them being independent) when $J>0$, and only one when $J=0$. From the physical amplitudes we construct the scattering matrix $M(\theta)$ in the channel spin representation:

$$\langle \Sigma_i \nu_i | M(\theta) | \Sigma_j \nu_j \rangle = \sum_{J \mu l_i l_j} X^J_{\tau_i \tau_j}(q_i, q_j, s) \mathcal{Y}^*_{(l_i \Sigma_i) J \mu}(\hat{q}_i) \mathcal{Y}_{(l_j \Sigma_j) J \mu}(\hat{q}_j), \tag{18}$$

where $\mathcal{Y}$ is the usual coupled spherical harmonic, and $q_i = q_j = [\frac{8}{3}(s+E_D)]^{1/2}$ (E_D is the deuteron binding energy).

The channel spin Σ is 1, and Eq. (18) thus defines a (3×3) matrix. For the practical calculation, we use the Madison convention[18] and take $l_i(l_j)=8$ as the upper limit. The differential cross section $\sigma(\theta)$ and analyzing powers T_{kq} are finally evaluated, using the following traces of matrix products:

$$\sigma(\theta) = \mathrm{Tr}\{M(\theta) M^+(\theta)\}, \qquad T_{kq} = \mathrm{Tr}\{M(\theta)\tau_{kq} M^+(\theta)\}/\sigma(\theta), \tag{19}$$

where the τ_{kq} are the spherical tensor operators relative to the deuteron.

III. TWO-BODY INTERACTIONS

The partial wave expansion of a two-body interaction V is given, for the coupled case, by

$$\langle \vec{p}| V | \vec{p}' \rangle = \sum_{cLL'\mu} i^{L'-L} \mathcal{Y}^*_{(LS)J\mu}(\hat{p}) V^c_{LL'}(p,p') \times \mathcal{Y}_{(L'S)J\mu}(\hat{p}') P_I . \tag{20}$$

Here, $c=\{JSI\}$ describes the quantum numbers and P_I projects onto the isospin subspace. The separable approximation concerns the "radial" part $V^c_{LL'}(p,p')$. For a rank-1 interaction, we write it

$$V_{LL'}(p,p') = g_L(p) C_L \Lambda C_{L'} g_{L'}(p'). \tag{21}$$

C_L and $C_{L'}$ are strength parameters, Λ is $+1$ (-1) when the corresponding partial wave is repulsive (attractive), and $g_L(p)$ and $g_{L'}(p')$ are the form factors. In the uncoupled case, we only set $L=L'$ in Eq. (20).

We perform a similar expansion for the two-body T matrix and, according to Eq. (21), we get

$$T_{LL'}(p,p',s) = g_L(p) C_L R(s) C_{L'} g_{L'}(p'). \tag{22}$$

The two-body propagator $R(s)$ has the form

$$R(s) = [\Lambda^{-1} - G(s)]^{-1} \tag{23}$$

TABLE I. Possible values of the quantum numbers for a given value of the total three-body angular momentum J.

	$(^3S_1$-$^3D_1)\alpha$		$(^1S_0)\alpha$	$(S_{1/2})N$		$(P_{1/2})N$			$(P_{3/2})N$		
τ	1	2	3	4	5	6	7	8	9	10	11
L'	0,2		0	0		1			1		
S'	1		0	$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{2}$		
J'	1		0	$\frac{1}{2}$		$\frac{1}{2}$			$\frac{3}{2}$		
I'	0		1	$\frac{1}{2}$		$\frac{1}{2}$			$\frac{1}{2}$		
Σ_A	1		0	0	1	1	1	1	1	2	2
l_A	J[a]		J	J	J[a]	$J+1$	$J-1$[a]	$J+1$	$J-1$[a]	$J+1$[a]	$J-1$[b]
Σ_B	1	1		1	1	0	1	1	2	2	2
l_B	$J-1$[a]	$J+1$		$J+1$	$J-1$[a]	J	J[a]	J[a]	$J+2$	J[a]	$J-2$[b]

[a] These cases do not appear for $J=0$.
[b] These cases do not appear for $J=0,1$.

with

$$G(s)=\sum_i C_i^2 \int_0^\infty dq\, q^2 \frac{g_i^2(q)}{s-q^2}. \quad (24)$$

We note that the above expressions can be easily extended to the case of rank >1 separable potentials by introducing a matrix formulation.[19]

A. Nucleon-α interactions

One can find many papers describing rank-1 separable potentials for $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves. All of them use form factors of the Yamaguchi type:

$$g_L(p)=p^L/(p^2+\beta^2)^{(L+m)/n} \quad (25)$$

with $(m,n)=(1,2)$, $(1,1)$, or $(2,2)$. These different forms are labeled A, B, and C in the Ghovanlou-Lehman paper,[1] where the situation is clearly restated.

In our calculations, we use two existing parametrizations elaborated by Shanley[2] (denoted S) and by Ghovanlou-Lehman[1] (GL), and a third one (CFL) which we have constructed, as will be described hereafter.

We give firstly some comments about the S and GL interactions. Shanley introduces a separable one-term interaction for each of the $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves, with parameters chosen to fit an effective range analysis of low energy n-α scattering carried out by Pearce and Swan.[20] The form factors are given by Eq. (25) with $(m,n)=(1,1)$. The $S_{1/2}$ interaction is repulsive so as to simulate the Pauli exclusion principle, and for P waves Shanley gives priority in fitting the lower energy in order to reproduce the $P_{3/2}$ resonance at 1.3 MeV.

In their ^{6}He ground-state calculations, Ghovanlou and Lehman use various combinations of existing interactions, and also construct new one-term $S_{1/2}$ and $P_{1/2}$ parametrizations referring to Arndt *et al.*[21,22] experimental phase shifts. We denote as GL the following combination: the $S_{1/2}$ wave is from Shanley,[2] the $P_{3/2}$ parameters are from Alessandrini *et al.*,[23] and the $P_{1/2}$ interaction is from Ghovanlou and Lehman[1] (Table II, form B in Ref. 1). In fact, it is clear that the S and GL parametrizations cannot give theoretical phase shifts which are in good agreement with n-α experimental data in all the energy range, because there are only two parameters in each partial wave. Moreover, a crucial problem consists of the choice among the existing experimental phase shifts which have to be fitted in order to get the potential parameters. We therefore construct new one-term separable interactions (denoted CFL) for $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ partial waves. The form factors are similar to those introduced by Doleschall[24] and have proper threshold and asymptotic behavior in momentum space:

$$g_L(p)=p^L(1+\gamma_L p^2)\Big/\prod_{j=0}^{L+1}(1+\beta_{jL}p^2). \quad (26)$$

The β and γ parameters (Table II) are determined by a least squares fit of the experimental data. We choose to fit the phase shifts obtained by Satchler *et al.*[25] from a real optical model potential of Woods-Saxon form fitting a large number of experimental neutron-α phase shifts below 20 MeV. Figure 1 shows the comparison between CFL, S, and GL phase shifts. Clearly, the CFL interaction, which is overparametrized, lead to the best fit of Satchler's data for each partial wave in all the energy range.

