## Comments

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# Elastic scattering of deuterons by carbon*

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Deuteron-carbon elastic scattering is studied within the framework of the Glauber approximation. The full Glauber multiple scattering series is evaluated with simplified nuclear wave functions. An approximation in which deuteron-nucleus scattering is expressed in terms of nucleon-nucleus scattering amplitudes is shown to be accurate for small momentum transfers. The effects of the Coulomb field and the deuteron $D$ state are investigated. The impressive fit to the 650 MeV $d$-$^{12}$C elastic scattering data, obtained in a recent calculation, is shown to be due to additional approximations made in that analysis.

[NUCLEAR REACTIONS $^{12}$C($d,d$), $E$=650 MeV; calculated $\sigma(E,\theta)$ and $\sigma_T(E)$, compared with measurements.]

### I. INTRODUCTION

The multiple diffraction theory due to Glauber has been successful[1,2] in describing hadron-nucleus scattering measurements at medium and high energies. The theory also describes well deuteron-deuteron elastic scattering.[3,4] A calculation has been recently presented[5] for 650 MeV $d$-$^{12}$C elastic scattering and compared with the data.[6] That calculation involves a number of additional approximations and simplifications within the framework of the Glauber theory. For example, only terms up to fourth order in multiple scattering were included. As pointed out by the authors themselves,[5] this is inadequate except at very small angles. The effects of the Coulomb interaction and the quadrupole deformation of the deuteron were neglected and an unrealistic deuteron form factor was used. The Coulomb interaction is important in collisions between nuclei[7,8] and the deuteron quadrupole form factor is known to have a significant effect in deuteron-nucleon collisions.[9] Mean values were used for $pp$ and $pn$ scattering amplitude parameters; this is justified only when the $pp$ and $pn$ parameters are roughly equal, which is not the case at nucleon energies of 325 MeV. In this work we examine the above-mentioned approximations and present results of somewhat more realistic calculations.

### II. SCATTERING AMPLITUDE

In the Glauber approximation, the scattering amplitude operator for collisions between the deuterons and nuclei with mass number $A$ can be written as

$$F_{dA}(\vec{q},\vec{r},\vec{r}_1,\ldots,\vec{r}_A) = \frac{ik}{2\pi}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\,\Gamma_{dA}(k,\vec{b},\vec{r},\vec{r}_1,\ldots,\vec{r}_A)\,, \quad (1)$$

where $\hbar k$ is the momentum of the incident deuteron, $\hbar\vec{q}$ is the momentum transferred to the target nucleus, $\vec{b}$ is the impact parameter between the deuteron and the target nucleus, $\vec{r}$ is the internal deuteron coordinate, and $\vec{r}_1,\ldots,\vec{r}_A$ are the nucleon coordinates of the target. The deuteron-nucleus profile function $\Gamma_{dA}$ is related to nucleon-nucleon profile functions $\Gamma_{NN}$ by

$$\Gamma_{dA}(k,\vec{b},\vec{r},\vec{r}_1,\ldots,\vec{r}_A) = 1-\prod_{j=1}^{A}(1-\Gamma_{pj}-\Gamma_{nj}+\Gamma_{pj}\Gamma_{nj})\,, \quad (2)$$

where $\Gamma_{pj}$ and $\Gamma_{nj}$ denote, respectively, $\Gamma_{pj}(\frac{1}{2}k,\vec{b}+\frac{1}{2}\vec{s}-\vec{s}_j)$ and $\Gamma_{nj}(\frac{1}{2}k,\vec{b}-\frac{1}{2}\vec{s}-\vec{s}_j)$, $\vec{s}$ and $\vec{s}_j$ being the projections of $\vec{r}$ and $\vec{r}_j$ on the impact parameter plane. The elastic scattering amplitude is obtained by taking the expectation value of the scattering amplitude operator in the ground states of the deuteron and the target nucleus. If the wave function of the target nucleus is described by an independent particle model, i.e.,

$$|\psi_A(\vec{r}_1,\ldots,\vec{r}_A)|^2=\prod_{j=1}^{A}|\phi_j(\vec{r}_j)|^2\,, \tag{3}$$

and if all nucleon-nucleon ($NN$) amplitudes, for simplicity, are taken to be equal, we obtain for the elastic scattering amplitude:

