PHYSICAL REVIEW C VOLUME 15, NUMBER 2 FEBRUARY 1977

Remarks on magnetic moments*

Larry Zamick
Physics Department, Rutgers University, New Brunswick, New Jersey 08903
(Received 13 September 1976; revised manuscript received 4 November 1976)

It is noted that the deviations of the isoscalar magnetic moments from Schmidt values behave in a systematic way, being positive for the jackknife cases and negative for the stretch cases. Other subjects of interest concerning magnetic moments are discussed.

[NUCLEAR STRUCTURE Systematics of isoscalar magnetic moments.]

In this note I first make some miscellaneous comments on isoscalar magnetic moments.

The magnetic moment operator can be written in terms of isoscalar and isovector parts

$$\mu = \mu_0 - \tau\mu_1 ,$$

where for the free neutron-proton system

$$\mu_0 = \tfrac{1}{2}(\mu_\pi + \mu_\nu) = 0.439\,81\mu_N ,$$

$$\mu_1 = \tfrac{1}{2}(\mu_\pi - \mu_\nu) = 2.352\,96\mu_N .$$

The Schmidt formula for the isoscalar ($T=0$) and isovector ($T=1$) g factor is

$$g_T = \left[1 \mp \frac{1}{(2l+1)}\right] g_{Tl} \pm \frac{g_{Ts}}{(2l+1)} , \quad j = l \pm \tfrac{1}{2}$$

with $g_{0l}=0.5$, $g_{0s}=0.8796$, $g_{1l}=0.5$, and $g_{1s}=4.7059$.

I would like to point out that the observed deviations from the Schmidt values, although very small, appear to behave in a systematic way. This is best demonstrated by limiting ourselves to a closed L-S jj shell plus or minus one nucleon. Hence, we consider the pairs ^{3}He-^{3}H, ^{15}N-^{15}O, ^{17}F-^{17}O, ^{39}Ca-^{39}K, and ^{41}Sc-^{41}Ca. These mirror pairs give us information concerning the orbitals $0s_{1/2}$, $0p_{1/2}$, $0d_{5/2}$, $0d_{3/2}$, and $0f_{7/2}$, respectively.

The results are listed in Table I, together with deviations from Schmidt values calculated by Shimizu, Ichimura, and Arima.[5]

Note that for the stretch cases, $j = l + \frac{1}{2}$, the deviation (experiment minus theory) is always *negative*, but for the jackknife cases, $j = l - \frac{1}{2}$, it is always *positive*. This was previously noted by Talmi.[9]

The most recent data in Table I is from Hanna[1] and co-workers. They measured $\mu(^{39}\text{Ca}) = 1.0216\mu_N$ which together with $\mu(^{39}\text{K}) = 0.392\mu_N$ gives the largest deviation from Schmidt yet measured. Previous to this measurement the $J=3^+$ of ^{38}K was known: $\mu = 1.374\mu_N$. In the Schmidt model

$$\tfrac{1}{2}[\mu(^{38}\text{K})^{3+}] = \tfrac{1}{2}\{[\mu(^{39}\text{Ca}) + \mu(^{39}\text{K})]^{3/2+}\} = \mu_0 = 0.6361\mu_N .$$

The deviation in mass 38 is $0.051\mu_N$, somewhat less than the value in mass 39 of $0.074\mu_N$. The difference between mass 38 and 39 was well explained by Gloeckner and Zamick,[2] and previously by Mavromatis[3] in his thesis, as being due to the admixture of the configuration $d_{3/2}^{-1}d_{5/2}^{-1}$ into the basic $d_{3/2}^{-2}$ configuration.

For the mirror pairs listed above there are no corrections to the magnetic moment in first order perturbation theory. Mavromatis and Zamick[4] showed that to get deviations from the Schmidt values in second order for the isoscalar moment one had to use a tensor interaction. They did the calculation and got the correct sign for the deviation but the magnitude was too small. As shown by Shimizu, Ichimura, and Arima[5] this is because the intermediate states were limited to $2\hbar\omega$. By allowing excitations up to $10\hbar\omega$ but otherwise doing the calculation the same way, the calculated deviation from Schmidt increased roughly a factor of 4 and as can be seen from Table I, is in good agreement with experiment.

Getting back to systematics we now ask if we can fit the deviation from the Schmidt values by an effective operator. We try

$$\langle j|\delta\mu_0|j\rangle = aj \pm \frac{bj}{(2l+1)} + \frac{c[1\mp(j+\frac{1}{2})]}{2(j+1)} ,$$

where a,b,c are constants. The first two terms

TABLE I. Deviations of isoscalar magnetic moments from the Schmidt value.