B. Nucleon-nucleon interactions

In our calculations, we only take into account the $^1S_0$ and coupled $^3S_1$-$^3D_1$ partial waves.

(i) In the $^1S_0$ partial wave, the separable potential must reproduce the experimental phase shift and the singlet low-energy parameters a_s (scattering length) and r_s (effective range). The usual one-term or two-term interactions use form factors of Yamaguchi type [Eq. (25)] and lead in fact to a very poor description of the $^1S_0$ channel. By example, the one-term Yamaguchi potential $(m,n=1,1)$ is purely attractive and the $^1S_0$ phase shift is always positive. Moreover, the two parameters C_L and β are fixed as soon as we choose the a_s and r_s values, and the phase shift is therefore also fixed and has correct values only at low energy. We will use a Yamaguchi n-p potential (denoted Y_{np}) corresponding to Shanley's singlet interaction with the values $a_s=-20.34$ fm and $r_s=2.7$ fm. Some people have then constructed two-term potentials as a superposition of attractive and repulsive parts with the corresponding form factors as given in Eq. (25). The four parameters are determined by minimization referring to the low-energy parameters and to the experimental phase shift. In fact, the number of parameters is still too small, and the results are better than Yamaguchi's only in the sense that the $^1S_0$ phase shift goes through the zero value at about 250 MeV (see for example Ref. 26).

TABLE II. CFL parameters for the N-α interaction. The strength C_L^2 is in MeV fm^{3+2L}, γ and β in fm^2.

	ΛC_L^2	γ_L	β_{1L}	β_{2L}	β_{3L}
$S_{1/2}$	8294	5.071	2.988	2.981	
$P_{1/2}$	−67.917	31.521	0.775	0.808	23.541
$P_{3/2}$	−43.540	6.245	1.535	0.001	4.985

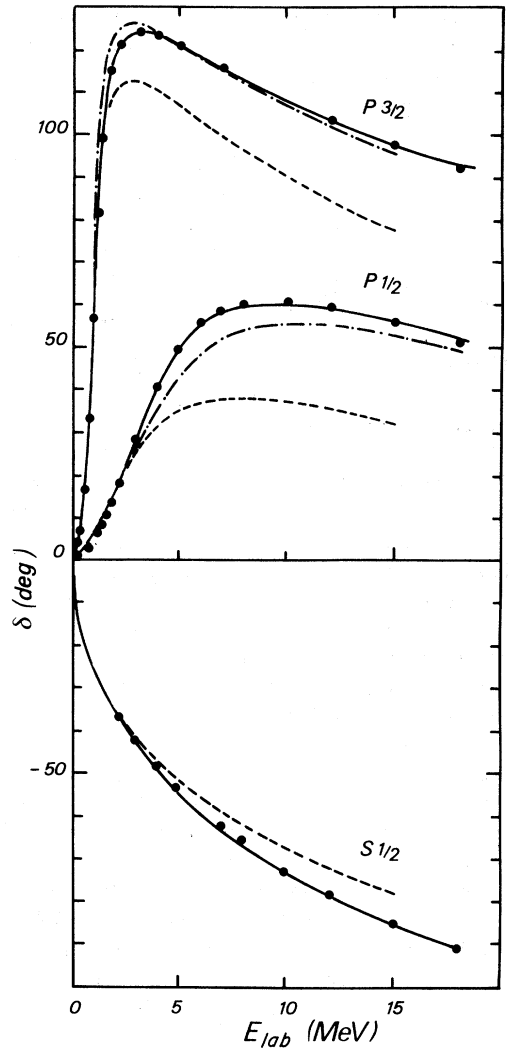

FIG. 1. Theoretical N-α phase shifts given by the parametrizations CFL (—), S (---), and GL (-·-·-). The experimental values are from Ref. 25.

In order to improve this situation, we have therefore constructed a new two-term potential (denoted CFL$_{np}$) with the following attractive (A) and repulsive (R) form factors:

$$g^A(p) = \frac{1+\gamma^A p^2}{(1+\beta_1^A p^2)(1+\beta_2^A p^2)}\,,$$
$$g^R(p) = \frac{\gamma^R p^2}{(1+\beta_1^R p^2)(1+\beta_2^R p^2)}\,. \qquad (27)$$

The parameters (Table III) are determined by minimization in such a way that the repulsive part of the interaction contributes only as a corrective factor at small p values where the attractive part must predominate. For the minimization procedure we refer to the McGregor, Arndt, and Wright[27] experimental phase shifts and we prescribe as constraints the n-p low-energy parameters $a_{np} = -23.671$ fm, $r_{np} = 2.74$ fm. We get a $^1S_0$ phase shift which is very close to experiment up to 200 MeV, as can be seen in Table IV.

TABLE III. CFL$_{np}$ parameters for the $^1S_0$ partial wave. The strength C^2 is in MeV fm^3, γ and β are in fm^2.

	ΛC^2	γ	β_1	β_2
Attractive	−38.651	24.420	0.4532	25.055
Repulsive	2.261	1	0.0692	0.0693

(ii) For the $^3S_1$-$^3D_1$ coupled partial waves, the situation is much more difficult, because the interaction must reproduce the $^3S_1$ and $^3D_1$ phase shifts, the mixing parameter ϵ_1, the deuteron properties and the triplet low-energy parameters a_t and r_t. In fact, separable interactions do not yet give a correct description simultaneously of all aspects of the $^3S_1$-$^3D_1$ state, and there only exist interactions describing some of the properties. In this paper, we will use the parametrizations of Doleschall[17] (rank-1, one term), Serduke[19] (rank-2, one term), and Mongan[26] (rank-2, two terms). We now briefly recall some properties of these rather simple interactions. All of them use form factors of Yamaguchi type [Eq. (25)]. The rank-1 parametrization is the most simple and gives therefore a rough description of the properties: the $^3S_1$ phase shift is always positive and agrees with experiment only at low energy, the $^3D_1$ phase shift has the wrong sign, and the mixing parameter is badly reproduced. The interaction denoted Y7, with D-state percentage value $P_D = 7\%$, is extracted from Ref. 17 and corresponds to Shanley's tensor potential. The rank-2, one-term parametrizations of Serduke give good deuteron properties and low-energy parameters, with P_D values in the range 1% to 7%. These potentials will be denoted ACSx

TABLE IV. $^1S_0$ phase shift for CFL$_{np}$ interaction ($a_{np} = -23.671$ fm, $r_{np} = 2.74$ fm). Experiment is from Ref. 27.