$$F_{dA}(\vec{q})=-K(q)\frac{ik}{2\pi}\int d^2b\, e^{i\vec{q}\cdot\vec{b}}\langle\psi_d|\left\{\sum_{j=1}^{A}\binom{A}{j}\sum_{k=0}^{j}\binom{j}{K}[-\langle\Gamma_{pj}\rangle]^{j-k}\sum_{l=0}^{k}\binom{k}{l}[-\langle\Gamma_{nj}\rangle]^{k-l}[\langle\Gamma_{pj}\Gamma_{nj}\rangle]^{l}\right\}|\psi_d\rangle\,, \tag{4}$$

with

$$\begin{aligned}\langle\Gamma_{Nj}\rangle&=\frac{2}{ik}\int_0^\infty J_0(q|\vec{b}_N|)f_{Nj}(\tfrac{1}{2}k,q)S_A(q)q\,dq,\quad N=p,n\\ \langle\Gamma_{pj}\,\Gamma_{nj}\rangle&=\frac{1}{(i\pi k)^2}\int d^2q'\exp[-i\vec{q}'\cdot(\vec{b}+\tfrac{1}{2}\vec{s})]f_{pj}(\tfrac{1}{2}k,\vec{q}')\int d^2q\exp[-i\vec{q}\cdot(\vec{b}-\tfrac{1}{2}\vec{s})]f_{nj}(\tfrac{1}{2}k,\vec{q})S_A(\vec{q}+\vec{q}')\,,\end{aligned} \tag{5}$$

where $K(q)$ is a center of mass correlation function and $S_A(q)$ is the form factor of the target nucleus. For harmonic oscillator target wave functions, $K(q)=\exp(q^2R^2/4A)$. In order to obtain analytic results, let us assume that the deuteron wave function can be approximated by[10]

$$|\psi_d(\vec{r})|^2=\sum_{j=1}^{N}\alpha_j(4\pi\beta_j)^{-3/2}e^{-r^2/4\beta_j}\,, \tag{6}$$

that the $NN$ amplitude may be written as

$$f(q)=\frac{k\sigma(i+\rho)}{4\pi}e^{-aq^2/2}\,, \tag{7}$$

and that the nuclear form factor may be represented by

$$S_A(q)=e^{-R^2q^2/4}\,. \tag{8}$$

Equation (4) may then be evaluated analytically with the result

$$F_{dA}(q)=-K(q)ik\sum_{j=1}^{A}\binom{A}{j}\sum_{k=0}^{j}\binom{j}{k}\sum_{l=0}^{k}\binom{k}{l}\left[\frac{-\sigma(1-i\rho)}{2\pi(R^2+2a)}\right]^{j-l}\left[\frac{\sigma^2(1-i\rho)^2}{16\pi^2a(R^2+a)}\right]^{l}\sum_{m=1}^{N}\frac{\alpha_m\exp[-q^2F(j,k,l,m)]}{A(j,l)E(j,k,l,m)}\,, \tag{9}$$

where

$$\begin{aligned}F(j,k,l,m)&=\frac{1}{2A(j,l)}+\frac{\beta_mD^2(j,k,l)}{E(j,k,l,m)},\qquad E(j,k,l,m)=1-2\beta_m\left[\frac{B^2(j,k,l)}{A(j,l)}-\frac{2(j-k)}{(R^2+2a)}-\frac{l(R^2+2a)}{2a(R^2+a)}\right],\\ D(j,k,l)&=\frac{1}{2}-\frac{B(j,k,l)}{A(j,l)}\,,\qquad B(j,k,l)=\frac{2(j-k)}{(R^2+2a)}+\frac{l}{(R^2+a)},\qquad A(j,l)=\frac{2(j-l)}{(R^2+2a)}\,.\end{aligned} \tag{10}$$

This simple expression for $F_{dA}$ is most useful for investigating the structure of the scattering amplitude and the significance (or insignificance) of the higher order terms in the multiple scattering series.