Orbit	$\mu_{0\ \text{Schmidt}}$	$(\mu-\mu_{0\ \text{Schmidt}})_{\text{exp}}$	$(\mu-\mu_0)_{\text{calc}}$ [a]
$0s_{1/2}$	0.4398	−0.0142	
$0p_{3/2}$	0.9398		
$0p_{1/2}$	0.1867	0.0312	0.040
$0d_{5/2}$	1.4398	−0.0258	−0.035
$0d_{3/2}$	0.6361	0.0704	0.063
$0f_{7/2}$	1.9398	−0.0226	−0.029
$0f_{5/2}$	1.1144		

[a]Reference 5.

correspond to an effective $g_l\vec{l}+g_s\vec{s}$; the last term comes from an induced l forbidden term $(y^2\sigma^1)^1$. There is no reason why a,b,c should be constants rather than functions of position, but because of the scarcity of data, I thought the simplest possible form should be tried. Let me emphasize that this analysis is crude and is just the beginning of an attempt to systematize the data.

Two fits were made. The first was to the five experimental data; the rest were to the four theoretical values of Shimizu *et al.*[5] The results are listed in Table II.

One interesting extrapolation of the fits is that other orbits with $j=l-\frac{1}{2}$ should have large deviations from Schmidt, e.g., $f_{5/2}$. Direct information on the $f_{5/2}$ moment is not readily available but perhaps it could be unfolded from moments of more complicated nuclei.

It appears from the fit that an l forbidden component is required. One word of caution here: For very large l the b and c terms behave nearly the same and can be lumped into one term. When this happens a fit with two parameters can be dangerous. However, we are here dealing with low l values.

The fits are far from impressive despite the few data points. The main difficulty seems to be in getting a large difference in the $d_{3/2}$ and $p_{1/2}$ data to come out. The fit values tend to be too close to each other. This would seem to indicate that the parameters b and c should depend on r, but we shall not attempt such an improvement here.

One may worry about deformed states. They appear to be very important for the isovector moments in mass 41 and mass 39. In the calculation of Erikson[6] in which 3p-2h deformed states are admixed into the ground states of ^{41}Ca and ^{41}Sc, and 3h-2p into the ground states of ^{39}K and ^{39}Ca, the correction is purely isovector. There is some confusion here because he has an approximate expression for the deformed moment

$$\mu_{3p-2h} \approx (g_s + g_v \tfrac{1}{4}\tau) + \tfrac{1}{2}(1+\tfrac{1}{10}\tau) l \cos\phi ,$$

where ϕ is a semiclassical angle. But I observe that one can get his exact results by using the same expression and setting $\cos\phi=1$. One does not need to introduce this angle at all. Only with $\cos\phi=1$ is the correction purely isovector.

On the other hand, Erikson seems to realize that the correction is purely isovector. In fact he explicitly states this in a different place.[6] Indeed his "exact" results seem fine.

In 1968, when the data were much sparser it was suggested by Leonardi and Rosa-Clot[7] that the isoscalar moments of all nuclei in a given shell with the same ground state angular momentum should be the same; for example, the $J=\frac{3}{2}$ states in the

TABLE II. Fit to isoscalar deviations from the Schmidt value.

Orbit	Fit to experiment[a]	Fit to theory[b]
$s_{1/2}$	−0.024	−0.024
$p_{3/2}$	−0.027	−0.032
$p_{1/2}$	0.046	0.050
$d_{5/2}$	−0.024	−0.035
$d_{3/2}$	0.057	0.056
$f_{7/2}$	−0.020	−0.034
$f_{5/2}$	0.066	0.060

[a] $a=0.007$; $b=-0.055$; $c=0.050$.
[b] $a=0.003$; $b=-0.050$; $c=0.060$.

s-d shell. ^{21}Na-^{21}Ne, ^{23}Na-^{23}Mg, ^{37}Ar-^{37}K, and ^{39}Ca-^{39}K. However, recent data do not support this suggestion. The value for mass 21 is $0.86\mu_N$ but for mass 39 it is $0.706\mu_N$ as indicated in Table I (and for mass 37 it is $0.58\mu_N$).

The difference in the two cases can easily be understood by doing a Nilsson model calculation for ^{21}Na-^{21}Ne. If the $K=\frac{3}{2}$ orbit is written as $C_{5/2}|d_{5/2}\rangle + C_{3/2}|d_{3/2}\rangle$ then the value of $\mu_0 = 0.0684 \times [C_{5/2}{}^2 - C_{3/2}{}^2] - 0.182\,25\, C_{5/2}C_{3/2} + 0.75$. This depends on the details, i.e., values of $C_{5/2}, C_{3/2}$. The mass 21 result of $0.86\mu_N$ is in between the $d_{3/2}$ value $0.63\mu_N$ and the $d_{5/2}$ value of $1.4\mu_N$.