E_{lab} (MeV)	$^1S_0$ phase shifts (deg) Experiment	CFL
1	62.41	62.10
10	60.85	61.44
20	54.79	55.53
30	50.10	50.66
50	43.02	43.01
100	30.83	30.14
160	20.04	20.04
200	13.96	14.91
280	3.43	6.73
360	−5.49	0.23
420	−11.37	−3.96

where x specifies the P_D value. Compared with Y7, the main advantage of the ACS parametrizations is to give a negative $^3D_1$ phase shift. At last, the rank-2, two term potentials of Mongan provide good $^3S_1$ and $^3D_1$ phase shifts in the energy range 0 to 200 MeV, but the mixing parameter is rather unrealistic since it becomes negative above 150 MeV, and the P_D value is too small ($<1.4\%$). We shall use the type IV parametrization of Ref. 26 denoted M1.4.

(iii) To check the influence of the tensor force in the d-α system, we also performed some calculations by neglecting the $^3D_1$ wave, the $^3S_1$ wave being described by a two-term Mongan interaction (Ref. 26). This parametrization (denoted M0) ensures the deuteron binding energy and triplet low-energy parameters, and gives good $^3S_1$ phase shift up to 200 MeV.

We end this section with Table V where we give the N-N and N-α interactions mainly used in our study. The name of the calculation is written as follows: the first label denotes the N-N parametrization and the second one refers to the N-α parametrization.

IV. RESULTS AND DISCUSSIONS

A. ^{6}Li levels

Our purpose is not to do an exhaustive study of the (N-N-α) bound system, but only to give results obtained with some interactions described in Sec. III. In our calculations, the energy reference is fixed at the breakup threshold of the three particles (α-n-p). We choose the following N-N and N-α parametrizations given in Table V: for $I=0$ levels, we use the Y7-S and ACS4-CFL potentials, and for the $I=1$ level, we consider the Y_{np}-S and CFL_{np}-CFL interactions. The bound states and continuum states are obtained as described in Sec. II C.

In a first step, we determine the binding energies of the $(1^+,0)$ ground state and of the $(0^+,1)$ excited state of ^{6}Li. In order to check our calculations, we use the Y7-S and Y_{np}-S parametrizations which are almost identical to those of Shanley. With Y7-S, we find the ground state at an energy of 0.68 MeV, and with Y_{np}-S, we obtain the $(0^+,1)$ level at 3.69 MeV relative to the experimental ground state, while the corresponding values of Shanley are 0.53 MeV and 3.59 MeV. Using the ACS4-CFL and CFL_{np}-CFL parametrizations, we obtain respectively 0.51 MeV for the ground state and 3.69 MeV for the $(0^+,1)$ state. A more detailed study concerning the sensitivity of these two levels to the N-N and N-α parametrizations can be found in Ref. 28. It is shown that the binding energy of the ground state decreases when the D-state percentage value increases, and that all N-α partial waves play an important role. In particular, the contribution of each N-α partial wave to the binding energy of the $(0^+,1)$ level is found to be similar to those given by Ghovanlou-Lehman for ^{6}He ground state (cf. Table V of Ref. 1). In a second step, we perform the phase shift analysis of d-α elastic scattering in order to get the levels of ^{6}Li with isospin $I=0$ located above the d-α elastic threshold. With the ACS4-CFL interaction, we find a triplet of resonances D_3, D_2, and D_1 at 2, 4.4, and 6.6 MeV deuteron energy values in the lab system. These values corre-

TABLE V. N-N and N-α parametrizations mainly used in our calculations. The parameters are given in the quoted references, and the labels are defined in the text.

Isospin of the system	Name of the calculation	$^1S_0$	$^3S_1$-$^3D_1$	$S_{1/2}$	$P_{1/2}$	$P_{3/2}$
$I=0$	Y7-S				Shanley (Ref. 2)	
	Y7-GL		Doleschall (Ref. 17)	Shanley (Ref. 2)	Ghovanlou -Lehman (Ref. 1)	Alessandrini *et al.* (Ref. 23)
	Y7-CFL					
	ACS4-CFL		Serduke (Ref. 19)		CFL	
	M0-CFL		Mongan (Ref. 26)			
$I=1$	Y_{np}-S	Yamaguchi			Shanley (Ref. 2)	
	CFL_{np}-CFL	CFL			CFL	

spond to levels located at 2.81, 4.41, and 5.88 MeV above the experimental ground state. The corresponding results obtained by Shanley without tensor force are 2.35, 5.8, and 7.5 MeV.

In Fig. 2, we compare the experimental low-lying levels of ^{6}Li with two theoretical spectra, one extracted from Shanley's paper (without tensor force), and the other corresponding to our ACS4-CFL calculation. We must point out the fact that the theoretical results do not take into account the Coulomb energy correction which is repulsive of about 0.8 MeV. Concerning the $(0^+, 1)$ level, our model gives a too small binding energy, and if we substract the Coulomb correction, the predicted level for ^{6}Li is not bound. In other words, the ground state binding energy of ^{6}He is predicted to be too small (~0.01 MeV) compared with experiment (0.969 MeV). The two theoretical spectra of the $I=0$ levels are shifted in the upward direction, and this will be enhanced by the Coulomb correction. Nevertheless, the energy spacing between the levels is correctly reproduced, except for the $(3^+, 0)$ and $(2^+, 0)$ states. However, the ACS4-CFL results for the $(2^+, 0)$ and $(1^+, 0)$ continuum states appear to be better than those of Shanley which are strongly shifted in the upward direction with a too large spacing. For this reason, we shall mainly use the ACS4-CFL interactions in the study of $(d$-$\alpha)$ elastic scattering observables.

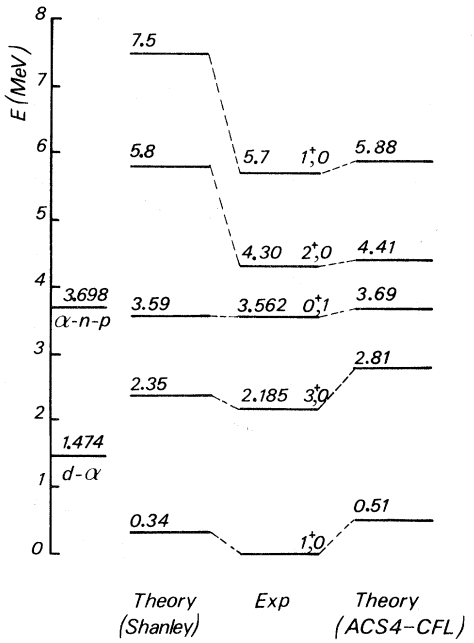

FIG. 2. Energy levels of ^{6}Li. The experimental levels are from Ref. 32. Shanley's results are without tensor force (Ref. 2).