The exact Glauber amplitude given by Eq. (4) is difficult to evaluate for general forms of nuclear wave functions and $NN$ amplitudes. We shall therefore, also consider an approximation in which the deuteron-nucleus scattering amplitude is expressed in terms of nucleon-nucleus amplitudes.[11] In this approximation the effects of the Coulomb field and of the deuteron $D$ state can also be included with relative ease. The deuteron-nucleus scattering amplitude operator, in this approximation, can be written as[8]

$$\begin{aligned}F_{dA}(\vec{q},\vec{s})=&\,2e^{-i\vec{q}\cdot\vec{s}/2}[F_C(\tfrac{1}{2}k,\vec{q})+e^{i\chi_{Cp}}F_{pA}(\tfrac{1}{2}k,\vec{q})]+2e^{i\vec{q}\cdot\vec{s}/2}e^{i\chi_{Cn}}F_{nA}(\tfrac{1}{2}k,\vec{q})+\frac{2i}{\pi k}e^{i\chi_{Cpn}}\\ &\times\int d^2q'\,e^{i\vec{q}'\cdot\vec{s}}F_{pA}(\tfrac{1}{2}k,\tfrac{1}{2}\vec{q}-\vec{q}')F_{nA}(\tfrac{1}{2}k,\tfrac{1}{2}\vec{q}+\vec{q}')\,,\end{aligned} \tag{11}$$

where $\chi_{Cp}$, $\chi_{Cn}$, and $\chi_{Cpn}$ are average Coulomb phase shift functions[12] and $F_C$, the proton-nucleus Coulomb amplitude, is given by[8]

$$F_C(k,\vec{q})=-\frac{2nk}{q^2}\exp\{-2i[n\ln(q/2k)-\arg\Gamma(1+in)]\}\,\mathfrak{F}_A(\vec{q})\mathfrak{F}_p(\vec{q})\,. \tag{12}$$

$\mathfrak{F}_A$ and $\mathfrak{F}_p$ are the charge form factors of the target nucleus and the proton and $n = Ze^2/\hbar v$ is the usual Coulomb parameter. The nucleon-nucleus scattering amplitude $F_{NA}$ is given by

$$F_{NA}(k,\vec{q}) = K(q) ik \int_0^\infty J_0(qb)\Gamma_{NA}(b) b\, db; \quad N=p,n, \tag{13}$$

$$\Gamma_{NA}(b) = \left\{1 - \left[1 - \frac{1}{ik}\int_0^\infty J_0(qb) f_{Np}(k,q) S_A(q) q\, dq\right]^Z \left[1 - \frac{1}{ik}\int_0^\infty J_0(qb) f_{Nn}(k,q) S_A(q) q\, dq\right]^{A-Z}\right\}. \tag{14}$$

If the $D$ state is included, the deuteron wave function can be written as[9]

$$\psi_d^M(\vec{r}) = (4\pi)^{-1/2} r^{-1}[u(r) + 8^{-1/2} S_{12}(\hat{r}) w(r)]\chi_1^M, \tag{15}$$

where $\chi_1^M$ are spin-one spinors and $S_{12}$ is the usual tensor operator. The differential cross section is then given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{3}\sum_{M,M'} |\langle \psi_d^{M'} | F_{dA}^{\rm av}(\vec{q},\vec{s}) | \psi_d^M\rangle|^2$$
$$= \left(\frac{d\sigma}{d\Omega}\right)_S + \left(\frac{d\sigma}{d\Omega}\right)_Q, \tag{16}$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_S = \left| F_{Cpn}(\vec{q}) S_S(\tfrac{1}{2}\vec{q}) + \frac{2i}{\pi k} e^{i\chi_{Cpn}} \int d^2q' S_S(\vec{q}\,') F_{pA}(\tfrac{1}{2}k, \tfrac{1}{2}\vec{q} - \vec{q}\,') F_{nA}(\tfrac{1}{2}k, \tfrac{1}{2}\vec{q} + \vec{q}\,')\right|^2,$$
$$\left(\frac{d\sigma}{d\Omega}\right)_Q = \tfrac{3}{4}\, |F_{Cpn}(\vec{q}) S_Q(\tfrac{1}{2}\vec{q})|^2 + \tfrac{1}{4}\left| F_{Cpn}(\vec{q}) S_Q(\tfrac{1}{2}\vec{q}) + \frac{2i}{\pi k} e^{i\chi_{Cpn}} \int d^2q' S_Q(\vec{q}\,') F_{pA}(\tfrac{1}{2}k, \tfrac{1}{2}\vec{q} - \vec{q}\,') F_{nA}(\tfrac{1}{2}k, \tfrac{1}{2}\vec{q} + \vec{q}\,')\right|^2, \tag{17}$$

with

$$F_{Cpn}(\vec{q}) = 2[F_C(\tfrac{1}{2}k,\vec{q}) + e^{i\chi_{Cp}} F_{pA}(\tfrac{1}{2}k,\vec{q}) + e^{i\chi_{Cn}} F_{nA}(\tfrac{1}{2}k,\vec{q})].$$