A point that has interested me is whether the magnetic moments of the $J=\frac{7}{2}$ ground states of the odd calcium isotopes lie on a straight line when plotted against neutron excess. This is the prediction of first order perturbation theory, in which the state $f_{7/2}{}^{n-1}f_{5/2}$ is admixed into the basic $f_{7/2}{}^n$ configuration.

From the known magnetic moments of ^{41}Ca and ^{43}Ca, −1.595 and −1.317, respectively, the predicted values for ^{45}Ca and ^{47}Ca would then be −1.04 and $-0.76\mu_N$. Unfortunately, the magnetic moments of mass 45 and 47 are difficult, if not impossible, to measure. But perhaps we can unfold the moments by using more complicated data.

For example, the measured moment of the $J=\frac{7}{2}^-$ ground state of ^{49}Ti is $=-1.1026\mu_N$. We can use the McCullen-Bayman-Zamick (MBZ) wave function[8]

$$\psi = \sum D(L_pL_N)[(f_{7/2}{}^2)_\pi^{L_p}(f_{7/2}{}^{-1})^{L_N}]^I$$

and the expression for the moment

$$\mu = \frac{(\mu_\pi+\mu_\nu)}{2j} I + \frac{(\mu_\pi-\mu_\nu)}{2j(I+1)} \times \sum_{L_pL_N} |D(L_pL_N)|^2 [L_p(L_p+1) - L_N(L_N+1)] ,$$

where the Schmidt limit $\mu_\nu = -1.913\mu_N$ and $\mu_\pi = 5.793\mu_N$, $j = \frac{7}{2}$, and I is the angular moment of the state. But rather than use the Schmidt values we attempt to fit $(\mu_\pi - \mu_\nu)$ from the data, keeping $(\mu_\pi + \mu_\nu)$ equal to the Schmidt value, i.e., we assume an isovector modification. With the MBZ wave function[8] $\sum |D(L_p L_N)|^2 \times [L_p(L_p+1) - L_N(L_N+1)] = 14.7427$ for the $J = \frac{7}{2}^-$ ground state of ^{49}Ti. We find then that the fit to the magnetic moment is achieved by having $\mu_\pi = 5.189$ and $\mu_\nu = -1.312$.

The neutron moment of ^{47}Ca as extrapolated from ^{49}Ti of -1.312 is very far away from the linear extrapolation from ^{41}Ca and ^{43}Ca, -0.761. In fact the former extraction gives a moment very close to ^{43}Ca.

I strongly believe that the moment obtained from ^{49}Ti is much sounder than the "linear extrapolation." This is because ^{41}Ca and ^{43}Ca ground states have significant deformed admixtures and this will surely obscure the analysis.

One last point, I strongly suspect that the quoted value for the magnetic moment of $^{47}_{21}\mathrm{Sc}_{26}$ of $5.34\mu_N$ is an error. The values of $D(L_N L_p)$ for $^{47}_{21}\mathrm{Sc}_{26}$ are the same as $D(L_p L_N)$ for ^{49}Ti. This tells us that the sum of the ground state moments of the two nuclei should equal $(\mu_\pi + \mu_\nu) = 3.88\mu_N$. If we believe ^{49}Ti this suggests that $\mu(^{47}\mathrm{Sc})$ should equal $4.98\mu_N$.

---

*Supported in part by the National Science Foundation.

[1]S. Hanna (private communication).

[2]D. Gloeckner (private communication).

[3]H. A. Mavromatis, Ph.D. thesis (unpublished).

[4]H. A. Mavromatis and L. Zamick, Phys. Lett. 20, 191 (1966).

[5]K. Shimizu, M. Ichimura, and A. Arima, Nucl. Phys. A226, 282 (1974).

[6]T. Erikson, Phys. Lett. 43B, 453 (1973); Nucl. Phys. A205, 593 (1973).

[7]R. Leonardi and M. Rosa-Clot, Phys. Rev. Lett. 21, 377 (1968).

[8]J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

[9]I. Talmi, in *Hyperfine Interactions in Excited Nuclei*, edited by G. Goldring and R. Kalish (Gordon and Breach, New York, 1971), p. 1144. Talmi states it thus: "the experimental values of $\langle \sum \sigma_z(i) \rangle$ seem to follow the simple rule: They are always *smaller in absolute value* than the values calculated from *jj*-coupling."