B. $(d$-$\alpha)$ scattering observables

To our knowledge, the only theoretical results concerning a three-body model of the $(d$-$\alpha)$ scattering are due to Shanley.[2] This calculation uses very simple separable interactions and does not clarify the influence of the parametrizations on the $(d$-$\alpha)$ observables, except for the effect of the tensor force. We have then focused our interest on the influence of the two-body description on the $(d$-$\alpha)$ observables calculated with 12 MeV incident deuterons. It is thus possible to choose a correct parametrization of N-α and N-N interactions. We then study the behavior of angular distribution and polarizations in the 0–25 MeV energy range where numerous experimental data are now available, and we examine some possible improvements.

1. Effects of two-body parametrizations

In a first step, we tried to clarify the effect of the tensor force on the d-α observables. For that purpose various calculations were performed with the same N-α interaction (CFL) and different $^3S_1$-$^3D_1$ potentials with various P_D values (cf. Table V): Y7, ACS1, ACS4, ACS7, and M1.4. In one calculation, we neglect the tensor force by considering the only $^3S_1$ wave (M0). All these calculations lead to results which appear to be not fundamentally different, the more sensitive observable being the iT_{11} vector analyzing power. The effect of the tensor force is small for the tensor polarizations, but is important in the forward part of iT_{11} and in the backward part of the angular distribution, where the amplitude increases with the P_D value, as was reported by Shanley. Moreover, ACS1 and M1.4 results (which are located between M0 and ACS4 results) are almost identical, and so are Y7 and ACS7 results, which indicates a small sensitivity of observables to the description of the tensor force for given P_D value. We give in Fig. 3 some significant results concerning the iT_{11} and T_{22} analyzing powers obtained with ACS7, ACS4, and M0 interactions. We conclude that the presence of tensor force is not essential to get the differential cross section and the analyzing powers in d-α scattering. Nevertheless, we noticed a greater sensitivity of the polarization transfer coefficients to the tensor force, and more experimental data would make a further study possible.

The second step of our study at 12 MeV concerned the influence of the parametrizations of $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ N-α partial waves. Three N-α parametrizations are tested (S, GL, and CFL) with the same N-N parametrization (Y7). The re-

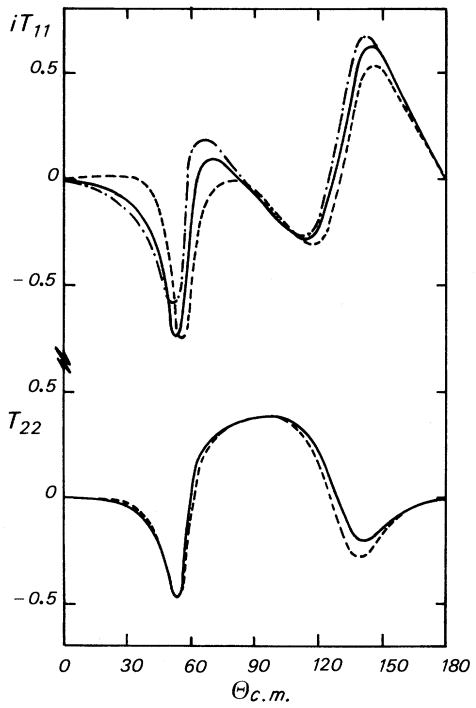

FIG. 3. Influence of the N-N tensor force on iT_{11} and T_{22} at 12 MeV. The N-α interaction is CFL, and the N-N parametrizations are ACS4 (—), ACS7 (---), and M0 (-•--•). For T_{22}, ACS4, and ACS7 results are almost identical.

sults concerning $d\sigma/d\Omega$, iT_{11}, and T_{22} are shown in Fig. 4. Important discrepancies appear in the results, the most obvious effect being exhibited on T_{22} between 40° and 90° (and so for T_{20}). As the tensor force is the same in the three calculations, we can attribute these discrepancies to the differences between the used N-α parametrizations. If we refer to Fig. 1, the large difference between Y7-S and Y7-CFL results is easily understood. However, even if the GL N-α phase shifts are closer to CFL phase shifts than S (Fig. 1), the Y7-GL and Y7-CFL results are still different, and this points out the great sensitivity of d-α elastic scattering observables to the description of N-α partial waves. Moreover, it appears in Figs. 5–9 that the calculation with ACS4-CFL at 12 MeV is in good agreement with experimental data and this is a good indication that CFL parametrizations are more suitable to d-α scattering calculation than S or GL.

2. *Evolution of observables with energy*

We have thus used these N-α interactions (CFL) and ACS4 for the $^3S_1$-$^3D_1$ wave. The calculation has been performed for eight energy values, and in Figs. 5 to 9 is reported a comparison of the theo-

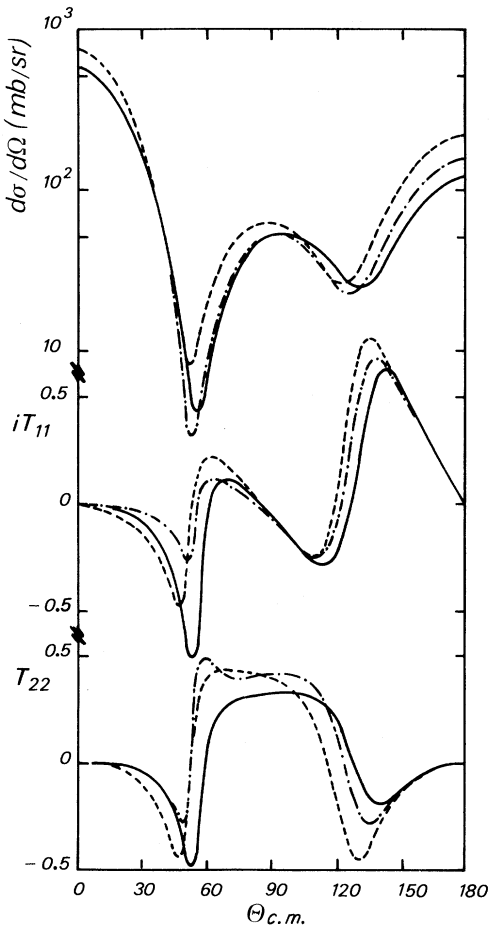

FIG. 4. Sensitivity of the d-α observables to the N-α interaction at 12 MeV. The tensor force is Y7, and the N-α parametrizations are CFL (—), S(---), and GL (-•--•).

retical values with the available experimental data.