$S_S$ and $S_Q$ are the deuteron spherical and quadrupole form factors defined by

$$S_S(q) = \int_0^\infty j_0(qr)[u^2(r) + w^2(r)]\, dr,$$
$$S_Q(q) = 2\int_0^\infty j_2(qr) w(r)[u(r) - 8^{-1/2} w(r)]\, dr. \tag{18}$$

## III. APPLICATION TO 650 MeV ELASTIC SCATTERING

In order to illustrate the accuracy of Eq. (11) we first neglect Coulomb and the deuteron $D$-state effects (by setting $F_C$, $\chi_{Cp}$, $\chi_{Cn}$, $\chi_{Cpn}$, and $S_Q$ equal to zero). We use a Gaussian wave function for carbon and a sum of Gaussians[10] for the deuteron and use average $NN$ parameters to be specified later. The results for the $d$-$^{12}$C elastic scattering angular distribution are shown in Fig. 1. The solid curve represents the full calculation. We see that the curve (dashed) obtained by retaining terms only up to fourth order is quite inaccurate at large angles ($\theta_{\rm lab} \gtrsim 7°$). The dot-dashed curve shows the result obtained from Eq. (9) by using the single Gaussian deuteron wave function of Ref. 5. These two simplifications used together (as done in Ref. 5) can lead to substantial error in the cross sections. Also shown in Fig. 1 are the predictions of Eq. (16), neglecting the Coulomb interaction and $S_Q$, and using the sum of Gaussians[10] for $S_S$. We see that in the angular region where data exist, i.e., $\theta_{\rm lab} \lesssim 13°$, Eq. (16) is a good approximation to the exact Glauber result obtained from Eq. (9), and can therefore be used to perform realistic calculations for $d$-$^{12}$C scattering and to examine the effects of the Coulomb field and the deuteron $D$ state.

In Fig. 2 we show the results for 650 MeV $d$-$^{12}$C elastic scattering as obtained from Eq. (16) through (18) by numerical integrations. For deuteron $S$ and $D$ states, we take the Gartenhaus-Moravcsik wave functions.[13] For the $^{12}$C form factor we use the harmonic oscillator result $(1 - q^2R^2/9)\exp(-q^2R^2/4)$ with $R = 1.59$ fm which is obtained from an rms radius of[14] 2.41 fm (upon making corrections for center of mass and finite proton size). For $NN$ amplitudes at 325 MeV, we have (interpolating between measured values when necessary) $\sigma_{pp} = 24.1$ mb (Refs. 15 and 16), $a_{pp} = 0.35$ $(\mathrm{GeV}/c)^{-2}$ (Ref. 15), $\rho_{pp} = 0.8$ (Ref. 17), $\sigma_{np} = 33$ mb (Refs. 17 and 16), $a_{np} = 2.8$ $(\mathrm{GeV}/c)^{-2}$ (Ref. 4), and for $\rho_{np}$ we use the value[18] 0.25. We see that, as expected, the Coulomb effects are important at small angles ($\theta \lesssim 2°$). They are also important at the minimum, even though the real parts $\rho_{NN}$ of the nucleon-nucleon amplitudes are quite large. (At energies where the $\rho_{NN}$ are small, the Coulomb effects become even more significant.) We also find that the contributions to the cross sections due to the deuteron $D$ state are small even near the minimum. The reason for this is that for $d$-$^{12}$C scattering the minimum occurs at $q^2 \sim 0.07$ $(\mathrm{GeV}/c)^2$ [as compared to $q^2 \sim 0.33$ $(\mathrm{GeV}/c)^2$ in $d$-$p$ scattering], and at this value of $q^2$, $S_Q{}^2(\frac{1}{2}q)$ is much smaller then $S_S{}^2(\frac{1}{2}q)$ (by a factor of ~60) and therefore its effect becomes noticeable only if a deep minimum would exist in the angular distributions due to the $S$-state alone. In Fig. 2 the minimum is already filled due to the large $\rho_{NN}$ and due to the presence of Coulomb interaction. The deuteron $D$ state therefore can be neglected for target nuclei heavier than carbon. (However, the $D$ state is still important for scattering by lighter nuclei, particularly if the $\rho_{NN}$ are not very large.) Also shown in Fig. 2 is the curve obtained by assuming $f_{pp} = f_{np}$ and using mean values for the $NN$ parameters. The error introduced by this procedure is small (~10%) but not negligible except at very small angles and such an averaging should not be done in any detailed analysis of the data. From Fig. 2, we notice that the calculations are systematically lower than the measurements at the larger angles. As pointed out in Ref. 5, there are uncertainties in the $NN$ parameters. But all the $d$-$^{12}$C data at 650 MeV cannot be fitted simultaneously by any reasonable variation of the $NN$ parameters. The seemingly good fit obtained in Ref. 5 is primarily the result of the increase in the calculated cross sections at larger angles due to the two simplifications mentioned earlier, i.e., retaining terms only up to fourth order and use of an unrealistic deu-