The differential cross section (Fig. 5) is in very good agreement with experiment at all energies. We note that the backward enhancement becomes too strong when energy increases. However, this defect is much smaller than in Shanley's calculation. We also note some deviations at small angles at low energy because of the lack of Coulomb interaction in our model.

The iT_{11} analyzing power (Fig. 6) is in general poorly reproduced, especially above 20 MeV where the calculation gives only the backward maximum. Below this energy the shape of theoretical curves corresponds with experimental data, but the extrema are shifted to the left about 10° and their amplitudes are incorrect.

The tensor analyzing powers (Figs. 7–9) present a good agreement with experiment (at our disposal up to 17 MeV). The experimental evolution of T_{20}, T_{21}, and T_{22} with energy is well reproduced, the

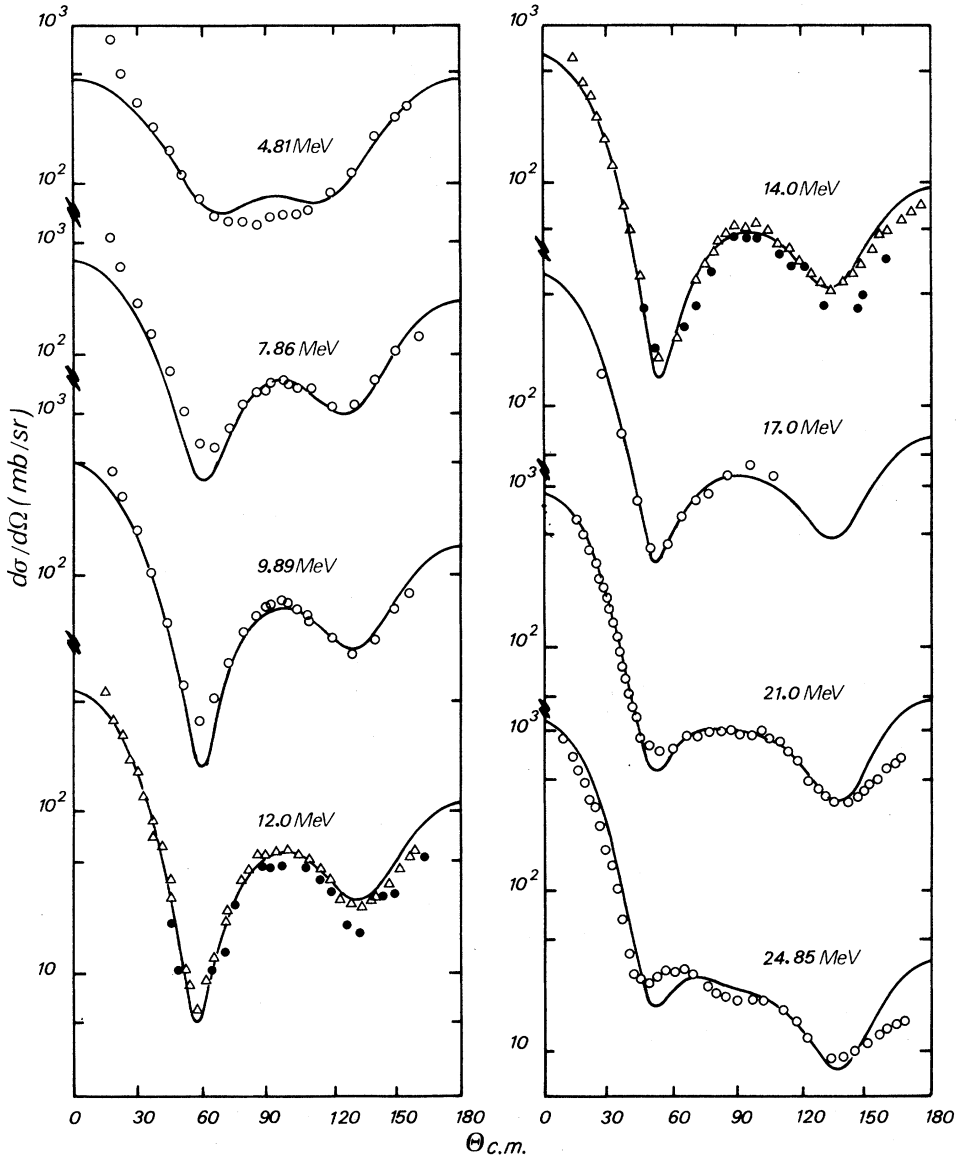

FIG. 5. d-α differential cross section with ACS4-CFL. The experimental data are from Ref. 33 (4.77 MeV), Ref. 34 (7.5, 10, 12 [Δ], 13.7 [Δ] MeV), Ref. 11 (12[●], 14[●], 17 MeV), Ref. 35 (21. MeV) and Ref. 36 (24.85 MeV).

only obvious defects being the lack of amplitude of the first minimum in T_{20} (except at 4.81 and 17 MeV) and the too high maximum in T_{20} and T_{22} below 12 MeV.

3. *Possible improvements*

First of all, we tried to simulate Coulomb effects. Following Shanley, we added Coulomb scattering amplitudes to the pure nuclear amplitudes. This improves the differential cross section up to 40° and brings some small changes for the analyzing powers in the range 0–90°.

In a second step, we have to consider the effect of higher two-body partial waves which were neglected in our calculations. Concerning the N-N interaction, we know from previous studies of n-d elastic scattering that all P partial waves must be included to get correct vector polarizations. Expecting such an effect in d-α scattering, we introduced in an exact way the $^1P_1$ N-N partial wave, the only one which occurs because of the conservation of total isospin. We thus get two coupled channels more for parity A and one for parity B. We performed the calculation at 12 MeV with the previous ACS4-CFL interactions and with a simple one-term $^1P_1$ interaction.[29] Only the $J=1^+$ three-body scattering amplitudes exhibit appreciable modification, leading to small changes in d-α scattering observables especially in the regions of extrema. In Fig. 10 the results for iT_{11}, T_{20}, and T_{22} are compared with the previous

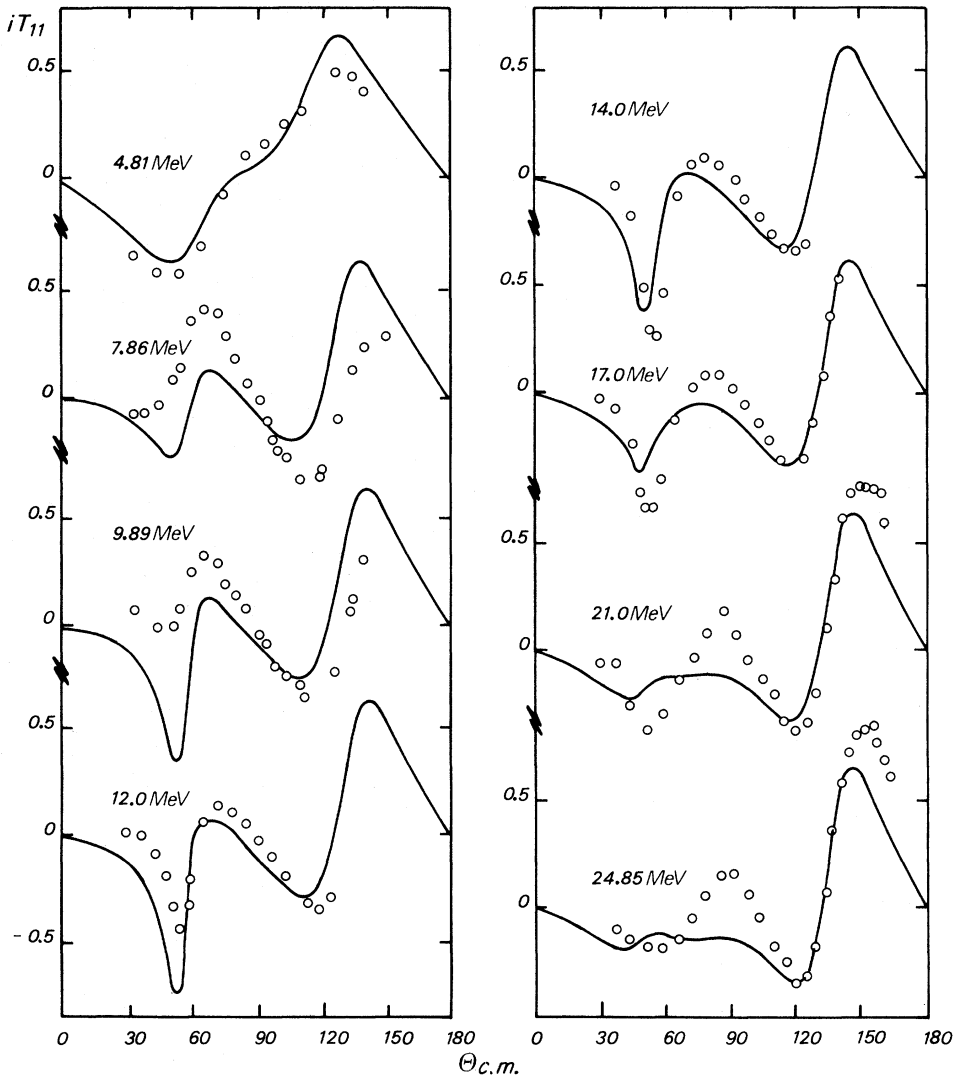

FIG. 6. iT_{11} vector analyzing power. The experimental data are from Ref. 6 (4.81, 7.86, and 9.89 MeV), Refs. 4 and 5 (12, 14, and 17 MeV) and Ref. 12 (19.8 and 24.9 MeV).

ACS4-CFL results. Even if the effect of the $^1P_1$ wave is small, we notice an obvious improvement of the results. However, as the computing time is increased by about 30%, it seems to be reasonable to neglect the $^1P_1$ contribution in a first approach. The $D_{3/2}$ and $D_{5/2}$ N-α partial waves may become important when energy increases. Nevertheless, they introduce at most 10 coupled channels in the system for each parity, and this leads to prohibitive computing time.

V. CONCLUSIONS

The results presented in Sec. IV show that a three-body model using separable N-N and N-α forces is able to predict diverse bound-state and scattering properties of the six-nucleon system. Concerning the bound states, the lack of Coulomb repulsion allows us to only compare with experiment the relative spacing of the ^{6}Li levels. An extensive study of ^{6}Li done in Ref. 28 shows the great sensitivity of the bound states to the N-N and N-α parametrizations, as it was found by Ghovanlou and Lehman for the ^{6}He ground state. For the scattering calculations, in order to get the d-α spin-dependent effects, such as polarizations, we have included spin-dependent N-N and N-α interactions. Nevertheless, the N-N tensor force is not essential, and the d-α polarizations arise solely from the N-α spin-dependence. The description of the N-α interaction is therefore of prime importance and it would be useful to have more unambiguous neutron-α experimental results. The main interest of this three-body approach is to reproduce quite well the evolution of the scattering observables with energy. The defects appearing in $d\sigma/d\Omega$ and iT_{11} at energies above 20 MeV clearly emphasize the limitation of a model supposing a

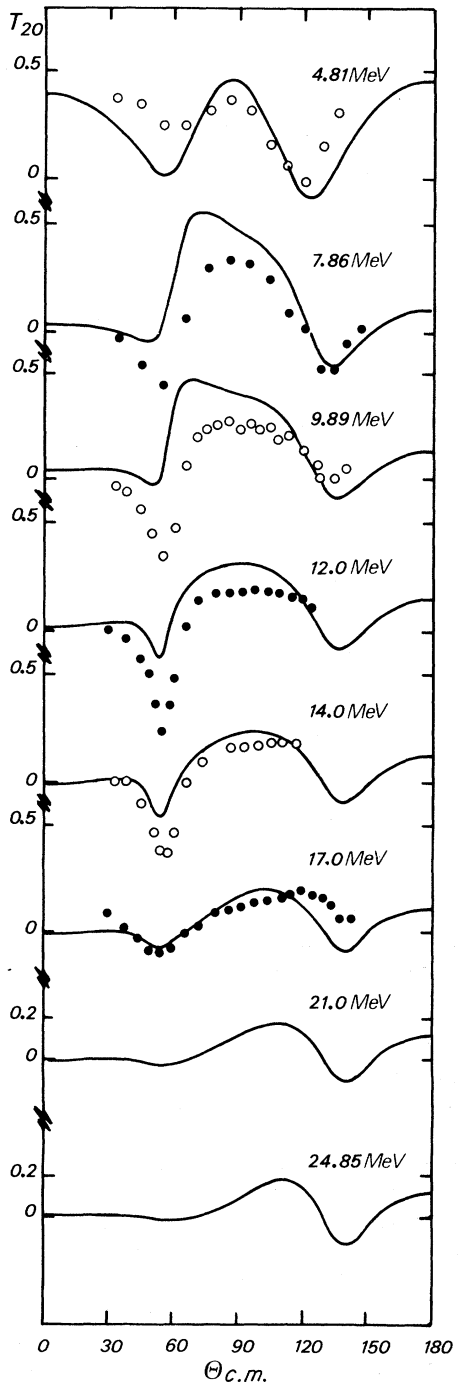

FIG. 7. T_{20} tensor analyzing power. The experimental data are from Ref. 7 (4.81, 7.86, and 9.89 MeV) and Refs. 4 and 5.