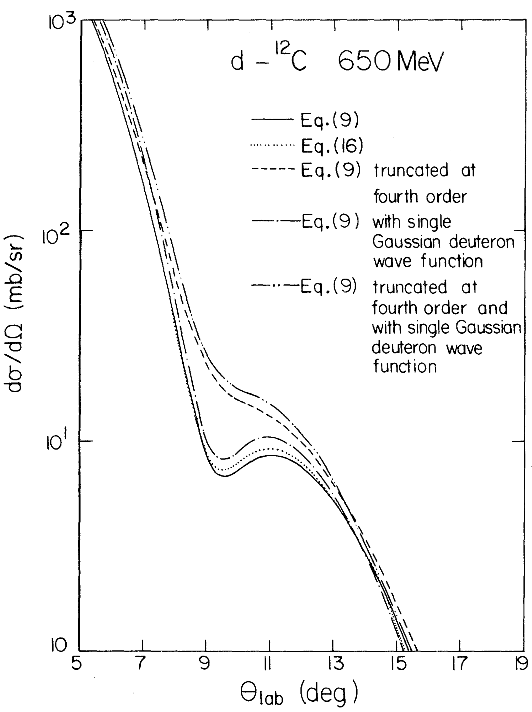


FIG. 1. $d$-$^{12}$C elastic scattering differential cross section at 650 MeV. The solid curve represents the full Glauber approximation result [Eq. (9)] with a realistic deuteron $s$-state wave function. The dashed curve is obtained by truncating the multiple scattering series at fourth order scattering. The dot-dashed curve represents the full Glauber approximation result with a single Gaussian deuteron $s$-state wave function. The dotted curve represents an approximation [Eq. (16)] to the full Glauber calculation (solid curve).

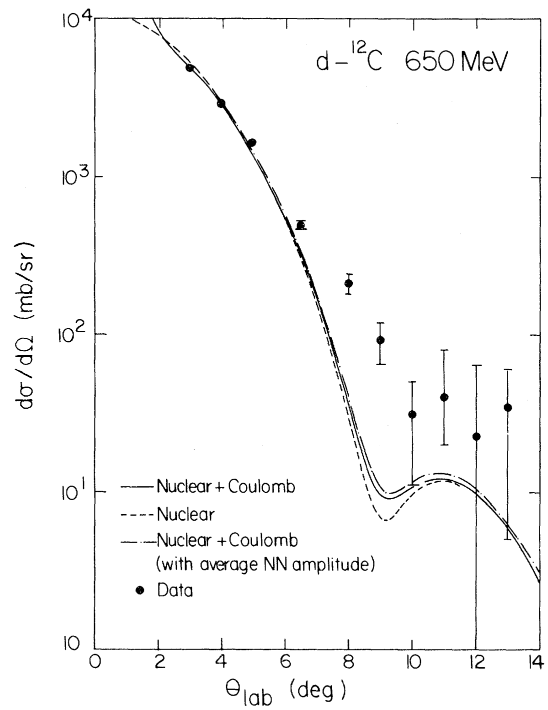


FIG. 2. $d$-$^{12}$C elastic scattering differential cross section at 650 MeV. All curves are obtained from Eqs. (16)–(18). The solid curve includes contributions from the Coulomb field. The dashed curve represents only the strong interaction contribution. The dot-dashed curve is obtained by assuming $f_{np} = f_{pp}$ and using mean values for the $NN$ parameters.

teron form factor.