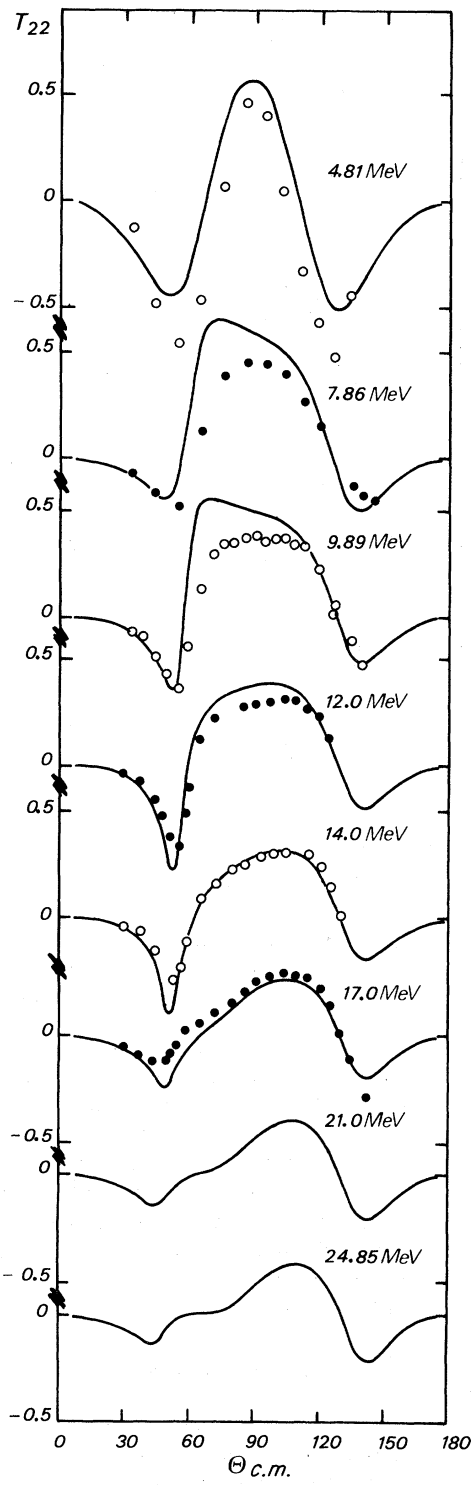

FIG. 8. T_{22} tensor analyzing power. The experimental data are from Ref. 8 (4.81, 7.86, and 9.89 MeV) and Refs. 4 and 5.

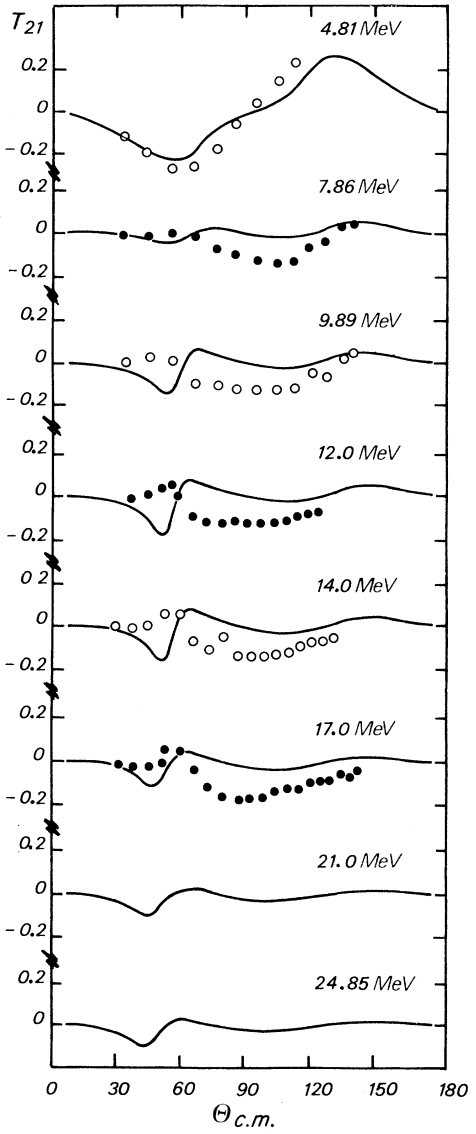

FIG. 9. T_{21} tensor analyzing power. Experiment as in Fig. 8.

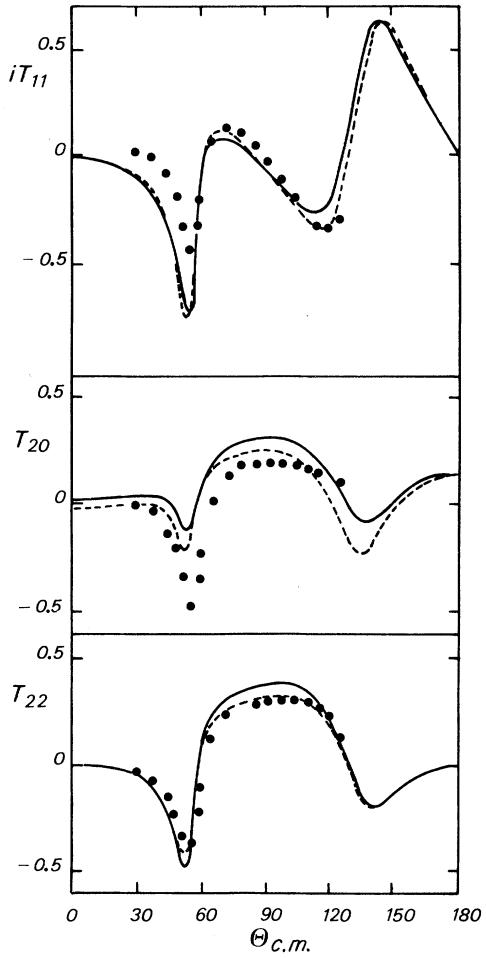

FIG. 10. Effect of the $^1P_1$ N-N partial wave on the analyzing powers at 12 MeV. The solid curve is the ACS4-CFL calculation, and the dotted curve is obtained when we add the $^1P_1$ wave.

structureless α particle (the inelastic channel $^4\mathrm{He}+d \rightarrow {}^3\mathrm{He}+t$ occurs at 21.5 MeV). The most striking fact is the systematic discrepancy between theory and experiment for the vector analyzing power iT_{11}. Here, we are in the same situation as for n-d elastic scattering where the deuteron vector polarization is badly reproduced (in the maximum and dip regions) with three-body calculations using separable as well as realistic local N-N interactions.

We would like to thank Professor E. Elbaz for many helpful and stimulating discussions and for his final reading of the manuscript.

APPENDIX A

The isospin coefficient is the usual "6j" coefficient:

$$C(I,J_iJ_j)=(-)^{2i_i+i_j+i_k+I_i}[I_iI_j]\begin{Bmatrix} i_i & I & I_i \\ i_j & i_k & I_j \end{Bmatrix}.$$

The function $G(q_i,q_j;u)$ has the following expression:

$$G(q_i,q_j;u)=\sum_{\Lambda_i\Lambda_j}[L_i-\Lambda_i\,L_j-\Lambda_j]\rho_i^{\Lambda_i}\rho_j^{L_j-\Lambda_j}q_i^{\Lambda_i+\Lambda_j}q_j^{L_i-\Lambda_i+L_j-\Lambda_j}\begin{pmatrix}2L_i+1\\ 2\Lambda_i\end{pmatrix}^{1/2}\begin{pmatrix}2L_j+1\\ 2\Lambda_j\end{pmatrix}^{1/2}H(u),$$

where

$$H(u)=\sum_f [f^2](-)^f \begin{Bmatrix} \Sigma_j & f & \Sigma_i \\ l_i & J & l_j \end{Bmatrix} \begin{Bmatrix} \Sigma_i & s_i & S_j & L_j \\ J_i & s_k & J_j & f \\ L_i & S_i & s_j & \Sigma_i \end{Bmatrix} 1 \Bigg\}$$

$$\times \sum_{L_1} [L_1^2] \begin{pmatrix} \Lambda_i & \Lambda_j & L_1 \\ 0 & 0 & 0 \end{pmatrix} \sum_{L_2} [L_2^2] \begin{pmatrix} L_i-\Lambda_i & L_j-\Lambda_j & L_2 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} L_i & L_i-\Lambda_i & \Lambda_i \\ L_j & L_j-\Lambda_j & \Lambda_j \\ f & L_2 & L_1 \end{Bmatrix}$$

$$\times \sum_{\mathfrak{L}} [\mathfrak{L}^2] \begin{pmatrix} L_1 & l_i & \mathfrak{L} \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_2 & l_j & \mathfrak{L} \\ 0 & 0 & 0 \end{pmatrix} (-)^{\mathfrak{L}} \begin{Bmatrix} L_2 & L_1 & f \\ l_i & l_j & \mathfrak{L} \end{Bmatrix} P_{\mathfrak{L}}(u).$$

In these relations, the coefficients for Racah algebra are defined as in Ref. 30, and $\binom{2L_\Lambda+1}{2\Lambda}$ is the binomial coefficient.

APPENDIX B

We have to evaluate the three Born amplitudes defined in Eq. (13), e.g., $Z_{1\alpha,2\beta}$, $Z_{3n,1\alpha}$, and $Z_{1\alpha,3m}$. We start with the cyclic Born amplitude $Z_{ij} = \langle i(jk)|\, G_0(s)\,|j(ki)\rangle$ which is given by Eq. (17) and Appendix A: (i) For $Z_{3n,1\alpha} = \langle 3(12)|\, G_0 |1(23)\rangle$, we have $i=3$, $j=1$ and thus $k=2$, so that we can use the cyclic form. (ii) For $Z_{1\alpha,2\beta} = \langle 1(23)|\, G_0|2(13)\rangle$, we have $i=2$, $j=2$ and thus $k=3$. We come to a cyclic form by writing

$$\langle 1(23)|\, G_0|2(13)\rangle = (-)^{\varphi}\langle 1(23)|\, G_0|2(31)\rangle .$$

The φ phase comes from the fact that the direction of the momentum $\vec{p}_2$ of the pair (13) and their spin-isospin coupling is reversed:

$$\varphi = L_2 + S_2 - s_1 - s_3 + I_2 - i_1 - i_3 .$$

As particle 1 is the nucleon and particle 3 the α, we get $(-)^{\varphi} = (-)^{L_2}$. (iii) For $Z_{1\alpha,3m} = \langle 1(23)|\, G_0|3(12)\rangle$, we have $i=1$, $j=3$ and thus $k=2$. We come in the same way to a cyclic form:

$$\langle 1(23)|\, G_0|3(12)\rangle = (-)^{\varphi_1+\varphi_2}\langle 1(32)|\, G_0|3(21)\rangle .$$

The φ_1 phase is due to exchange of particles 2 and 3, and we get as in (ii) $(-)^{\varphi_1} = (-)^{L_1}$. Finally, the φ_2 phase comes from the exchange of nucleons 1 and 2, and $(-)^{\varphi_2} = -1$.

APPENDIX C

The coupled system (11) can be written in the abstract form:

$$\begin{aligned} f &= K_1 g, \\ g &= g_0 + K_2 g + K_3 f . \end{aligned} \qquad \text{(C1)}$$

The f are the physical amplitudes for $d-\alpha$ elastic scattering.

To deal with the singularities of the kernels, we use the method of contour deformation.[31] The integrals are evaluated with a Gauss-Gegenbauer $\frac{5}{2}$ quadrature with 16 or 20 mesh points. The linear system can be solved in two ways:

(i) Equation (C1) are written on matrix form and the solution of the system is given by:

$$\begin{bmatrix} f \\ g \end{bmatrix} = \left[1 - \begin{pmatrix} 0 & K_1 \\ K_2 & K_3 \end{pmatrix}\right]^{-1} \begin{bmatrix} 0 \\ g_0 \end{bmatrix} .$$

However, the use of this method is constrained by the dimension of the matrix to be inverted.

(ii) The system (C1) can be also solved by iteration with the method of Padé. Nevertheless the (C1) form is ill-conditioned because of the lack of an inhomogeneous term in the first equation. This fact yields the nonconvergence of the Padé approximants, and we must reduce Eq. (C1) to a single equation by reporting $f = K_1 g$ into the second equation:

$$g = g_0 + K_2 g + K_3 K_1 g . \qquad \text{(C2)}$$

The Padé process applied to Eq. (C2) is now convergent and leads to g, so that we obtain the physical amplitudes $f = K_1 g$. For the practical calculation, we use diagonal Padé $[N/N]$. To obtain a good convergence, we must choose $N=7$ for the lowest values of the total spin $J=0$, 1, and for higher J, we can decrease the N value. The system is so solved up to $J=6$, and for $J>6$ we take the Born approximation $f = K_1 g_0$. The calculation is stopped at $J=9$, so we get the physical amplitudes up to $l=8$.

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