Deuteron-nucleus total cross sections can be obtained from Eq. (11) (neglecting the Coulomb interactions) by means of the optical theorem. We obtain $\sigma_{tot} = 470$ mb using the same parameters as used for the differential cross sections. Total cross sections can also be calculated from the exact Glauber amplitude, i.e., Eq. (9) using a Gaussian wave function for $^{12}C$ (with the correct rms radius), a realistic deuteron form factor given by the sum of Gaussians,[10] and using average $NN$ amplitudes. Equation (9) then yields $\sigma_{tot} = 466$ mb. The experimental value[6] is $456 \pm 18$ mb.

In conclusion, we find that the 650 MeV $d$-$^{12}C$ total cross section measurement is well described by the Glauber approximation. However, the predictions of the theory for elastic scattering angular distributions do not agree well with the data near and beyond the minimum if nuclear correlations and spin-dependent effects effects are neglected. We should point out that before any attempt is made to extract information about nuclear correlations, spin effects should be included. It is quite difficult to perform realistic calculations for the exact Glauber amplitude which include spin effects. However, such a calculation can be carried out using the approximate Eq. (11), at least for angles out to the secondary maximum where Eq. (11) is a good approximation to the full Glauber multiple scattering series.

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[1]R. J. Glauber, in *High Energy Physics and Nuclear Structure*, edited by S. Devons (Plenum, New York, 1970), p. 207.

[2]J. Saudinos and C. Wilkin, Annu. Rev. Nucl. Sci. 24, 341 (1974).

[3]V. Franco, Phys. Rev. 175, 1376 (1968).

[4]G. Alberi, L. Bertocchi, and G. Bialkowski, Lett. Nuovo Cimento 3, 108 (1970); Nucl. Phys. B17, 621 (1970).

[5]K. S. Chadha and V. S. Varma, Phys. Rev. C 13, 715 (1976).

[6]L. M. C. Dutton *et al.*, Phys. Lett. 16, 331 (1965).

[7]W. Czyż and L. C. Maximon, Ann. Phys. (N.Y.) 60, 484 (1970).

[8]V. Franco and G. K. Varma, Phys. Rev. C 12, 225 (1975).

[9]D. R. Harrington, Phys. Rev. Lett. 21, 1496 (1968); V. Franco and R. J. Glauber *ibid.* 22, 370 (1969).

[10]The wave function of Eq. (6) corresponds to a deuteron form factor given by a sum of Gaussians. A sum of three Gaussians which is fitted to realistic deuteron form factors is given in Ref. 8 [Eq. (49)]. The parameters are $\alpha_j = 0.34$, 0.58, 0.08 and $\beta_j = 141.5$, 26.1, and 15.5 $(\mathrm{GeV}/c)^{-2}$, respectively.

[11]V. Franco, Ph.D. thesis, Harvard University, 1963 (unpublished); L. Zamick, Ann. Phys. (N.Y.) 21, 550 (1963); M. L. Rustgi, Nucl. Phys. 59, 460 (1964); A. Tekou, *ibid.* B46, 141 (1972).

[12]The average Coulomb phase shift functions are described in Appendix B of Ref. 8 for the case of $N$-$d$ scattering. The formulas are applicable to $d$-nucleus scattering if nucleon-nucleon profile functions at momentum $\hbar k$ are replaced by nucleon-nucleus profile functions at momentum $\hbar k/2$.

[13]M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).

[14]H. Crannell, Phys. Rev. 148, 1107 (1966).

[15]O. Benary *et al.*, UCRL Report No. UCRL-20000NN, 1970 (unpublished).

[16]W. N. Hess, Rev. Mod. Phys. 30, 368 (1958); P. Schwaller *et al.*, Phys. Lett. 35B, 243 (1971).

[17]L. M. C. Dutton *et al.*, Nucl. Phys. B9, 594 (1969).

[18]A value $\rho_{pn} = 0.05 \pm 0.20$ was obtained in Ref. 17. However, the formula used in Ref. 17 for the analyzing of $d$-$p$ scattering is erroneous for small $q$. We have, therefore, used $\rho_{pn} = 0.25$ as predicted by dispersion relations. See, A. A. Carter and D. V. Bugg, Phys. Lett. 20, 203 (1966).